Mobile Capital, International Inequalities, and the Welfare Gains from Trade

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Abstract

This paper explores the impact of mobile capital and firm heterogeneity on international inequalities and a country's welfare gains from trade integration. We show that, when trade is liberalized, the international inequality of wages is either bell-shaped or increases monotonically. With firm selection, however, the international inequality of firm shares is always magnified due to a strong within-industry reallocation effect in favor of exporting firms within the small country. Our model reveals that a country's gains from trade depend not only on changes in domestic expenditure share and trade elasticity, but also on the relative factor price that adjusts with capital reallocation across countries. Numerical exercises are provided to show how relative factor price adjustment is quantitatively relevant for welfare changes.

Keywords: firm heterogeneity, mobile capital, international inequalities, gains from trade

JEL Classification: F12, R12

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1 Introduction

With globalization, countries have become more interrelated, as evidenced by the significantly increasing scale of international goods and capital flows across countries. This trend has prompted increased interest in the analysis of the impact of trade integration on inequalities between different countries, as well as the extent to which a country can gain from trade. One general belief is that globalization might result in the de-industrialization of small countries because of their size disadvantage. Meanwhile, international inequalities can be multifaceted, as represented by cross-country differences in wages, firm productivity, and standards of living. Does trade integration affect these different aspects of international inequalities in the same way or not? And if not, in what ways are they affected?

Moreover, although a recent paper by Arkolakis, Costinot, and Rodríguez-Clare (2012, henceforth ACR) proposes two variables—domestic expenditure share and elasticity of trade with respect to variable trade costs—to measure a country's gains from trade, the effect of factor mobility on welfare changes is not yet sufficiently known. In particular, along with capital flows across countries, if there is an adjustment of either the international trade pattern or the factor prices in a country, then can new channels that shape a country's gains from trade be identified?

This paper develops a highly tractable model that explores the determination of international inequalities and a country's welfare gains from trade integration with mobile capital. As deeply discussed in Piketty (2014), capital plays an important role in inequality studies. Here we embed capital movement together with firm delocation across countries into a standard trade model with firm heterogeneity and country asymmetry in population size. Specifically, we consider a world economy that consists of two countries and one sector with monopolistic competition and firm heterogeneity. For simplicity, in the first step, we adopt a production function in which capital and labor are used for distinct purposes. Capital, employed by either entrant or surviving firms, is used as the sunk cost of entry and the fixed costs of domestic production and exporting. Meanwhile, labor is used by surviving firms as the variable input in production. This setup allows us to identify two important forces that may jointly shape international inequalities and a country's welfare gains from trade. The first force works through firm location *across* countries, whereas the second works by reallocating profits among heterogeneous firms *within* each country.

We obtain two sets of results from our setup. Regarding the impact of trade integration on international inequalities, we show that the large country provides a higher wage rate, and that the cross-country wage differential is either bell-shaped or increases monotonically when variable trade costs decrease. This implies that cross-country wage rates do not automatically converge with trade liberalization. The presence of capital movement works in favor of the large country by engendering a net inflow of capital and greater labor demand to support its exports. Moreover, with firm selection in exporting, the effect of lower variable trade costs on wage rates relies on structural parameters, such as the fixed cost of exporting or the degree of firm heterogeneity. We find that the international wage differential increases when only a small fraction of firms can export. This is likely if variable trade costs are high or a firm cannot easily get a high productivity level to cover its fixed costs in exporting.

We also observe more uneven firm distribution across countries. With firm heterogeneity, the cross-country difference in firm shares adjusts with changes in both the mass of entrants and the cutoff productivities. At the entrant margin, the large country's share is positively related to its relative wage rate. It is thus possible that entrants relocate out of the large country if trade liberalization shrinks the international wage gap. However, this effect on firm shares is dominated by the within-industry reallocation effect that changes each country's cutoff productivities. When trade is liberalized, exporters in the small country can more easily export to the large market at a lower cost. As a result, there is a larger increase in the small country's domestic cutoff productivity, indicating stronger within-industry reallocation at work. Our model therefore highlights firm heterogeneity as a key factor that may outweigh the effect of entrant delocation on firm distribution across countries.

Our second set of results reveals a measure of each country's welfare changes if international trade occurs with mobile capital that is used for firm production, both with and without firm heterogeneity. In addition to the impact of changes in the domestic expenditure share, we find that any change in a country's relative factor price also matters for its welfare gains from trade. Within our context, the international flow of capital plays two important roles. First, it counteracts any imbalance of trade in goods; second, it engenders a general equilibrium effect on each country's ratio of wage to rental rate through an adjustment in both the number of entrants and the labor demand within each country. We show how this additional margin of welfare adjustment is affected by parameters such as population size or the degree of firm heterogeneity. We also evaluate the quantitative relevance of the relative factor price to the size of welfare changes. Numerical results indicate that, holding the welfare adjustment to changes in the domestic expenditure share fixed, the large country receives additional gains from trade as it experiences a net capital inflow, whereas the opposite holds for the small country.

As a complement, our second step of analysis considers an alternative cost function. Following Bernard et al. (2007b), we let capital and labor be jointly used to produce a composite good in a Cobb-Douglas production function, which is then used in both entry and firm production. Comparing the impact of trade liberalization in these different settings, we find that altering the model setup does not qualitatively change the effect of trade liberalization on international inequalities in either wage rates or firm shares. In addition, the quantitative relevance of relative factor price for welfare change increases under the Cobb-Douglas production function.

Our analysis is related to the literature that explores the determination of wage rates and firm productivities when trade occurs between asymmetric countries. In the case of country asymmetry in population size,[1] one approach is to allow for inelastic labor supply where market size affects the outcome of firm selection and wage rate (see, for example, Arkolakis et al., 2008; Demidova and Rodríguez-Clare, 2013; Felbermayr et al., 2013).[2] A common feature in this literature is that trade should be balanced between countries since each country consists of only one sector and only labor is used in firm production. In contrast, by allowing for capital that is internationally mobile as a new production factor, the trade balance condition is relaxed in our model. This enables us to capture the behavior of firm entry and its association with the cross-country trade pattern, which then feeds back into the determination of wage rates and cutoff productivities. In this sense, our model is also related to Takahashi et al. (2013) that allows for wage rate changes while excluding firm selection. With firm heterogeneity, the effect of within-industry reallocation plays a key role in our setup since it may become the dominating force in the change of cross-country inequalities.

Our paper also contributes to the literature that studies spatial configuration under firm heterogeneity. Since a firm cannot observe its productivity unless it has made the location decision and paid the sunk cost of entry, our model explores the effect of firm selection on agglomeration. This is related to Ottaviano (2012), von Ehrlich and Seidel (2013), and Behrens and Robert-Nicoud (2014).[3] One key result in this line of research is that firm heterogeneity magnifies spatial inequalities. When forces such as the pro-competitive effect are at work, the strength of agglomeration forces may also depend on the average productivity of firms in an industry. We differ by providing a fully general equilibrium setup that captures the interaction between firm heterogeneity and factor price adjustment. We also show that when it comes to the effects of trade integration,

[1] In the standard one-factor two-sector trade models with free trade in homogeneous goods and consumer demand being iso-elastic, country size has no role in either wages or cutoff firm productivities (Helpman et al., 2004; Baldwin and Forslid, 2010). Demidova (2008) considers country asymmetry in the firm productivity distribution while keeping the wage rate equalized between countries.

[2] Another approach allows for the pro-competitive effect which yields a positive correlation between country size and the cutoff firm productivity (Melitz and Ottaviano, 2008; Zhelobodko et al., 2012; Behrens et al., 2014).

[3] Baldwin and Okubo (2006) and Okubo et al. (2010) examine firm sorting issues. Productivity is revealed before a firm relocates to a different region.

spatial inequalities in wage rates and firm shares may exhibit different patterns of change if factor mobility and firm heterogeneity are at work simultaneously.

Finally, our welfare discussion relates to the literature that examines gains from trade in the presence of mobile production factors, including Redding (2012), Allen and Arkolakis (2014), and Caliendo et al. (2014). This literature quantitatively evaluates the impact of factor reallocation on welfare changes. Our model is closer to Redding (2012) in the sense that both allow for increasing returns to scale as an agglomeration force. We depart by including mobile capital across countries which relaxes the goods trade balance condition imposed in the previous literature. We show that the welfare impact of factor reallocation works via changes in the ratio of wage to rental rate, where structural parameters such as the country size and firm heterogeneity matter.

The remainder of this paper is organized as follows. Section 2 presents the basic model and Section 3 discusses the equilibrium. Section 4 examines the impacts of trade liberalization on wages, firm shares and cutoff productivities. We also present an alternative setting with different input usage. The implications of mobile capital and firm heterogeneity on each country's welfare gains from trade is studied in Section 5. Section 6 concludes.

2 The model

2.1 Preferences

Consider a world with two countries indexed by $i = 1, 2$. Country i is populated by L_i workers. Each worker supplies one unit of labor inelastically and earns wage w_i in country i. Meanwhile, each worker is endowed with one unit of capital. Country i's endowment of capital is written as $K_i = L_i$.[4] Workers cannot move across countries, whereas capital is perfectly footloose. With two sources of income, wage and capital investment revenue, each worker consumes differentiated goods provided by domestic and foreign firms. Preferences are given by

$$U = \left(\int_{\omega \in \Omega} m(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1,$$

where Ω is the set of varieties for consumption, $m(\omega)$ is the quantity of variety ω consumed, and $\sigma > 1$ is the elasticity of substitution. Define $p(\omega)$ as the price of variety ω. Given the utility function, the price index is given by $P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}$. Utility

[4] This setup omits country differences in the relative factor endowment. Thus, trade is not driven by Heckscher-Ohlin (HO) comparative advantage. See Bernard et al. (2007b) for a discussion about the joint impact of HO comparative advantage and firm heterogeneity on trade and welfare.

maximization indicates that for a firm that produces variety ω in country i, the total demand in country j is

$$q_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} X_j P_j^{\sigma-1}, \tag{1}$$

where $p_{ij}(\omega)$ is the price of variety ω produced in i and consumed in j, and X_j is the total expenditure in j.

2.2 Production

There is a continuum of monopolistically competitive firms in the differentiated goods sector. Two factors of production, labor and capital, are used in production. Following Melitz (2003), firms are ex-ante identical. After renting f_e units of capital as the sunk cost of entry in country i, a firm observes its productivity φ, which is randomly drawn from a cumulative distribution $G(\varphi)$. Each firm then decides whether to produce or not, and makes a decision on exporting. Once a firm produces, technology is such that for a firm that sells from country i to country j, $f_{ij} > 0$ units of capital and $\ell_{ij} = \tau_{ij} q_{ij}/\varphi$ units of labor are used as the fixed inputs and the variable inputs, respectively, where

$$f_{ij} = \begin{cases} f_d & \text{if } i = j \\ f_x & \text{if } i \neq j \end{cases}$$

is the amount required to serve the domestic market and foreign market, respectively. Assume that the variable trade costs, being of the iceberg type, are symmetric between countries: $\tau_{ij} = \tau_{ji} \geq 1$ $(j \neq i)$ and $\tau_{ii} = 1$. Thus, for a firm with productivity φ located in country i, the total cost of selling in country j is $TC_{ij}(\varphi) = r_i f_{ij} + w_i \tau_{ij} q_{ij}(\varphi)/\varphi$, where a larger φ indicates a smaller marginal cost.

Given the total demand function (1), the profit-maximizing prices are a constant markup over the delivered marginal costs in different destinations: $p_{ij}(\varphi) = \sigma w_i \tau_{ij}/[(\sigma-1)\varphi]$. The net profit for selling from country i to country j is $\sigma^{-1} X_j P_j^{\sigma-1}(p_{ij}(\varphi))^{1-\sigma} - r_i f_{ij}$. This determines the productivity cutoff φ_{ij}^* above which a firm in country i makes a positive profit in country j:

$$\varphi_{ij}^* = \left[\frac{\sigma r_i f_{ij} P_j^{1-\sigma}}{X_j} \left(\frac{\sigma w_i \tau_{ij}}{\sigma - 1}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \tag{2}$$

Following the literature, we restrict exogenous parameters to ensure that $\varphi_{ij}^* > \varphi_{ii}^* (j \neq i)$ always holds.[5] That is, exporters are always relatively more productive than are firms

[5] When country asymmetry is sufficiently large or the costs related to exporting are sufficiently low, it is possible that firms in the small country can easily export goods to the large foreign market. In this case, the cutoff productivity of exporting is smaller than that of domestic production. We simplify the analysis by focusing on cases where exporters in *both* countries are always more productive than firms that only sell in the domestic market. As an example of parameter restrictions, Appendix F presents the condition of country size asymmetry that ensures $\varphi_{ij}^* > \varphi_{ii}^*$ for any given cost of exporting.

that serve domestic market only. In addition, due to the positive amount of fixed inputs $f_{ii} = f_d > 0$, there exists a cutoff level φ_{ii}^* below which a firm cannot make a profit even in the domestic market and thus must exit. Defining M_i^e as the mass of entrant firms in country i, the mass of surviving firms is $M_i = M_i^e[1 - G(\varphi_{ii}^*)]$ and the mass of exporters is $M_{ij} = M_i^e[1 - G(\varphi_{ij}^*)]$.

Selection among heterogeneous firms indicates the following *ex-post* productivity density distribution $\mu_{ij}(\varphi)$:

$$\mu_{ij}(\varphi) = \begin{cases} \dfrac{g(\varphi)}{1 - G(\varphi_{ij}^*)}, & \text{if } \varphi \geq \varphi_{ij}^*, \\ 0, & \text{otherwise,} \end{cases}$$

where $g(\cdot)$ is the density function of φ. For convenience, the *ex-post* weighted average productivity is defined as

$$\tilde{\varphi}_{ij} = \Big[\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \mu_{ij}(\varphi) d\varphi \Big]^{\frac{1}{\sigma-1}}. \tag{3}$$

3 Market equilibrium

We now turn to the discussion of market equilibrium. First, free entry in the differentiated goods sector equates the entry costs to expected profit, leaving a zero expected net profit of entry. Specifically, for a firm in country i, this free entry condition can be expressed as

$$r_i f_e = \sum_j \left[1 - G(\varphi_{ij}^*)\right] r_i f_{ij} \left[\left(\frac{\tilde{\varphi}_{ij}}{\varphi_{ij}^*} \right)^{\sigma-1} - 1 \right], \tag{4}$$

in which $r_i f_{ij} \left[(\tilde{\varphi}_{ij}/\varphi_{ij}^*)^{\sigma-1} - 1 \right]$ is the *ex-post* expected net profit under the CES demand conditional on firm selling from country i to country j.

Next, consider the capital market equilibrium. The usage of capital indicates that

$$\sum_i K_i = \sum_i M_i^e f_e + \sum_i M_i^e \sum_j \left[1 - G(\varphi_{ij}^*)\right] f_{ij}. \tag{5}$$

In other words, both countries consume capital as the sunk cost of entry as well as the fixed costs of domestic production and exporting.

In an interior equilibrium of firm shares, capital movement equalizes the rental rates in both countries such that $r_i = \bar{r}$, where $\bar{r}$ is endogenously determined. The iso-elastic demand and the linearity of the cost functions yield a constant ratio, equal to $\sigma - 1$, between aggregate wage payment $\sum_i w_i L_i$ and aggregate rental payment of capital $\bar{r} \sum_i K_i$.[6]

[6]See Appendix A for details.

Thus, the equilibrium rental payment $\bar{r}$ is

$$\bar{r} = \frac{\sum_i w_i L_i}{(\sigma - 1) \sum_i K_i}. \tag{6}$$

In the following analysis, we introduce some notations to examine the role of country asymmetry in population size. First, the total labor force is $L \equiv \sum_i L_i$, and country 1's share of workers is $\theta \equiv L_1/L$. Suppose that $\theta > 1/2$, i.e., country 1 is larger than country 2 in terms of population size. Second, we set labor in country 2 as numeraire ($w_2 = 1$). country 1's relative wage rate is $w = w_1/w_2 = w_1$. In addition, let the total endowment of capital be $K = \sum_i K_i$ and the total mass of entrants be $M^e = \sum_i M_i^e$. country 1's share of entrants is denoted by $\lambda^e = M_1^e/M^e$.

We also follow the literature by assuming a Pareto distribution of φ such that $G(\varphi) = 1 - \varphi^{-\kappa}$ where $\varphi > \varphi_{i\min} = 1$, $\kappa > 1$ and $\kappa > \sigma - 1$. Parameter κ represents the degree of firm heterogeneity. Note that the weighted average productivity in (3) becomes

$$\tilde{\varphi}_{ij} = \left(\frac{\kappa}{\kappa - \sigma + 1} \right)^{\frac{1}{\sigma - 1}} \varphi_{ij}^*, \tag{7}$$

condition $\kappa > \sigma - 1$ ensures that $\tilde{\varphi}_{ij}$ is well-defined. We then express (4) as

$$\frac{\kappa - \sigma + 1}{\sigma - 1} f_e = \sum_j (\varphi_{ij}^*)^{-\kappa} f_{ij} \tag{8}$$

such that (5) yields the total mass of entrants

$$M^e = \frac{(\sigma - 1)K}{\kappa f_e}. \tag{9}$$

In addition, the total payment for capital input in country i equals $M_i^e \bar{r} \kappa f_e/(\sigma - 1)$ indicating that $w_i L_i/(w_j L_j) = M_i^e/M_j^e$ and that

$$M_i^e = \frac{w_i L_i}{\bar{r} \kappa f_e}. \tag{10}$$

This equation links M_i^e with the country asymmetry in population size and wage rate w_i. Specifically, the ratio of total wage payment between countries is equal to that of the mass of entrants. We can then write country 1's share of entrants (λ^e) as a function of its relative wage rate and its share of population size:

$$\lambda^e = \frac{w\theta}{w\theta + 1 - \theta}. \tag{11}$$

An immediate conclusion from (11) is that

$$\lambda^e - \theta = \frac{\theta(1 - \theta)(w - 1)}{w\theta + (1 - \theta)}$$

becomes positive iff $w > 1$. At the entrant margin, this reproduces the result of Takahashi et al. (2013) with firm homogeneity in production technology.

When there is firm heterogeneity, market size is of more importance to the mass of surviving firms and to aggregate productivity in each country. To see this, we examine the relationship between different productivity cutoffs. First, with the equalization of rental rates, Equation (2) establishes the following relationship between φ_{ij}^* $(j \neq i)$ and φ_{ii}^* :

$$\varphi_{12}^* = \varphi_{22}^* \Lambda w, \qquad \varphi_{21}^* = \varphi_{11}^* (\frac{\Lambda}{w}), \tag{12}$$

where $\Lambda = \tau (f_x/f_d)^{1/(\sigma-1)}$. Second, by including (12) in the free entry condition (4), the ratio of domestic productivity cutoffs satisfies

$$\left(\frac{\varphi_{11}^*}{\varphi_{22}^*}\right)^{\kappa} = \frac{1 - w^{\kappa}\Delta}{1 - w^{-\kappa}\Delta}, \tag{13}$$

where $\Delta = \tau^{-\kappa}(f_x/f_d)^{(\sigma-1-\kappa)/(\sigma-1)}$. As in the literature, we focus on parameters such that $\Delta \in [0,1]$ holds, which is automatically satisfied if $f_x \geq f_d$, namely, exporting incurs more fixed costs than would domestic production. This Δ can be taken as a measure of trade freeness. For any given κ and σ, a smaller value of either τ or f_x/f_d indicates a larger Δ.

In (13), $\varphi_{11}^*/\varphi_{22}^*$ decreases with w and $\varphi_{11}^* < \varphi_{22}^*$ holds iff $w > 1$. A higher wage rate in country 1 increases local income. The larger market demand overcompensates for the higher wage costs disadvantage and increases profitability for firms that enter country 1. Thus, less-productive firms are more likely to survive. This result arises from the CES setup that captures the income effect and excludes the presence of a pro-competitive effect.

Define λ as country 1's share of surviving firms. If the size asymmetry between countries is not sufficiently large such that $\varphi_{22}^* < \varphi_{21}^*$ always holds, we can write λ as

$$\lambda = \frac{w\theta(1 - w^{-\kappa}\Delta)}{w\theta(1 - w^{-\kappa}\Delta) + (1-\theta)(1 - w^{\kappa}\Delta)}. \tag{14}$$

As shown in Appendix E, λ increases with w in (14). Meanwhile, comparing (11) and (14), we see that $\lambda > \lambda^e > \theta$ holds if and only if $w > 1$.

To complete the model, we now derive an equation to pin down the equilibrium wage rate w^*. The *wage equation* is obtained by combining (2) with (13):

$$\mathcal{F}(w, \Delta) \equiv \mathcal{A}_2(w)\Delta^2 + \mathcal{A}_1(w)\Delta + \mathcal{A}_0(w) = 0, \tag{15}$$

where

$$\mathcal{A}_2(w) \equiv \theta(\sigma - 1 + \theta)w - (1-\theta)(\sigma - \theta),$$

$$\mathcal{A}_1(w) \equiv \sigma[(1-\theta)w^{\kappa} - \theta w^{1-\kappa}],$$
$$\mathcal{A}_0(w) \equiv \theta(1-\theta)(w-1).$$

Several properties are known from (15) and we summarize them in Lemma 1.

Lemma 1 *$\mathcal{F}(w, \Delta)$ increases with w. For any given $\Delta \in [0,1]$, Equation (15) gives a unique equilibrium wage rate w^* satisfying $w^* \in [1, \min\{\Delta^{-\frac{1}{\kappa}},\ (\frac{\theta}{1-\theta})^{\frac{1}{2\kappa-1}}\}]$. Furthermore, w^* increases with θ and decreases with κ.*

Proof. See Appendix B. ■

It is interesting that if $\kappa \to \sigma - 1$, then Equation (15) degenerates to the wage equation in Takahashi et al. (2013) where no firm selection is at work. In particular, parameters f_x, f_d, and f_e become irrelevant. To capture the intuition, di Giovanni and Levchenko (2013) show that firm revenue $R(\varphi)$ follows a power law distribution because $\Pr(R(\varphi) > y) = \zeta y^{-\delta}$ where ζ is a country-specific parameter of market access and the exponent parameter $\delta = \kappa/(\sigma - 1)$ decreases with κ. When $\kappa \to \sigma - 1$, the degree of firm heterogeneity is significantly large and δ is close to 1. With very fat-tailed firm size distribution, Zipf's law applies, implying that a tiny mass of very productive firms now accounts for the majority of production in the industry.[7] In this situation, the effect of firm selection on w^* is subdued for two reasons: first, the firms with cutoff productivities take an extremely small share of revenue and labor demand in each country; second, any change in f_e, f_d, f_x, and τ now imposes a marginal impact on the very productive firms that always export.[8] This is strikingly different from the situation where $\kappa > \sigma - 1$ and firms with cutoff productivities account for a larger proportion of the aggregate labor demand, which is further affected by the parameters $\{f_e, f_d, f_x, \tau\}$.

The proof of Lemma 1 also demonstrates that $w^* > 1$ for $\Delta \in (0,1)$. Therefore, the following inequalities are all true: (a) $w^* > 1$, (b) $\lambda > \lambda^e > \theta > 1/2$, and (c) $\varphi_{11}^* < \varphi_{22}^*$. Results (a) and (b) are closely related to the widely discussed HME without heterogeneous productivity, which indicates that the large country obtains a more-than-proportionate share of firms and a higher wage rate. Result (c) arises from firm heterogeneity and is derived from the general equilibrium framework with iso-elastic demand.

[7]In the case where not all firms export, di Giovanni et al. (2011) suggest using domestic sales data to calibrate firm size distribution parameters. Their results indicate that Zipf's law applies generally with δ estimates quite close to 1. Similar empirical results are found in other papers including Okuyama et al. (1999), Axtell (2001), and Fujiwara et al. (2004). However, the condition $\kappa > \sigma - 1$ is generally assumed in theoretical models, which ensures that the average firm revenue has a finite value.

[8]Regarding the welfare effect of changes in f_x or τ, di Giovanni and Levchenko (2013) show further that the outcome is significantly different between the cases of $\kappa \to \sigma - 1$ and $\kappa > \sigma - 1$.

Equipped with these results, we can further discuss the implications of firm selection and capital mobility on the trade pattern between countries. There is a negative pattern between the productivity cutoff for exporting and market size, i.e., $\varphi^*_{12}/\varphi^*_{21} = w^2(\varphi^*_{22}/\varphi^*_{11}) > 1$. This indicates a smaller share of exporting firms as well as a larger share of expenditures on imported varieties in the large country. However, capital movement works as a dominating force and makes the large country the net exporter of the differentiated goods. Appendix C provides the proof.

4 Impact of trade liberalization

Because our model allows for interdependence of wage rates, cutoff productivities and firm locations, this section discusses the impact of trade liberalization on each variable. We start by using (15) to explore the change in w^*. We then present graphical analysis to demonstrate how the equilibrium cutoff productivities and country 1's equilibrium share of surviving firms λ^* may change, keeping in mind that a larger trade freeness measure Δ may affect $w^*$and the variables concerned in different directions.

4.1 Change in wages

Note that (15) is well-defined for $\Delta \in [0, 1]$. By taking the partial derivatives of (15) with respect to Δ, we find that

$$\left.\frac{dw^*}{d\Delta}\right|_{\Delta=0} = \frac{\sigma(2\theta-1)}{\theta(1-\theta)} > 0, \quad \left.\frac{dw^*}{d\Delta}\right|_{\Delta=1} = \frac{1-2\theta}{\kappa} < 0. \tag{16}$$

Because (15) is a quadratic function of Δ, it has at most two roots of Δ for a given w^*. In other words, any horizontal line crosses wage curve $w^*(\Delta)$ at most twice in the Δ-w^* plane. Given (16), wage curve $w^*(\Delta)$ has a bell shape. Therefore, there exists a threshold level $\widehat{\Delta} \in (0, 1)$ at which $\partial w^*/\partial\Delta$ changes its sign from positive to negative.

Since Δ is related to f_x/f_d and κ for any given σ, we are then left with two possible patterns between τ and w^*. For large values of f_x/f_d and κ such that $(f_x/f_d)^{(\sigma-1-\kappa)/(\sigma-1)} < \widehat{\Delta}$, a lower τ always magnifies the wage inequality between countries that are asymmetric in population size. In contrast, for small values of f_x/f_d and κ such that $(f_x/f_d)^{(\sigma-1-\kappa)/(\sigma-1)} > \widehat{\Delta}$, there is a bell-shaped curve between τ and international wage inequality.

To explain the result, we consider a situation in which τ increases from $\tau = 1$. If $f_x > f_d$, selection into exporting still works. An increase in τ generates two opposing forces on the labor demand in each country. On the one hand, exporting becomes more difficult. This raises the cutoff productivity for exporting and decreases the mass of exporters. Given the asymmetry of w and P between countries for $\Delta < 1$, firms in the small country are more negatively affected by the contraction of foreign market access. Accordingly, a larger drop in labor demand occurs in country 1. Moreover, this effect is strong when a large share of firms engage in exporting, which occurs if f_x/f_d is small or if firms in a country are more likely to attain a high productivity level for exporting (i.e., when κ is small). Market competition, on the other hand, affects labor demand in the opposite direction because it is more difficult for imported varieties to penetrate the domestic market as τ increases. The resulting attenuation of foreign competition raises domestic firms' profits and expands their labor demand. Given f_x and the asymmetry of productivity cutoffs, the small country is more protected from the attenuation of foreign competition. The weakening of market competition results in a larger increase in labor demand within the small country. This effect is strong if exporting is initially difficult or if competition among firms is not intense, corresponding to a larger f_x/f_d and κ, respectively.[9]

Our result reveals that wages in the two countries do not necessarily converge monotonically, implying that the existing worldwide income inequality does not shrink automatically through trade integration. This result does not appear in a one-sector one-factor setup in which balanced trade and factor immobility always result in wage rate convergence between countries. Indeed, the result of wage convergence is widely observed in various trade models, irrespective of the existence of firm heterogeneity or the elasticity of consumer demand (Krugman, 1980; Arkolakis et al., 2008; Behrens et al., 2014). With labor immobility and one input in production, the mass of entrant firms remains fixed in both countries, indicating no changes in labor demand at the entrant margin. The trade balance condition then ensures that the small country benefits more from improved foreign market access when trade is liberalized. This is represented by a greater reallocation that benefits exporting firms and generates a larger increase in labor demand in the small country.

In contrast, the possibility of non-converging wage inequalities in our model can be explained by two important facts. First, capital movement across countries is likely to magnify the large country's size advantage by creating both a more-than-proportionate share of entrants and higher aggregate productivity among the surviving firms. This effect is strong if τ is initially large. Second, and more importantly, even though a gradual decrease in τ benefits the small country through greater expansion of foreign market access, there always exists self-selection of exporting and hence the parameters of f_x/f_d and κ play a key role. When firm selection into exporting is intense, as shown by a higher productivity cutoff for exporting or a small percentage of exporters in the economy, even

[9] In contrast, if all firms are able to export, when $f_x = f_d$ or $\kappa \to \sigma - 1$, then the countries are affected symmetrically by weaker market competition.

a sufficiently large decrease in τ cannot subvert the small country's size disadvantage, and thus, international wage inequality may continue increasing when τ decreases.

4.2 Change in productivity cutoffs

4.2.1 Local productivity cutoffs

This subsection studies how Δ affects the levels of different cutoff productivities. We first examine the change in country 2's equilibrium cutoff φ_{22}^*.

Because w is a key endogenous variable in our setup, Figure 1 puts w and φ_{22} in a plane and links both variables through two curves, representing the free entry condition and the zero-profit cutoff condition respectively. The equilibrium variables of $\{w^*, \varphi_{22}^*\}$ correspond to the intersection of both curves. Since we showed analytically in Section 4.1 that a larger Δ may either increase or decrease the equilibrium wage rate w^*, we focus on the direction of movement of φ_{22}^* on the vertical axis.

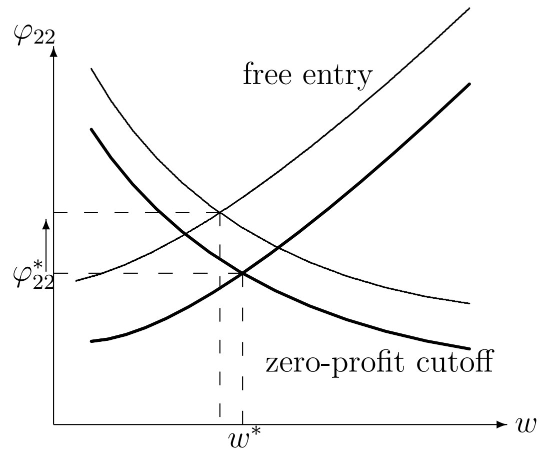

Figure 1: φ_{22}^* increases with Δ

The free entry condition reveals how w and φ_{22} interact such that an entrant receives a zero net expected profit of entry in country 2. Combining (8) with (12) and (13), this condition for country 2 is written as

$$\frac{\kappa - \sigma + 1}{\sigma - 1} \frac{f_e}{f_d} = \varphi_{22}^{-\kappa} \frac{1 - \Delta^2}{1 - w^\kappa \Delta}. \tag{17}$$

The RHS of (17) decreases with φ_{22} and increases with w. It yields a positively sloped *free entry curve* that links w with φ_{22}. An increase in country 1's wage rate indicates greater local demand, which is more attractive to firms in country 2. Some less-productive firms of country 2 can cover the fixed cost of exporting and sell in the larger foreign market. The resulting reallocation effect works in favor of the more productive exporters resulting in an increase in φ_{22}. Thus, an increase in Δ shifts the free entry curve upward from the thick to the thin curve.

Meanwhile, after substituting the expressions of r_i, E_i and P_i and $G(\varphi)$ into (2), the following equation holds for country 2:

$$\varphi_{22}^{\kappa} = \frac{\sigma(\sigma-1)f_d}{(\kappa-\sigma+1)f_e}\,\frac{(1-\theta)+\theta w^{1-\kappa}\Delta}{(1-\theta)(\theta w+\sigma-\theta)}. \tag{18}$$

The RHS of (18) decreases with w. Hence, we can plot a negatively sloped *zero-profit cutoff curve* in Figure 1. A higher wage rate in country 1 indicates a cost disadvantage for its exporters. The weakened competition from imported varieties then allows less productive firms to stay in country 2.

In addition, an increase in Δ shifts both the free entry curve and zero-profit cutoff curve upward. Regarding the free entry curve, an increase in Δ makes it easier to export from country 2 to country 1 for any given w. This lowers φ_{21} and increases country 2's average profit from exporting. With zero expected profit of entry, the resulting reallocation in favor of exporters indicates a profit loss for firms that serve the domestic market only, causing a decrease in φ_{22} . On the other hand, for firms in country 2, as Δ increases, more varieties are imported from country 1 and the imported varieties are less expensive for any given w. The resulting intensification of import competition decreases firm profit and prompts the least productive firms to exit the market. Thus, the zero-profit cutoff curve also moves upward when Δ increases.

Since the equilibrium cutoff φ_{22}^* is always related to the intersection of the free entry curve and the zero-profit cutoff curve, we conclude that a larger Δ always raises country 2's equilibrium cutoff φ_{22}^*. This change is irrelevant to the direction of movement of w^*.

A similar exercise shows that cutoff productivity φ_{11}^* also increases with Δ. Furthermore, since Δ does not occur in the free entry condition (8), there is a negative relationship between φ_{ii}^* and $\varphi_{ij}^*(j \neq i)$. That is, φ_{ii}^* and φ_{ij}^* always move in opposite directions following a change in Δ. Therefore, both φ_{12}^* and φ_{21}^* decreases with Δ.

4.2.2 Cutoff productivity ratio

Because our model features interaction between country size asymmetry and capital mobility, another important question is raised about the effect of trade liberalization on the ratio of productivity cutoffs. As shown in Figure 2, we can use the *free entry* curve and the *balance of payment* curve to examine the impact of Δ on $\varphi_{11}^*/\varphi_{22}^*$ in equilibrium.

The free entry curve is drawn from (13), showing a negatively sloped relationship between $\varphi_{11}/\varphi_{22}$ and w. An increase in country 1's relative wage expands the local market, which makes it easier for relatively unproductive entrants to survive. The balance

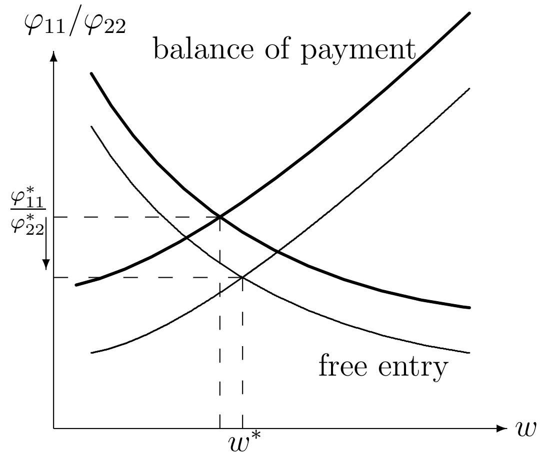

Figure 2: $\varphi^*_{11}/\varphi^*_{22}$ decreases with Δ

of payment condition for country 1, on the other hand, states that its net export volume of differentiated goods equals the amount of net capital inflow:

$$X_{12} - X_{21} = \bar{r}(M_1^e f_e + M_{11} f_d + M_{12} f_x) - \bar{r} K_1, \tag{19}$$

where X_{ij} $(i, j = 1, 2)$ is the total value of export from country i to country j, and the RHS measures the difference between the demand for capital from local firms and the endowment of capital owned by local workers. Appendix D shows that this equation can be rewritten as

$$\Delta \Big[\theta w^{1-\kappa} - (1-\theta) w^{\kappa} \Big(\frac{\varphi_{11}}{\varphi_{22}} \Big)^{-\kappa} \Big] = \theta (w-1) \, \frac{1 - \theta + \theta w^{1-\kappa} \Delta}{\theta w + \sigma - \theta}. \tag{20}$$

As shown in Appendix D, $\varphi_{11}/\varphi_{22}$ and w are positively related according to (20). So the balance of payment curve in Figure 2 is positively sloped. Intuitively, since the net inflow of capital into country 1 is equal to $\bar{r}K(\lambda_e - \theta)$, where both $\bar{r}$ and λ_e increase with w, country 1 should export more as it receives more capital inflow. Responding to this change in trade pattern, the within-industry reallocation effect in favor of exporters is stronger in the country that sends more goods abroad. As a result, $\varphi_{11}/\varphi_{22}$ increases with w.

Turning to the effect of Δ on the equilibrium values of $\varphi^*_{11}/\varphi^*_{22}$, the RHS of (13) decreases with Δ and thus the free entry curve shifts down if Δ increases. Holding $w > 1$ fixed, trade liberalization makes it easier for country 2's exporters to access the larger foreign market. This results in a smaller φ^*_{21} but a larger φ^*_{22}. In contrast, since the extent of foreign market access expansion is smaller for the large country, the increase in φ^*_{12} is smaller than that for φ^*_{21}. As a result, $\varphi^*_{11}/\varphi^*_{22}$ decreases if Δ increases. With regard to the balance of payment curve, Appendix D shows that it also shifts downward when

Δ increases. Country 1's wage cost disadvantage is strengthened by trade liberalization. Therefore, holding w constant, a larger Δ decreases the net export volume from country 1 to country 2. Within-industry reallocation then works to shape a lower value of $\varphi_{11}^*/\varphi_{22}^*$.

In sum, when Δ increases, both curves in Figure 2 shift downward from the thick to the thin curves. At the intersection of the free entry curve and the balance of payment curve, the equilibrium cutoff productivity ratio $\varphi_{11}^*/\varphi_{22}^*$ always decreases. Although the equilibrium wage rate w^* may either rise or fall, the monotonic decrease of $\varphi_{11}^*/\varphi_{22}^*$ implies that a stronger within-industry reallocation effect always occurs in the small country irrespective of the general equilibrium outcome for changes in w^*.

More specifically, when Δ is initially small, trade liberalization increases competition in the small country, even though the outflow of capital shrinks the mass of domestic entrants. When Δ is initially large, trade liberalization generates a greater benefit for the exporters in the small country. This effect causes a larger increase in labor demand among local exporters, which could reduce the wage gap across countries.

4.3 Changes in firm share

We now turn to the effect of Δ on country 1's equilibrium share of surviving firms λ^*. Since both the productivity cutoffs and λ^e are related to w, we follow the previous analysis by utilizing another graphical analysis that plots the relationship between w and λ. First, Equation (14), derived from the *capital mobility* condition, provides a positively sloped curve between w and λ. See Appendix E for the proof that λ increases with w. This is straightforward because a higher w induces a more than proportionate share of entrants in country 1 and firm selection magnifies country 1's size advantage ($\varphi_{11}^* < \varphi_{22}^*$).

Second, rewriting the balance of payment condition in country 1 as a function of w and λ yields

$$\Delta\left[\theta w^{1-\kappa} - (1-\theta)w^{\kappa-1}\left(\frac{1-\theta}{\theta}\right)\left(\frac{\lambda}{1-\lambda}\right)\right] = \theta(w-1)\left(\frac{1-\theta+\theta w^{1-\kappa}\Delta}{\theta w+\sigma-\theta}\right). \quad (21)$$

Appendix E shows that λ decreases with w in (21). Thus, there is a negatively sloped *balance of payment* curve in Figure 3. Since λ is positively related to λ^e but negatively related to $\varphi_{11}/\varphi_{22}$, the negative slope of the balance of payment curve suggests that with a higher relative wage rate, the large country exports more, and the resulting within-industry reallocation effect is sufficiently strong that it dominates the increased λ^e and leads to a smaller λ.

Appendix E also demonstrates that both the capital mobility curve and balance of payment curve shift upward with increasing Δ. Figure 3 reveals this fact with two thick curves for a smaller Δ and two thin curves for a larger Δ. The intuition is similar to the analysis in Section 4.2.2 given that for any w and λ^e, λ is negatively related to $\varphi_{11}/\varphi_{22}$.

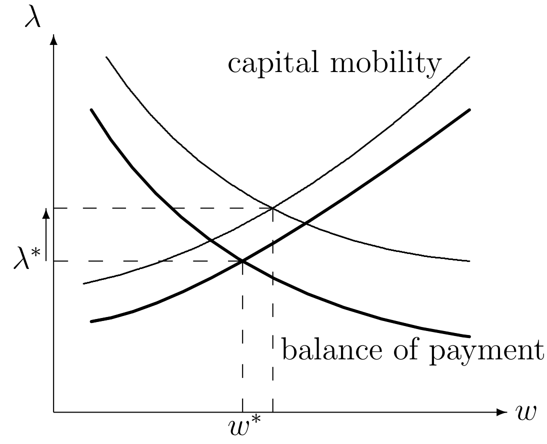

Figure 3: λ^* increases with Δ

As the intersection of the capital mobility curve and the balance of payment curve, country 1's equilibrium share of surviving firms (λ^*) always increases with Δ. This result differs from the change in firm share at the entrant margin: when Δ is large, further trade liberalization always generates a larger share of surviving firms in the large country even though fewer entrants are present in the local market. Thus, our model highlights firm heterogeneity as a factor that may dominate the role of capital mobility in determining firm distribution.

We summarize our findings as follows:

Proposition 1 *When trade freeness measure Δ increases, country 1's equilibrium wage rate w^* exhibits two possible patterns of change: either it increases if f_x/f_d or κ is large or it is bell-shaped when f_x/f_d or κ is small. The equilibrium cutoff productivities φ_{11}^* and φ_{22}^* increase, whereas φ_{12}^*, φ_{21}^* and $\varphi_{11}^*/\varphi_{22}^*$ decrease. In addition, country 1's equilibrium share of producing firms λ^* always increases.*

4.4 A joint-input setup

We now employ a setup in which capital and labor are jointly used as the inputs of production. Following Bernard et al. (2007b), we assume that each unit of input is represented by a composite good comprising capital and labor via a Cobb-Douglas function. Specifically, the sunk cost of entry is denoted by $w_i^{\alpha} r_i^{1-\alpha} f_e$ $(0 < \alpha < 1)$, and the cost function for each incumbent firm that sells from country i to j is rewritten as $TC_{ij}(\varphi) = w_i^{\alpha} r_i^{1-\alpha}(f_{ij} + \tau_{ij} q_{ij}(\varphi)/\varphi)$. We still assume that capital is perfectly mobile and labor supply is inelastic. Thus, $r_i = \bar{r} (i = 1, 2)$ holds in equilibrium. Although α can take any value in $(0,1)$, the fixed and variable inputs use the same composite good. This differs from our previous setting, in which the production factors for fixed and variable inputs are distinct.

Solving the model, we first obtain the following set of equilibrium variables for any given value of w:

$$M^e = \frac{(\sigma-1)}{\alpha\sigma\kappa}\frac{\left[w^{1-\alpha}\theta + (1-\theta)\right]}{\bar{r}^{1-\alpha} f_e},$$
$$\lambda^e = \frac{w^{1-\alpha}\theta}{w^{1-\alpha}\theta + (1-\theta)}, \quad \left(\frac{\varphi_{11}^*}{\varphi_{22}^*}\right)^{-\kappa} = \frac{1-\Delta w^{-\frac{\alpha\kappa\sigma}{\sigma-1}}}{1-\Delta w^{\frac{\alpha\kappa\sigma}{\sigma-1}}}, \text{ and}$$
$$\bar{r} = \frac{(1-\alpha)\left[w\theta + (1-\theta)\right]}{\alpha}.$$

Moreover, the equilibrium wage rate, w^*, is uniquely determined by

$$w^{\frac{\alpha\kappa\sigma}{\sigma-1}}[1-2\theta+\theta(w-1)(\alpha(\theta-1)-\theta)]\Delta^2 + [w\theta - w^{\frac{2\alpha\kappa\sigma}{\sigma-1}}(1-\theta)]\Delta + w^{\frac{\alpha\kappa\sigma}{\sigma-1}}(w-1)(1-\alpha)\theta(\theta-1) = 0. \tag{22}$$

Partial derivatives reveal that

$$\left.\frac{dw^*}{d\Delta}\right|_{\Delta=0} = \frac{2\theta-1}{\theta(1-\theta)(1-\alpha)} > 0, \quad \text{and} \quad \left.\frac{dw^*}{d\Delta}\right|_{\Delta=1} = \frac{1-2\theta}{\kappa} < 0.$$

In other words, the bell-shaped curve between international wage inequality and Δ is preserved. This also indicates two possible international wage differential patterns when τ decreases: either it monotonically increases if f_x/f_d or κ is large or it is bell-shaped if either f_x/f_d or κ is small.

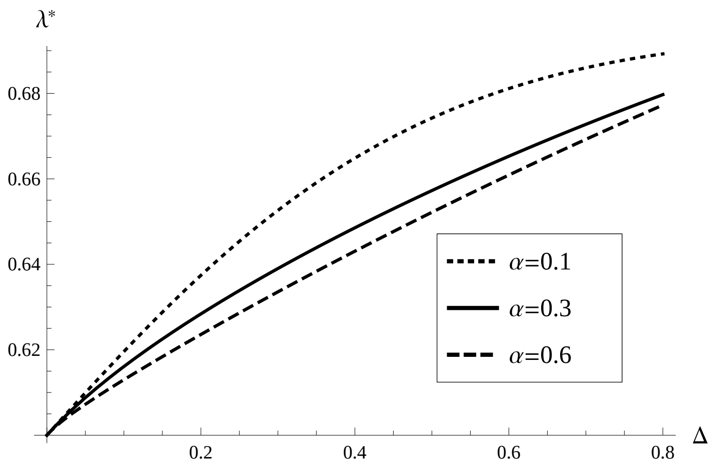

Figure 4: The effects of Δ in the joint-input setup

We now examine the response of the equilibrium share of producing firms λ^*. Unlike the previous setup, we rely on a numerical simulation to establish how λ^* responds to a larger Δ. Figure 4 presents the results under different parameter settings of α when other parameters are set as $\theta = 0.6$, $\kappa = 4$, and $\sigma = 3.8$. As found in Section 4.3, λ^* increases monotonically with Δ. The simulation results suggest that the previous conclusions on international wage and firm share differential are qualitatively true.

5 Gains from trade integration

This section examines the implications of mobile capital for the welfare gains from trade integration in each country. We first derive an expression that accurately measures a country's per-capita welfare, with and without firm heterogeneity. We then compare it with the measure of ACR that only allows for the effect of changes in the domestic trade share. We also explore how the direction and the extent of the deviation between both measures are related to structural parameters. Finally, we present a numerical exercise that quantitatively shows the extent of a country's gains from trade integration under different measures.

5.1 Derivation of consumer welfare

Recall that $X_j = \sum_i X_{ij}$ is the total expenditure of country j. Aggregate accounting indicates that $X_j = w_j L_j + \bar{r} K_j$. Define $\pi_{ij} = X_{ij}/X_j$ as country j's share of expenditure on goods from country i. Substituting the mass of firms and average firm productivities, π_{ij} is written as:

$$\pi_{ij} = \frac{w_i L_i (w_i \tau_{ij})^{-\kappa} (f_{ij})^{\frac{\sigma-1-\kappa}{\sigma-1}}}{\sum_{s=1}^{2} w_s L_s (w_s \tau_{sj})^{-\kappa} (f_{sj})^{\frac{\sigma-1-\kappa}{\sigma-1}}} \tag{23}$$

Following ACR, the (partial) elasticity of trade volume with respect to the variable trade cost τ_{ij} is denoted by $\varepsilon = -\partial \ln(X_{ij}/X_{jj})/\partial \ln \tau_{ij}$. In the case of firm heterogeneity with an untruncated Pareto distribution of $G(\varphi)$, $\varepsilon^{\text{het}} = \kappa > \sigma - 1$ holds.[10] Keeping the wage rates in each country fixed, $\kappa > \sigma - 1$ indicates that changes of τ_{ij} affect both the mass of exporters and the average value of exports (Chaney, 2008).

With mobile capital, $r_j = \bar{r}$ always holds where $\bar{r}$ is the equilibrium rental rate of capital given by (6). According to the zero-profit cutoff condition in (2), the price index

[10] A recent paper of Melitz and Redding (2015) explores the welfare implications of firm heterogeneity. When the distribution of firm productivity $G(\varphi)$ is truncated, it is shown that the ACR term should be adjusted since the welfare gains term depends on any changes in the number of entrant firms. See also Melitz and Redding (2014) for related discussions. For the sake of comparison with ACR, we let $G(\varphi)$ be untruncated. We will see that allowing for unbalanced goods trade generates country-specific adjustment in the relative factor prices, which affects the welfare changes and reflects the impact of entrant delocation across countries.

in country j is

$$P_j = \frac{\sigma w_j}{(\sigma-1)\varphi_{jj}^*}\left(\frac{X_j}{\bar{r}\sigma f_{jj}}\right)^{\frac{1}{1-\sigma}} \tag{24}$$

where φ_{jj}^* reflects the role of firm heterogeneity and its value varies with τ_{ij}. Thus, we obtain

$$X_{jj} = \frac{\sigma}{(\kappa-\sigma+1)}\frac{f_{jj}}{f_e}w_j L_j(\varphi_{jj}^*)^{-\kappa}, \tag{25}$$

where the last equality is from (7), (10) and (24). Expression (25) indicates that country j's expenditure on domestic goods increases with its total income that is proportional to $w_j L_j$, and decreases with domestic cutoff productivity φ_{jj}^*. Using $X_{jj} = \pi_{jj}X_j$, (25) gives cutoff productivity φ_{jj}^* as

$$\varphi_{jj}^* = \left[\frac{\sigma f_{jj}}{(\kappa-\sigma+1)}\frac{w_j L_j}{f_e}\frac{1}{\pi_{jj}X_j}\right]^{\frac{1}{\kappa}},$$

where π_{jj} is given by (23). Country j's domestic cutoff productivity φ_{jj}^* increases with the local mass of entrants, which is proportional to $w_j L_j/f_e$, and decreases with the domestic expenditure on domestic goods ($\pi_{jj}X_j$). Holding w_j and X_j fixed, any trade liberalization that decreases country j's share of domestic expenditure generates a within-industry reallocation effect that raises the cutoff productivity for domestic production φ_{jj}^*.

Substituting φ_{jj}^* into the price index term (24), the per-capita welfare in country j becomes

$$V_j = \psi L_j^{\frac{1}{\sigma-1}}\left(1+\frac{\bar{r}}{w_j}\right)^{\frac{\sigma\kappa-\sigma+1}{(\sigma-1)\kappa}}\left(\frac{w_j}{\bar{r}}\right)^{\frac{1}{\sigma-1}}\pi_{jj}^{-\frac{1}{\kappa}}, \tag{26}$$

where $\psi \equiv (\sigma-1)\sigma^{-1}(\sigma f_{jj})^{\frac{1}{\kappa}-\frac{1}{\sigma-1}}[f_e(\kappa-\sigma+1)]^{-\frac{1}{\kappa}}$. Equation (26) says that the country with a larger population size (L_j) and a more open domestic market (smaller π_{jj}) ends up with a higher level of consumer welfare. Moreover, by relaxing the trade balance condition and capturing the linkage between mobile capital and wage rate adjustment, $w_j/\bar{r}$ becomes important in determining a country's welfare. Although a higher wage rate directly indicates a higher welfare level, the increase of wage cost negatively affects consumer welfare. Holding country j's domestic expenditure share (π_{jj}) fixed, the net effect of any change in the ratio of local wage to rental rate ($w_j/\bar{r}$) on welfare is ambiguous and relies on κ and σ. Note also that the equilibrium variables $\{w_j, \bar{r}, \pi_{jj}\}$ are all shaped by country j's size parameter (L_j) and the level of trade cost (τ_{ij}).

Similar to firm heterogeneity, when firms are identical in productivity, we rewrite country j's welfare as a function of its domestic expenditure share and the ratio of wage to rental rate. Specifically, country j's share of expenditure on goods from i becomes

$$\pi_{ij}^{\text{hom}} = \frac{w_i^{\text{hom}}L_i(w_i^{\text{hom}}\tau_{ij})^{1-\sigma}}{\sum_{s=1}^{2}w_s^{\text{hom}}L_s(w_s^{\text{hom}}\tau_{sj})^{1-\sigma}}, \tag{27}$$

where the notations with a superscript of "hom" denote the corresponding values when there is firm homogeneity in productivity. Partial trade cost elasticity becomes $\varepsilon^{\text{hom}} = \sigma - 1$. Any change in τ_{ij} does not affect the number of exporters. Consumer welfare in country j then equals

$$V_j = \psi^{\text{hom}} L_j^{\frac{1}{\sigma-1}} \left(1 + \frac{\bar{r}^{\text{hom}}}{w_j^{\text{hom}}}\right) \left(\frac{w_j^{\text{hom}}}{\bar{r}^{\text{hom}}}\right)^{\frac{1}{\sigma-1}} \left(\pi_{jj}^{\text{hom}}\right)^{\frac{1}{1-\sigma}}, \tag{28}$$

where $\psi^{\text{hom}} = (\sigma - 1)^{\frac{2-\sigma}{1-\sigma}} f^{\frac{1}{1-\sigma}}/\sigma$, f is the amount of capital used as fixed inputs in production, and equilibrium variables $\{w_j^{\text{hom}}, \bar{r}^{\text{hom}}\}$ are related to both L_j and τ_{ij}. Since we showed that when $\kappa \to \sigma - 1$, the equilibrium wage rate in (15) equals to that in the case without firm heterogeneity, w_j^{hom} can be obtained from (15) by letting $\kappa \to \sigma - 1$, and $\bar{r}^{\text{hom}}$is easily obtained since (6) always holds regardless of firm heterogeneity.

Combining (26) and (28), we summarize country j's welfare as

$$V_j = \text{constant} \times \pi_{jj}^{-1/\varepsilon} \times \Psi_j, \tag{29}$$

where the constant term summarizes all effects from country size L_j and the other exogenous parameters of $\{f_x, f_d, f_e, f, \kappa, \sigma\}$. The term of domestic expenditure share π_{jj} is given by (23) under firm heterogeneity and by (27) under firm homogeneity. Partial trade elasticity ε equals

$$\varepsilon = \begin{cases} \sigma - 1 & \text{with firm homogeneity} \\ \kappa & \text{with firm heterogeneity.} \end{cases} \tag{30}$$

Compared to the formula in ACR that links a country's welfare with its share of domestic expenditure, our model yields the additional term Ψ_j in (29). Since mobile capital is introduced as a production factor, we relax the macro restriction of balanced trade that is necessary for the formula in ACR to hold.[11] The additional margin of welfare adjustment under mobile capital is represented by any change in Ψ_j that is related to the relative factor price $w_j/\bar{r}$ and the exogenous parameters of κ and σ:

$$\Psi_j = \begin{cases} \left(1 + \frac{\bar{r}^{\text{hom}}}{w_j^{\text{hom}}}\right) \left(\frac{w_j^{\text{hom}}}{\bar{r}^{\text{hom}}}\right)^{\frac{1}{\sigma-1}} & \text{with firm homogeneity,} \\ \left(1 + \frac{\bar{r}}{w_j}\right) \left(\frac{w_j}{\bar{r}}\right)^{\frac{1}{\sigma-1}} \left(1 + \frac{\bar{r}}{w_j}\right)^{\frac{\kappa-(\sigma-1)}{(\sigma-1)\kappa}} & \text{with firm heterogeneity.} \end{cases} \tag{31}$$

[11] The additional two macro restrictions for the ACR formula to hold include: (i) a constant ratio between the total firm profit and total firm revenue, and (ii) a CES import demand system (ACR, 102-103). These two macro restrictions hold in our setup.

5.2 Welfare effects from trade integration

5.2.1 Gains from trade

We now discuss how mobile capital affects a country's gains from trade. Equation (29) links country j's welfare (V_j) with its share of domestic expenditure (π_{jj}) and the ratio of wage to rental rate ($w_j/\bar{r}$). Compared to the autarky case where $\pi_{jj}^A = 1$, two sources of welfare adjustment arise when each country is open to trade. The first is from the drop in the domestic expenditure share ($\pi_{jj}^T < \pi_{jj}^A = 1$), and the second reflects the impact of changes in the country's relative factor prices:

$$\frac{V_j^T}{V_j^A} = \left(\frac{\Psi_j^T}{\Psi_j^A}\right)\left(\pi_{jj}^T\right)^{-\frac{1}{\varepsilon}}. \tag{32}$$

With country size asymmetry, the additional margin of welfare adjustment further relies on country specific parameters. Returning to the benchmark setup with two countries where country 1 is larger in size ($\theta > 1/2$), the following property always holds: $w_1^A = w_2^A = 1$ and $\bar{r}^A = 1/(\sigma-1)$ under autarky while $w_1^T > 1 = w_2^T$ and $\bar{r}^T = (\theta w_1^T + 1 - \theta)/(\sigma - 1)$ under trade. Regarding the common terms in (31), we prove in Appendix G that

$$\left(1+\frac{\bar{r}^T}{w_1^T}\right)\left(\frac{w_1^T}{\bar{r}^T}\right)^{\frac{1}{\sigma-1}} \geq \left(1+\frac{\bar{r}^A}{w_1^A}\right)\left(\frac{w_1^A}{\bar{r}^A}\right)^{\frac{1}{\sigma-1}}. \tag{33}$$

This property reflects that, besides the decrease of π_{11} under trade, country 1 receives additional welfare gains, which are attributed to the net inflow of capital. Furthermore, in the case of heterogeneity, an extra term $(1+\bar{r}/w_1)^{[\kappa-(\sigma-1)]/[(\sigma-1)\kappa]}$ occurs in country 1's welfare expression. With $\kappa > \sigma - 1$, an opposite inequality always holds in the large country:

$$\left(1+\frac{\bar{r}^T}{w_1^T}\right)^{\frac{\kappa-(\sigma-1)}{(\sigma-1)\kappa}} \leq \left(1+\frac{\bar{r}^A}{w_1^A}\right)^{\frac{\kappa-(\sigma-1)}{(\sigma-1)\kappa}}.$$

Compared to the case without firm heterogeneity, firm selection magnifies the large country's size advantage by further increasing its share of both entrants and surviving firms. Accordingly, there is a greater increase in labor demand in the large country, which further rises its wage cost and shrinks its benefits from the capital inflow.

Considering country 2's change in Ψ_2, the direction of changes in $(1+\bar{r}/w_2)(w_2/\bar{r})^{1/(\sigma-1)}$ relies on the values of σ and θ. The total effect is thus indefinite.[12] More specifically, if firm heterogeneity is excluded, then after controlling for its domestic expenditure share change, the small country may either lose or gain when firm delocation and trade pattern

[12]The numerical exercise presented in Subsection 5.3 shows that Ψ_2 is larger under trade than under autarky.

adjust with changes in the factor prices. This is different from the additional benefit that a small country could receive when firm heterogeneity is at work. To see this, note that for country 2, the extra welfare term in (31) satisfies

$$\left(1+\frac{\bar{r}^T}{w_2^T}\right)^{\frac{\kappa-(\sigma-1)}{(\sigma-1)\kappa}} \geq \left(1+\frac{\bar{r}^A}{w_2^A}\right)^{\frac{\kappa-(\sigma-1)}{(\sigma-1)\kappa}}.$$

Since firm heterogeneity always magnifies the large country's size advantage, capital becomes more expensive relative to labor in the small country. Meanwhile, the within-industry reallocation effect works stronger if trade occurs. Both generate an increase of the small country's consumer welfare under firm heterogeneity.

The result is summarized as follows.

Proposition 2 *In the presence of mobile capital, goods trade is not necessarily balanced. A country's welfare is measured by (29), and its gains from trade rely on the change in its relative factor prices, in addition to the trade elasticity and the change in its domestic expenditure share.*

5.2.2 Effects of trade liberalization

We next consider the welfare implications of any trade liberalization that changes the variable trade cost from an initial value, τ_{ij}, to a counterfactual value, τ'_{ij}. The change in consumer welfare is measured by the values of all the endogenous variables in the initial equilibrium and the changes in these endogenous variables. Let $\hat{x} = x'/x$ denote the change in a variable, where x' represents the variable in the counterfactual equilibrium and x represents the variable in the initial equilibrium.[13]

Note that consumer welfare in our setup is determined by $w_i/\bar{r}$ and π_{ii}, where $\bar{r}$ is given by (6). We can measure the welfare effect of a change in τ_{ij} ($\hat{\tau}_{ij} = \tau'_{ij}/\tau_{ij}$) via an equation system that summarizes the changes in wage rate $\hat{w}_i$ and the expenditure shares $\hat{\pi}_{ij}$.[14] Since the labor market clearing condition and the goods market clearing condition indicate that $w_i L_i = \frac{\sigma-1}{\sigma}\sum_{i=1}^{2}\pi_{ij}(w_j L_j + \bar{r}K_j)$, we obtain an equation that links $\hat{w}_i$ with $\hat{\pi}_{ij}$:

$$\hat{w}_i = \frac{\sigma-1}{\sigma w_i L_i}\sum_{j=1}^{2}\left[\hat{w}_j w_j L_j + \bar{r}K_j\sum_{i=1}^{2}\hat{w}_i\Big(\frac{w_i L_i}{\sum_{i=1}^{2} w_i L_i}\Big)\right]\hat{\pi}_{ij}\pi_{ij}. \tag{34}$$

Meanwhile, responding to any trade cost shock $\hat{\tau}_{ij}$, $\hat{\pi}_{ij}$ becomes

$$\hat{\pi}_{ij} = \frac{(\hat{w}_i)^{1-\kappa}(\hat{\tau}_{ij})^{-\kappa}}{\sum_{s=1}^{2}\pi_{sj}(\hat{w}_s)^{1-\kappa}(\hat{\tau}_{sj})^{-\kappa}}. \tag{35}$$

[13]This is referred to as the "exact hat algebra" in Costinot and Rodríguez-Clare (2014).

[14]Holding the world endowment of capital fixed, the change of equilibrium rental rate of capital is represented by $\hat{\bar{r}} = \bar{r}'/\bar{r} = \sum_{i=1}^{2}\hat{w}_i\left(\frac{w_i L_i}{\sum_i w_i L_i}\right)$.

Equations (34) and (35) then provide an equation system that solves $\{\hat{w}_i, \hat{\pi}_{ij}\}$ for any given $\hat{\tau}_{ij}$. Correspondingly, the change of country j's welfare is $\hat{V}_j = \hat{\pi}_{jj}^{-\varepsilon}\hat{\Psi}_j$ where $\hat{\Psi}_j$ is related to the change of country j's relative factor price, $\hat{w}_j/\hat{\bar{r}}$.

5.3 Numerical exercise

In this subsection, we quantitatively examine the implications of mobile capital and firm heterogeneity for the welfare gains from trade integration between asymmetric countries. To simplify matters, we assume two countries in the economy, as we did in the theoretical model. Country 1 is assumed to be the US while country 2 is a hypothetical country which is assumed to be identical to the US except for population size. We set $\theta > 1/2$ so that the population size in the US is larger than the other country.

We start by setting the elasticity of substitution between varieties as $\sigma = 4$, which is consistent with the estimates reported in Bernard et al. (2003). Following Costinot and Rodríguez-Clare (2014), the shape parameter of the Pareto distribution is set as $\kappa = 5$. In the case of firm heterogeneity, κ reflects the partial elasticity of trade flows with respect to the variable trade costs. We thus adopt an alternative value of trade elasticity, i.e., $\kappa = 4.12$ from Simonovska and Waugh (2014).

The untruncated Pareto distribution of firm productivity is symmetric between countries while US population share θ has different values. The fixed cost of domestic production parameter, f_d, is normalized to one. The sunk entry cost parameter, f_e, is also set as one since it does not affect the ratio of cutoff productivities $(\varphi^*_{ij}/\varphi^*_{ii})$ when the productivity distribution is untruncated Pareto. We then solve f_x and τ to match two stylized facts for firm exporting in the US manufacturing industry reported in Bernard et al. (2007a): around 18% of US manufacturing firms export, and around 14% of the total value of shipments is made by exporters. For each value of θ and κ, the implied value of the US wage ratio (w) is also derived from the equation system.

Table 1 reports the implied values of $\{w, f_x, \tau\}$ that match the US manufacturing data for the given values of θ and κ. The implied values of τ range between 1.62 and 1.78, which are similar to the estimate of 1.74 in Anderson and van Wincoop (2004). The implied values of f_x are between 0.30 and 0.57 if the share of US exporters is matched to be near 18%. The US implied wage rate ranges from 1.02 to 1.09. As expected, w increases if either θ is larger or the extent of firm heterogeneity is greater (a smaller κ).

Having obtained these implied values, country j's gains from trade are calculated as $(V_j^T - V_j^A)/V_j^T$. This term describes (the negative of) the percentage change in real income needed to bring consumers in country j back to their initial utility level under autarky. Table 1 reports the gains from trade under different cost function settings. Panel A corresponds to the benchmark cost setting while Panel B shows the welfare results in

Table 1: Gains from trade

parameters		implied value			Gains from trade					
					Country 1			Country 2		
θ	κ	w	f_x	τ	ACR	Actual	% devi	ACR	Actual	% devi
Panel A: Benchmark setup ($TC_{ij} = r_i f_{ij} + w_i \tau_{ij} x_{ij}/\varphi$)										
0.6	4.12	1.04	0.57	1.73	2.23%	2.34%	−4.38%	3.71%	3.56%	4.04%
0.7	4.12	1.09	0.57	1.62	2.18%	2.34%	−6.65%	6.21%	5.90%	5.31%
0.6	5	1.04	0.46	1.76	1.50%	1.57%	−4.53%	2.47%	2.36%	4.21%
0.7	5	1.08	0.46	1.66	1.46%	1.57%	−7.02%	4.07%	3.86%	5.74%
Panel B: Joint-input setup ($TC_{ij} = w_i^\alpha r_i^{1-\alpha}(f_{ij} + \tau_{ij} x_{ij}/\varphi)$)										
0.6	4.12	1.05	0.57	1.74	2.19%	2.34%	−6.20%	3.71%	3.50%	5.84%
0.7	4.12	1.10	0.57	1.63	2.12%	2.35%	−9.55%	6.22%	5.76%	7.88%
0.6	5	1.04	0.46	1.76	1.47%	1.57%	−6.39%	2.46%	2.32%	6.08%
0.7	5	1.08	0.46	1.67	1.41%	1.57%	−10.03%	4.08%	3.76%	8.49%

a joint-input setup. We choose exogenous parameters with two important considerations. First, the equilibrium cutoff productivities (φ_{ij}^*) are always above the lower bound of the productivity distribution ($\varphi_{i\,\min} = 1$). Second, the equilibrium cutoff for exporting, φ_{ij}^* ($j \neq i$), is always above that for domestic production, φ_{ii}^*, which is likely to be violated in the small country if the extent of country asymmetry is sufficiently large or the variable trade cost is sufficiently low.

For each country, the ACR columns in Table 1 report the gains from trade if using the formula in ACR with term $(\pi_{jj}^T)^{-1/\kappa}$. The gains from trade range between 1.46% and 2.23% for the US (country 1) while the values are larger for the small country (country 2), ranging between 2.47% and 6.2%. The actual values of a country's gains from trade, however, are different because unbalanced trade also shapes changes in the relative factor prices. The columns labeled "Actual" for each country show the values of gains from trade if one takes into account both the change in π_{jj} and the change in $w_j/\bar{r}$ from autarky to trade.

The results reveal that, compared to the values derived from the ACR formula, the larger country receives larger gains from trade if the actual welfare measure is used, whereas an opposite result holds for the small country. In the benchmark cost function with $\theta = 0.7$ and $\kappa = 5$, the predicted gains from trade value is around 1.46% for country 1 if using the ACR formula for evaluation while the actual value is about 1.57%. The ACR formula thus underestimates the actual gains from trade by around 7%. The extent of underestimation increases to around 10% in the joint-input setup. For the small country, the sign of deviation reverses since using the ACR formula overestimates country 2's gains from trade by about 6% in both setups. Even though the percentage of the ACR formula's deviation is somewhat modest in our simplified one-sector, two-country setup, the effect of capital movement on a country's gains from trade is still quantitatively relevant through the changes in $w_j/\bar{r}$. With $w_j^A/\bar{r}^A$ being symmetric under autarky while $w_1^T/\bar{r}^T > w_2^T/\bar{r}^T$ holding under trade, there is a larger deviation if the ACR formula is used to measure the gains from trade for the large country.

In the same vein, we consider a counterfactual exercise that explores the welfare effects of trade liberalization. Table 2 calculates the changes of consumer welfare for different values of variable trade costs. Given country size asymmetry, the values of τ are chosen to ensure that the cutoff productivity for exporting is always larger than that for domestic production. We report the change of consumer welfare in country j ($\hat{V}_j$) when the variable trade costs change from τ to a smaller value of τ' using either method: evaluating $(\hat{\pi}_{jj})^{-1/\kappa}$ in the ACR formula, or calculating the actual welfare change by additionally taking account of the change in $w_j/\bar{r}$.

In addition to the findings in Table 2, which show larger benefits from trade liberalization for the small country, we report the deviation between the welfare change predicted using the ACR formula and that derived from our theoretical model. We do not find a monotonic pattern between the direction of deviation and country size θ. This is related to the wage effect of trade liberalization. Since we showed analytically in Section 4.1 that, as trade liberalizes, the large country's equilibrium wage ratio is either an inverted-U shape or is monotonically increasing, the direction of change in $w_j/\bar{r}$ not only is country-specific, but also depends on the values of τ before and after the trade shock. Within our two-country context, Table 2 shows that, for the large country, the actual value of gains from trade liberalization tends to be larger than the value predicted from the ACR formula if τ falls from a large initial value. The opposite pattern holds for the small country. Finally, as τ decreases, the predicted values of welfare gains become close to each other in both methods. Referring to (26), the term related to the change in $\pi_{jj}^{-1/\kappa}$ gradually has a greater impact on a country's welfare adjustment when trade is liberalized.

6 Conclusion

A recent book of Piketty (2014) attracts attention to the role of capital in income inequality, which is an important production factor not sufficiently investigated in recent trade literature. This paper presents a highly tractable model that embeds firm heterogeneity and mobile capital in a trade model with country asymmetry in population size.

We reach two important results regarding the effects of trade integration. The first

Table 2: Welfare effects of trade liberalization

Trade costs	Country 1			Country 2		
	ACR	Actual	% devi.	ACR	Actual	%devi.
Panel A: Benchmark setup ($TC_{ij} = r_i f_{ij} + w_i \tau_{ij} x_{ij}/\varphi$)						
$\theta = 0.6,\ \kappa = 5,\ f_x = 0.455$						
$\tau = 3,\ \tau' = 1.5$	103.07%	103.10%	−1.12%	105.07%	105.02%	1.02%
$\tau = 3,\ \tau' = 2$	100.69%	100.73%	−5.25%	101.15%	101.1%	4.88%
$\tau = 2,\ \tau' = 1.5$	102.356%	102.353%	0.14%	103.879%	103.883%	−0.11%
$\theta = 0.7,\ \kappa = 5,\ f_x = 0.455$						
$\tau = 3,\ \tau' = 1.5$	102.31%	102.36%	−2.26%	106.61%	106.5%	1.66%
$\tau = 3,\ \tau' = 2$	100.53%	100.59%	−10.09%	101.5%	101.38%	8.36%
$\tau = 2,\ \tau' = 1.5$	101.773%	101.766%	0.38%	105.04%	105.05%	−0.25%
Panel B: Joint-input setup ($TC_{ij} = w_i^{\alpha} r_i^{1-\alpha}(f_{ij} + \tau_{ij} x_{ij}/\varphi)$)						
$\theta = 0.6,\ \kappa = 5,\ f_x = 0.455$						
$\tau = 3,\ \tau' = 1.5$	103.05%	103.11%	−2.02%	105.10%	105.01%	1.79%
$\tau = 3,\ \tau' = 2$	100.68%	100.74%	−7.97%	101.17%	101.1%	7.71%
$\tau = 2,\ \tau' = 1.5$	102.35%	102.352%	−0.09%	103.887%	103.884%	0.08%
$\theta = 0.7,\ \kappa = 5,\ f_x = 0.455$						
$\tau = 3,\ \tau' = 1.5$	102.29%	102.38%	−3.95%	106.67%	106.48%	3.02%
$\tau = 3,\ \tau' = 2$	100.51%	100.60%	−15.22%	101.54%	101.36%	13.68%
$\tau = 2,\ \tau' = 1.5$	101.764%	101.764%	0.02%	105.053%	105.052%	−0.01%

relates to changes in international inequalities. We show that trade liberalization, as represented by smaller variable trade costs, generates either a bell-shaped or a monotonically increasing wage differential, relying on the exogenous parameters of firm heterogeneity and the fixed cost of exporting, both of which affect the share of exporting firms in a trade economy.

In contrast, the cross-country firm share differential is always magnified. Starting from autarky, the large country's firm share increases because more capital flows out of the small country and the within-industry reallocation effect magnifies the large country's size advantage. When the variable trade costs continue to decrease, the higher wage cost may become a disadvantage for the large country, which leads to an outward reallocation of capital. With firm selection, however, the effect of entrant delocation is outweighed by the within-industry reallocation effect that works stronger in the small country.

Our second result measures a country's gains from trade in the presence of mobile capital and firm heterogeneity. In addition to the ACR term that captures the welfare effect of any change in a country's domestic expenditure share, we show that there is another margin of welfare adjustment working through the change in a country's ratio of wage to rental rate. Since the ACR formula is based on one macro restriction that requires balanced trade between countries, our model attempts an extension that allows for imbalanced trade driven by country size asymmetry and international capital movement.

Our numerical results reveal that holding the effect of any change in the domestic expenditure share fixed, the large country obtains an additional benefit from trade as it receives more capital inflow whereas the opposite holds for the small country. These results, however, should be treated carefully since we limit our analysis to the case where two countries differ only in population size. One possible direction of future work could be to consider a multi-country setup where countries may also differ in productivity distribution or access to world trade. This would allow us to calculate each country's welfare gains from cross-country data when any changes in the trade pattern interact with capital reallocation across countries.

Appendix A: Aggregate payment of capital and labor

The total wage cost in country i equals the payment to workers who are employed as flexible inputs in production:

$$\begin{aligned}
\text{wage payment in } i &= w_i \sum_j M_i^e [1 - G(\varphi_{ij}^*)] \int_{\varphi_{ij}^*}^{\infty} \ell_{ij}(\varphi) \mu_{ij}(\varphi) d\varphi \\
&= w_i \sum_j M_i^e [1 - G(\varphi_{ij}^*)] \frac{(\sigma - 1) r_i f_{ij}}{w_i} \int_{\varphi_{ij}^*}^{\infty} \left(\frac{\varphi}{\varphi_{ij}^*}\right)^{\sigma-1} \mu_{ij}(\varphi) d\varphi \\
&= \frac{(\sigma - 1)\kappa}{\kappa - (\sigma - 1)} M_i^e \sum_j [1 - G(\varphi_{ij}^*)] r_i f_{ij} \\
&= M_i^e \kappa r_i f_e, \qquad \text{(A1)}
\end{aligned}$$

where the third and last equalities are derived from the free entry condition and the Pareto distribution.

Payment to capital arises from two sources: the sunk costs paid by all entrants and the fixed costs for all survivors. The aggregate payment to capital in country i equals

$$\text{capital payment in } i = M_i^e r_i \Big[f_e + \sum_j (1 - G(\varphi_{ij}^*)) f_{ij} \Big] = \frac{1}{\sigma - 1} M_i^e \kappa r_i f_e. \qquad \text{(A2)}$$

Equations (A1) and (A2) then imply that the ratio between aggregate wage payment and aggregate capital payment is fixed at $\sigma - 1$ for each country.

Appendix B: Proof of Lemma 1

Functions $\mathcal{A}_2(w)$, $\mathcal{A}_0(w)$ are evidently increasing functions of w that are positive when $w > 1$. By the assumption of $\kappa > 1$, function $\mathcal{A}_1(w)$ is also increasing with $\mathcal{A}_1((\frac{\theta}{1-\theta})^{\frac{1}{2\kappa-1}}) = 0$. Furthermore, $\mathcal{F}(1, \Delta) = (2\theta - 1)\sigma\Delta(\Delta - 1) \leq 0$. Therefore, $\mathcal{F}(w, \Delta)$ increases with w, and the wage equation has a unique solution w^* in $[1, (\frac{\theta}{1-\theta})^{\frac{1}{2\kappa-1}}]$ for any given $\Delta \in [0, 1]$. Another bound of $\Delta^{-\frac{1}{\kappa}}$ comes from the positiveness of (13). Note that $w^* > 1$ holds when $\Delta \in (0, 1)$.

To prove the monotone properties regarding θ and κ, we calculate the partial derivatives of $\mathcal{F}$ with respect to θ and κ. We have

$$\begin{aligned}\frac{\partial \mathcal{F}}{\partial \theta} &= -(1 - \Delta^2)(2\theta - 1)(w - 1) - \Delta\sigma[w(w^{-\kappa} - \Delta) + w^{\kappa} - \Delta] \leq 0, \\ \frac{\partial \mathcal{F}}{\partial \kappa} &= w^{1-\kappa}\sigma[(1 - \theta)w^{2\kappa-1} + \theta] \ln w \geq 0,\end{aligned}$$

where the inequalities hold strictly when $\Delta \in (0, 1)$ so that $w > 1$.

Appendix C: Trade pattern

The equilibrium wage rate $w^* > 1$ holds when $0 < \Delta < 1$. Note that

$$\begin{aligned}&\Big(\frac{\theta}{1-\theta} w^{1-2\kappa} \frac{1 - \Delta w^{\kappa}}{1 - \Delta w^{-\kappa}} - 1\Big)\Big[\frac{\theta w^{1-\kappa}\Delta + (1-\theta)\Delta w^{-\kappa}}{1 - \theta + \theta w^{1-\kappa}\Delta}\Big] \\ &\qquad = \frac{\theta}{1-\theta} w^{1-2\kappa} \frac{1 - \Delta w^{\kappa}}{1 - \Delta w^{-\kappa}} - \frac{\theta w^{1-2\kappa} + (1-\theta)w^{-\kappa}\Delta}{1 - \theta + \theta w^{1-\kappa}\Delta} \\ &\qquad = \left[\frac{\theta w^{1-2\kappa} + (1-\theta)w^{-\kappa}\Delta}{1 - \theta + \theta w^{1-\kappa}\Delta}\right] \left(\frac{(w-1)(1 - \theta + w\theta)}{w(\sigma - 1 + \theta) + 1 - \theta}\right) > 0,\end{aligned}$$

where the second equality is from (15). Thus,

$$\frac{\theta}{1-\theta} w^{1-2\kappa} \frac{1 - \Delta w^{\kappa}}{1 - \Delta w^{-\kappa}} > 1. \tag{C1}$$

The ratio of aggregate export volume equals

$$\frac{X_{12}}{X_{21}} = \Big(\frac{\theta}{1-\theta}\Big) w^{1-2\kappa} \Big(\frac{\varphi_{11}^*}{\varphi_{22}^*}\Big)^{\kappa} = \Big(\frac{\theta}{1-\theta}\Big) w^{1-2\kappa} \frac{1 - w^{\kappa}\Delta}{1 - w^{-\kappa}\Delta} > 1,$$

where the inequality results from (C1).

Appendix D: Relationship between w and $\varphi_{11}^*/\varphi_{22}^*$

Derivation of Equation (20)

The LHS of (19) is

$$X_{12} - X_{21} = M^e \frac{\sigma\kappa}{\kappa-\sigma+1}\bar{r} f_x \Big[\lambda^e (\Lambda w)^{-\kappa} (\varphi_{22}^*)^{-\kappa} - (1-\lambda^e)(\frac{\Lambda}{w})^{-\kappa}(\varphi_{11}^*)^{-\kappa}\Big], \tag{D1}$$

where the equality is from (12). The RHS of (19) is

$$M^e \lambda^e \bar{r}[f_e + (\varphi_{11}^*)^{-\kappa} f_d + (\varphi_{12}^*)^{-\kappa} f_x] - \bar{r} K_1 = M^e \lambda^e \frac{\kappa}{\sigma-1}\bar{r} f_e - \bar{r}\theta K, \tag{D2}$$

according to country 1's free entry condition under the Pareto distribution.

Plugging (9) and (11) into (D1) and (D2), we obtain

$$\frac{\sigma(\sigma-1)}{\kappa-\sigma+1}\frac{f_x}{f_e}(\varphi_{22}^*)^\kappa \Big[\theta w (\Lambda w)^{-\kappa} - (1-\theta)\Big(\frac{\Lambda}{w}\Big)^{-\kappa}\Big(\frac{\varphi_{11}^*}{\varphi_{22}^*}\Big)^{-\kappa}\Big] = \theta(1-\theta)(w-1).$$

Substituting the expression for φ_{22}^* into this equation gives the balance of payment condition (20) which should be met by the equilibrium cutoffs and wages.

The balance of payment curve in Figure 2

Let $\Phi = \varphi_{11}^*/\varphi_{22}^*$. Equation (20) is equivalent to $\mathcal{I}(w, \Phi, \Delta) = 0$ where

$$\mathcal{I}(w, \Phi, \Delta) = \Delta[\theta w^{1-\kappa} - (1-\theta) w^\kappa \Phi^{-\kappa}] - \theta(w-1)\Big(\frac{1-\theta+\theta w^{1-\kappa}\Delta}{\theta w + 1 - \theta}\Big),$$

which yields the following results:

$$\begin{aligned}
\frac{\partial \mathcal{I}(w,\Phi,\Delta)}{\partial w} &= -\Delta(1-\theta)\kappa w^{\kappa-1}\Phi^{-\kappa} - \frac{\Delta\theta w^{-\kappa}\sigma(\kappa-1)}{\theta w + \sigma - \theta} - \frac{\theta\sigma(1-\theta+\theta\Delta w^{1-\kappa})}{(\theta w+\sigma-\theta)^2} \\
&< 0, \\
\frac{\partial \mathcal{I}(w,\Phi,\Delta)}{\partial \Phi} &= \Delta(1-\theta) w^\kappa \kappa \Phi^{-\kappa-1} > 0, \\
\frac{\partial \mathcal{I}(w,\Phi,\Delta)}{\partial \Delta} &= \theta w^{1-\kappa} - (1-\theta) w^\kappa \Phi^{-\kappa} - \theta(w-1)\frac{\theta w^{1-\kappa}}{\theta w + 1 - \theta} = \frac{\theta(w-1)(1-\theta)}{\Delta(\theta w + 1 - \theta)} \\
&> 0.
\end{aligned} \tag{D3}$$

Furthermore, because

$$\frac{d\Phi}{dw} = -\frac{\partial\mathcal{I}/\partial w}{\partial\mathcal{I}/\partial\Phi} > 0, \text{ and } \frac{d\Phi}{d\Delta} = -\frac{\partial\mathcal{I}/\partial\Delta}{\partial\mathcal{I}/\partial\Phi} < 0,$$

the balance of payment curve in Figure 2 increases with w and decreases with Δ.

Appendix E: The curves of Figure 3

The capital mobility curve

We calculate the derivatives of (14) as follows.

$$\begin{aligned}
\frac{\partial\lambda}{\partial w} =& \frac{\theta(1-\theta)w^{-\kappa}\{(\kappa-1)\Delta(1+w^{-2\kappa})+w^{-\kappa}[1+(1-2\kappa)\Delta^2]\}}{[\Delta+\theta w^{1-2\kappa}\Delta-\theta\Delta-w^{-\kappa}(1-\theta+\theta w)]^2} \\
\geq& \frac{\theta(1-\theta)w^{-\kappa}\{2(\kappa-1)\Delta w^{-\kappa}+w^{-\kappa}[1+(1-2\kappa)\Delta^2]\}}{[\Delta+\theta w^{1-2\kappa}\Delta-\theta\Delta-w^{-\kappa}(1-\theta+\theta w)]^2} \\
=& \frac{\theta(1-\theta)w^{-\kappa}\{2(\kappa-1)\Delta w^{-\kappa}(1-\Delta)+w^{-\kappa}(1-\Delta^2)\}}{[\Delta+\theta w^{1-2\kappa}\Delta-\theta\Delta-w^{-\kappa}(1-\theta+\theta w)]^2} \\
\geq& 0, \\
\frac{\partial\lambda}{\partial\Delta} =& \frac{\theta(1-\theta)w^{\kappa+1}(w^{2\kappa}-1)}{[w^{2\kappa}(\theta-1)\Delta+w^{\kappa}(1+\theta(w-1))-w\Delta\theta]^2} > 0.
\end{aligned}$$

The second inequality holds strictly as long as $\Delta < 1$. These inequalities show that the capital mobility curve increases with both w and Δ.

The balance of payment curve

By using (11) and (14), we have

$$\left(\frac{\varphi_{11}^*}{\varphi_{22}^*}\right)^{-\kappa} = \frac{1-\theta}{w\theta}\;\frac{\lambda}{1-\lambda},$$

and thus, we can rewrite (21) as $\mathcal{J}(w,\lambda,\Delta)=0$ where

$$\mathcal{J}(w,\lambda,\Delta) \equiv \Delta\Big[\theta w^{1-\kappa} - w^{\kappa-1}\frac{(1-\theta)^2}{\theta}\frac{\lambda}{1-\lambda}\Big] - \theta(w-1)\frac{1-\theta+\theta w^{1-\kappa}\Delta}{\theta w+1-\theta}.$$

It follows immediately that

$$\begin{aligned}
\frac{\partial\mathcal{J}(w,\lambda,\Delta)}{\partial\lambda} =& -\Delta w^{\kappa-1}(\theta w+\sigma-\theta)\frac{(1-\theta)^2}{\theta(1-\lambda)^2} < 0, \\
\frac{\partial\mathcal{J}(w,\lambda,\Delta)}{\partial\Delta} >& 0
\end{aligned}$$

where the second inequality is exactly the same as (D3). Meanwhile,

$$\begin{aligned}
\frac{\partial}{\partial w}\mathcal{J}(w,\lambda,\Delta) =& \Delta\theta(1-\kappa)w^{-\kappa}[\theta w+\sigma-\theta-\theta(w-1)] \\
&- \Delta(k-1)w^{\kappa-2}(\theta w+\sigma-\theta)\frac{(1-\theta)^2\lambda}{\theta(1-\lambda)} \\
&- \theta(1-\theta+\theta w^{1-\kappa}\Delta) + \Delta\theta\Big[\theta w^{1-\kappa} - w^{\kappa-1}\frac{(1-\theta)^2\lambda}{\theta(1-\lambda)}\Big].
\end{aligned}$$

The first and second terms are negative. Because $\mathcal{F}(w,\lambda)=0$, the last two terms can be rewritten as

$$\begin{aligned}-\theta[(1-\theta)+&\theta\Delta w^{1-\kappa}]+\theta^2(w-1)\frac{1-\theta+\theta\Delta w^{1-\kappa}}{\theta w+\sigma-\theta}\\ &=-\left(1-\theta+\theta\Delta w^{1-\kappa}\right)\left(\frac{\theta\sigma}{\theta w+\sigma-\theta}\right)<0.\end{aligned}$$

Therefore, it holds that

$$\frac{d\lambda}{dw}=-\frac{\partial\mathcal{J}/\partial w}{\partial\mathcal{J}/\partial\lambda}<0,\ \text{and}\ \frac{d\lambda}{d\Delta}=-\frac{\partial\mathcal{J}/\partial\Delta}{\partial\mathcal{J}/\partial\lambda}>0.$$

Appendix F: The relationship between θ and productivity cutoffs

For the large country, exporting always requires a higher cutoff productivity than does domestic production ($\varphi^*_{11}<\varphi^*_{12}$). However, for the small country, when market size asymmetry is sufficiently large, the cutoff productivity level for exporting may be lower than that for domestic production, i.e., $\varphi^*_{22}>\varphi^*_{21}$. From (12) and (13),

$$\left(\frac{\varphi^*_{22}}{\varphi^*_{21}}\right)^{\kappa}=\frac{1-w^{-\kappa}\Delta}{1-w^{\kappa}\Delta}w^{\kappa}\Lambda^{-\kappa}. \tag{F1}$$

The RHS of (F1) increases with w because

$$\frac{d}{dw}\left(\frac{1-w^{-\kappa}\Delta}{1-w^{\kappa}\Delta}\right)=\frac{\kappa w^{\kappa-1}(1-\Delta^2)}{(1-w^{\kappa}\Delta)^2}\geq 0.$$

It also increases with θ by Lemma 1. Therefore, $\varphi^*_{22}>\varphi^*_{21}$ holds when $w(\theta)$ is above the threshold level of $\hat{w}(\hat{\theta})$, which is solved by

$$1=\left[\frac{1-\hat{w}(\hat{\theta})^{-\kappa}\Delta}{1-\hat{w}(\hat{\theta})^{\kappa}\Delta}\right]\hat{w}(\hat{\theta})^{\kappa}\Lambda^{-\kappa}. \tag{F2}$$

In summary, (F2) and (15) determine the threshold level of $(\hat{\theta},\hat{w}(\theta))$ above which $\varphi^*_{22}>\varphi^*_{21}$ holds.

Appendix G: Welfare terms of country 1

Substituting the terms $w_1^T=w^*>1$, $\bar{r}_1^T$ and $\bar{r}_1^A$ into the terms in Ψ_1, inequality (33) holds if the following inequality

$$\sigma w-(1-\theta)(w-1)-\sigma w\left(\frac{w\theta+1-\theta}{w}\right)^{\frac{1}{\sigma-1}}>0 \tag{G1}$$

holds for any $w > 1$, $\sigma > 1$ and $\theta \in [1/2, 1)$. To see this, consider two cases. If $\sigma \in (1, 2]$, then $1/(\sigma - 1) \geq 1$ and

$$\begin{aligned}\text{the LHS of (G1)} \geq & \sigma w - (1-\theta)(w-1) - \sigma w \left(\frac{w\theta + 1 - \theta}{w} \right) \\ = & (w-1)(1-\theta)(\sigma - 1) > 0.\end{aligned}$$

On the other hand, if $\sigma > 2$, then

$$0 < \frac{1}{\sigma - 1} < 1 \quad \text{and} \quad 0 < \frac{(w-1)(1-\theta)}{w} < 1$$

hold. Thus,

$$\left(\frac{w\theta + 1 - \theta}{w} \right)^{\frac{1}{\sigma - 1}} = \left[1 - \frac{(w-1)(1-\theta)}{w} \right]^{\frac{1}{\sigma - 1}} < 1 - \frac{(w-1)(1-\theta)}{w(\sigma - 1)}.$$

Therefore,

$$\begin{aligned}\text{the LHS of (G1)} > & \sigma w - (1-\theta)(w-1) - \sigma w \left[1 - \frac{(w-1)(1-\theta)}{w(\sigma - 1)} \right] \\ = & \frac{(w-1)(1-\theta)}{\sigma - 1} > 0.\end{aligned}$$

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