The Aggregate Effects of the Great Black Migration

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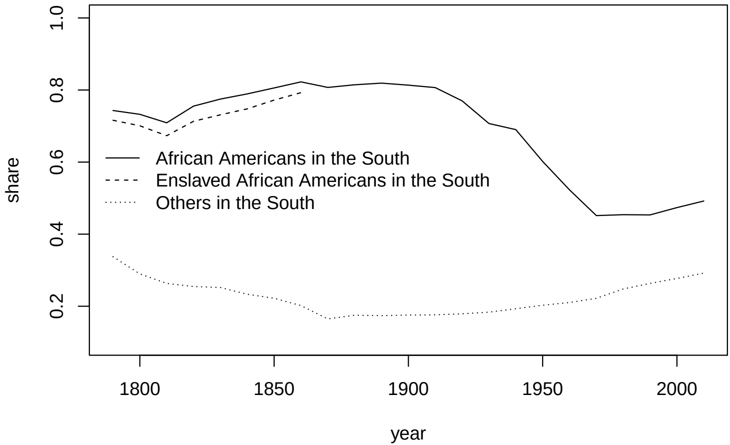
Population Shares in the South by Race



South: confederate states

plus border states

Outline

- Four million African Americans moved from the South to the North of the US between 1940 and 1970.
- How did it impact aggregate US output and the welfare of cohorts of African Americans and others?
- I quantify a dynamic general equilibrium model that comprises migration behavior of African Americans and others.

Preview

- Shutting down the migration of African Americans across the North and the South between 1940 and 1970
  - decreases aggregate US output in 1970 by 0.7%,
  - decreases the welfare of African Americans born in Mississippi in the 1930s by 3.5%,
  - increases the welfare of African Americans born in Illinois in the 1930s by 0.2%.

- Shutting down the migration of others across the North and the South for the same period
  - decreases aggregate US output in 1970 by 0.3%.

Contribution to Literature

1. Economic geography of African Americans
   - Myrdal (1944)
   - **Boustan** (**2009**, 2010, 2017), Derenoncourt (2022), Althoff and Reichardt (2022)
2. Dynamic spatial models
   - Caliendo, Dvorkin, and Parro (2019), **Allen and Donaldson (2022)**, Kleinman, Liu, and Redding (2022)

- This paper is the first to quantify the aggregate, general equilibrium effects of the great Black migration.

Empirical Facts

1. The migration rate of African Americans from the South was higher than any other group of people. details

2. African Americans who moved from the South to the North earned much higher wages than African Americans who stayed in the South. details

3. Only one-fourth of the mover-stayer wage gap was absorbed by the mover-stayer rent gap for African Americans from the South. details

Model

Environment

- Time $t = 0, 1, \cdots$
- There are J locations.
- Individuals of cohort c are born in period c and live through at most period $c + \bar{a}$.
- Ages range from 0 to $\bar{a}$.

Preferences and Location Choices

- The period utility of individuals is

$$u^i_{r,a,t} = \begin{cases} 0 & \text{for } a = 0, \\ \log\left(\frac{w^i_{r,a,t}}{(r^i_t)^\gamma}\right) + \log B^i_{r,a,t}, & \text{for } a = 1, \cdots, \bar{a}. \end{cases}$$

- For $a \leq \bar{a} - 1$, the value is

$$v^i_{r,a,t} = u^i_{r,a,t} + \max_{j=1,\cdots,J} \left\{ s_{r,a,t} E[v^j_{r,a+1,t+1}] - \tau^{j,i}_{r,a,t} + \nu \varepsilon^j_{r,a,t} \right\}.$$

- For $a = \bar{a}$, the value is

$$v^i_{r,a,t} = u^i_{r,a,t}.$$

- Assuming $\varepsilon^j_{r,a,t}$ draws a type-I extreme value, for $a \leq \bar{a} - 1$, the expected value is

$$V^i_{r,a,t} = u^i_{r,a,t} + \nu \log\left(\sum_{j=1}^{J} \exp(s_{r,a,t} V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t})^{1/\nu}\right). \tag{1}$$

Migration Flows and Populations

- The migration share of (r, a, t) from i to j is

$$\mu^{j,i}_{r,a,t} = \frac{\exp(s_{r,a,t} V^{j}_{r,a+1,t+1} - \tau^{j,i}_{r,a,t})^{1/\nu}}{\sum_{k=1}^{J} \exp\left(s_{r,a,t} V^{k}_{r,a+1,t+1} - \tau^{k,i}_{r,a,t}\right)^{1/\nu}}. \qquad (2)$$

- Population in each demographic group next period is

$$L^{j}_{r,a+1,t+1} = \sum_{i=1}^{J} \mu^{j,i}_{r,a,t} s_{r,a,t} L^{i}_{r,a,t} + I^{j}_{r,a+1,t+1}. \qquad (3)$$

Production

- Output is
$$Y_t^i = A_t^i L_t^i.$$
- L_t^i aggregates labor of various ages
$$L_t^i = \left(\sum_a (\kappa_{a,t}^i)^{\frac{1}{\sigma_a}} (L_{a,t}^i)^{\frac{\sigma_a - 1}{\sigma_a}} \right)^{\frac{\sigma_a}{\sigma_a - 1}}.$$
- $L_{a,t}^i$ aggregates labor of different races
$$L_{a,t}^i = \left(\sum_r (\kappa_{r,a,t}^i)^{\frac{1}{\sigma_r}} (L_{r,a,t}^i)^{\frac{\sigma_r - 1}{\sigma_r}} \right)^{\frac{\sigma_r}{\sigma_r - 1}}.$$
- Wages are priced at the marginal product of labor
$$w_{r,a,t}^i = A_t^i (L_t^i)^{\frac{1}{\sigma_a}} (\kappa_{a,t}^i)^{\frac{1}{\sigma_a}} (L_{a,t}^i)^{-\frac{1}{\sigma_a} + \frac{1}{\sigma_r}} (\kappa_{r,a,t}^i)^{\frac{1}{\sigma_r}} (L_{r,a,t}^i)^{-\frac{1}{\sigma_r}}. \quad (4)$$

wage ratios across races

Fertility

- Newborns in period t are

$$L^{i}_{r,0,t} = \sum_{a=1}^{\bar{a}} \alpha_{r,a,t} L^{i}_{r,a,t}. \tag{5}$$

- $\alpha_{r,a,t}$: how many babies are born per one person of (r, a, t).

Rent

- Rent depends on a location-specific shifter and local income

$$r_t^i = \bar{r}^i \left(\gamma \sum_r \sum_a L_{r,a,t}^i w_{r,a,t}^i \right)^\eta . \tag{6}$$

- Absentee landlords receive rent (or rent is dumped).

Equilibrium

Given $\{L^i_{r,a,0}\}$, an equilibrium is

- $\{V^i_{r,a,t}\}$ such that (1),
- $\{w^i_{r,a,t}\}$ such that (4),
- $\{L^i_{r,a,t}\}$ such that (3) and (5),
- $\{\mu^{i,j}_{r,a,t}\}$ such that (2),
- $\{r^i_t\}$ such that (6).

Steady State

A steady state is an equilibrium in which all endogenous variables are time-invariant:

- $\{V^i_{r,a}\}$ such that (1),
- $\{w^i_{r,a}\}$ such that (4),
- $\{L^i_{r,a}\}$ such that (3) and (5),
- $\{\mu^{i,j}_{r,a}\}$ such that (2),
- $\{r^i\}$ such that (6),

dropping time subscripts t from the equations.

Quantification

Data and Units of Observations

- I obtain wages, populations, and migration shares from US censuses 1940-2000 and American Community Survey 2010.

- Races are African Americans and others.
- Age bins are:

model	0	1	$\cdots$	6
data	1-10	11-20	$\cdots$	61-70

- Locations are 36 US states, DC, and the constructed rest of the North.
  - The rest of the North accounts for
    - 0.1% of the Black population in 1940.
    - 1% of the Black population in 2010.

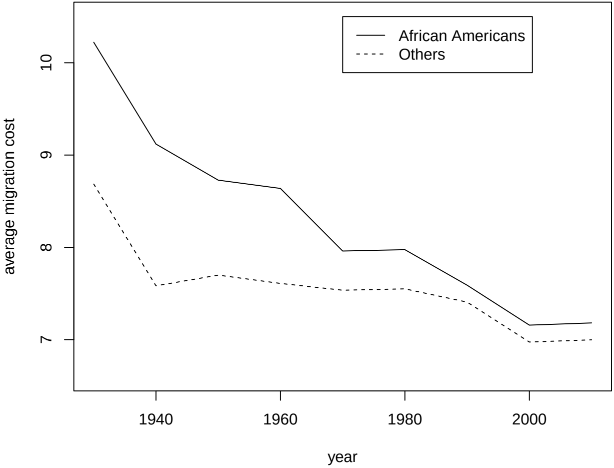
Elasticities

$\sigma_r = 9.0$	substitutability across races	details
$\sigma_a = 2.3$	substitutability across ages	details
$1/\nu = 0.8$	migration elasticity	details
$\eta = 0.4$	rent elasticity	details

Other Parameters

- Given the elasticities, inverting the model yields productivity, amenities, and migration costs.
- Fertility $\alpha_{r,a,t}$ is directly observed in census/ACS data.
- Survival probabilities $s_{r,a,t}$ are taken from life tables of CDC.

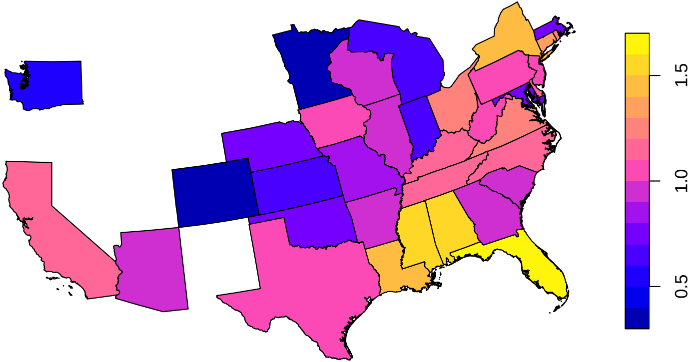
Migration Costs by Year



migration costs by ages

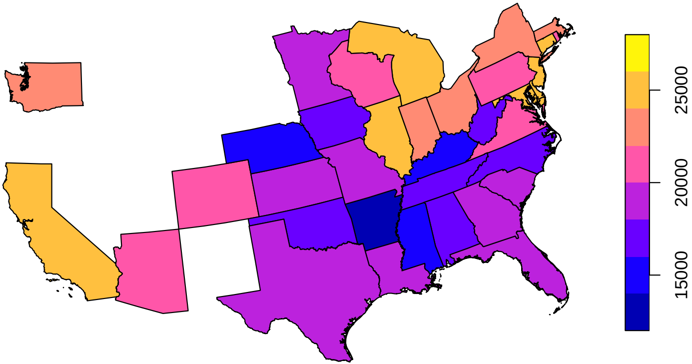
Amenities in 1960

African Americans



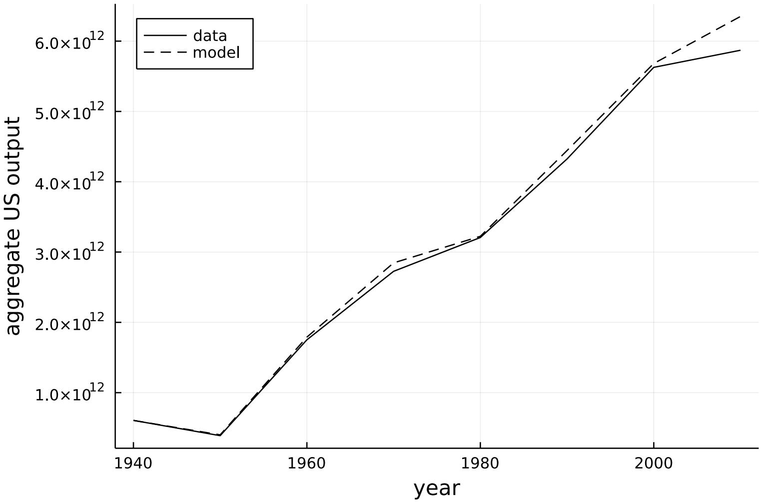
amenities for others in 1960

Productivity in 1960

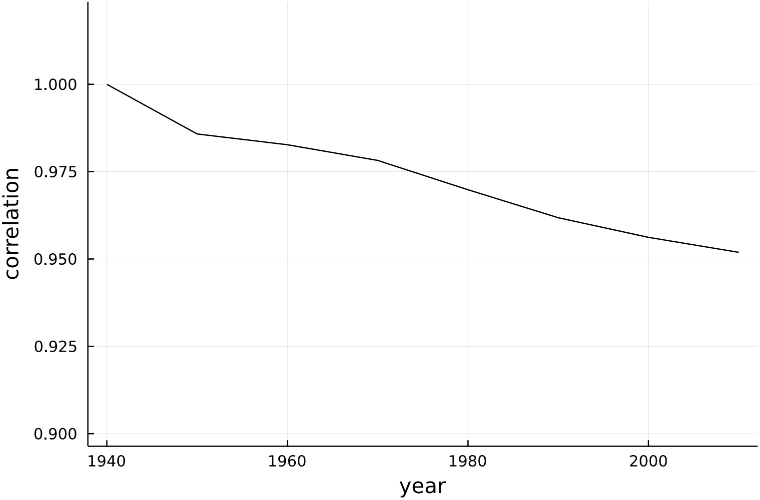


Model Fit

US output: Model vs Data



Populations of Race-Age-Locations: Model vs Data



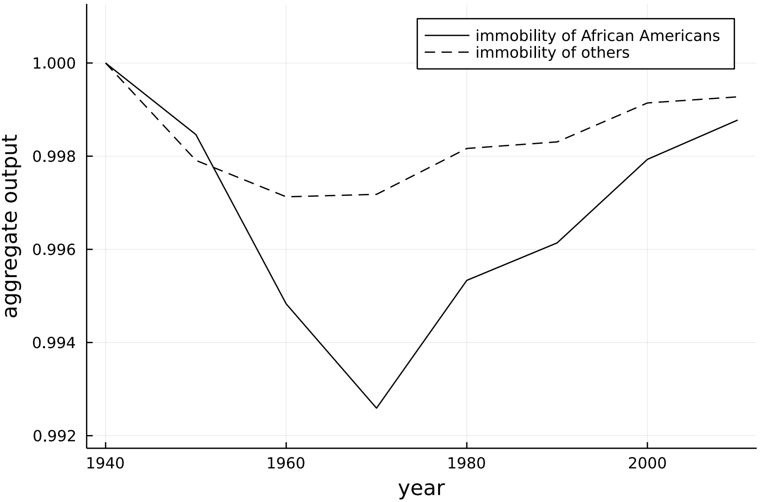
Counterfactuals

Counterfactuals

1. African Americans cannot move across the North and the South from 1940 to 1960.

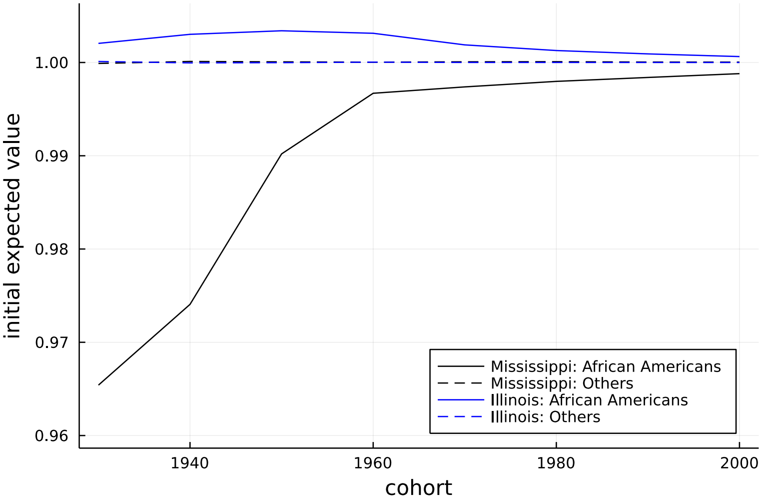
2. Others cannot move across the North and the South for the same period.

US output relative to the Baseline Equilibrium



Initial Expected Values

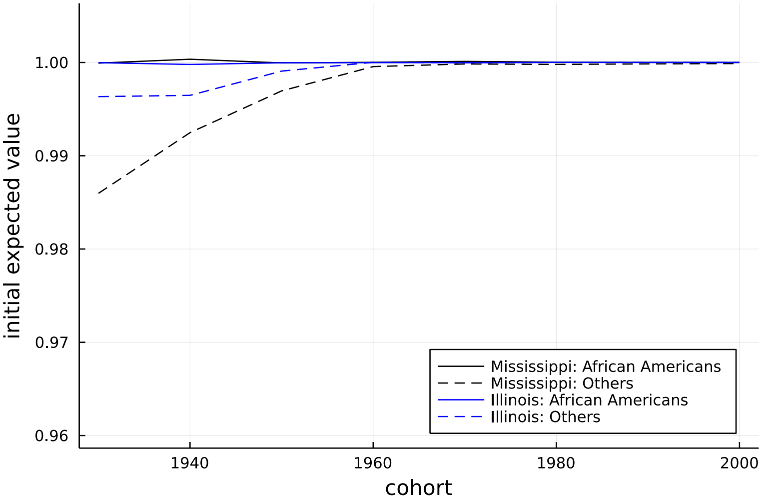
Immobility of African Americans Relative to the Baseline



map: African Americans map: others

Initial Expected Values

Immobility of Others Relative to the Baseline

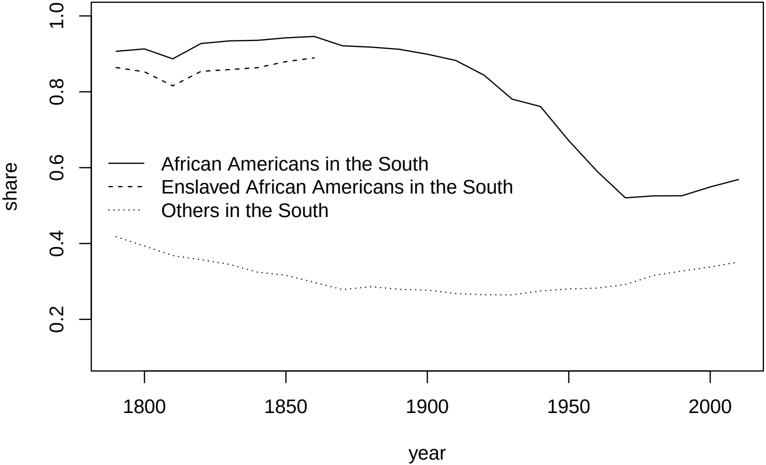


map: African Americans map: others

Conclusion

- I quantify the aggregate effects of the great Black migration with a dynamic spatial model.
- African Americans migrated from the South to the North for higher wages despite their high migration costs and low amenities in the North.
- The mobility of African Americans and others increased aggregate output in 1970 by 0.7 and 0.3%, respectively.
- The mobility of African Americans induced
  - a large increase in the welfare of African Americans in the South,
  - a small decrease in the welfare of African Americans in the North.

Population Shares in the South by Race



South: confederate states + 4 border states back

Movers and Stayers

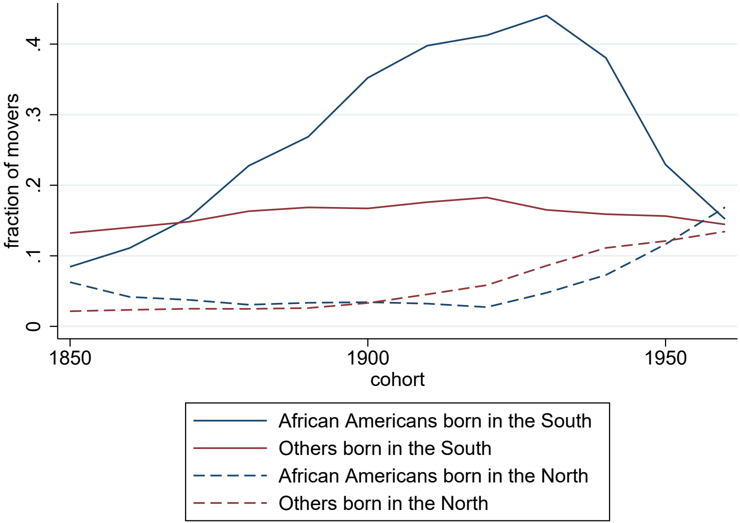
For each cohort c, birthplace (the North or the South), race (African Americans or others),

- stayers live in the birthplace as of year $c + 50$,
- movers live in the other place than the birthplace as of year $c + 50$.

back

Fractions of Movers

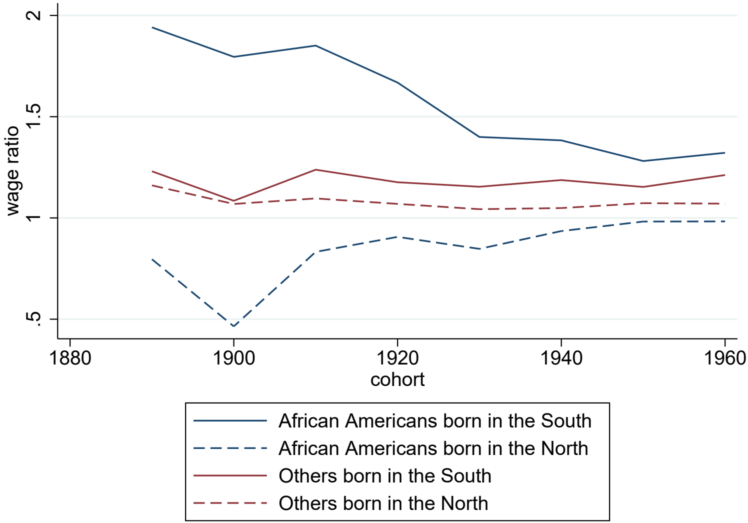
for Cohort c as of Year $c + 50$



back

Mover-Stayer Wage Ratios

Cohort c as of year $c+50$



gaps in dollar value | ratios in per capita payrolls | back

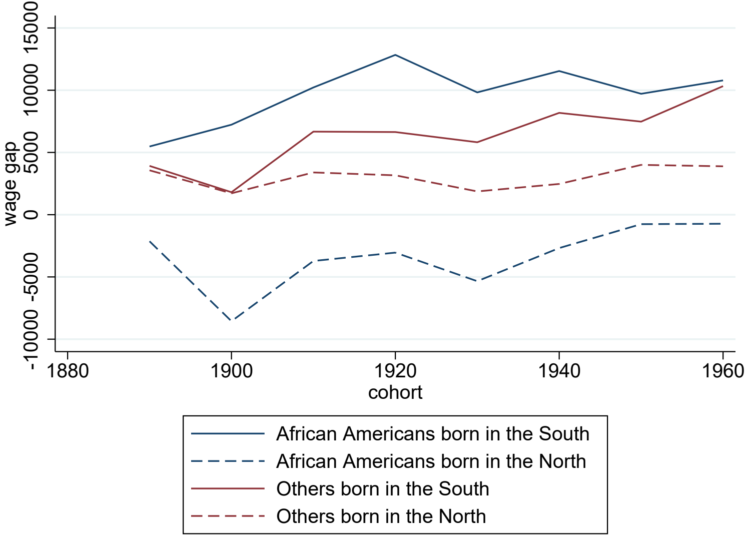
Wage and Rent Gaps between Movers and Stayers

for African Americans from the South

gaps in per capita payrolls and rent | back

Mover-Stayer Wage Gaps

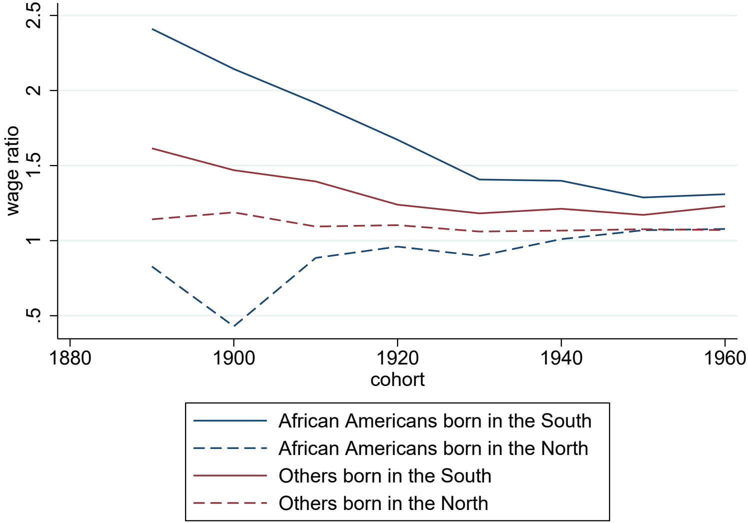
Cohort x as of year $x + 50$



Back

Mover-Stayer Ratios in Per Capita Payroll

Cohort x as of year $x + 50$



Back

Gaps in Per Capita Payroll and Rent

for African Americans from the South

Back

Relative Wages of Races within Ages

- The relative wages within cohorts are

$$\frac{w^n_{b,a,t}}{w^n_{o,a,t}} = \frac{\left(\kappa^n_{b,a,t}\right)^{\frac{1}{\sigma_r}}\left(L^n_{b,a,t}\right)^{-\frac{1}{\sigma_r}}}{\left(\kappa^n_{o,a,t}\right)^{\frac{1}{\sigma_r}}\left(L^n_{o,a,t}\right)^{-\frac{1}{\sigma_r}}}$$

back

Elasticity of Substitution across Races

- For location n, age a, period t, the CES production function implies

$$\frac{w^n_{b,a,t}}{w^n_{o,a,t}} = \frac{\left(\kappa^n_{b,a,t}\right)^{\frac{1}{\sigma_r}}\left(L^n_{b,a,t}\right)^{-\frac{1}{\sigma_r}}}{\left(\kappa^n_{o,a,t}\right)^{\frac{1}{\sigma_r}}\left(L^n_{o,a,t}\right)^{-\frac{1}{\sigma_r}}}.$$

- Taking logs of both sides,

$$\log\left(\frac{w^n_{b,a,t}}{w^n_{o,a,t}}\right) = -\frac{1}{\sigma_r}\log\left(\frac{L^n_{b,a,t}}{L^n_{o,a,t}}\right) + \frac{1}{\sigma_r}\log\left(\frac{\kappa^n_{b,a,t}}{\kappa^n_{o,a,t}}\right).$$

back

Estimation

Following Card (2009)

- The main specification is

$$\log\left(\frac{w^n_{b,a,t}}{w^n_{o,a,t}}\right) = -\frac{1}{\sigma_r}\log\left(\frac{L^n_{b,a,t}}{L^n_{o,a,t}}\right) + f_a + f_t + f_{a,t} + \varepsilon^n_{a,t}.$$

- Construct an IV using shift-share predicted populations

$$\hat{L}^n_{r,a,t} = \sum_{j=1}^{J} \mu^{n,j}_{r,a-1,t-1-X} \cdot s_{r,a-1,t-1} L^j_{r,a-1,t-1}.$$

- I set $X = 2$: the migration shares 20 years before.

Results

Dependent variable:	$\log(w^n_{b,a,t}/w^n_{o,a,t})$	
Model:	OLS	IV
$\log(L^n_{b,a,t}/L^n_{o,a,t})$	-0.1154***	-0.1108***
	(0.0120)	(0.0127)
fixed effects:		
year	✓	✓
age	✓	✓
year-age	✓	✓
Observations	1,368	1,368
First-stage F-statistic		3,112.5

Block bootstrap standard errors are in parentheses. ***: 0.01.

back

Migration Elasticity

- If real wage$^{j}_{r,a,t+1}$ increases by 1% *ceteris paribus*, $\mu^{j,i}_{r,a,t}$ increases by $\frac{1}{\nu}$%.

back

Rewriting Expected Values

Toward the estimation of the migration elasticity

- The expected value is the period utility plus the option value.

$$
\begin{aligned}
V^i_{r,a,t} &= u^i_{r,a,t} + \nu \log \left(\sum_{j=1}^{J} \exp(s_{r,a,t} V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t})^{1/\nu} \right) \\
&= u^i_{r,a,t} + \Omega^i_{r,a,t}.
\end{aligned}
$$

Decomposing Migration

- Using $\Omega^{j}_{r,a,t}$, I can write migrants of (r,a,t) from i to j as

$$L^{i}_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\left\{\frac{1}{\nu}(s_{r,a,t}V^{j}_{r,a+1,t+1} - \tau^{j,i}_{r,a,t}) - \frac{1}{\nu}\Omega^{i}_{r,a,t} + \log(L^{i}_{r,a,t})\right\}$$

- Destination and origin fixed effects capture $V^{j}_{r,a+1,t+1}$ and $\Omega^{i}_{r,a,t}$ respectively:

$$L^{i}_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\{v^{j}_{r,a,t} + \omega^{i}_{r,a,t} + \tilde{\tau}^{j,i}_{r,a,t}\},$$

where

$$v^{j}_{r,a,t} = \frac{1}{\nu}s_{r,a,t}V^{j}_{r,a+1,t+1},$$

$$\omega^{i}_{r,a,t} = -\frac{1}{\nu}\Omega^{i}_{r,a,t} + \log(L^{i}_{r,a,t}),$$

$$\tilde{\tau}^{j,i}_{r,a,t} = -\frac{1}{\nu}\tau^{j,i}_{r,a,t}.$$

Recovering Period Utility

- Arranging destination and origin fixed effects backs out period utilities

$$
\begin{aligned}
& \frac{v^j_{r,a,t}}{s_{r,a,t}} + \omega^j_{r,a+1,t+1} - \log(L^j_{r,a+1,t+1}) \\
= & \frac{1}{\nu} u^j_{r,a,t} \\
= & \frac{1}{\nu} \left\{ \log\left(\frac{w^j_{r,a+1,t+1}}{(r^j_{t+1})^{\gamma}} \right) + \log(B^j_{r,a+1,t+1}) \right\}.
\end{aligned}
$$

Two-Step Estimation of $1/\nu$

Following Artuc and McLaren (2015)

1. Regress the number of migrants on the destination and origin fixed effects and the terms capturing migration costs

$$L^{i}_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\left\{ v^{j}_{r,a,t} + \omega^{i}_{r,a,t} + \tilde{\tau}^{j\neq i}_{t} + \tilde{\tau}^{\{i,j\}}_{r,G(t)} + \tilde{\tau}^{\{i,j\}}_{a,G(t)} \right\} + \varepsilon^{j,i}_{r,a,t}.$$

   - $G(\cdot)$ classifies years to groups.

2. Regress the induced period utilities times the migration elasticity on wages and the terms capturing amenities

$$\begin{aligned} &\frac{\hat{v}^{j}_{r,a,t}}{s_{r,a,t}} + \hat{\omega}^{j}_{r,a+1,t+1} - \log(L^{j}_{r,a+1,t+1}) \\ =&\frac{1}{\nu}\log(w^{j}_{r,a+1,t+1}) + \tilde{B}^{j}_{r,a+1} + \tilde{B}^{j}_{r,t+1} + \varepsilon^{j}_{r,a,t}. \end{aligned}$$

   - I instrument $w^{j}_{r,a+1,t+1}$ by $w^{j}_{r,a+1,t}$.

Estimates of Migration Elasticity

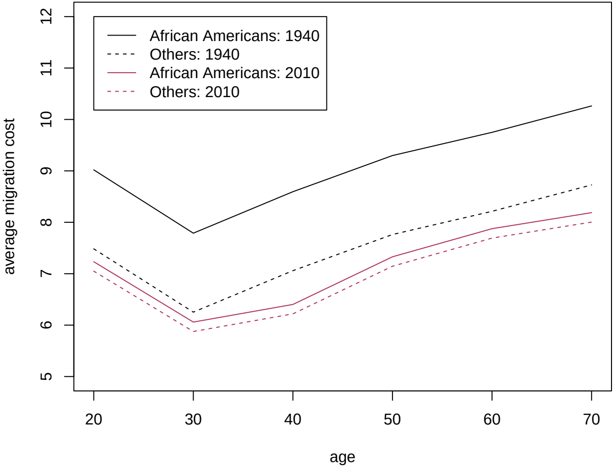
Dependent variable:	period utility $\times$ migration elasticity		
	(1)	(2)	(3)
log(real wage)	0.4976***	0.6129***	0.7676***
	(0.1323)	(0.1665)	(0.1952)
fixed effects:			
race-location	✓	✓	✓
age-location	✓	✓	✓
year-location	✓	✓	✓
age-race	✓	✓	✓
year-race	✓	✓	✓
age-race-location		✓	✓
year-race-location			✓
Observations	2,660	2,660	2,660

Robust standard errors clustered at locations. ***: 0.01.

Migration Elasticities in Literature

	location	value
Bryan and Morten	Indonesia	3.18
	US	2.69
Tombe and Zhu	China	1.50
Fajgelbaum, Morales, Suarez Serrato, and Zider	US	2.10
Caliendo, Opromolla Parro, and Sforza	EU	0.50
Suzuki	Japan	2.01 (1.57$\sim$3.32)

Migration Costs by Age



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First-Difference Estimation

Following Monras (2019)

- Taking the first differences of the relative wage equation

$$\Delta \log\left(\frac{w^n_{b,a}}{w^n_{o,a}}\right) = -\frac{1}{\sigma_r}\Delta \log\left(\frac{L^n_{b,a}}{L_{o,a}}\right) + \frac{1}{\sigma_r}\Delta \log\left(\frac{\kappa^n_{b,a}}{\kappa^n_{nb,a}}\right).$$

- The main specification is

$$\Delta \log\left(\frac{w^n_{b,a}}{w^n_{o,a}}\right) = -\frac{1}{\sigma_r}\Delta \log\left(\frac{L^n_{b,a}}{L_{o,a}}\right) + f_a + \varepsilon^n_a.$$

  - The time differences are taken between 1940 and 2010.
- The first-step IV specification is

$$\Delta \log\left(\frac{L^n_{b,a}}{L_{o,a}}\right) = \beta \log\left(\frac{L^n_{b,a,1930}}{L^n_{o,a,1930}}\right) + f_a + \varepsilon^n_a.$$

Results of the First-Difference Estimation

Dependent variable:	$\Delta \log(w^n_{b,a}/w^n_{nb,a})$	
Model:	OLS	IV
$\Delta \log(L^n_{b,a}/L^n_{nb,a})$	-0.1551***	-0.2043***
	(0.0241)	(0.0294)
fixed effects:		
age	✓	✓
Observations	266	266
First-stage F-statistic		661.8

Heteroskedasticity robust standard errors in parentheses. ***: 0.01.

Estimates of Elasticity of Substitution across Races

	$-1/\sigma_r$	implied σ_r
Level	-0.111	9.0
FD	-0.204	4.9
Boustan (2009)	-0.120	8.3
	(-0.186$\sim$-0.090)	(5.38$\sim$11.11)

Elasticity of Substitution across Ages

- The nested CES production function implies

$$\frac{w^n_{a,t}}{w^n_{a',t}} = \frac{\left(\kappa^n_{a,t}\right)^{\frac{1}{\sigma_a}}\left(L^n_{a,t}\right)^{-\frac{1}{\sigma_a}}}{\left(\kappa^n_{a,t}\right)^{\frac{1}{\sigma_a}}\left(L^n_{a',t}\right)^{-\frac{1}{\sigma_a}}},$$

where

$$w^n_{a,t} = \left[\sum_{r'} \kappa^n_{r',a,t}\left(w^n_{r',a,t}\right)^{1-\sigma_r}\right]^{\frac{1}{1-\sigma_r}},$$

$$L^i_{a,t} = \left[\sum_{r'}\left(\kappa^i_{r',a,t}\right)^{\frac{1}{\sigma_r}}\left(L^i_{r',a,t}\right)^{\frac{\sigma_r-1}{\sigma_r}}\right]^{\frac{\sigma_r}{\sigma_r-1}}.$$

back

Estimation of Elasticity of Substitution across Ages

- Fix age bin a'.
- The main specification is, for any $a \neq a'$,

$$\log\left(\frac{w^n_{a,t}}{w^n_{a',t}}\right) = -\frac{1}{\sigma_0}\log\left(\frac{L^n_{a,t}}{L^n_{a',t}}\right) + f_a + f_t + f_{a,t} + \varepsilon^n_{a,t}.$$

- $\hat{L}^n_{a,t}$ aggregates the shift-share predicted populations for (r', a, t, n)

$$\hat{L}^n_{a,t} = \left[\sum_{r'} (\kappa^n_{r',a,t})^{\frac{1}{\sigma_r}} (\hat{L}^n_{r',a,t})^{\frac{\sigma_r - 1}{\sigma_r}}\right]^{\frac{\sigma_r}{\sigma_r - 1}}.$$

- Construct an IV using $\hat{L}^n_{a,t}$.

Elasticity of Substitution across Ages

Dependent variable:	$\log(w^n_{a,t}/w^n_{a',t})$	
Model:	OLS	IV
$\log(L^n_{a,t}/L^n_{a',t})$	-0.3108***	-0.4322**
	(0.0751)	(0.1859)
fixed effects		
year	✓	✓
age	✓	✓
year-age	✓	✓
Weights	-	-
Observations	1,824	1,368
First-stage F-statistic		1,756.0

Block bootstrap standard errors are in parentheses. ***: 0.01, **: 0.05.

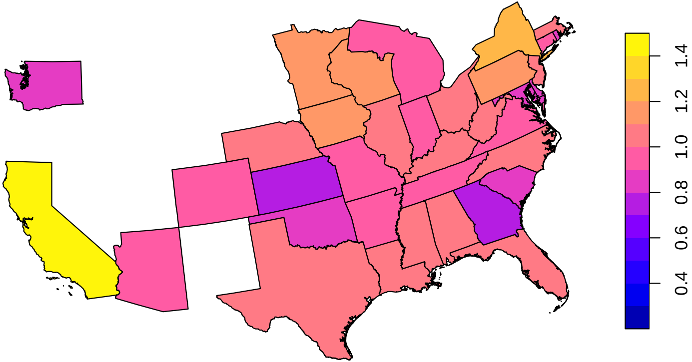
Estimates of Elasticity of Substitution across Ages

	-1/σ_0	implied σ_1
my estimate	-0.432	2.3
Card and Lemieux	-0.203	4.9
	(-0.233$\sim$-0.165)	(4.3$\sim$6.1)

- Ottaviano and Peri (2012) and Manacorda et. al. (2012) found estimates similar to Card and Lemieux (2001).
- My age bin is 10 years but the literature's age bin is 5 years.

Amenities in 1960

Others



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Estimation: Rent Elasticity η

- Assume that the rent elasticity η is common in all locations.
- Taking logs of the rent equation:

$$\log r_t^i = \log \bar{r}^i + \eta \log \left(\gamma \sum_r \sum_c L_{r,c,t}^i w_{r,c,t}^i \right).$$

- Take time differences:

$$\Delta \log r^i = \eta \Delta \log(\text{income}^i).$$

- Then I can use states as a sample.
- For state i, the econometric specificaiton is

$$\Delta \log r^i = \eta \Delta \log(\text{income}^i) + \varepsilon_i.$$

  - The time differences are taken between 1970 and 2010.
- I instrument $\Delta \log(\text{income}^i)$ by the manufacturing shares and college graduates shares as of 1950.

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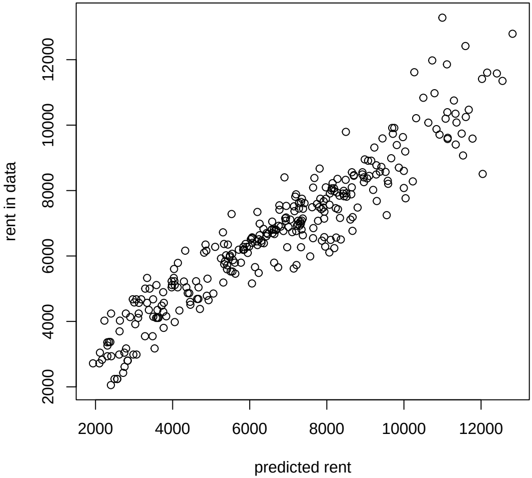
Estimates: Rent Elasticity η

Dependent variable:	$\Delta \log r^i$	
Model:	OLS	IV
$\Delta \log(\text{income}^i)$	0.3948***	0.4092***
	(0.0254)	(0.0264)
Weights	L^i_{1970}	L^i_{1970}
Observations	38	38
First-stage F-statistic		162.4

Robust standard errors are in parentheses. ***: 0.01.

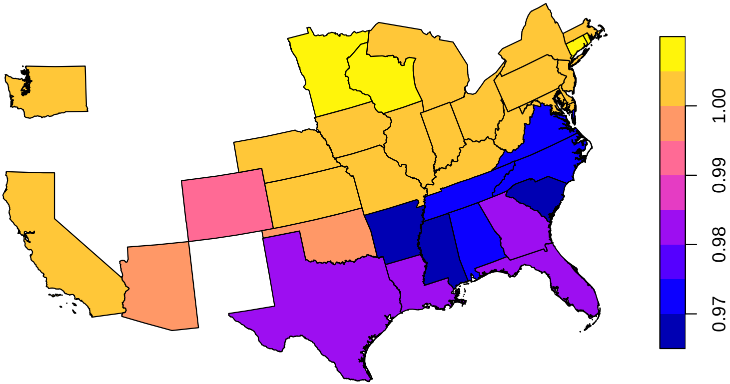
Goodness of Fit: Nation-wide Rent Elasticity



correlation: 0.944

Expected Values of African Americans Born in the 1930s

Immobility of African Americans relative to the baseline



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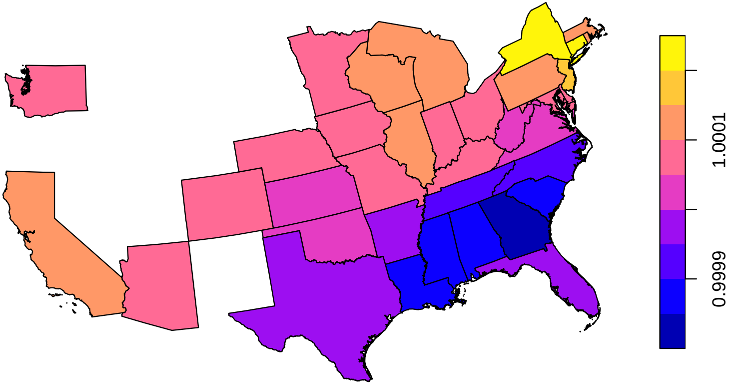
Expected Values of Others Born in the 1930s

Immobility of African Americans relative to the baseline



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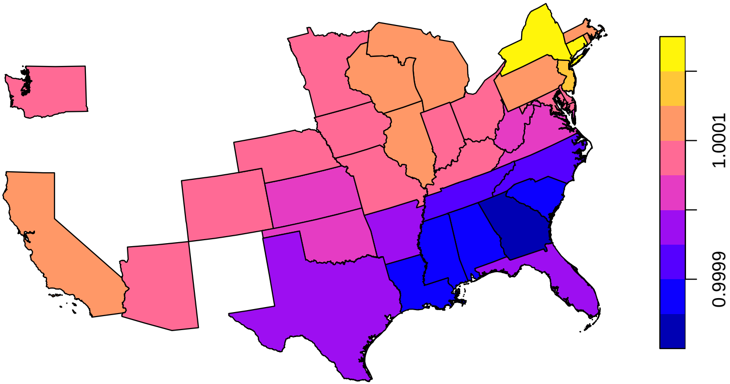
Expected Values of African Americans Born in the 1930s

Immobility of others relative to the baseline



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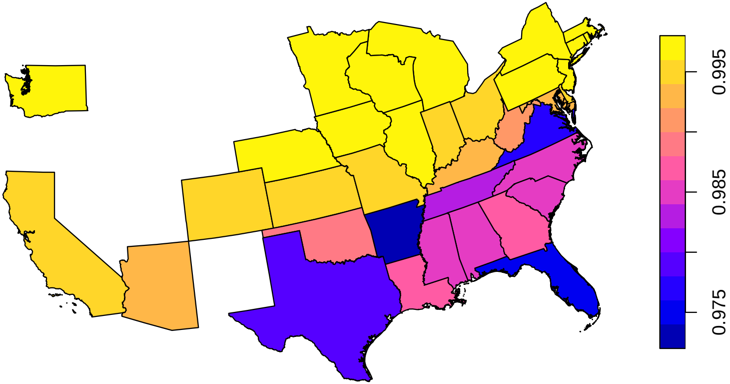
Expected Values of Others Born in the 1930s

Immobility of others relative to the baseline



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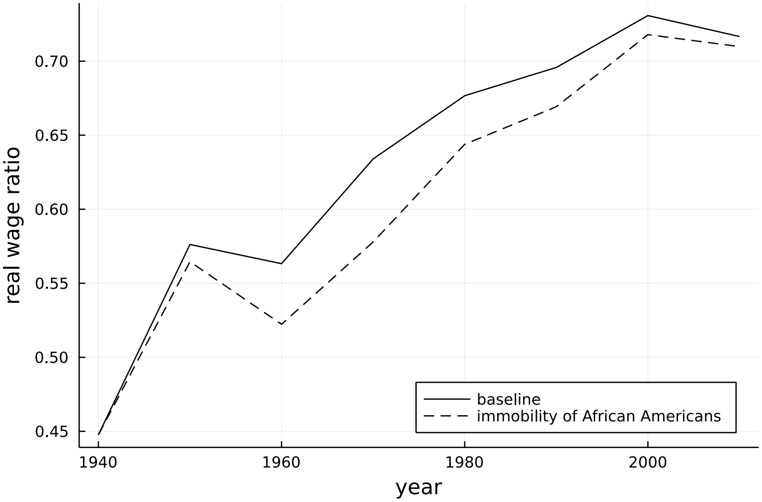
Average Real Wage Ratios

between African Americans and others



Parameters in the Baseline Equilibrium

- I have parameter values from 1940 to 2010.
- From 2020 onward, I assume all parameters are as of 2010.
- But I use fertility such that the populations of African Americans and others will be constant from 2010.
- So that the economy will converge to the steady state.

Value Function Iteration

1. Load the expected values of the final steady state $V^i_{r,a,\infty}$. Assume the economy converges to the steady state in period T: $V^i_{r,a,T} = V^i_{r,a,\infty}$.
2. Load the populations in the initial period $L^i_{r,a,0}$.
3. Guess the expected values from period 0 to $T-1$ $V^i_{r,a,t}$ for $t = 0, \cdots, T-1$.
4. Compute migration shares $\mu^{j,i}_{r,a,t}$ given the guessed expected values $V^i_{r,a,t}$.
5. Compute the populations $L^i_{r,a,t}$ forward given the migration shares $\mu^{j,i}_{r,a,t}$.
6. Compute wages $w^i_{r,a,t}$, rent r^i_t, and eventually period utility $u^i_{r,a,t}$ given the populations $L^i_{r,a,t}$.
7. Compute the expected values $V^i_{r,a,t}$ backward given the period utility $u^i_{r,a,t}$.