Money and Nontraded Goods in an Open Economy: A Unified Money-Search Approach

Tetsuya Saito
Department of Economics, Lehigh University
612 Taylor Street, Bethlehem, PA 18015
Email: tsaito@lehigh.edu

Very Preliminary and Incomplete

April 5, 2012

Abstract

This paper considers the third-generation money-search model (Lagos and Wright, 2005, *JPE*) with an extension with a standard Ricardian international trade model. In the model, international traded goods are traded in the centralized market and nontraded goods are traded in the decentralized market. By this extension, we can consider money, nontraded goods, and traded goods in a unified framework. We then find that an improvement in the productivity of the exporting good reduces trading partner's domestic goods prices while the impact in one's own is conditional on the relative wage rate (whence, the Balassa-Samuelson effect is conditionally observed); an increase in the real balances per capita raises the value of one's currency while it reduces the value of other's currency. As an extension, we consider foreign exchange market interventions. We then find that foreign exchange interventions aiming at depreciation may result in further appreciation after local adjustments. We also find an *á la* overshooting effect of Dornbusch.

1 Introduction

The search-theoretic model of money that is also known as the money-search or the Kiyotaki-Wright model is classified into three generations by divisibility of money and goods. The first generation model initiated by Kiyotaki and Wright [19] uses indivisible money and indivisible goods. The indivisibility of goods is eased by Shi [35] and Trejos and Wright [38], which is called the second generation model. Further, the indivisibility of money is eased by Shi [36] and Lagos and Wright [22], which is called the third generation model.

The extension from the first generation model to the second generation model did not bring too much of difficulty, or it brought some simplicity in the model as the first generation model had to solve discrete choice or mixed-strategy equilibrium due to the indivisibility of goods. The extension then enabled us to introduce price and bargaining power considerations in the money-search model. However, the extension from the second generation model to the one with divisible money was not easy, since distribution of money holdings may not be stationary (or degenerate). If the distribution of money holdings is nondegenerate, we may not be able to compute the Bellman equation in the model.

In the standard third generation model, the problem in the distribution of money holdings is resolved by the centralized market just after the decentralized transactions, where the distribution of money holdings changes due to the random matching process. In Lagos and Wright [22], agents participate in the decentralized market (say, during daytime) and then they go to the centralized market (say, at night). In Shi [36], we consider a household consisting of two types of family members: ones participating in the centralized market and the others participating in the decentralized market. In such models, randomized money holdings are sorted immediately after the random matching process and then the distribution is degenerate; whence, we can easily solve the model.

By divisibility both in money and goods, the third generation model could be received by broader audience than the first and second generation models. Actually, there are several interesting applications for the third generation model. In some sense, it is natural to have such two markets in one economy. However, we may still wonder how to interpret the division of the centralized and the decentralized markets if we want to have positive meanings in the existence of the two markets.

To resolve the annoyance, there are two ways: (i) solving a nondegenarate model or (ii) giving positive meanings to the two markets other than sorting the money holdings. The first option is actually taken, for example, by Molico [30] with numerical simulations and by Menzio *et al.* [28] with a recently developed mathematical technique in the labor-search theory (*i.e.*, Gonzalez and Shi [12]). The second option is taken by several works applying the third generation model. For example, Shi and Wang consider the centralized market as the bond market. This interpretation seems quite natural as bonds adjust the balance of money holdings in the real economy. This paper is also the one taking the second option. In this paper, we interpret the centralized market is the market for internationally traded goods. We integrate the centralized market with a standard Ricardian model. The Ricardian model turns out consistent with the structure of the centralized market in Lagos and Wright [22]. In the model, the decentralized market is assumed to be the nontraded goods sector. The integration of money-search and international trade models then enables us to analyze nontraded goods and monetary aspect of an economy within a unified framework.

The analysis of nontraded goods in international trade caught some attentions of researchers after the 1960s (for example, Balassa [2], Samuelson [33], Komiya [20] and a much earlier work in the 1950s by McKenzie [27]) and the earlier 1970s (for example, Ethier [9, 10], Jones [17], and Dornbusch *et al.* [7]). After the late 1970s, the focus went to the "new trade theory" and less attentions were made on nontraded goods in classical and neoclassical frameworks. However, there are some interesting studies on nontraded goods, such as Sanyal and Jones [34] that considers internationally traded goods are middle products and nontraded goods are final products; hence, in their model, the main body of consumptions is nontraded goods. Recently, the focus on nontraded goods is again getting attentions of researchers as "new new trade theory" enables us to deal with multiple outputs and factors easily, which is also easily integrated into the contemporary open macroeconomics framework (for example, Ghironi and Melitz [11]).

This paper tries to deliver analyses on money and nontraded goods in an open economy model using a unified search-theoretic framework. In the analysis, we will notice that the extension of the third-generation money-search model using a very basic Ricardian international trade framework simplifies the analysis of international trade model with money as well as it generates some classical well-known results in this field. We then find that this work is not merely a justification of the separation of the markets (centralized and decentralized), but it also provides a simple framework for international trade with money in a unified and manner (between real and monetary side).

In the search-theoretic framework developed by Lagos and Wright [22], the centralized market (night market) is naturally integrated into a classical theory of international trade (Ricardian model). We can then discuss basic properties discussed in trade theory with existence of money. The main findings of the analysis are that the Balassa-Samuelson effect (Balassa [2] and Samuelson [33]) is conditional on the relative productivity of the exporting sector; an advancement in the productivity of the exporting sector may deteriorate the social welfare level if the country is under influence of the Balassa-Samuelson effect; and foreign exchange interventions aiming at export promotion (depreciation) may result in appreciation after price adjustments (*á la* Dornbusch [6]).

The discussion is developed as follows. We setup the basic framework in Section 2 following the development of the model in Lagos and Wright [22] to find the difference in my extended model. We then continue on the setup of the international market in Section 3. The model is analyzed numerically in Section 4 to confirm some algebraic findings in the previous two sections as well as confirming some basic properties of the social welfare. The focus in this section is on the price levels. We further extend our analysis to see the impact of foreign exchange interventions. The discussion is concluded in Section 6. Some technical issues are placed in the appendix.

2 Money-Search Model

We consider a two-country model with a standard third-generation money-search model structure (Lagos and Wright [22]), so that there are decentralized and centralized markets. We then assume that internationally nontraded goods are traded in the decentralized market, say local market, and internationally traded goods in the centralized market, say world market. We suppose that there are two traded goods indexed by $i \in \{1,2\}$. The traded goods are homogeneous goods while the nontraded goods are differentiated-variety goods. The basic idea in Sanyal and Jones [34] and the basic structure of my work may have potentially large connections. In this paper, nontraded goods are traded in the decentralized market while traded goods are traded in the centralized market. Transactions in the centralized market can be considered as transactions by professionals, such as buyers and sellers of middle products in the final goods sectors, and transactions in the decentralized market can be considered as retail sales. Yet, nontraded and traded goods are treated as if both of them are final goods.

Each period is divided into two subperiods. In the first subperiod, agents are randomly paired with other agents to produce and consume internationally nontraded goods while in the second period, they produce and consume internationally traded goods in the centralized market. The two markets in a country, say Country $j \in \{1,2\}$, are as described as follows. In this section, as we focus on one country, we drop the subscript to identify countries for simple notations. After this section, we indicate variables of each country by adding subscript j between i (goods indicator) and t (period indicator). For example, variable z regarding Country j and Good i in period t is denoted as z_{ijt} after this section while we drop j from this notation as z_{it} in this section.

There is a continuum of infinitely lived agents with measure $N > 0$ and money stock is $\mathscr{M} > 0$. Agents are specialized in producing a certain product type and randomly paired with other agents bilaterally. The random matching process confirms to a Poisson process of arrival rate α. Without loss of generality, the length of the subperiod is supposed to be sufficiently short to have $0 < \alpha \leq 1$. At probability $0 < \sigma \leq 1/2$ for each moment, one meets someone who wishes to consume one's product. At the same probability, σ, one meets someone who produces a product that one wishes to consume. In other words, one is a seller in a transaction at probability σ and a buyer at probability σ. Let $m_t \geq 0$ be real money holdings of an agent in the beginning of the first subperiod within Period t. Let $V_t(m_t)$ and $W_t(\mu_t)$ be value functions of the agent holding m_t in the local market and μ_t in the world market within period t, respectively. If there is a change in the real money holdings by d_t (either payment or income), for example, we find that the real money holdings after the first subperiod is given by $\mu_t = m_t \pm d_t$.

Let $\mathscr{F}_t(\zeta)$ be the cumulative distribution function of money holdings ζ in period t that satisfies

$$\mathscr{M} \equiv \int_0^\infty \zeta dF_t(\zeta). \tag{1}$$

Let $d(m,m')$ be the real transfer in exchange for the good when buyer holds m and seller holds m'. Let $q(m,m')$ be the quantity of good exchanged with $d(m,m')$. The value function is then computed as

$$\begin{aligned} V_t(m_t) \;=\; & \alpha\sigma \int_\zeta \{u[q(m_t,\zeta)] + W_t[m_t - d(m_t,\zeta)]\} d\mathscr{F}_t(\zeta) \\ & + \alpha\sigma \int_\zeta \{W_t[m_t + d(\zeta,m_t)] - c[q(\zeta,m_t)]\} d\mathscr{F}_t(\zeta) \\ & + \alpha\delta \int_\zeta B_t(m_t,\zeta) dF(\zeta) + (1 - 2\alpha\sigma - \alpha\delta) W_t(m_t), \end{aligned} \tag{2}$$

where δ is the probability of double coincidence of wants. In barter trade, it is known that the social optimum $q^* = \arg\max\{u(q) - c(q)\}$ is traded,[1] so that $B_t(m_t,\zeta)$ should be given by

$$B_t(m_t,\zeta) = u(q^*) - c(q^*) + W_t(m_t). \tag{3}$$

Let the bargaining power of buyer be $0 < \theta \le 1$. Let m_t' be the money holdings of a paired seller. The Nash bargaining problem maximizes the Nash product given by

$$\{u(q_t) + W(m_t - d_t) - W(m_t)\}^\theta \left\{-c(q_t) + W\left(m_t' + d_t\right) - W\left(m_t'\right)\right\}^{1-\theta}, \tag{4}$$

with respect to q_t and d_t subject to $d_t \le m_t$ and $q_t \ge 0$. To solve the Nash bargaining solution, we now consider the world market (centralized market). Without loss of generality, we assume agents living in Country i has natural comparative advantage in producing Good i.

$$W_t(\mu_t) = \max_{X,L,m_{t+1}} \{U(x_1,x_2) - \ell + \beta V_{t+1}(m_{t+1})\}. \tag{5}$$

[1]For instance, the instantaneous gain from barter is identicall for the two agents as $u(q) - c(q)$. After the transaction, the money holdings are unaltered from $W(m)$. Thence, agents maximizes the net- change $u(q) - c(q)$ to get the social optimum $q^* = \arg\max\{u(q) - c(q)\}$. It is also easily shown from the first order condition that the social optimum is independent of money holdings. From the Nash bargaining problem in a double-coincidence matching, we find the following equation to solve the first order condition:

$$\frac{\partial q/\partial m}{\partial q/\partial \zeta} = \frac{u'[q(m,\zeta)]}{c'(q(\zeta,q))} = \frac{u'[q(\zeta,m)]}{c'[q(m,\zeta)]},$$

which implies $u'(q) = c'(q)$ and then $q(m,\zeta) = q(\zeta,m) = q^*$.

The budget constraint of an individual agent in the world market is given by

$$\underbrace{\mathscr{P}_t m_{t+1}}_{\text{Balance after transactions}} = \underbrace{(\mathscr{W}_{it}\ell + \mathscr{P}_t\mu_t)}_{\text{Income and initial money holdings}} - \underbrace{(P_{1t}x_1 + P_{2t}x_2)}_{\text{Consumptions of traded goods}}, \tag{6}$$

where $\mathscr{W}_{it}$ is the wage rate paid by the sector producing Good $i \in \{1,2\}$ in Country $j \in \{1,2\}$.

$$W_t(\mu_t) = \phi_t^i \mu_t + \max_{X,m'} \left\{ U(x_1,x_2) - \left(p_{1t}^i x_1 + p_{2t}^i x_2\right) - \phi_t^i m_{t+1} + \beta V_{t+1}(m_{t+1}) \right\}, \tag{7}$$

where $\phi_t^i \equiv \mathscr{P}_t / \mathscr{W}_{it} > 0$, $p_{1t}^i \equiv P_{1t}/\mathscr{W}_{it} > 0$, and $p_{2t}^i \equiv P_{2t}/\mathscr{W}_{it} > 0$.

Remark 1 *In the equilibrium, $\mathscr{W}_{1t} = \mathscr{W}_{2t} = \mathscr{W}_t$ or there is no labor supply in the sector that pays a lower wage than the other sector. This equilibrium condition implies that $p_{it}^1 \equiv p_{it}^2 \equiv p_{it}$ and $\phi_t^1 \equiv \phi_t^2 \equiv \phi_t$.*

Proof. The statement of this remark follows from the basic property of competitive long-run labor market equilibrium. The detail is discussed in Section 3. ■

The consumption of the two internationally traded goods satisfies the first order condition:

$$\frac{p_{1t}}{p_{2t}} = MRS_{12}(x_{1t}, x_{2t}), \tag{8}$$

where $MRS_{12}(x_{1t}, x_{2t})$ is the marginal rate of substitution between Goods 1 and 2. There is only one good (composite goods) traded in the centralized market in the standard third generation model *á la* Lagos and Wright [22]; thence, $W_t(\mu_t)$ is linear in μ_t with slope ϕ_t, as the consumption in the centralized market is determined without money holdings. If there is more than two goods traded in the centralized market, we cannot show the same linearity of $W_t(\mu_t)$ in μ_t. For instance, (8) derived the relationship between x_1 and x_2 in the equilibrium as $\tilde{x}_{1t} = \tilde{x}_1(\tilde{x}_{2t})$, which is inserted into the budget constraint to solve the equilibrium as $\tilde{x}_{it} = \tilde{x}_{it}\left(\mu_t, \mu_{t+1}, \phi_t, p_{1t}, p_{2t}, \ell\right)$ for Good $k \in \{1,2\}$ (further details are discussed in Section 3).

Remark 2 *Suppose the utility function is homogeneous of degree one in consumption levels. The value function in the centralized market W_t is then linear in money holdings of the current period m and labor hours ℓ with slope $\Phi_t = \phi_t + \chi_t > 0$, where χ_t is a derived component relating with internationally traded goods prices. W_t is also linear in money holdings of the beginning of the next period m' with slope $-\Phi_t$.*

Remark 2 implies $W_t(m_t \pm d_t) \equiv W_t(m_t) \pm \Phi_t d_t$. The Nash product (4) is then simplified as

$$\{u(q_t) - \Phi_t d_t\}^{\theta} \{\Phi_t d_t - c(q_t)\}^{1-\theta}, \tag{9}$$

which is an analogue of Lagos and Wright [22]. The first order conditions (Karush-Kuhn-Tucker conditions) are given as follows. The first order condition with respect to q_t is given by

$$\theta u'(q_t) \left(\frac{\Phi_t d_t - c(q_t)}{u(q_t) - \Phi_t d_t}\right)^{1-\theta} - (1-\theta) c'(q_t) \left(\frac{\Phi_t d_t - c(q_t)}{u(q_t) - \Phi_t d_t}\right)^{-\theta} + \lambda_{qt} = 0, \tag{10}$$

where $\lambda_{qt} \geq 0$ is the Lagrange multiplier for the nonnegative condition for q. For $q \to 0$, if incentive rationality conditions are held, the above first order condition approaches positive infinity that is a contradiction for $q_t \to 0$. Thus, $q_t > 0$ must hold and then $\lambda_{qt} = 0$. From (10) and $\lambda_{qt} = 0$, we find

$$\frac{\Phi_t d_t - c(q_t)}{u(q_t) - \Phi_t d_t} = \frac{(1-\theta) c'(q_t)}{\theta u'(q_t)}, \tag{11}$$

which solves

$$\Phi_t d_t = Z(q_t) \equiv \frac{\theta u'(q_t) c(q_t) + (1-\theta) c'(q_t) u(q_t)}{\theta u'(q_t) + (1-\theta) c'(q_t)}. \tag{12}$$

The first order condition with respect to d_t is given by

$$-\theta \Phi_t \left(\frac{\Phi_t d_t - c(q_t)}{u(q_t) - \Phi_t d_t}\right)^{1-\theta} + (1-\theta) \Phi_t \left(\frac{\Phi_t d_t - c(q_t)}{u(q_t) - \Phi_t d_t}\right)^{-\theta} - \lambda_{dt} = 0, \tag{13}$$

where $\lambda_{dt} \geq 0$ is the Lagrange multiplier for the budget constraint $m_t \geq d_t$. For this condition, we need to evaluate the complementary slackness condition:

$$(m_t - d_t) \lambda_{dt} = 0. \tag{14}$$

If $\lambda_{dt} = 0$, from (11) and (13), we find that the Nash bargaining solution is social optimum $q^* = \arg\max \{u(q) - c(q)\}$, as $u'(q_t) = c'(q)$; whence, the corresponding payment is written as m^*. If $\lambda_{dt} > 0$, we find that the payment in the Nash bargaining solution is $d_t = m_t$ and the corresponding quantity of trade is $\tilde{q}(\Phi_t m_t)$. Therefore, we will find that the Nash bargaining solution $\tilde{q}_t = \tilde{q}(\Phi_t m_t)$ and $\tilde{d}_t = \tilde{d}(m_t)$ is characterized by

$$\tilde{q}(\Phi_t m_t) = \begin{cases} \tilde{q}(\Phi_t m_t) & (m_t < m^*) \\ q^* & (m_t \geq m^*) \end{cases}, \tag{15}$$

$$\text{and} \quad \tilde{d}(m_t) = \begin{cases} m_t & (m_t < m^*) \\ m^* & (m_t \geq m^*) \end{cases}. \tag{16}$$

For later use, we remark the following.

Remark 3 *To be consistent with an interior solution for the optimum money holdings carried into the next period, we have to have* $\phi_t/\Phi_{t+1} \geq \beta$.

Proof. Suppose $\phi_t/\Phi_{t+1} < \beta$. In the value function (7), the first order derivative with respect to the money holdings carried into the next period m' is computed as

$$-\phi_t + \beta\Phi_{t+1}. \tag{17}$$

If (17) is strictly positive, there is no solution for m' satisfying $m' \leq m^*$. Therefore, it must be weakly negative and that implies that $\phi_t/\Phi_{t+1} \geq \beta$ must hold. ■

Price of a good is the terms of trade between money and the good, $\tilde{d}_t/\tilde{q}_t$. The Nash bargaining solution (15) refers Φ_t in order to determine $\tilde{q}$ instead of ϕ_t. Thus, the price level of the entire economy is measured by $1/\phi_t$ by definition while the price level of the local market by $1/\Phi_t$. This difference emanates from the existence of the world market.

The analogue holds for the trading partner (seller).

$$V_t(m_t) = \mathscr{V}_t(m_t) + \max_{m_{t+1}} \{-\phi_t m_{t+1} + \beta V_{t+1}(m_{t+1})\}, \tag{18}$$

where $\mathscr{V}_t(m_t)$ is defined to be

$$\begin{aligned} \mathscr{V}_t(m_t) \quad = \quad & \alpha\sigma\left\{u[\tilde{q}(\Phi_t m_t)] - \Phi_t \tilde{d}(m_t)\right\} + \alpha\sigma \int_\zeta \left\{\Phi_t \tilde{d}(\zeta) - c[\tilde{q}(\Phi_t \zeta)]\right\} d\mathscr{F}_t(\zeta), \\ & + \alpha\delta\{u(q^*) - c(q^*)\} + \phi_t m_t + U(\tilde{x}_{1t}, \tilde{x}_{2t}) - (p_{1t}\tilde{x}_{1t} + p_{2t}\tilde{x}_{2t}). \end{aligned} \tag{19}$$

Note, by the above definition, $\mathscr{V}_t(m_t)$ is linear in m_t with slope Φ_{it}. To solve the model, we iterate (18) recursively to get

$$V_t(m_t) = \mathscr{V}_t(m_t) + \sum_{s=t}^{\infty} \beta^{s-t} \max_{m_{s+1}} \{-\phi_s m_{s+1} + \beta \mathscr{V}_{s+1}(m_{s+1})\}. \tag{20}$$

We then find the first order condition to maximize the sequence:

$$\phi_t = \beta\left[\Phi_{t+1} + \alpha\sigma\left\{\Phi_{t+1} u'[\tilde{q}(\Phi_{t+1} m_{t+1})]\tilde{q}'(\Phi_{t+1} m_{t+1}) - \Phi_{t+1}\tilde{d}'(m_{t+1})\right\}\right]. \tag{21}$$

From (12), we find

$$\Phi_{t+1}\tilde{d}(m_{t+1}) = Z[\tilde{q}(\Phi_{t+1} m_{t+1})], \tag{22}$$

and its derivative with respect to q

$$\Phi_{t+1}\tilde{d}'(m_{t+1}) = \Phi_{t+1} Z'[\tilde{q}(\Phi_{t+1} m_{t+1})]\tilde{q}'(\Phi_{t+1} m_{t+1}), \tag{23}$$

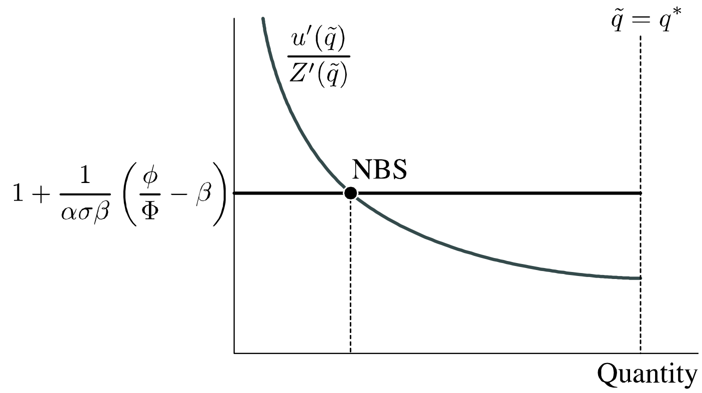

Figure 1: Equilibrium in the non-traded goods sector

where $\tilde{d}(m) = m$ and $\tilde{d}'(m) = 1$ for $m \leq m^*$: so that $\tilde{d}'(m) = 1$, and we further find

$$\tilde{q}'(\Phi_{t+1}m_{t+1}) = \frac{1}{Z'[\tilde{q}(\Phi_{t+1}m_{t+1})]}. \tag{24}$$

We substitute (24) into (21) and rearrange terms to get

$$\phi_t = \beta\Phi_{t+1}\left\{\alpha\sigma\frac{u'[\tilde{q}(\Phi_t m_{t+1})]}{Z'[\tilde{q}(\Phi_t m_{t+1})]} + 1 - \alpha\sigma\right\}. \tag{25}$$

We then focus on the stationary equilibrium in the following analysis. The stationary equilibrium satisfies $m_{t+1} \equiv m_t \equiv m$ and subscript t (period indicator) is dropped from all variables; whence, we reach an equilibrium condition for $\tilde{q}$ (and $\tilde{m}$):

$$\frac{u'(\tilde{q}_{t+1})}{Z'(\tilde{q}_{t+1})} = 1 + \frac{1}{\alpha\sigma\beta}\left(\frac{\phi_t}{\Phi_{t+1}} - \beta\right). \tag{26}$$

We assume conditions for monotonicity of $u'(q)/Z'(q)$ are satisfied, as discussed in Lagos and Wright [21] This assumption is a sufficient condition to have a degenerate money holdings distribution in the equilibrium.

Proposition 1 *There exists an equilibrium $\tilde{m}_t$ satisfying $\tilde{q}(\tilde{m}_t) \leq q^*$; whence, $\tilde{m}_t < m^*$.*

Proof. See Appendix A. ■

Proposition 1 only states that any equilibrium money holdings (that can be time dependent) must be less than the level that enables the buyer to purchase the social optimum consumption level. For further analysis, we may want to focus on the stationary equilibrium. However, we cannot discuss the stationarity here, as $\Phi_{t+1} \equiv \phi_{t+1} + \chi_{t+1}$ and the equilibrium condition (26) involves χ_{t+1}. The stationarity is shown in the next section by proving expectation of the expected value of χ_{t+1} has a degenerate distribution; whence, we can show stationarity by (21) and (22).

3 International Trade

3.1 Real Side

We consider a Ricardian model, as it is consistent with the production function in Lagos and Wright [22]: linear production function for the centralized market goods. Without loss of generality, we assume Country j has comparative advantage in Good j; hence, Country 1 exports Good 1 in exchange for importing Good 2 and *vice versa* for Country 2. Letting $a_{ij} > 0$ be input coefficient of Good i in Country j (equivalently, we are letting a_{ij}^{-1} be the marginal product of labor), we then find that $a_{11} \leq a_{12}$ and $a_{21} \geq a_{12}$ with at least one strict inequality.

The labor supply in the internationally traded goods sector is derived from the budget constraint:

$$\tilde{\ell}_{ijt} = \phi_{jt}\tilde{m}_{jt+1} - \phi_{jt}\mu_t + \left(p_{1jt}\tilde{x}_{1jt} + p_{2jt}\tilde{x}_{2jt}\right), \tag{27}$$

where μ_t represents real money holdings after the first subperiod within period t. Since preference is represented by a utility function of homogeneous of degree one, the optimum consumption level $\tilde{x}_{ijt}$ is linear in real income level $I_{it}/p_{ijt} \equiv (\ell_t + \phi\mu_t - \phi\tilde{m})/p_{ijt}$. Without loss of generality, we can choose p_{ijt} to be the price of Good j for Country j, so that $p_{ijt} \equiv p_{jjt}$. Good j is produced by Country j at any event so long as the price is nonzero. The price of Good i in Country j is then determined by the equilibrium wage rate that is equal to the marginal product value of labor:

$$\mathscr{W}_{jt} = a_{jj}^{-1}P_{jjt} \quad \Rightarrow \quad \frac{P_{jjt}}{\mathscr{W}_{jt}} \equiv p_{jjt} = a_{jj}, \tag{28}$$

which implies that p_{jjt} is stationary.

The price of each traded good in the world market is measured by P_i^W in a settlement currency unit (*i.e.*, U.S. dollar, euro, gold, *etc.*). The relative world price of Good 1 in terms of Good 2 is denoted as $p_t^W \equiv P_{1t}^W/P_{2t}^W$. Under free trade, $p_{ijt} = P_{it}^W/\mathscr{W}_{jt}$, so that $p_{1jt}/p_{2jt} = p_t^W$. By linearity of $\tilde{x}_{ijt}$ in the real income level, we can then write the total spending on the consumption of internationally traded goods as

$$p_{1jt}\tilde{x}_{1jt} + p_{2jt}\tilde{x}_{2jt} = \mathscr{X}_{jt}I_t, \tag{29}$$

where $\mathscr{X}_{jt} = X_j(p_t^W; a_{jj})$ is appropriately defined aggregate demand function when the real income level is unity. We then rewrite and rearrange (27) to get

$$(\mathscr{X}_j - 1)\ell_t = (\mathscr{X}_j - 1)(\phi_{jt}\tilde{m}_{jt+1} - \phi_{jt}\mu_t). \tag{30}$$

Remark 4 $\mathscr{X}_j \equiv 1$.

Proof. Let us suppose $\mathscr{X}_{jt} \neq 1$. We then find $p_{1jt}\tilde{x}_{1jt} + p_{2jt}\tilde{x}_{2jt} \lesseqgtr I_t$. This implies the budget constraint is not binding. Therefore, $\mathscr{X}_j \equiv 1$ must hold. ■

If $\mathscr{X}_{jt} = 1$, so long as the world market adjusts the money holdings carried over the periods being unique across agents as $\tilde{m}_{t+1}$, any ℓ_{ijt} can be a candidate for the equilibrium value. Thus, now (**??**) is not binding and we then find that agents having more than $\tilde{m}_{t+1}$ after the first subperiod can participate in production to consume more than $\tilde{m}_{t+1} - \mu_t$; agents having less than $\tilde{m}_{t+1}$ after the first subperiod can consume the internationally traded goods; and agents ended up with $\mu_t = \tilde{m}_{t+1}$ can participate in the world market.

Proposition 2 $\tilde{\ell}_{ijt} = 1$ *for* $\mathscr{X}_{jt} = 1$ *if the participation constraint for the world market is satisfied.*

Proof. In the world market, we derive optimum consumptions from maximizing $U\left(x_{1jt}, x_{2jt}\right) - \ell_{ijt}$ in the value function (7) subject to the budget constraint. We then find that $\tilde{x}_{ijt}$ is linear in ℓ_{ijt}, as it is linear in the real income level, and so is the indirect utility function, so that the net utility from the internationally traded goods is also linear in ℓ_{ijt}. This implies that $U\left(x_{1jt}, x_{2jt}\right) - \ell_{ijt} \geq 0$ must hold if the participation constraint is satisfied. Therefore, the optimum labor supply is a corner solution: $\ell_{ijt} = 1$ for $\mathscr{X}_{jt} = 1$. ■

Proposition 2 indicates that the individual labor supply is stationary and $\tilde{\ell}_{ijt} \equiv 1$; hence, the nation-wide labor supply is N_j. The aggregate supply of Good i in Country j is then given by

$$Y_{ijt} = n_{ijt}b_{ij}N_j. \tag{31}$$

The aggregate demand is subject to the budget constraint for $\tilde{\ell}_{ijt} = 1$; hence, the individual demand for Good i in Country j is

$$\tilde{x}_{ijt} = \mathscr{X}_{ijt}\left(1 + \mu_t - \tilde{m}_{t+1}\right). \tag{32}$$

Aggregating the individual demands provides

$$X_{ijt} = \mathscr{X}_{ijt}\left(N_{jt} + \int_0^{\infty} \phi_{jt}\left(\zeta - \tilde{m}_{jt+1}\right) d\mathscr{G}_{jt}\left(\zeta\right)\right). \tag{33}$$

For the monetary balance across agents to maintain $\tilde{m}_{jt+1}$ (e.g. the distribution to be degenerate), we must have

$$\int_0^{\tilde{m}_{jt}} \phi_{jt}\left(\tilde{m}_{jt+1}-\zeta\right) d\mathscr{G}_{jt}(\zeta) \equiv \int_{\tilde{m}_{jt}}^{\infty} \phi_{jt}\left(\zeta-\tilde{m}_{jt+1}\right) d\mathscr{G}_{jt}(\zeta), \tag{34}$$

which implies

$$\int_0^{\infty} \phi_{jt}\left(\zeta-\tilde{m}_{jt+1}\right) d\mathscr{G}_{jt}(\zeta) \equiv 0. \tag{35}$$

We then further find that the aggregate demand is computed as

$$X_{ijt} = \mathscr{X}_{ijt} N_j. \tag{36}$$

Proposition 3 *The world market is independent of $\mathscr{G}_{jt}(\zeta)$ if $\mathscr{X}_{ijt} = 1$, so that the world relative price is deterministic.*

We define the relative demand and supply functions as

$$D(p_t^W) = \frac{\sum_{j=1}^2 X_{1jt}}{\sum_{j=1}^2 X_{2jt}} \quad \text{and} \quad S(p_t^W) = \frac{\sum_{j=1}^2 Y_{1jt}}{\sum_{j=1}^2 Y_{2jt}}. \tag{37}$$

The relative aggregate demand function is a downward-sloping function and the relative aggregate supply function is a step function as depicted in Figure 2. The property of the relative aggregate supply function follows from the basic property of the demand function: an increase in the relative price of Good 1 in terms of Good 2 decreases the demand for Good 1 while it increases the demand for Good 2 in both countries (hence, in the world market). The property of the relative aggregate supply curve follows from the properties of specialization and nonspecialization in productions. For instance, as a_{ij}^{-1} represents marginal product of labor, there is no supply of Good 1 since both countries specialize in Good 2 (*e.g.*, $N_{11} = N_{12} = 0$, and $N_{21} = N_1$ and $N_{22} = N_2$) if the relative price of Good 1 is less than the bottom threshold value, $p_t^W < a_{11}/a_{21}$; there is no supply of Good 2 since both countries specialize in Good 1 (*e.g.*, $N_{11} = N_1$ and $N_{12} = N_2$, and $N_{21} = N_{22} = 0$) if the relative price of Good 1 is larger than the upper threshold value $p_t^W > a_{12}/a_{22}$; and there are fixed quantities of supplies of Goods 1 and 2 (*e.g.*, $N_{11} = N_1$ and $N_{22} = N_2$, and $N_{12} = N_{21} = 0$) since Country j completely specializes in Good j if the relative price of Good 1 is between the two threshold values, $a_{11}/a_{21} < p_t^W < a_{12}/a_{22}$. If $p_t^W = a_{11}/a_{21}$, Country 2 specializes in Good 2 (*e.g.*, $N_{12} = 0$ and $N_{22} = N_2$) while Country 1 may not completely specialize in Good 1; hence, Country 1 may produces the both goods (*e.g.*, $N_{11} \in [0, N_1]$). Similarly, if $p_t^W = a_{12}/a_{22}$, Country 1 specializes in Good 1 (*e.g.*, $N_{11} = N_1$ and $N_{21} = 0$) while Country 2 may not completely specialize in Good 2; hence, Country 2 may produce the both goods (*e.g.*, $N_{22} \in [0, N_2]$).

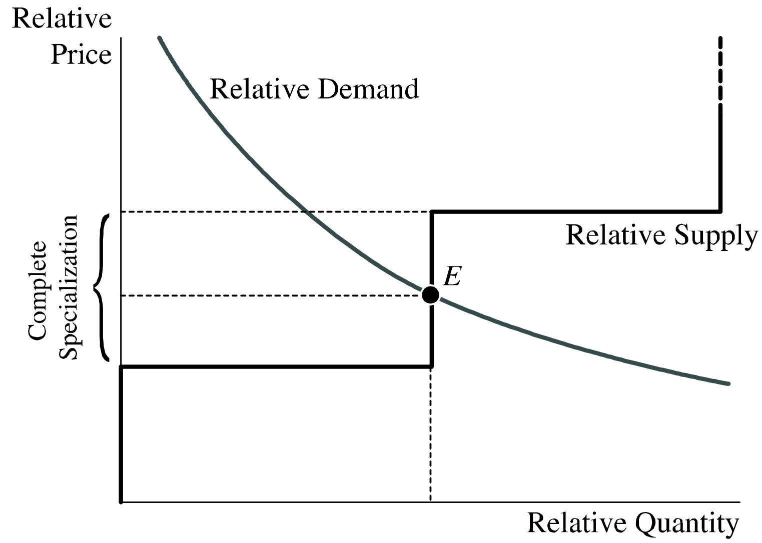

Figure 2: Euqilibrium in the Ricardian model

The equilibrium in the world market is given by the intersection between the relative aggregate demand and supply (for example, E in Figure 2). We then determine the equilibrium consumption and production levels from (37) and subsequently the value of money and the price level of nontraded goods from (26).

Remark 5 *The world market is stationary, so that stationarity in ϕ_t implies stationarity in Φ_t.*

Let us drop t from the notations in the stationary state (*i.e.*, $\Phi_t \to \Phi$). We then find the next proposition from (64) and Remark 5.

Proposition 4 *The price level of nontraded goods $1/\Phi$ is lowered by an improvement of terms of trade in the world market.*

In the following analysis, for simplicity, we restrict our attentions to the complete specialization case: $a_{11}/a_{21} < p_t^W < a_{12}/a_{22}$.

3.2 Monetary Side

Since we have money in our analysis, additional variables are needed to accurately describe the model. For simplicity, we assume there is no vehicle currency (or the currency of one of the two countries is the vehicle currency in international settlements). The quantities of imports (Good $k \neq j$) and exports (Good j) of Country j are represented by IM_j and EX_j, respectively: the quantities of transactions of traded goods are determined in the later part of this section. Let ψ_j

be a foreign exchange rate that convert Country j's currency into the trading partner's currency (say, Country k's). Let M_j be the amount of money spent on international trade (measured by local currency unit). Exports from Country j to Country k bring $\psi_k \mathscr{P}_k M_k$ to Country j. Imports from Country k to Country j require Country j to pay $\psi_j \mathscr{P}_j M_j$. These conditions, say payment flows, are equivalent to the market clearing conditions in the international trade market: $EX_j \equiv IM_k$. Mathematically, the payment flows are expressed as

$$P_1^W EX_1 \equiv \psi_2 \mathscr{P}_2 M_2 \quad \text{and} \quad P_2^W IM_1 \equiv \psi_1 \mathscr{P}_1 M_1, \tag{38}$$

$$\text{and} \quad P_2^W EX_2 \equiv \psi_1 \mathscr{P}_1 M_1 \quad \text{and} \quad P_1^W IM_2 \equiv \psi_2 \mathscr{P}_2 M_2. \tag{39}$$

These conditions are arranged to get

$$p^W EX_1 \equiv p^W IM_2 \equiv \frac{\psi_2 \phi_2 M_2}{p_{22}} \quad \text{and} \quad \frac{EX_2}{p^W} \equiv \frac{IM_1}{p^W} \equiv \frac{\psi_1 \phi_1 M_1}{p_{11}}. \tag{40}$$

where $j \neq k$. Using the labor market equilibrium condition (28), we then find M_j as

$$M_1 = \frac{a_{11}/p^W}{\psi_1 \phi_1} \quad \text{and} \quad M_2 = \frac{a_{22} p^W}{\psi_2 \phi_2}. \tag{41}$$

The two equations then imply one of the two expressions for the exchange rate:

$$\psi_1^{-1} \equiv \psi_2 = p^W \cdot \left(\frac{\phi_1}{\phi_2} \cdot \frac{M_1/M_2}{a_{11}/a_{22}} \right)^{1/2}. \tag{42}$$

The exchange rate, by definition, converts the value of two currencies into each other; hence, by $\phi_j \equiv \mathscr{P}_j / \mathscr{W}_j$, we have

$$\psi_1^{-1} \equiv \psi_2 \equiv \frac{\mathscr{P}_1}{\mathscr{P}_2} \equiv \frac{\phi_1}{\phi_2} \cdot \frac{\mathscr{W}_1}{\mathscr{W}_2}. \tag{43}$$

By definition, depreciation and appreciation of the currency of Country j correspond to an increase and a decrease in ψ_j^{-1}, respectively. From (28), we know $\mathscr{W}_j = a_{jj}^{-1} P_{jj}$, so that the exchange rate of Country 1's currency in terms of Country 2's currency ($\psi_2 = \psi_1^{-1}$) is rearranged as

$$\psi_1^{-1} \equiv \frac{\phi_1}{\phi_2} \cdot \frac{P_{11}/a_{11}}{P_{22}/a_{22}} \equiv \frac{\phi_1}{\phi_2} \cdot \frac{p^W}{a_{11}/a_{22}}. \tag{44}$$

We can then determine the exchange rate using already determined variables and given parameters.

There is an immediate result following from (44) that is an analogue of the Balassa-Samuelson effect (Balassa [2] and Samuelson [33]).

Proposition 5 *The currency of the relatively productive country is overvalued if and only if the* *relative productivity of exporting sector is in a certain range:* $p^W < a_{11}/a_{22}$.

We usually apply the equation of exchange (or quantity theory of money) to introduce money in international trade (for example, see Dornbusch *et al.* [7] that also includes nontraded goods in the analysis). In our model, however, we do not use the quantity theory of money. We actually distinguish the money in use for nontraded goods transactions from the money in use for traded goods transactions, as we consider the decentralized market as nontraded goods market and the centralized market as the traded goods market. Yet, there is a close relationship between the quantity theory and the money-search (*i.e.*, arrival rate and velocity of money), we are not totally away from the traditional modeling style in international trade theory.

The exchange rate given by (44) states that the currency exchange rate is affected by values of the two currencies determined in the nontraded goods transactions and the relative of internationally traded goods in the world market; hence, the currency exchange rate is augmented by the nontraded goods sector (for example, see Dixit and Norman [4, pp. 208-11] for the basic arguments). Dornbusch *et al.* [7] derive their flexible exchange rate as a function of pecuniary wages of the two countries. The wages are affected by money supply, velocity of money, and labor supply. In our model, as we will see sooner, ϕ_j and M_j are affected by nontraded goods transactions and money supply, and nontraded transactions determine labor supply in the internationally traded goods sector. Thus, we can say that our exchange rate is consistent with the flexible exchange rate found in the very basic model such as Dornbusch *et al.* [7]. The augmentation by the nontraded goods sector is not a new finding, but it is still a major discussion point. Recently, for example, Dotsey and Duarte [8] argue this point using a DSGE model and calibrations to find the augmentation effect.

4 Price Levels and Comparative Advantage

4.1 Numerical Model

For the decentralized market, we consider a *constant relative risk aversion* preference and a *linear* cost functions:

$$u(q) = \frac{q^{1-\rho}}{1-\rho} \quad \text{and} \quad c(q) = q, \tag{45}$$

where $\rho \in (0,1)$ is the rate of risk aversion. For the world market, we assume a *constant elasticity of substitution* preference (CES utility function):

$$U(x_1, x_2) = \left(x_1^{(s-1)/s} + x_2^{(s-1)/s} \right)^{s/(1-s)},$$

Table 1: Parameters

Arrival Rate (α)	0.5
Probability of Coincidence (σ)	1
Bargaining Power of the Buyer (θ)	1
Discount Factor (β)	0.95
Relative Rate of Risk Aversion (ρ)	0.15
Elasticity of Substitution (s)	2

where $s > 0$ is a preference parameter.[2] The elasticity of substitution is then given by $s = 1/(1-r) \in (0, +\infty)$. The production technology is *classical*, so that the aggregate production of Good i in Country j is $N_{ij} = a_{ij} Y_{ij}$, where a_{ij} is the input coefficient.

For simulation, we assume the input coefficient matrix is given by

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} a_{11} & 1 \\ 1 & 0.5 \end{pmatrix}, \tag{46}$$

where the input coefficient of Good 1 in Country 1 is variable: $a_{11} \in (0,1)$. Thus, Country 1 and Country 2 exhibit comparative advantages in Goods 1 and 2, respectively. Some parameters used in the simulation are shown in Table 1. As this table shows, we use $\theta = 1$ (buyers make take-it-or-leave-it offers). This assumption does not affect the basic properties of the model while it affects the socisl welfare level: as it is well-known, the efficiency is reached when the buyers make take-it-or-leave-it offers (see, for example, Lagos and Wright [22]).[3]

The remaining parameters are N_1 and N_2, and $\mathscr{M}_1$ and $\mathscr{M}_2$. These parameters are set for each simulation. The money stocks are considered as policy parameters. To see the country size, we see relative population size defined by

$$n \equiv \frac{N_1}{N_2}. \tag{47}$$

By definition, we then find

$$\frac{\mathscr{M}_1}{\mathscr{M}_2} \equiv \frac{n\tilde{m}_1}{\tilde{m}_2}. \tag{48}$$

4.2 Local Market

The price level of nontraded goods is $1/\Phi$ and the price of money is $\phi \equiv \mathscr{P}/\mathscr{W}$, so that $1/\phi$ indicates the real wage rate of the internationally traded good sector. We then focus on the two price levels $1/\phi$ and $1/\Phi$ to watch the local economy.

[2] The function approaches linear form as $\rho \to 0$, Cobb-Douglas as $\rho \to 1$, and Leontief as $\rho \to -\infty$.

[3] The welfare level is affected also by the form of bargaining as shown by Aruoba et al. [1]. In their analysis, Nash bargaining and Kalai bargaining solutions suggest different welfare values and different social optimum real balance levels.

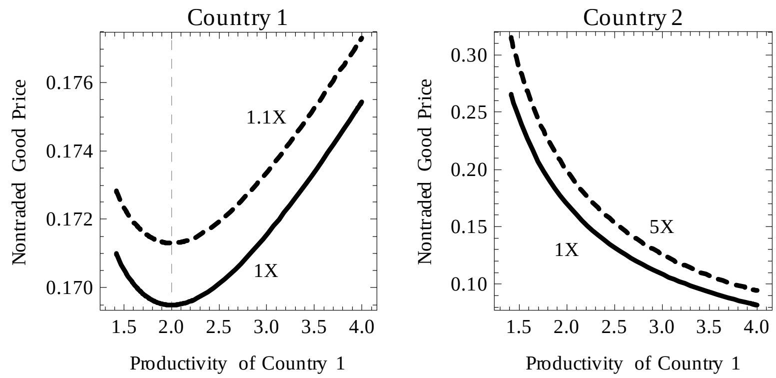

Figure 3: Nontraded good price and an increase in the productivity of Country 1

The simulation results for the nontraded goods price levels $1/\Phi$ when the productivity of Country 1 is improving are shown in Figure 3. As this figure shows, the price levels of the nontraded goods are affected by the productivity in the internationally traded goods sector. In the figure, $\tilde{m}_1 \in \{1, 1.1\}$ and $\tilde{m}_2 \in \{1, 5\}$ are shown. As it shows, an increase in the optimum money holdings per capita must be accompanied by a higher price level. The improvement of Country 1's exporting good sector reduces the prices of nontraded goods in Country 2 (foreign country). In Country 1 (home country), the nontraded goods prices are decreasing if the productivity level of home's exporting sector is relatively less than that of the foreign's exporting sector (e.g. $a_{11}^{-1} < a_{22}^{-1} = 2$). If the productivity of home's exporting sector is better than that of the foreign's, the nontraded goods prices are increasing in the improvement of the productivity. As we will discuss in later (Section 4.3), this result is an analogue of the Balassa-Samuelson effect, as stated in Proposition 5.

The simulation results for the wages $1/\phi$ when the productivity of Country 1 is improving are shown in Figure 4 for $\tilde{m}_1 \in \{1, 1.1\}$ and $\tilde{m}_2 \in \{1, 5\}$. The wages show analogous movements as the nontraded goods prices. In this figure, we can see that larger optimum money holdings must be accompanied by a higher wage rate. In Country 2 (foreign country), the improvement of the productivity in Country 1 (home country) reduces the wage rate. In home country, the improvement of the exporting sector reduces the wage rate if the productivity of own exporting sector is relatively less than that of foreign's. However, the improvement of the productivity in the exporting sector increases the wage rate if the productivity of the exporting sector is better than that of foreign's. The result for Country 1 (home country) shows *á la* Balassa-Samuelson effect as seen in the observation of the nontraded good price in Country 1 (Figure 3).

To see the changes in wage rates in each country in response to a change in the relative country size, we consider a symmetric case $a_{11} = a_{22} = 0.5$. An increase in n indicates that there is a relative increase in labor supply in Country 1 while a relative decline in labor supply in Country 2.

Thus, the real wage rate in Country 1 should decline while the real wage rate in Country 2 should increase. This notion is confirmed by Figure 5. In this figure, we can also confirm that an increase in the optimum money holdings per capita must be accompanied by an increase in the wage rate.

A similar movement as wages is seen in nontraded goods prices, as depicted in Figure 6. We then find that an increase in the relative country size of Country 1 reduces the nontraded goods price level in Country 1 as it indicates that an increase in the labor supply in the exporting good sector in Country 1 affects the production of nontraded goods. The reverse is also true: in Country 2, the nontraded goods prices go up as the labor supply in the exporting good sector in Country 2 declines to affect the nontraded goods.

4.3 International Market

We look at the currency exchange market. Figure 7 shows changes in exchange rate ψ_1^{-1} for each relative country size $N_1/N_2 \equiv n \in \{0.8, 1, 1.5\}$ and optimum per-capita money holdings (optimum real balances per capita) $(\tilde{m}_1, \tilde{m}_2) \in \{(1,1), (5,1)\}$. From these simulations, we can find that improvements of the productivity of Country 1's exporting sector unambiguously appreciate Country 1's currency, and an increase in the optimum money holdings is associated with appreciation and an increase in the relative country size is associated with devaluation.

An increase in the relative country size implies that the aggregate supply of currency of Country 1 increases in the exchange market, so that it reduces the value of the currency of Country 1 (devaluation) as it is well-known in international finance. An increase in the per-capita real balances $\tilde{m}_1$ also increases the aggregate supply of currency of Country 1. However, the increase in $\tilde{m}_1$ is an increase in the *optimum* money holdings per capita. Therefore, an increase in $\tilde{m}_1$ must be accompanied by an increase in the value of money and that results in the appreciation of the currency. From an alternative view, Figure 8 depicts the fact that an increase in the relative country size of Country 1 depreciates one's currency while an increase in the optimum real balances per capita appreciates one's currency.

Next, we consider the Balassa-Samuelson effect. By definition, ϕ_1/ϕ_2 represents the purchasing power parity (PPP) of the currency of Country 1 in terms of the currency of Country 2. The comparison between the PPP and the exchange rate is given in Figure 9. As suggested by Proposition 5, the currency in the foreign exchange is overvalued if $p^W < a_{11}/a_{22}$. This segment in the figure corresponds to the segment in Figures 3 and 4 where the price level of nontraded goods and the wage rate increase as the productivity of the exporting sector in Country 1 improves: the Balassa-Samuelson effect is observed here.

A new finding in this paper is the existence of the segment where the Balassa-Samuelson effect does not exist (*e.g.*, $p^W \geq a_{11}/a_{22}$). In this segment, the improvement of the productivity in the ex-

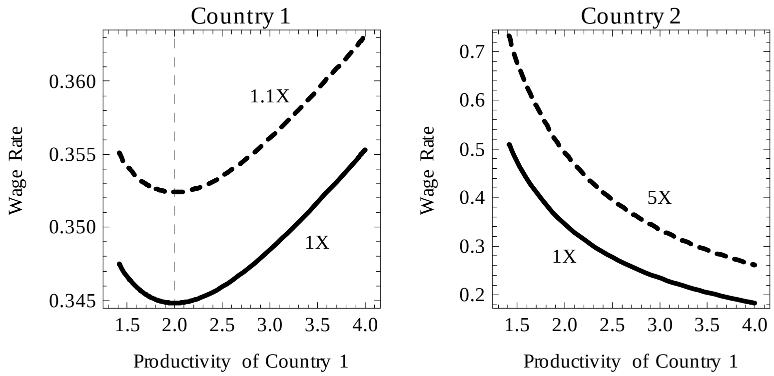

Figure 4: Wage rate of traded goods sectors and an increase in the productivity in Country 1

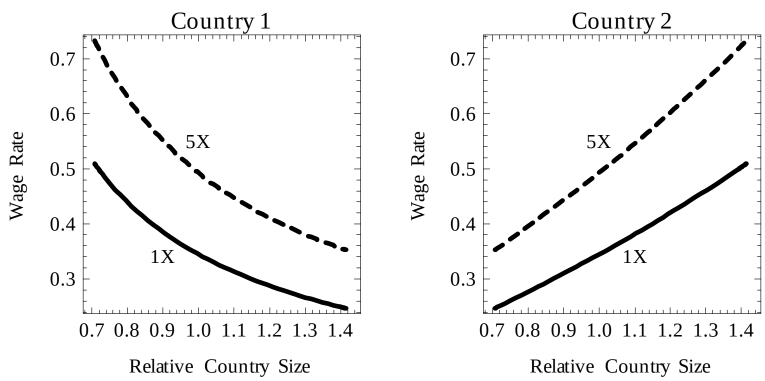

Figure 5: Relative country size and wage rate ($a_{11} \equiv 0.5$)

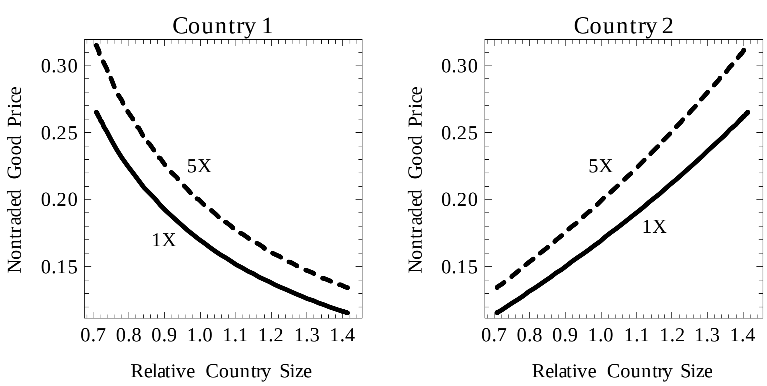

Figure 6: Relative country size and price level of nontraded goods ($a_{11} \equiv 0.5$)

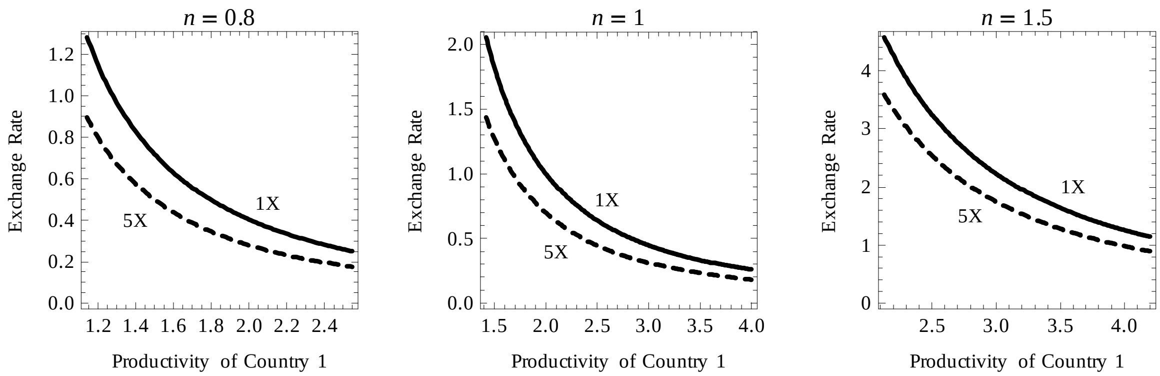

Figure 7: Exchange rate and comparative advantage

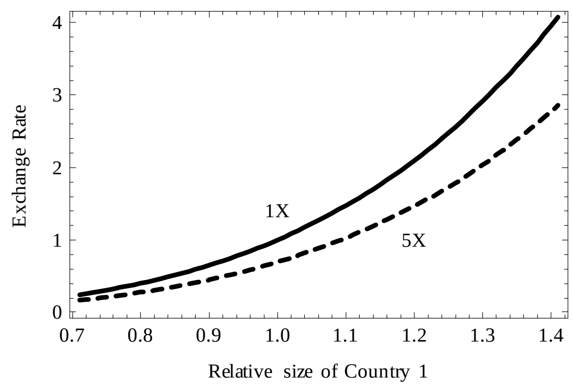

Figure 8: Relative country size and exchange rate

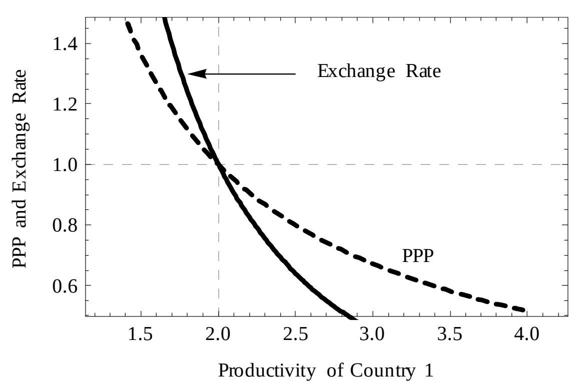

Figure 9: Purchasing power parity and exchange rate: the Balassa-Samuelson effect

porting sector in Country 1 generates an undervaluation of the currency of Country 1. In addition, it reduces domestic price levels (nontraded goods prices and wage rate). In some sense, we can say the system described by this model has a built-in subsidy scheme for a country that has just started experiencing technological progress in its exporting sector.

4.4 Social Welfare

The social welfare is derived from the individual value function (2). Since there will be no confusion, let us drop the subscripts from the most of notations. We use tilde to denote the variables and values of functions in the stationary equilibrium; so that the value function in the equilibrium, for example, is denoted as $V(\tilde{m}) = \tilde{V}$.

At the equilibrium, we find

$$\begin{aligned} \tilde{V} &= \alpha\sigma\{u(\tilde{q}) - c(\tilde{q})\} + \alpha\delta\{u(q^*) - c(q^*)\} + \tilde{W}, && (49) \\ \text{where} \quad \tilde{W} &= U(\tilde{x}_1, \tilde{x}_2) - (p_1\tilde{x}_1 + p_2\tilde{x}_2). && (50) \end{aligned}$$

To derive the social welfare function, we take the average of the all individuals in this economy. We then find

$$\begin{aligned} \frac{1}{N}\int_\zeta U(\tilde{x}_1, \tilde{x}_2)\, d\mathscr{G}(\zeta) &= \frac{\chi}{N}\int_\zeta (\ell + \phi_{it}\zeta - \phi_{it}\tilde{m})\, d\mathscr{G}(\zeta) = \chi, && (51) \\ \text{and} \quad \frac{1}{N}\int_\zeta (p_1\tilde{x}_1 + p_2\tilde{x}_2)\, d\mathscr{G}(\zeta) &= \frac{\mathscr{X}}{N}\int_\zeta (\ell + \phi_{it}\zeta - \phi_{it}\tilde{m})\, d\mathscr{G}(\zeta) = 1, && (52) \end{aligned}$$

where the last equation follows from Remark 4. The above equations imply

$$\tilde{W} = \tilde{\chi} - 1. \qquad (53)$$

Therefore, the social welfare is represented by

$$\tilde{V} = \alpha\sigma\{u(\tilde{q}) - c(\tilde{q})\} + \alpha\delta\{u(q^*) - c(q^*)\} + (\tilde{\chi} - 1). \qquad (54)$$

To process numerical simulations, we use the same parameters as in Section 4.3.[4]

One of the very basic properties of the social welfare in money-search model with respect to the per-capita real balances is shown in Figure 10. As it is well-known, the social welfare function depicts a hump-shaped pattern in real balances; hence, there is a social optimum real balance level. The impact of relative country size of Country 1 is depicted in Figure 10. We can then see an

[4] Since $\delta = \sigma^2$, we then find $\delta = 0.25$. In addition, the social optimum q^* is given by solving $u'(q) = c'(q)$. Since the utility and the cost functions are given by (45), we can easily find $q^* = 1$.

increase in own population increases own welfare level while it decreases the welfare level of the other.

The impact of the improvement in the relative productivity of the exporting sector in Country 1 is shown in Figure 12. The technological advancement in Country 1 is unambiguously profitable for the foreign country (Country 2). It is also profitable for the home country (Country 1) if the relative productivity level does not exceed a certain level. In particular, as the threshold value coincides with the one for the Balassa-Samuelson effect, the technological advancement is beneficial for the home country so long as the currency is not over valued. If the currency is overvalues (Balassa-Samuelson effect), the technological advancement negatively affects the welfare level of the home country.

5 An Application to Foreign Exchange Interventions

It is noteworthy that the *purchasing power parity* ϕ_1/ϕ_2 in the money-search model is given by equating (42) and (44) to get

$$\frac{\phi_1}{\phi_2} = \frac{a_{11}M_1}{a_{22}M_2}. \tag{55}$$

This equation implies that there is a relationship between money and exchange rate as

$$\psi_1^{-1} = \frac{p^W M_1}{M_2}\left(\frac{\phi_1}{\phi_2}\right)^2. \tag{56}$$

We can then consider the foreign exchange interventions ΔM_j by Country j:

$$\psi_1^{-1} = \frac{p^W M_1\left(1+\hat{M}_1\right)}{M_2\left(1+\hat{M}_2\right)}\left(\frac{\phi_1}{\phi_2}\right)^2, \tag{57}$$

where $\hat{M}_j \equiv \Delta M_j/M_j$ is the scale of foreign exchange intervention.

Typically the foreign exchange interventions aim at depreciation of one's currency to promote export and/or to protect importing sector (in this model, however, import industry protections cannot be included due to complete specialization assumption). This may not be rational from an orthodox economic perspective, but it is justified by politically augmented models. For example, political motivations in formations of international trade policies are discussed in Mayer [25, 26], Hillman [15], Hillman and Urpsrung [16], and Grossman and Helpman [14]. These studies especially focus on the formations of import tariffs, but the logic is applied to discuss export promotion policies. Corden [3] also introduces a politically augmented social welfare function as a weight-sum of economic surplus, where the weights are augmented by political motives. In the money-search model, as it is well-known, we observe a hump-shaped social welfare function with

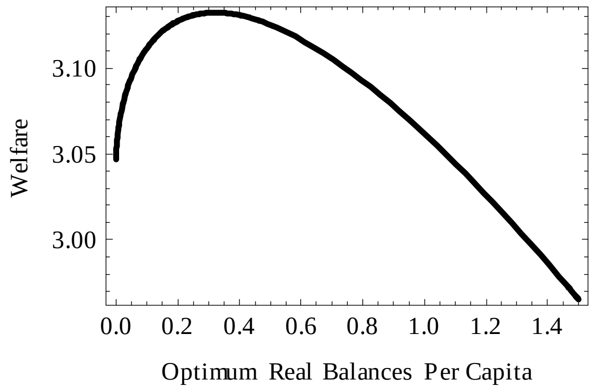

Figure 10: Social welfare and real balances

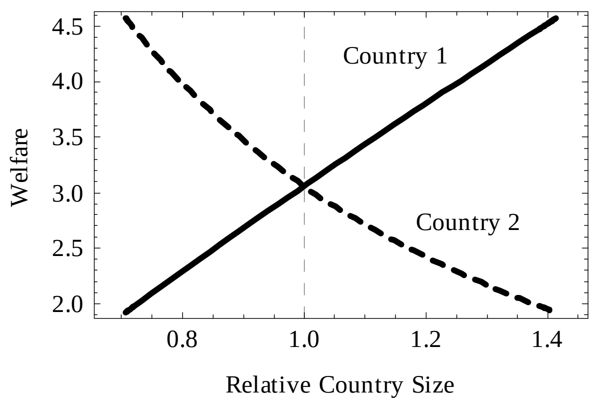

Figure 11: Relative country size of Country 1 and social welfare of the two countries

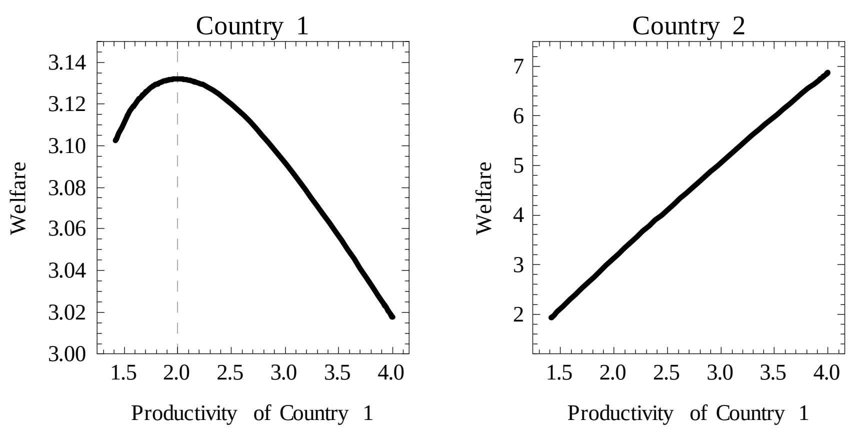

Figure 12: Producitvity of Country 1's exporting sector and social welfare

respect to the real balances (cf. Figures 10). Therefore, the foreign exchange interventions may or may not improve own social welfare conditional on the *status quo* real balances. In the following analysis, we only focus on the exchange rate after the interventions and the following adjustment process.

From (40) and (57), we find

$$EX_1 \equiv IM_2 \equiv \frac{\phi_1}{\phi_2} \cdot \frac{\phi_1 M_1 \left(1 + \Delta \hat{M}_1\right)}{a_{22}\left(1+\hat{M}_2\right)} \tag{58}$$

$$\text{and} \quad EX_2 \equiv IM_1 \equiv \frac{\phi_2}{\phi_1} \cdot \frac{\phi_2 M_2 \left(1 + \hat{M}_2\right)}{a_{11}\left(1+\hat{M}_1\right)}, \tag{59}$$

where $p_{jj} = a_{jj}$ is applied. We suppose the government aims at export promotions, so that it tends trying to depreciate one's currency: for Country 1, see (57) and (58). At the point of the intervention, ϕ's and opponent's money in the international currency market are taken as given. The foreign exchange intervention by Country 1 to induce depreciation then increases the supply of money of Country 1's currency in the foreign exchange market.

The instantaneous effect is the same as an increase in the relative country size in the exporting good market (an increase in the relative size of Country 1 increases M_1 if the absolute size of Country 2 is fixed). The share of Good 1 in the world allocated to Country 1 then declines (hence, the export increases). By the intervention, the import decreases. The balance of payment is then adjusted by the change in the exchange rate (depreciation).

After the second subperiod of the current period, if no *sterilization* action is taken, the next first subperiod begins. In the local market, the increase in the real balances by the intervention must be adjusted in the local market. In Country 1, the wage rate $\phi_1^{-1} \equiv \mathscr{W}_1/\mathscr{P}_1$ increases as the real balances per capita increases, as we have seen in the previous section: for example, upward shifts in the curves in Figures 4 and 5. This implies that ϕ_1 decreases in the adjustment process and it becomes an appreciation pressure in the foreign exchange market in the next subperiod. We then also find from (55) that M_1 decreases by the same percentage as ϕ_1 in this adjustment process. In Country 2, the adjustment should take place in χ_2. That is, however, a constant as the aggregate demand and supply in the world do not change to alter p^W. As consequence, the exchange rate appreciates and the currency is valued higher than the level before the intervention if the wage rate is affected so largely.

Proposition 6 *Foreign exchange interventions aiming at devaluating one's currency increases the exports in the current period, but it may result in a decline of the exports below the level before the intervention if the adjustment is so large, as the exchange rate appreciates more than the level before the intervention.*

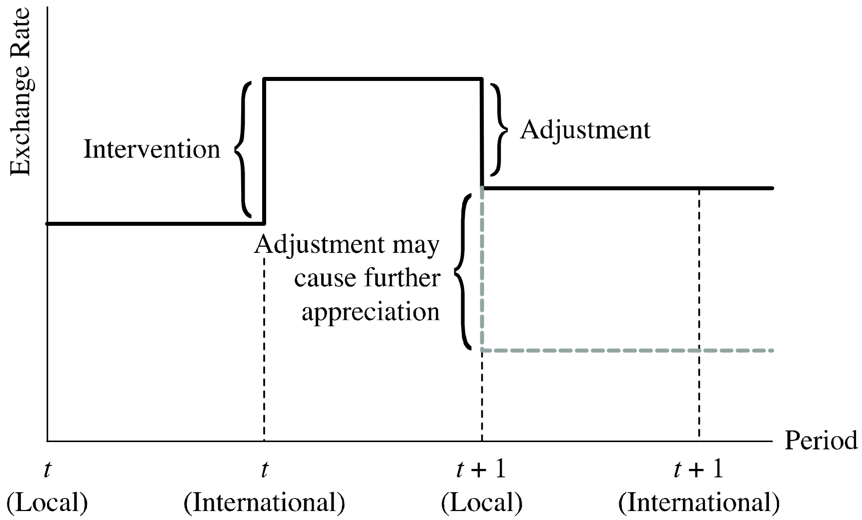

Figure 13: An illustration of a foreign exchange intervention

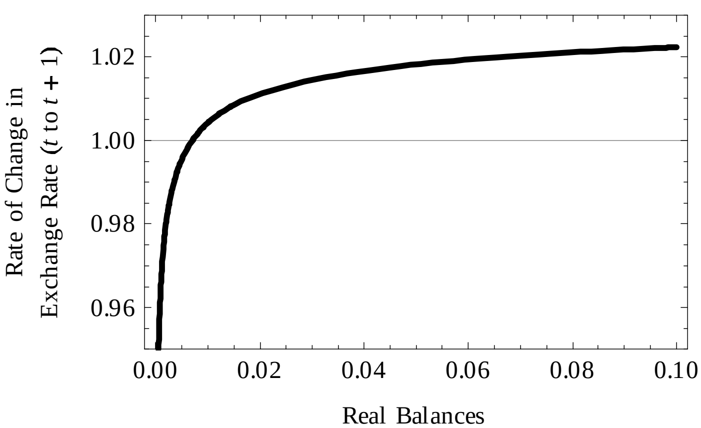

Figure 14: An example for the post-adjustment exchange rate

The summary of the exchange rate dynamics after the intervention and the following adjustment is illustrated in Figure 13. In addition a numerical example is also shown in Figure 14. In the example, the government of Country 1 intends to depreciate own currency by 10% in period t. In this figure, the vertical axis measures the ratio of exchange rate before and after the intervention and the following adjustment process; hence, below one indicates an appreciation of the exchange rate while less than one indicates a depreciation. In this example, if the real balances are sufficiently small, we observe that the exchange rate appreciates more than the level before the intervention after the adjustment. In some sense, we have found an analogous effect as the overshooting effect (Dornbusch [6]).

6 Conclusion

This paper extended the third-generation money-search model invented by Lagos and Wright [22] with international trade. The extension simplifies the analysis of international trade with money and it also justifies the separation of the two markets (centralized and decentralized). In the centralized market, there are international transactions, which are considered usually for wholesale merchants. In the decentralized market, there are local transactions, which are considered usually for retail sales. The notion that the international trade provides intermediate products to produce retail products sold in the local market is also applied by Sanyal and Jones [34].

Using the extended framework, we have seen some classical results, such as the Balassa-Samuelson effect (Balassa [2] and Samuelson [33]) and the overshooting effect of Dornbusch [6]. We have found that the Balassa-Samuelson effect is conditional on the relative productivity of the exporting sectors and the existence of the Balassa-Samuelson effect causes some new findings, such as the negative effect of the advancement of the technology in the exporting sector.

As an extension, we have considered the impact of foreign exchange interventions. Although it is conditional on the *status quo* real balances, we have found that the intervention aiming at depreciation may result in further appreciation than the *status quo* level after the adjustment followed by the intervention. If this result is observed in the real world, it warns that the foreign exchange interventions only have a limited impact in the current period while it worsens the situation for the monetary authority (or the government) intending to promoting exports in the long run.

For further extensions, we can easily include the trade policies such as tariffs and subsidies. In addition, as it is analyzed by Saito [32] in an autarky economy, we can easily include the banking sector. The banking sector can further be extended to include some parameters for central banking, such as reserve ratio and reserve fund rate. We can then extend the model discussed in this paper to include the banking sector and the central bank to analyze the impacts of banking regulations in the world economy.

Appendix

A Proof of Proposition 1

For simple notations, for $u(q)$ and $c(q)$, let us write $u(q)$ as u, $u'(q)$ as u', *etc*... With these notations, $Z'(\tilde{q})$ is computed as

$$Z'(\tilde{q}) = \frac{u'c'\{\theta u' + (1-\theta)c'\} + \theta(1-\theta)(u-c)(u'c'' - u''c')}{\{\theta u' + (1-\theta)c'\}^2}, \tag{60}$$

where variable q is dropped from the expression for simple notations. At the social optimum $\tilde{q} = q^*$, we find $u' = c'$, so that

$$Z'(q^*) = \frac{u'^2 + \theta(1-\theta)(u-c)(c'' - u'')}{u'}. \tag{61}$$

Since $c''(q) > 0$ and $u''(q) < 0$, we find

$$\frac{u'(q^*)}{Z'(q^*)} = \frac{u'^2}{u'^2 + \theta(1-\theta)(u-c)(c'' - u'')} \leq 1, \tag{62}$$

where the equality holds only if $\theta = 1$.

Since $u'(0) = +\infty$, we find

$$\frac{u'(q^*)}{Z'(q^*)} = 1 \leq \frac{u'(\tilde{q})}{Z'(\tilde{q})} \leq \frac{u'(0)}{Z'(0)} \rightarrow +\infty. \tag{63}$$

In addition, Remark 3 implies

$$1 + \frac{1}{\alpha\sigma\beta}\left(\frac{\phi_t}{\Phi_{t+1}} - \beta\right) \geq 1, \tag{64}$$

where the equality holds only if at $\phi_t/\Phi_{t+1} = \beta$. Therefore, from inequalities (63) and (64) and continuity of $u'(q)$ and $Z'(q)$, we prove that there is at least one equilibrium satisfying (26).

B Numerical Model

B.1 Local Market (Decentralized)

For the local market, we assume a constant relative risk aversion (CRRA) preference:

$$u(q) = \frac{q^{1-\rho}}{1-\rho}. \tag{65}$$

As $u'(q) = q^{-\rho}$, $u''(q) = -\rho q^{-\rho-1}$, and $u'''(q) = \rho(\rho+1) q^{-\rho-2}$, we find

$$u'(q) u'''(q) = \rho(\rho+1) q^{-2\rho-2} > \{u''(q)\}^2 = \rho^2 q^{-2\rho-2}. \tag{66}$$

The CRRA function thus does not satisfy the log-concave property; hence, we may not have uniqueness in $\tilde{m}$ unless we apply a linear cost function, as notified in Lagos and Wright [21, 22]. We the use a linear production technology:

$$c(q) = q. \tag{67}$$

From the utility and cost functions given above, we find

$$Z(q) = \frac{\theta q^{1-\rho} + (1-\theta) q^{1-\rho}/(1-\rho)}{\theta q^{-\rho} + (1-\theta)}. \tag{68}$$

B.2 World Market (Centralized)

In order to analyze the model, we assume the preference of each agent over the internationally traded goods is represented by a CES function as

$$U(x_1, x_2) = \left(x_1^{(s-1)/s} + x_2^{(s-1)/s}\right)^{s/(s-1)}, \tag{69}$$

where $s > 0$ is the elasticity of substitution. The first order condition (8) is then given by

$$\frac{p_{1t}}{p_{2t}} = \left(\frac{x_1}{x_2}\right)^{-s} \quad \Rightarrow \quad x_2^* = x_1^* \left(\frac{p_{1t}}{p_{2t}}\right)^{1/s}. \tag{70}$$

Rearranging the budget constraint (6), we obtain

$$x_1^* = \frac{\ell + \phi_{it} m - \phi_{it} \tilde{m} - p_{2t} x_2^*}{p_{1t}}, \tag{71}$$

which is further arranged to get

$$x_1^* = \frac{\ell + \phi_{it} m - \phi_{it} \tilde{m}}{p_{1t}} - x_1^* \left(\frac{p_{1t}}{p_{2t}}\right)^{(1-s)/s}. \tag{72}$$

We then solve this equation to get

$$x_{1j}^* = \frac{\ell + \phi_{it} m - \phi_{it} \tilde{m}}{\Pi_{1j}} \quad \text{and} \quad x_{2j}^* = \frac{\ell + \phi_{it} m - \phi_{it} \tilde{m}}{\Pi_{2j}}, \tag{73}$$

where Π_1 and Π_2 represent

$$\Pi_{1j} = p_{1j} \left\{ 1 + \left(\frac{p_{1t}}{p_{2t}} \right)^{(1-s)/s} \right\}, \tag{74}$$

$$\text{and} \quad \Pi_{2j} = p_{2j} \left\{ \left(\frac{p_{1t}}{p_{2t}} \right)^{(s-1)/s} + 1 \right\}. \tag{75}$$

From (73), the relative aggregate demand is computed as

$$\frac{\sum_{j=1}^{2} X_{1j}}{\sum_{j=1}^{2} X_{2j}} = \frac{\frac{1}{p_{11}} \left\{ 1 + \left(\frac{p_{11}}{p_{21}} \right)^{(1-s)/s} \right\}^{-1} N_1 + \frac{1}{p_{12}} \left\{ 1 + \left(\frac{p_{12}}{p_{22}} \right)^{(1-s)/s} \right\}^{-1} N_2}{\frac{1}{p_{12}} \left\{ 1 + \left(\frac{p_{11}}{p_{21}} \right)^{(s-1)/s} \right\}^{-1} N_1 + \frac{1}{p_{22}} \left\{ 1 + \left(\frac{p_{12}}{p_{22}} \right)^{(s-1)/s} \right\}^{-1} N_2} \tag{76}$$

$$= \frac{\left\{ 1 + \left(\frac{p_{11}}{p_{21}} \right)^{(1-s)/s} \right\}^{-1} + \left\{ 1 + \left(\frac{p_{12}}{p_{22}} \right)^{(1-s)/s} \right\}^{-1} \frac{p_{11}}{p_{12}} \frac{N_2}{N_1}}{\left\{ 1 + \left(\frac{p_{11}}{p_{21}} \right)^{(s-1)/s} \right\}^{-1} \frac{p_{11}}{p_{21}} + \left\{ 1 + \left(\frac{p_{12}}{p_{22}} \right)^{(s-1)/s} \right\}^{-1} \frac{p_{11}}{p_{22}} \frac{N_2}{N_1}}. \tag{77}$$

Under free trade, $p_{i1} \equiv p_{i2}$; whence, for $p^W \equiv p_{1j}/p_{2j}$, we find

$$\frac{\sum_{j=1}^{2} X_{1j}}{\sum_{j=1}^{2} X_{2j}} = \frac{\left(1 + p^{W(1-s)/s} \right)^{-1} n + \left(1 + p^{W(1-s)/s} \right)^{-1}}{\left\{ \left(1 + p^{W(s-1)/s} \right)^{-1} n + \left(1 + p^{W(s-1)/s} \right)^{-1} \right\} p^W}. \tag{78}$$

The net indirect utility function is represented as $\chi_j \left(\ell + \phi_{it} m - \phi_{it} \tilde{m} \right)$ with χ_j given by

$$\chi_j = \left(\Pi_{1j}^{(1-s)/s} + \Pi_{2j}^{(1-s)/s} \right)^{s/(s-1)} - \left(\frac{p_{ij}}{\Pi_{1j}} + \frac{p_{ij}}{\Pi_{2j}} \right). \tag{79}$$

We can then easily confirm Remark 4:

$$\mathscr{X}_j = \frac{p_{1j}}{\Pi_{1j}} + \frac{p_{2j}}{\Pi_{2j}} \equiv 1. \tag{80}$$

As we want to express the net indirect utility function in terms of the world price and the locally determined price. For each country, we arrange χ_j as

$$\chi_1 = \frac{1}{\Pi_{11}}\left\{1+\left(\frac{\Pi_{1j}}{\Pi_{2j}}\right)^{(s-1)/s}\right\}^{s/(s-1)} - 1, \tag{81}$$

$$\text{and} \quad \chi_2 = \frac{1}{\Pi_{22}}\left\{\left(\frac{\Pi_{1j}}{\Pi_{2j}}\right)^{(1-s)/s} + 1\right\}^{s/(s-1)} - 1. \tag{82}$$

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