

# Gains from Trade with Footloose R&D

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# 1. Overview

## 2. Model Environment

## 3. Equilibrium

## 4. Results

# Motivation & Research Questions

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- **Motivating Fact:** Top R&D talent is highly mobile.
  - e.g., Brain Drain in Tech: Brilliant researchers educated in Europe (such as top French engineers) often relocate to the US, seeking larger markets and capital
- **Research Questions:**
  - **RQ1:** When R&D investment is internationally mobile (“Footloose”), which countries attract innovation and accumulate higher technology levels?
  - **RQ2:** How is this international concentration of technology induced by existing trade patterns, and how does it reshape them?
  - **RQ3:** How do counterfactual changes in trade barriers affect welfare once this endogenous reallocation of R&D is taken into account?

# Methodology

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- Quantitative trade model (Eaton and Kortum, 2002)
- Endogenous technological level
  - Expanding variety model (Romer, 1990)
  - (Interpretable as a dynamic version of Krugman, 1980)
- Footloose R&D and Directed Technological Change
  - Extends Acemoglu (2002), where innovation is directed toward the most profitable sector, to a spatial setting driven by local market size
- Exact Hat Algebra
  - Allows for counterfactual analysis by estimating only a small set of elasticity parameters

# Results (Preview) & Contributions

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- **Theoretical Result: A New Welfare Formula**

- Welfare gains are determined by the standard ACR term (Arkolakis et al., 2012) augmented by a new term capturing the R&D reallocation:

$$\widehat{W}_n = \underbrace{\widehat{\pi}_{nn}^{-1/\beta\theta}}_{\text{Standard ACR}} \times \underbrace{\Phi(\{\widehat{\pi}_{kk}\}, \dots)}_{\text{Endogenous Tech. Change}}$$

- **Quantitative Result: Amplified but Uneven Gains**

- Compared to the ACR benchmark, the gains from trade (GFT) are significantly amplified for countries that attract innovation
- However, some countries experience *smaller* welfare gains than the benchmark due to the endogenous outflow of R&D talent

- **Contribution to the Literature**

- Existing trade-and-innovation models typically focus on the *domestic* reallocation of R&D across sectors (e.g., Somale, 2021)
- This paper highlights the **international spatial reallocation** of R&D

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## Basic Setup

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- Based on EK + Krugman + Romer + DTC (Acemoglu)
- $J$  countries,  $n, i, j, k \in \{1, \dots, J\}$
- Time is continuous and indexed by  $t \in [0, \infty)$
- **Production Labor** ( $L_n$ ): immobile and country-specific
- **Researchers** ( $R$ ): A fixed global supply that is internationally mobile (“Footloose”)
- The household optimization follows the standard Ramsey-style setup, yielding the familiar Euler equation:

$$\dot{c}_{n,t}/c_{n,t} = r_{n,t} - \rho$$

# International Trade

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- Adopt the multi-country Ricardian framework a lá Eaton & Kortum (2002)
- Continuum of tradable goods  $\omega \in [0, 1]$
- $\tilde{z}_i(\omega)$ : country-specific productivity in producing good  $\omega$ 
  - $\tilde{z}_i(\omega) \sim$  Fréchet dist.  $\tilde{\mathcal{F}}_i(\tilde{z}) = \exp(-\tilde{T}_i \tilde{z}^{-\theta})$
- $\Psi_{i,t}$ : input cost in country  $i$
- $d_{ni}$ : iceberg trade cost
- Price level in country  $n$  is given by:

$$P_{n,t} \propto \left( \sum_{i=1}^J \tilde{T}_i (d_{ni} \Psi_{i,t})^{-\theta} \right)^{-1/\theta},$$

# Production Structure

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- Tradable goods are produced using labor and a continuum of endogenous intermediate varieties,  $v \in [0, N_{i,t}]$ 
  - $N_{i,t}$  grows through R&D (Romer, 1990)
- Intermediate firms engage in **monopolistic competition** and charge a constant markup  $1/(1 - \beta)$
- Each intermediate firm's profit  $\Pi_{i,t}$  is proportional to the local market size:

$$\Pi_{i,t} = (1 - \beta) \frac{w_{i,t} L_i}{N_{i,t}}$$

# Innovation

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- Inputting a unit of researcher creates  $\eta_i N_{i,t}^\chi N_{w,t}^{1-\chi}$  units of new varieties, i.e.,

$$\dot{N}_{i,t} = \eta_i N_{i,t}^\chi N_{w,t}^{1-\chi} R_{i,t},$$

where  $N_{w,t}$  is the world technological frontier that grows at an exogenous rate  $g$

- The free entry condition requires:

$$\underbrace{\eta_i N_{i,t}^\chi N_{w,t}^{1-\chi} V_{i,t}}_{\text{the value of innovation}} = w_{i,t}^R \quad \text{if innovation occurs in country } i$$

where  $V_{i,t}$  denotes the value of the intermediate firm, which is characterized by the following Hamilton-Jacobi-Bellman equation:

$$r_{i,t} V_{i,t} - \dot{V}_{i,t} = \Pi_{i,t}$$

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## Effective Productivity & Detrended Variables

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- On the BGP, each good's price is rewritten as:

$$\frac{d_{ni} \bar{w}_i^\beta P_i^{1-\beta}}{\bar{N}_i^\beta \tilde{z}_i(\omega)}$$

where  $\bar{w}_i \equiv w_{i,t} e^{-gt}$  and  $\bar{N}_i \equiv N_{i,t} e^{-gt}$

- Define *effective* productivity as  $z_i(\omega) \equiv \bar{N}_i^\beta \tilde{z}_i(\omega)$
- $z_i(\omega)$  follows Fréchet distribution:

$$\mathcal{F}_i(z) = \Pr\left(\bar{N}_i^\beta \tilde{z}_i(\omega) < z\right) = \exp\left(-\tilde{T}_i (z \bar{N}_i^{-\beta})^{-\theta}\right) = \exp\left(-T_i z^{-\theta}\right),$$

where  $T_i \equiv \tilde{T}_i \bar{N}_i^{\beta\theta}$

## R&D Allocation

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- Researchers relocate to equalize real wages globally ( $w_{i,t}^R/P_i = w_{j,t}^R/P_j$ ).
- From the free entry condition, country  $i$ 's share of global researchers is given by:

$$\frac{R_i}{R} = \frac{w_{i,t}L_i/P_i}{\sum_k w_{k,t}L_k/P_k} \equiv s_i$$

- This and  $\dot{N}_{i,t} = gN_{i,t} = \eta_i N_{i,t}^\chi N_{w,t}^{1-\chi} R_i$  yield:

$$N_{i,t} = \left( \frac{\eta_i}{g} s_i R \right)^{1/(1-\chi)} N_{w,t} \implies T_i = \tilde{T}_i \left( \frac{\eta_i}{g} s_i \bar{R} \right)^{\beta\theta/(1-\chi)} \bar{N}_w^{\beta\theta}$$

# Equilibrium Conditions (Hat Algebra)

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1. Expenditures

$$\widehat{X}_n = \widehat{w}_n$$

2. Price levels

$$\widehat{P}_n = \left( \sum_{i=1}^N \pi_{ni} \widehat{T}_i \left( \widehat{d}_{ni} \widehat{w}_i^\beta \widehat{P}_i^{1-\beta} \right)^{-\theta} \right)^{-1/\theta}$$

3. Bilateral trade shares

$$\widehat{\pi}_{ni} = \widehat{T}_i \left( \frac{\widehat{d}_{ni} \widehat{w}_i^\beta \widehat{P}_i^{1-\beta}}{\widehat{P}_n} \right)^{-\theta}$$

4. Labor incomes

$$\bar{w}_i \bar{L}_i \widehat{w}_i = \beta \sum_{n=1}^N \pi_{ni} \bar{X}_n \widehat{\pi}_{ni} \widehat{X}_n$$

# Equilibrium Conditions (Hat Algebra)

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## 5. Technology levels

$$\hat{T}_i = \left( \frac{\hat{w}_i / \hat{P}_i}{\sum_j s_j \hat{w}_j / \hat{P}_j} \right)^{\beta\theta / (1-\chi)}$$

where

$$s_j = \frac{\bar{w}_j \bar{L}_j / P_j}{\sum_k \bar{w}_k \bar{L}_k / P_k}$$

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## Relative Change in Real Wages

- We have

$$\frac{\widehat{w}_n}{\widehat{P}_n} = \underbrace{\widehat{\pi}_{nn}^{-1/\beta\theta}}_{\text{ACR}} \times \underbrace{\widehat{T}_n^{1/\beta\theta}}_{\text{DTC}}$$

- But now,  $\widehat{T}_n$  is determined endogenously
- Can we get a closed-form formula?  $\rightarrow$  Yes

### The New Welfare Formula with Footloose R&D

$$\frac{\widehat{w}_n}{\widehat{P}_n} = \widehat{\pi}_{nn}^{-\frac{1}{\beta\theta}} \left( \frac{\widehat{\pi}_{nn}^{\frac{1-\chi}{\chi\beta\theta}}}{\sum_k s_k \widehat{\pi}_{kk}^{\frac{1-\chi}{\chi\beta\theta}}} \right)^{\frac{1}{1-\chi}}$$

# Gains from Trade

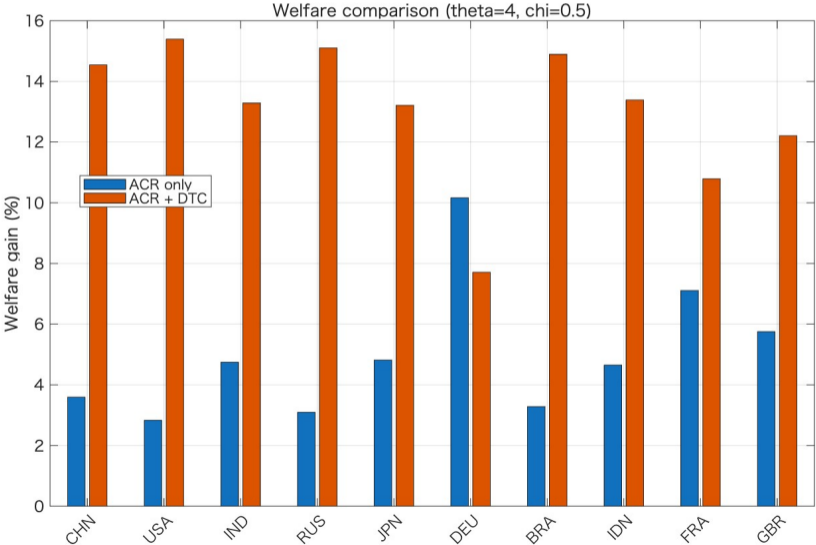
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- Consider the case where  $\hat{d}_{ni} \rightarrow \infty$  ( $n \neq i$ )
- We have

$$\text{GFT}_n = \left( \frac{\hat{w}_n}{\hat{P}_n} \right)^{-1} - 1 = \pi_{nn}^{-\frac{1}{\beta\theta}} \left( \frac{\pi_{nn}^{-\frac{1-\chi}{\chi\beta\theta}}}{\sum_k s_k \pi_{kk}^{-\frac{1-\chi}{\chi\beta\theta}}} \right)^{-\frac{1}{1-\chi}} - 1$$

- Numerical exercise
  - Set  $\beta\theta = 4$  and  $\chi = 0.5$
  - Data sources:
    - Home trade shares ( $\pi_{nn}$ ): UN Comtrade & World Bank WDI
    - R&D shares ( $s_n$ ): Proxied by Real GDP from Penn World Table (PWT)

# Gains from Trade



# Mechanism: The Brain Drain Effect

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- **Data: Home Trade Shares ( $\pi_{nn}$ )**
  - **Mega-Markets:** USA (0.89), CHN (0.87)
  - **Highly Open Economies:** DEU (0.68), FRA (0.76)
- **Standard ACR Logic (The Benefit of Trade):**
  - Highly open economies (low  $\pi_{nn}$  like DEU) typically enjoy larger standard gains from trade
- **New Logic: “Home Market Effect” & “Brain Drain”**
  - To avoid iceberg trade costs, footloose R&D talent relocates to absolute mega-markets where they can serve the largest customer base directly
  - While free trade lowers prices, highly open small and mid-sized economies suffer a “brain drain”

# Conclusion & Future Work

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- **Conclusion**

- Developed a quantitative trade model where the international allocation of R&D talent is endogenously determined
- Derived a tractable welfare formula highlighting that endogenous R&D reallocation significantly alters the standard gains from trade

- **Plan: a Full Quantitative Model**

- Replace strict real wage equalization with logit-based researcher mobility to govern bilateral talent flows
- Incorporate tradable/non-tradable sectors and trade imbalances for richer calibration
- Solve the extended full model to evaluate rich policy scenarios (e.g., the welfare impact of US-China decoupling).