Defence sector, armaments-labor ratio and national security

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Abstract

This paper analyzes a national defense economy in which the army reduces the risk of attack and damage. The results show that it is important how countries or people feel about damage from attack to military personnel, citizens and wealth. The feeling determines the optimal arms procurement and army personnel. It affects also international trade. It is found that labor (armaments) input into the military sector is not always decreased for the rise of wage (armaments price). The model suggests that conscription affects army expenditure and international trade.

Key word: Military expenditure; Armaments-labor ratio; Willingness-to-pay for safety; Arms import: International trade
JEL Codes: H56, F19

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0. Introduction

A lot of studies have analyzed military expenditure and its effects. However, the procurement-personnel ratio in the military sector has not been focused on. From the view point of economics or international trade theory, the capital-labor ratio is a key variable and it is dependent on the factor price ratio. Generally speaking, we may think that the procurement-personnel ratio is low in developing countries. In the private sector, the cost of labor is only the wage. But in the military economy, the expected sacrifice of personnel, which includes army persons, is added to the wage as a cost. This paper shows effects of the expected sacrifice on an economy with a military sector.

Army personnel and procurement reduce the risk in which the country is attacked so that civilians and soldiers fall victim. Under the cost minimization, the government will pay national defence expenditure for safety. Then the optimal army personnel and weapon can be shown by using the notion of risk reduction. This type of analysis is called willingness-to-pay approach (Jones-Lee (1990)). Although his paper is pioneer, only the expenditure is treated and the sacrifice is not separated into citizens, soldiers and wealth. Also in other papers, the relationship between defense sector and international trade has not been analyzed.

This paper shows that the optimal procurement-labor ratio in the military sector depends on how people feel about the damage from attack. The feelings are revealed as victim or damage evaluation parameters. These parameters are classified into four, those of military personnel, civilians, armaments and wealth. For example, it is shown how the attitude to the victim of soldiers affects the optimal inputs.

This paper shows that military personnel are not always reduced when the parameter for soldier's damage is increased or the wage is raised. Because the feelings determine the employment in the defence sector, the labor in the rest of the military sector is affected. Thus, the capital-labor ratio in the private sector is also affected so that the structure of private productions is also changed according to the damage attitude. Thus, the feelings determine the comparative advantage. Moreover, this paper suggests that conscription could affect army expenditure and international trade by changing the damage attitude of the ruling class and policy makers.

The next section presents the model in which there are two tradable sectors and one military sector. An economy with the notions of victim or damage parameters is introduced. Section 2 presents optimal inputs of army personnel and armaments into military sector. Optimization of national welfare including expected damage of attack is considered. In section 3, some comparative statistics are analyzed. In section 3.1, the effects of these parameters on optimal inputs are analyzed. The effects of wage and factor endowments are considered respectively in section 3.1 and 3.2. The effects of armament price and factor endowment are considered respectively in section 3.3 and 3.4.

1. The model

This paper considers a small country model which has two private sectors and one national defense sector[1]. The private sectors are produced by labor and capital. Defense services are produced by military personnel and arms imports[2]. Arms are not produced in the home country for simplicity. The international transfer of weapons and military aid are not considered[3]. The national income constraint is expressed as:

$$p_1D_1 + p_2D_2 = w(L_1 + L_2 + L_A) + r\ (K_1 + K_2) - T,$$

where D_i is the demand for consumer goods i (i = 1, 2), p_i is the price for goods i, w is the wage rate, L_j is the labor input into the j-th sector (j = 1, 2, A), r is the rental rate, K_i is the capital input into the i-th sector, T is the amount of tax revenue. Labor input, L_A, is military personnel which includes non-combatant as well as soldiers. Government budget constraint is :

$$T = wL_A + p_MM,$$

where M is the volume of procurement imports and p_M is the unit or average price of M. From the assumption of small country, p_1, p_2 and p_M are fixed.

Full employment conditions for factor endowments are:

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[1] A small country model is often used in the international trade theory since it is very convenient to approach a lot of problems. In this model the prices of tradable goods are constant.

[2] It is assumed that capital is not used in the sector. It is said that the share of the procurement and personnel expenditure in the defense sector is very large (Sandler and Hartley, 1995).

[3] Taking it into consideration is easy. But the results are not changed basically. Realistically, after the cold war, the arms trade has become more commercial (Brzoska , 2004).

$\bar{L} = L_1 + L_2 + L_A$,
$\bar{K} = K_1 + K_2$,

where $\bar{L}$ and $\bar{K}$ are, respectively, labor and capital endowments and they are exogenously determined.

National defence service production function is defined as:

$$F = F(L_A, M). \qquad (1)$$

It is assumed that F_L, $F_M > 0$, $F_{ij} > 0$, $F_{ii} < 0$ (i = L, M). In this paper, the quality of defence equipment is not considered. Actually, developed countries use high quality armament. But in the model, it can be interpreted that such countries use a lot of armaments in terms of efficiency units, and that national defence service is armament-intensive in developed countries. On the other hand, it would be labor-intensive in developing countries.[4]

The perceived probability of being attacked is defined as:

$$P = P(F, F^*), \qquad (2)$$

where F* is military expenditure of foreign countries and P is perceived probability, $0 < P < 1$. It is assumed that $P_F < 0$, $P_{FF} > 0$, and $P_{F*} > 0$. If there is no defense sector (F = 0), the perceived probability of being attacked becomes a value, θ ($0 < \theta < 1$), for a F*. $P_{FF} > 0$ means that the marginal effect of defense service is decreased. Figure 1 shows it.

Let V be an evaluation parameter of damage to soldiers, weapon and civilians. It is the evaluation per unit attack. It is defined as:

$$V = [\beta\rho_1 L_A + \alpha\rho_2 (L_1 + L_2)] + \beta\rho_3 M p_M + \alpha\rho_4 (K_1 + K_2) p_K, \qquad (3)$$

where β is a damage rate to military sector, α (= $(1 - \beta)$) is a damage rate to private

[4] Some developing countries use secondhand tanks which were made long ago in the Soviet Union. In the case the tanks have small M. On the other hand, newly made and expensive tanks have large M.

sector, ρ_1 is an evaluation parameter for the damage of military persons, ρ_2 of civilian damage, ρ_3 for arms procurement damage and ρ_4 for capital stock damage. Although the value of life may be infinity, ρ_1 and ρ_2 are assumed to be finite. ρ_i (w) (i = 1, 2, 3, 4) is a function of wage. It is assumed that $\rho_i' \geqq 0$.

Let A* be the rate of damage and γ be the amount of being attacked by foreign. γ may be considered a function of foreign military expenditure and $A^* = A^*(\gamma)$. When the country is attacked by foreign countries, the victim value for the soldiers etc. is $A^* \beta \rho_1 L_A$. If $A^* = 5\%$, $\beta = 80\%$, $L_A = 100000$, the casualties in the battle is $A^* \beta L_A = 4000$ people. When ρ_1 is ten thousand dollars per capita, the evaluation victim for L_A becomes forty million dollars. From equations (2) and (3), the expected damage under constant F* is defined by:

$$A^*VP = A^*V\,P(F) = E(L_A, M), \tag{4}$$

where $A^*VP = E(L_A, M)$ is the expected damage. It may be interpreted that the unit of A*V is life. For example, a seriously injured person by attack is counted as half. We can think that the victim is counted as death unit. By the national defense service, the perceived probability of being attacked is reduced ($P'(F) < 0$)and the expected volume of damage is also decreased. In the following we assume that the two factors are complement, that is:

$$-E_{LAM} > 0, \; -E_{MLA} > 0.$$

This means that the marginal national defense effects, $(-E_{LA})$ and $(-E_M)$, are increased by the other factor, M or L_A. This assumption could be considered to be plausible. To obtain our results, the assumption is important.

From the budget constraints, private expenditure to consumer goods can be written as:

$$p_1D_1 + p_2D_2 = w(L - L_A) + rK - p_M M.$$

Then, welfare can be defined as private expenditure minus expected damage by foreign attack:

$W = w(L - L_A) + rK - p_M M - A^* V P$[5]. (5)

In the model, exogenous variables are p_1, p_2, L, K, A*, p_M and endogenous are L_1 , L_2, K_1, K_2 ,L_A, M, W.

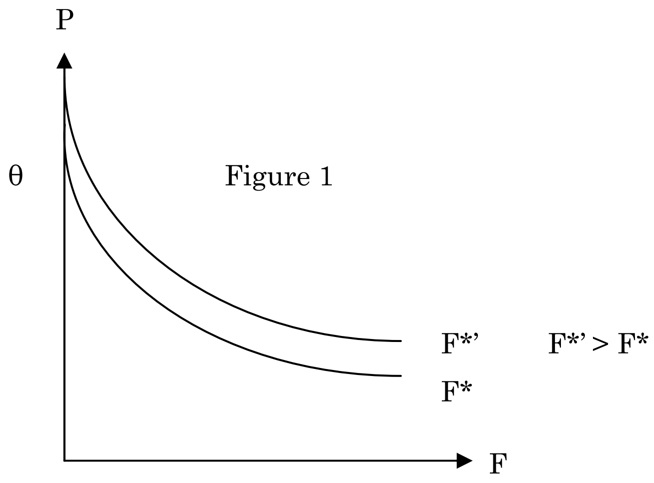

Figure 1

2. Optimization

2.1. Optimal M

In the section, the optimal arms import is considered. From equation (5), welfare effect of army imports is expressed as:

$$W_M = \partial W/\partial M = - p_M - \partial E/\partial M. \quad (6)$$

The first term ($- p_M$) is direct cost and the term ($- \partial E/\partial M$) is a national defense effect of

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[5] We can define welfare by using the expected utility function with risk aversion. For example, it is represented as: $(1 - P)U(p_1D_1 + p_2D_2) - PU(p_1D_1 + p_2D_2 - A^*V)$. I don't treat the expected utility function for simplicity. A kind of risk-neutral and linear utility function is used in this paper since I want to focus on the attitude to the victims. It is obvious that army expenditure is increased when we are more interested in victim of attack and/or the utility function becomes risk averse from risk-neutral.

an increase in armaments. From $\partial V/\partial M = \beta\rho_3 p_M$ and $\partial P/\partial M = P'F_M$, the national defense effect is divided into two terms:

$$\partial E/\partial M = A^*P(\partial V/\partial M) + A^*V(\partial P/\partial M) = A^*P\beta\rho_3 p_M + A^*V P'F_M$$

The first term ($A^*P(\partial V/\partial M)$) is an indirect victim cost through increase in M. The second term ($A^*V(\partial P/\partial M)$) is a direct defense service effect.

The optimal condition is shown by:

$$p_M = -\partial E/\partial M = -A^*P\beta\rho_3 p_M - A^*V P'F_M$$

At optimal, it is necessary that ($-\partial E/\partial M$) is positive, that is,

$$P\beta\rho_3 p_M < -V P'F_M.$$

Although the sign of the second derivative of ($\partial W/\partial M$) is undetermined, it is negative from the stability condition, $E_{MM} < 0$[6]. In Figure 2 the optimal M* is shown.

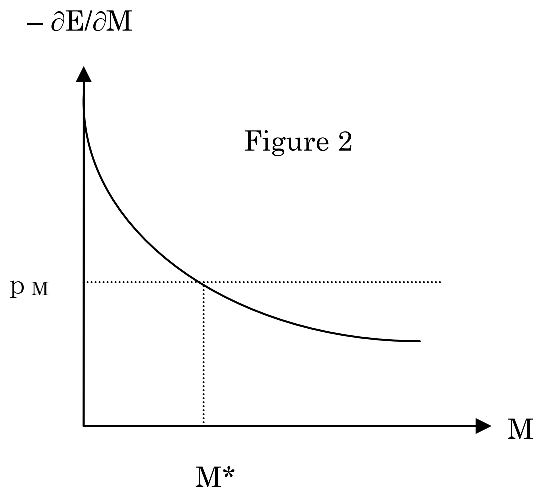

Figure 2

2.2. Optimal L_A

[6] $W_{MM} = \partial(\partial W/\partial M)/\partial M = \partial(\partial E/\partial M)/\partial M = -[2\beta\rho_3 p_M P'F_M + V(P''(F_M)^2 + P'F_{MM})]A^*$.

In the section, the optimal personnel input to military sector are considered. Welfare effect of labor input is expressed as:

$$\partial W/\partial L_A = - \ w \ - \partial E/\partial L_A \quad (7)$$

The first term, w, is a direct cost and the term $(-\partial E/\partial L_A)$ is a national defense effect of an increase in labor input. The national defense effect is divided into two terms:

$$-\partial E/\partial L_A = -A^*P(\partial V/\partial L_A) - A^*V(\partial P/\partial L_A)$$
$$= -A^*P(\beta\rho_1 - \alpha\rho_2) - A^*V\,P'F_L.$$

The first term $-A^*(\beta\rho_1-\alpha\rho_2)$ is an indirect victim cost through increase in L_A. The term, $-A^*P\beta\rho_1$, is the victim effect because employment in the military sector increases. The term, $A^*P\alpha\rho_2$, is the victim reduction effect because citizens are decreased by an increase in L_A. Although the sign of $(\beta\rho_1-\alpha\rho_2)$ is undetermined, it depends on the attitude of policy makers or the preferences of the countries. The second term, $-(A^*VP'F_L)$, is a direct national defense effect of military personnel input. By augmenting soldiers people worry about them. But the purpose of this augmenting is to defense the country. The former is the term, $-A^*P\beta\rho_1$, the later is the term, $-A^*VP'F_L$.

The optimal condition is shown by:

$$w = -\partial E/\partial L_A = -A^*[P(\beta\rho_1-\alpha\rho_2) + V\,P'F_L].$$

At optimal, it is necessary that $(-\partial E/\partial L_A)$ is positive, that is, $P(\beta\rho_1-\alpha\rho_2) < -V\,P'F_L$.

Although the sign of the second derivative of $(\partial W/\partial L_A)$ is undetermined, it is negative from the stability condition $(W_{LL} = E_{LL} < 0)$[7].

3. Comparative Statistics

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[7] $W_{LL} = \partial(\partial W/\partial L_A)/\partial L_A = E_{LL} - [2V_{LA}\,P'F_L + V(\partial P'F_L/\partial L_A)]A^*$
$= -[2(\beta\rho_1-\alpha\rho_2)P'F_L + V(P''(F_L)^2 + P'F_{LL})]A^*.$

3.1. The attitude to victim

When the attitude to victim is changed because of some reasons such as the event of Sep. 9.11 and varying the view or the culture, how are personnel input and armaments imports affected?

3.2.1 The attitude to personnel victim in the military sector

What is the effect of evaluation parameter of personnel victim in the military sector, ρ_1? By using the functions of W_M and W_{LA}, the partial effects are:

$W_{M\rho1} = \partial(\partial E/\partial M)/\partial\rho_1 = -A^*V_{\rho1}P'F_M > 0$,
$W_{L\rho1} = \partial(\partial E/\partial L_A)/\partial\rho_1 = -A^*P\beta(1 + L_A F_L P'/P)$.

From $-W_{M\rho1} > 0$, the marginal welfare effect of M is increased by $d\rho_1 > 0$. On the other hand, the sign of $W_{L\rho1}$ is undetermined. $W_{L\rho1}$ is separated into two terms. The first term of $W_{L\rho1}$, $-A^*P\beta < 0$ means that the marginal welfare effect of L_A is decreased for $d\rho_1$. Augmenting soldiers raises the expected victim since people becomes sensitive for the victim of soldiers since ρ_1 is raised. We call this term victim effect of ρ_1. The second term, $-A^*(\partial V/\partial\rho_1) P' F_L > 0$ means that the marginal welfare effect of L_A is increased for $d\rho_1$. We call this the partial defence effect of ρ_1 since larger ρ_1 increases the marginal defence effect of L_A.

From equations (5) and (6), the total differentiations of ρ_1 are given by:

$$\begin{pmatrix} dM \\ dL_A \end{pmatrix} = \begin{pmatrix} W_{LL} & -W_{ML} \\ -W_{LM} & W_{MM} \end{pmatrix} \begin{pmatrix} -W_{M\rho1} \\ -W_{L\rho1} \end{pmatrix} d\rho_1 (1/\Delta)^8. \qquad (8)$$

From the assumption of partial derivatives of the defense service function, E(M, L_A), we have:

$W_{LM} = -\partial(\partial E/\partial L)/\partial M > 0$,
$W_{ML} = -\partial(\partial E/\partial M)/\partial L_A > 0$.

From the stability condition, it is shown:

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[8] where $W_{ij} = \partial(\partial W/\partial i)/\partial j$ (i, j=M, L_A, w), $W_{LM} = -A^*[P'F_M(\beta\rho_1 - \alpha\rho_2) + V_M P'F_L + V(\partial(P'F_L)/\partial M)]$, $W_{ML} = -A^*[P'F_L\beta\rho_4 p_M + P'F_M V_{LA} + V(\partial(P'F_M)/\partial L_A)]$.

$$\Delta \equiv \begin{vmatrix} W_{MM} & W_{ML} \\ W_{LM} & W_{LL} \end{vmatrix} > 0.$$

By using the equation (8), the effects of $d\rho_1$ on dM and dL_A are shown as:

$$dM/d\rho_1 = (-W_{LL}W_{M\rho 1} + W_{ML}W_{L\rho 1})/\Delta, \qquad (8)'$$
$$dL_A/d\rho_1 = (W_{LM}W_{M\rho 1} - W_{MM}W_{L\rho 1})/\Delta.$$

Then we can say that $dM/d\rho_1 > 0$ and $dL_A/d\rho_1 > 0$ if $W_{L\rho 1} > 0$. $W_{L\rho 1}$ is rewritten as

$$W_{L\rho} = -A^*P\beta(1 - \eta\gamma),$$

where $\eta = -FP'/P = -(\partial P/\partial F)(F/P)$ and $\delta = L_AF_L/F$. η is a probability elasticity of defense expenditure. δ is a kind of a labor share in the military sector. By using η and δ, the condition for the sign of $W_{L\rho 1}$ is obtained: $W_{L\rho 1} \lesseqgtr 0 \Leftrightarrow 1 \gtreqless \eta\delta$.

Then, Proposition 1 is obtained:

Proposition 1.

If $W_{L\rho 1} > 0$, that is, $1 < \eta\delta$ or the victim effect of ρ_1 is smaller than the partial defence effect, the optimal employment and procurement in the military sector are both increased for the high evaluation parameter of victim or life. If $W_{L\rho 1} < 0$, there can be the case in which $dM/d\rho_1 < 0$ and $dL_A/d\rho_1 < 0$. The sign of $W_{L\rho 1}$ depends also on $\eta\delta$.

In the Proposition1, we find that if the elasticity of defense expenditure is large and/or the labor share in the military sector is large, $W_{L\rho 1}$ tends to be positive so that worrying about soldiers let the military sector expand.

3.2.2. The attitude for civilian victim

What is the effect of evaluation parameter of civilian victim, ρ_2? By using the functions of W_M and W_{LA}, the partial effects are:

$$W_{M\rho 2} = -A^*V_{\rho 2}P'F_M > 0,$$
$$W_{L\rho 2} = -A^*\{-\alpha P + (L_1 + L_2)\alpha P'F_L\} > 0.$$

Then it is obtained:

$$\begin{pmatrix} dM \\ dL_A \end{pmatrix} = \begin{pmatrix} W_{LL} & -W_{ML} \\ -W_{LM} & W_{MM} \end{pmatrix} \begin{pmatrix} -W_{M\rho 2} \\ -W_{L\rho 2} \end{pmatrix} d\rho_2(1/\Delta),$$

$$dM/d\rho_2 = (-W_{LL}W_{M\rho 2} + W_{ML}W_{L\rho 2})/\Delta > 0,$$
$$dL_A/d\rho_2 = (W_{LM}W_{M\rho 2} - W_{MM}W_{L\rho 2})/\Delta > 0. \qquad (9)$$

Similarly, we have:

$$W_{M\rho k} = -A^*V_{\rho k}P'F_M > 0, \quad W_{L\rho k} = -A^*V_{\rho k}\,\alpha\, P'F_L > 0,$$
$$dM/d\rho_k > 0, \quad dL_A/d\rho_k > 0. \qquad (k = 3, 4)$$

This result is shown in Proposition 2.

Proposition 2.

The increase in victim or life parameter to citizen, armaments and capital stock, always raises armaments imports and military employment.

We can have a proposition related to international trade. From Proposition 1, higher ρ_1 country tends to use labor in the military sector from $dL_A/d\rho_1 > 0$ if $W_{L\rho 1} > 0$. Then employment in the two private sectors is reduced from $d(L_1 + L_2)/d\rho_1 = -dL_A/\rho_1 < 0$. Since the private sectors are constructed from two-goods and two-factors, we can apply the standard Heckscher-Ohlin theorem to the two private sectors. Then higher ρ_1 country doesn't tend to use labor in the private sector so that such countries product and export capital-intensive goods for $W_{L\rho 1} > 0$. Similarly from Proposition 2, higher ρ_j (j = 2, 3, 4) countries use military labor, L_A, more than lower ρ_j countries. Then the two private sectors in high ρ_j countries use labor more and such countries tend to export capital-intensive goods from factor endowments theorem. From Proposition 1 and 2, the proposition which is related to international trade is obtained:

Proposition 3.

If $dL_A/d\rho_1 > 0$, higher ρ_1 countries tend to export capital-intensive goods. The countries which have high victim evaluations to citizen (ρ_2), armaments (ρ_3) and capital stock (ρ_4), tend to export capital-intensive goods.

3.2. Wage (or per capita growth)

When wage rate is raised, how are personnel inputs and armaments imports affected? From equations (5) and (6), the total differentiations of dw are obtained by:

$$\begin{pmatrix} W_{MM} & W_{ML} \\ W_{LM} & W_{LL} \end{pmatrix} \begin{pmatrix} dM \\ dL_A \end{pmatrix} = \begin{pmatrix} -W_{Mw} \\ -W_{Lw} \end{pmatrix} \mathrm{dw}, \qquad (10)$$

Using the functions of W_M and W_{LA}, and the assumptions of E_{ij}, the second derivatives are respectively given by:

$$W_{Mw} = -\partial(\partial E/\partial M)/\partial w = -A^*V_wP'F_M > 0,$$
$$W_{Lw} = -1 - \partial(\partial E/\partial L_A)/\partial w = -1 - A^*P(\beta\rho'_1 - \alpha\rho'_2) - A^*V_wP'F_L, \qquad (11)$$

where V_w is an effect of wage on evaluation of victim and it is positive[9].

By using Δ, equations (10) are re-written to:

$$\begin{pmatrix} dM \\ dL_A \end{pmatrix} = \begin{pmatrix} W_{LL} & -W_{ML} \\ -W_{LM} & W_{MM} \end{pmatrix} \begin{pmatrix} -W_{Mw} \\ -W_{Lw} \end{pmatrix} \mathrm{dw}(1/\Delta). \qquad (10)'$$

Then the effects of dw on dM and dL_A are shown as:

$$(\Delta)dM/dw = -W_{LL}W_{M_w} + W_{ML}W_{Lw},$$
$$(\Delta)dL_A/dw = W_{LM}W_{M_w} - W_{MM}W_{L_w}. \qquad (12)$$

Thus, it is obtained:

If $W_{LW} > 0$, $dM/dw > 0$ and $dL_A/dw > 0$.

Intuitively, when W_{LW} is positive, both marginal welfare effects of M and L_A are increased since W_{MW} is also positive. Then, M and L_A are both raised. From (11) we find that W_{LW} tends to be positive if ρ'_j(j = 2,3,4) and V_w is larger. In this case, the marginal national defense effect of L_A is increased so that policy maker would raise the employment of the military sector. The sign of $d(M/L_A)/dw$ is undetermined[10]. If people consider the victim same, that is, ρ'_i(i = 1,2,3,4) = 0, the cost is wage only, and the

[9] $V_w = \beta\rho'_1L_A + \alpha\rho'_2(L_1 + L_2) + \beta\rho'_3Mp_M + \alpha\rho'_4(K_1 + K_2)p_K$.

[10] Detail is omitted for simplicity.

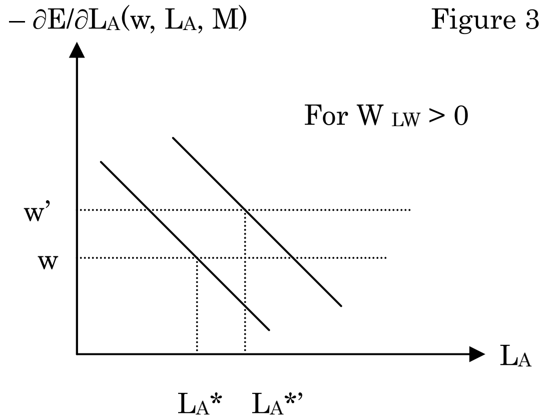
military sector reduces. Then Proposition 4 is obtained:

Proposition 4.
A rise in wage rate does not always decrease the military personnel. If $\beta\rho'_1$ is sufficiently small and $\alpha\rho'_j$(j = 2,3,4) is large, both the factor inputs into the military sector are raised and the sector is expanded. We can't exclude the case in which the armaments-labor (M/L_A) is decreased by a wage rise.

If a rise in wage is interpreted as economic growth, Proposition 4 is relevant to the problem whether the military sector is expanded or shrunk as the economy grows. For per-capita growth, the military sector is increased if we think the victim of citizen more important than that of soldiers.

Figure 3 shows the case in which the military sector is expanded. In the diagram, w' is a new high wage and L_A *' is a corresponding equilibrium.

Figure 3



3.3. The price of armaments

Are military personnel decreased or not when the price of armaments is raised? We can treat this problem similarly to section 3.2. W_{Mw} ,W_{Lw} and eq.(12) are respectively rewritten to:

$(\Delta)dM/dp_M = -W_{LL}W_{Mp_M} + W_{ML}W_{Lp_M}$,

$(\Delta)dL_A/dp_M = W_{LM}W_{Mp_M} - W_{MM}W_{Lp_M}$, (13)

where $W_{Mp_M} = -1 - A^* V_{p_M} P' F_M$ ($V_{p_M} = \partial V/\partial p_M = \beta\rho_3 M$),
and $W_{Lp_M} = -A^* V_{p_M} P' F_L > 0$.

If $\beta\rho_3 M$ is sufficiently large so that W_{Mp_M} is positive, the imports of armaments and personnel in the military sector are both increased in spite of the rise in the armaments price. It would not coincide with our intuition. The reason is that we usually think the armaments victim unimportant, that is to say ρ_3 is very small in our mind. In this case, W_{Mp_M} becomes negative and dM can be also negative. Moreover, if the marginal effect of military expenditure on the probability is sufficiently large and W_{Mp_M} is positive, dM can be positive whereas armaments price is raised. Then Proposition 5 is obtained:

Proposition 5.
The armament input is not always decreased by a rise in its price. If people think the value of the armaments more important, that is, ρ_3 is sufficiently large, it is possible that the armaments import is increased to protect itself for a rise in the price. For sufficient small ρ_3, the result can coincide with our intuition, that is, $dM/dp_M < 0$. For sufficiently small P', armament imports are reduced. Similarly labor in the military sector is not always increased for dp_M.

3.4. Factor endowments

In this section it is analyzed whether the Rybczynski theorem is valid or not. From eqs. (3), (6) and (7), and $L_1 + L_2 = \bar{L} - L_A$, we have:

$W_{L\bar{L}} = -A^* V_{\bar{L}} P' F_L = -A^* \alpha\rho_2 P' F_L > 0$,
$W_{M\bar{L}} = -A^* V_{\bar{L}} P' F_M = -A^* \alpha\rho_2 P' F_M > 0$.

Then from $\begin{pmatrix} dM \\ dL_A \end{pmatrix} = \begin{pmatrix} W_{LL} & -W_{ML} \\ -W_{LM} & W_{MM} \end{pmatrix} \begin{pmatrix} -W_{M\bar{L}} \\ -W_{L\bar{L}} \end{pmatrix} d\bar{L}$, the effects are shown by:

$dM/d\bar{L} = (-W_{LL}W_{M\bar{L}} + W_{ML}W_{L\bar{L}})/\Delta > 0$,
$dL_A/d\bar{L} = (W_{LM}W_{M\bar{L}} - W_{MM}W_{L\bar{L}})/\Delta > 0$.

Now, how about the sign of $d(L_A - \bar{L})/d\bar{L}$? It is given by:

$$\Delta d(L_A - \bar{L})/d\bar{L} = (W_{LM}W_{M\bar{L}} - W_{MM}W_{L\bar{L}}) - \Delta$$
$$= (W_{LM}W_{M\bar{L}} - W_{MM}W_{L\bar{L}}) - (W_{LL}W_{MM} - W_{ML}W_{LM}).$$

Since the sign of $d(\bar{L} - L_A)/d\bar{L}$ is ambiguous, it is found that the Rybczynski theorem is not always valid. If the increase in the marginal welfare effect of M or L_A by another factor M, L_A and $\bar{L}$ become large, that is, W_{LM}, $W_{M\bar{L}}$, $W_{L\bar{L}}$ and W_{ML} are large, then $d(L_A - \bar{L})/d\bar{L}$ is also large. In this case, the marginal effects of army's input, M and L_A, increase so that labor is more employed in the military sector.

For dK, similarly we can obtain:

$$W_{L\bar{K}} = -A^*V_KP'F_L = -A^*\beta\rho_3p_MP'F_L > 0,$$
$$W_{M\bar{K}} = -A^*V_KP'F_M = -A^*\beta\rho_3p_MP'F_M > 0,$$
$$dL_A/d\bar{K} = (W_{LM}W_{M\bar{K}} - W_{MM}W_{L\bar{K}})/\Delta > 0,$$
$$dM/d\bar{K} = (-W_{LL}W_{M\bar{K}} + W_{ML}W_{L\bar{K}})/\Delta > 0.$$

From the above equations, we find that an increase in factor endowment, $\bar{K}$, expands the military sector. Thus, it is obtained:

$$d(\bar{L} - L_A)/d\bar{K} = dL_A/d\bar{K} < 0 .$$

It means that capital–labor ratio in the private sector, $(\bar{L} - L_A)/\bar{K}$, is reduced for dK. We can say that relative capital abundant country is also abundant in the tradable sectors. By the increase in capital stock, capital-intensive goods tend to be exportable. We find that the Rybczynski theorem is valid for dK.

Then Proposition 6 is obtained:

Proposition 6.

For an increase in capital stock, the trade pattern is consistent to the Rybczynski theorem and Heckscher-Ohlin Theorem. However, for an increase in labor endowments, it is not always valid since the increase raises the marginal welfare effect of labor in the military sector so that more persons are employed in the military sector and less persons in the tradable sectors.

The asymmetry of the results comes from the asymmetric production functions between private sector and military sector. The former uses capital while the latter not capital. Thus, the persuasion of this Proposition depends on the degree of the use of capital in the military sector[11].

Concluding Remarks

This paper theoretically shows that expected damage affects the optimal defence expenditure, the weapon-soldier ratio, the structure of production and international trade. Although the followings are easy analyses, there are some problems.

First, who feels damage? It may be related to the political system. When politics are autocratic and a governing class is not worried about being conscripted, the ruling class may not think that the damage or the loss of soldiers is serious. In the case, the weapon-soldier ratio would remain low whereas wage–weapon price ratio is raised. If the armed forces are volunteers and they are from the specific class in the country, the same would take place. On the other hand, if people are equally conscripted to join the armed forces, the Government's attitude would change. Thus, the conscription affects the weapon-personnel ratio and industrial structure.

Second is related to the problem of imperfect information. In our model, defence service is determined by 'perceived' probability and volume of being attacked. Even in the recent Iraq War, it is said that correct information of Iraqi armament was not obtained before the war. We determine defence expenditure and expect attack of foreign country under vague information and popular belief. Moreover, to what extent can we know the true victim? For example, under news black, we can't know the real of war and we are not sensitive about the victim of war. After visiting the Hiroshima Peace Memorial Museum, the attitude may change.

Third, this paper doesn't consider the strategy of attack. Using our model, we can conjecture that the home country will attack the other country if the attack reduces the damage risk, if the expected damage by foreign military is small and if a revenge attack is not anticipated. However, even in this case, the country may hesitate to attack if enemy's casualties are included in the damage evaluation. The determinant and the content of attack depend also on the attitude of government or people to the damage of an attacked country. According to the attitude, attack policy could be minimum of 'only domestic damage' or 'domestic and foreign damage'. When the foreign damage is

[11] See footnote 2.

included in the evaluation, the scope of attack may be only military base and/or the attack points are carefully determined. When a minimum-policy of domestic and foreign damage is adopted, the domestic damage itself can't be a minimum. But, because of the reduction of foreign victim, there is the case in which the domestic victim could be minimized in the long run since the revenge is decreased. The feeling of revenge is important since total friction between attack country and attacked country becomes small as the feeling of revenge is weaker. When we think that total cost which include not only victim of attack but also the friction cost after occupation or attack, a policy of minimization foreign victim is not so bad.

This paper is relevant to the question when people cease war and when disarmament realizes. The history of the human is also that of war. We may think that economic growth lets people stop the war and lead them to disarmament since the value of human life rises and the victims of war are also increased. But according to our analysis, it is not always true. Economic growth increases the valuation of national defence and it causes expansion of the military expenditure because of protecting the rich economy. Moreover if the relative victim evaluation of soldiers is low in a country, the country would want to attack since the expected damage of soldiers is small.

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