The Dynamic Heckscher-Ohlin Model:
A Diagrammatic Analysis

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Plan of my talk

- Settings of dynamic H-O models
- Motivation of our studies
- Results in H-O models including our recent studies
- Purpose of this paper
- Findings
- Settings of our model
- Diagrammatic explanations for results in literature and our models
- Conclusions

Dynamic versions of the H-O model

- Each country has access to the same technology for producing two goods using a fixed factor (labor) and a reproducible factor (capital) under conditions of perfect competition and constant returns to scale.

- Factors of production are assumed to be mobile between sectors within a country, but immobile internationally, and there are no markets for international borrowing and lending.

- Countries (Home and Foreign) have identical preferences with homotheticity and a constant intertemporal elasticity of substitution (CIES).

$$\dot{E}/E = \dot{E}^*/E^*$$

Motivation of our previous studies

- Houthakker and Taylor (1970) find that the share of income spent on food declines and the share spent on services rises with household income.

- Hunter (1991) estimated a linear expenditure system for 11 product categories across 34 countries, and found that departures from homotheticity have a significant impact on trade volumes.

- Ogaki and Atkeson (1997) find evidence of differences in the IES with the level of household wealth.

- Jensen and Miller (2008) provide evidence that two staple commodities, rice and wheat, are Giffen goods in China.

Motivation of our previous studies (cont.)

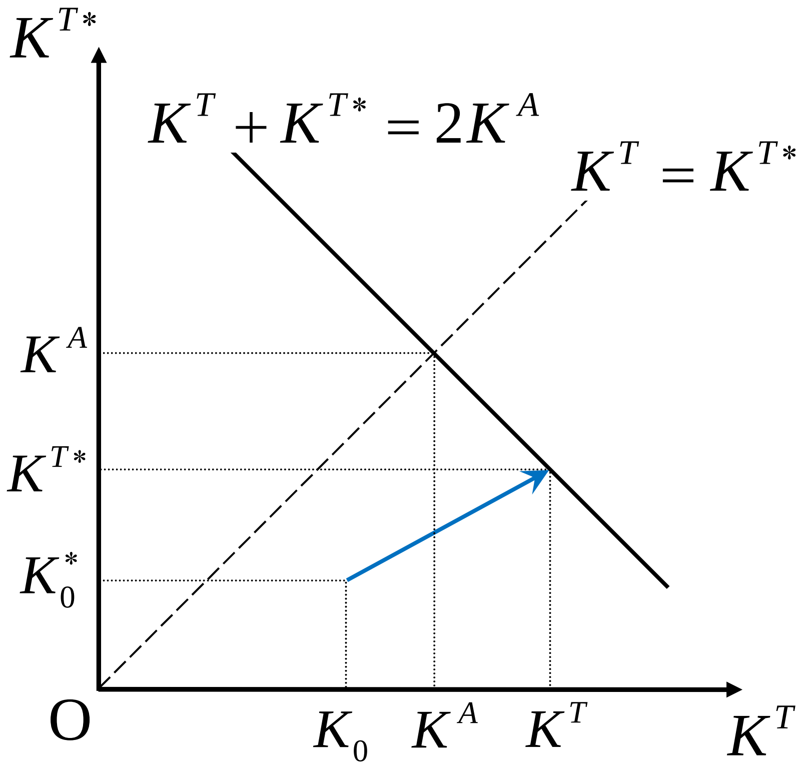
How do departures from the assumption of homothetic preferences and CIES affect the pattern of trade and the dynamics around the steady states of dynamic H-O models?

Results in H-O models with homothetic preferences

(i) There is a continuum of steady state equilibria under free trade, each of which is a saddle point characterized by incomplete specialization in production, and $K^T + K^{T*}$ is constant.

(ii) The static H-O theorem holds: the country that is capital abundant in the steady state exports the capital intensive good.

(iii) The initially capital abundant country remains capital abundant, and will export the capital intensive good on the path to the steady state. (The dynamic H-O theorem holds: The future trade patterns are determined by the initial relative factor endowments.)

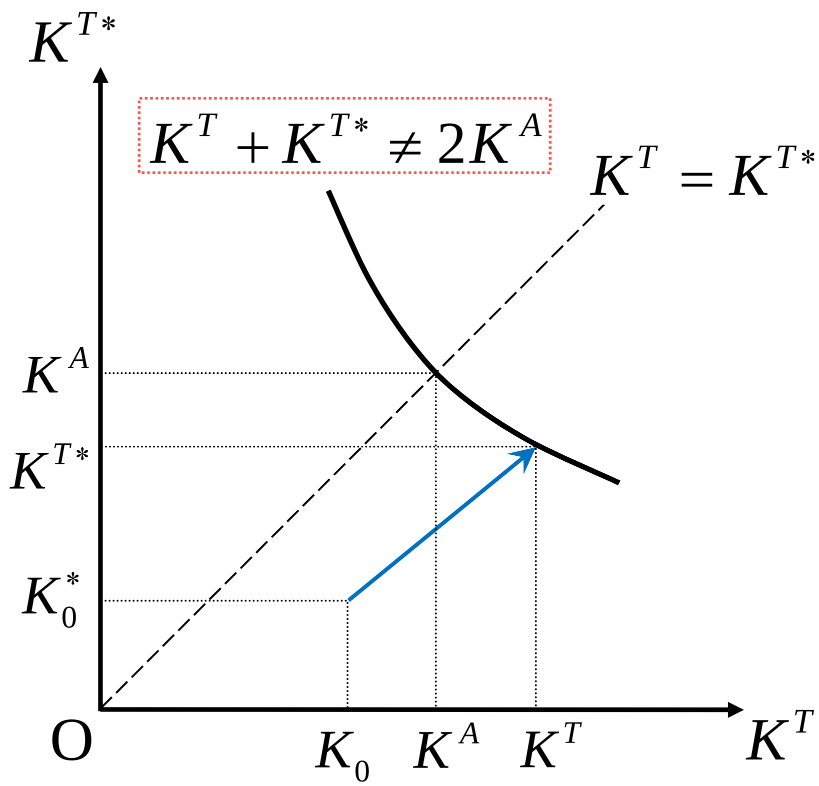


cf. Atkeson and Kehoe (2000) discuss the possibility that countries that start the development process later end up with a lower capital labor ratio and level of income than countries that develop earlier.

Results in our previous studies with normal goods

If labor productivities and discount factors are the same across countries, then the main results of the benchmark H-O model will hold as long as goods are normal in consumption.

The primary difference introduced in this case is that the world capital stock in the steady state will depend on the distribution of income across countries.



Results in our previous studies with normal goods (cont.)

The case of $L > L^* = \mu L$ (μ: labor productivity in Foreign)

In autarky, Home is the capital abundant (scarce) country

in the steady state: $\dfrac{K^A}{L} > \dfrac{K^{A*}}{L^*} \quad \left(\dfrac{K^A}{L} < \dfrac{K^{A*}}{L^*} \right),$

iff labor-intensive good 1 is a necessity (luxury).

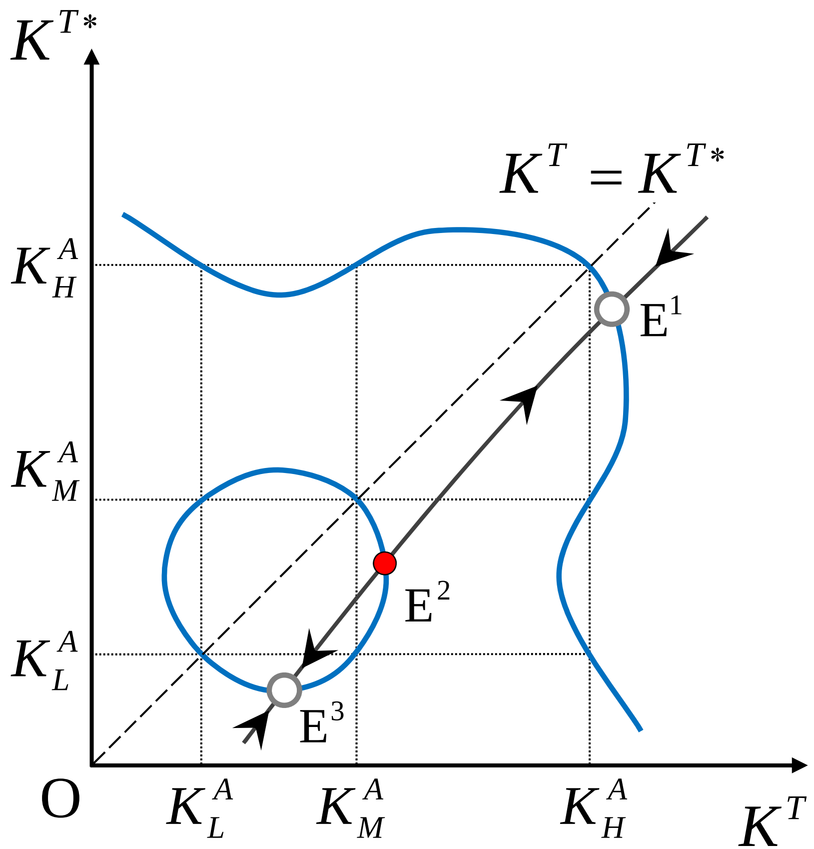
This can yield the violation of the static H-O theorem.

The dynamic H-O theorem will hold if preferences are

homothetic with CIES. $\dfrac{K_0}{L} > \dfrac{K_0^*}{L^*} \Rightarrow \dfrac{K^T}{L} > \dfrac{K^{T*}}{L^*}$

Results in our previous studies with an inferior good

In the case where the labor intensive good is inferior, there may be multiple steady states in autarky and free-trade steady states where the static H-O theorem is violated and/or the saddle-point stability does not hold.



Purpose of this paper

To show that main results in dynamic H-O models (with non-homothetic preferences) can be derived from diagrams that represent the basic functions in static models such as

(i) the Rybczynski line, ········ Production side

(ii) an income expansion path, ········ Demand side

(iii) an excess demand function.

Findings

We can define their steady state versions and show that

(i) the “steady state” Rybczynski line and (ii) the income expansion path evaluated at the “steady state prices” yield (iii) the “steady state” excess demand function that specifies the country's excess demand as a function of its capital stock.

Using the excess demand functions for each of the countries, we can derive the locus of home and foreign capital stocks that are consistent with a steady state equilibrium with free trade.

Also, we can see the stability of steady states and the steady state trade pattern only from their shapes.

The production side

Let

$L\ (L^*)$: labor endowment in Home (Foreign),

$K\ (K^*)$: captital stock in Home (Foreign),

$a_i(w,r)$: unit cost function in sector i.

Under incomplete specialization,

the competitive profit conditions require that

$$a_1(w,r) = p,$$

$$a_2(w,r) = 1,$$

and we obtain factor prices, $w(p)$ and $r(p)$.

The Stolper-Samuelson theorem

Totally differentiating the competitive profit conditions yield

$$a_{1w}(w,r)dw + a_{1r}(w,r)dr = dp,$$

$$a_{2w}(w,r)dw + a_{2r}(w,r)dr = 0,$$

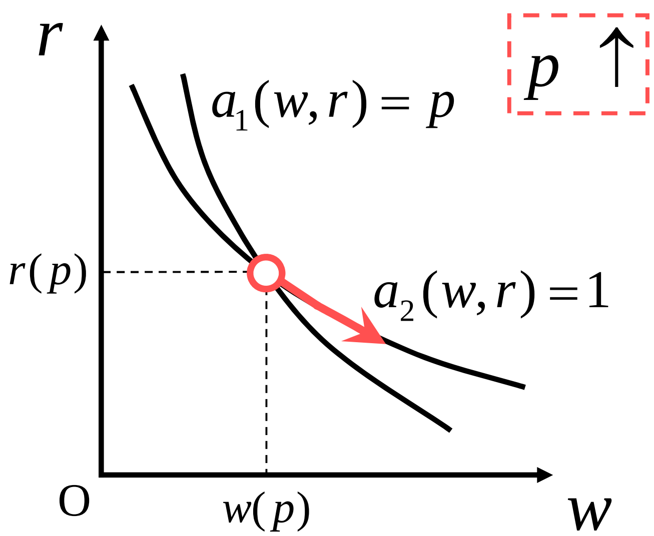
where a_{iw} and a_{ir} are equal to labor and capital input coefficients in sector i, respectively, and we assume they are all positive.

Let good 1 be labor intensive:

$$\frac{a_{1r}}{a_{1w}} < \frac{a_{2r}}{a_{2w}} \quad \Leftrightarrow \quad \Delta \equiv a_{1w}a_{2r} - a_{2w}a_{1r} > 0.$$

Then, $\dfrac{pw'(p)}{w(p)} = \dfrac{a_{2r}}{\Delta}\left(a_{1w} + \dfrac{r}{w}a_{1r}\right) > 1$

and $r'(p) = -\dfrac{a_{2w}}{\Delta} < 0.$

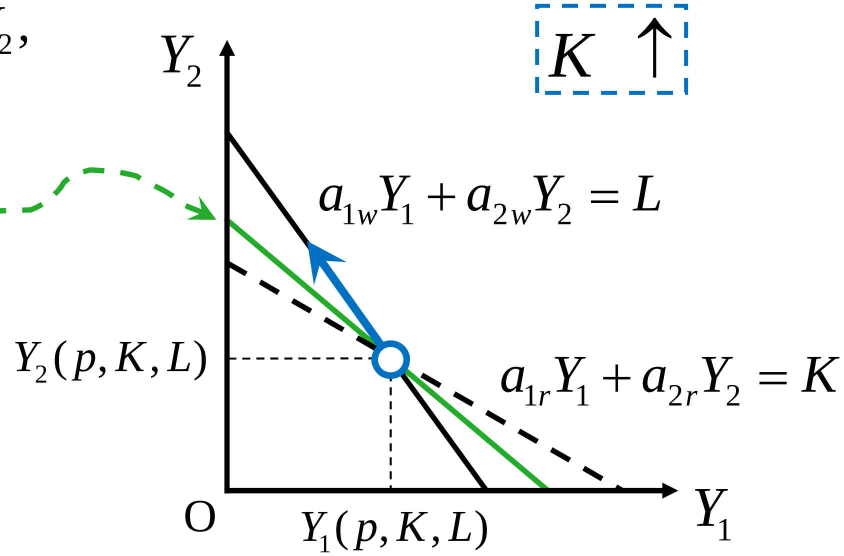


The Rybczynski theorem

Factor market equilibrium requires that

$$L = a_{1w}Y_1 + a_{2w}Y_2 \quad \text{and} \quad K = a_{1r}Y_1 + a_{2r}Y_2,$$

$$\left(\Rightarrow w(p)L + r(p)K = pY_1 + Y_2 \right)$$



which yield

$$Y_1(p,K,L) = \frac{a_{2r}L - a_{2w}K}{\Delta}$$

$$= w'(p)L + r'(p)K,$$

$$Y_2(p,K,L) = \frac{-a_{1r}L + a_{1w}K}{\Delta}$$

$$= w(p)L + r(p)K - p[w'(p)L + r'(p)K].$$

Then, $\dfrac{\partial Y_1}{\partial K} = r' < 0$ and $\dfrac{\partial Y_2}{\partial K} \cdot \dfrac{K}{Y_2} = \dfrac{(r - pr')K}{(w - pw')L + (r - pr')K} > 1.$

The consumption side

$$\max \int_0^\infty u(C_1, C_2)\exp(-\rho t)dt,$$

subject to

$$w(p)L + r(p)K = pC_1 + C_2 + \dot{K} + \delta K, \quad K_0 \text{ given.}$$

$$\left(\Leftrightarrow \; pY_1 + Y_2 = pC_1 + C_2 + \dot{K} + \delta K \right)$$

ρ : the discount rate

δ : the rate of depreciation on capital

λ : the shadow value of capital

Let $\mathcal{H} = u(C_1, C_2) + \lambda[w(p)L + r(p)K - pC_1 - C_2 - \delta K]$.

The necessary conditions for optimality are

$$u_1(C_1, C_2) = \lambda p, \quad u_2(C_1, C_2) = \lambda, \quad \boxed{\dot{\lambda} = \lambda[\rho + \delta - r(p)],} \text{ and } \lim_{t\to\infty} K(t)\lambda(t)\exp(-\rho t) = 0.$$

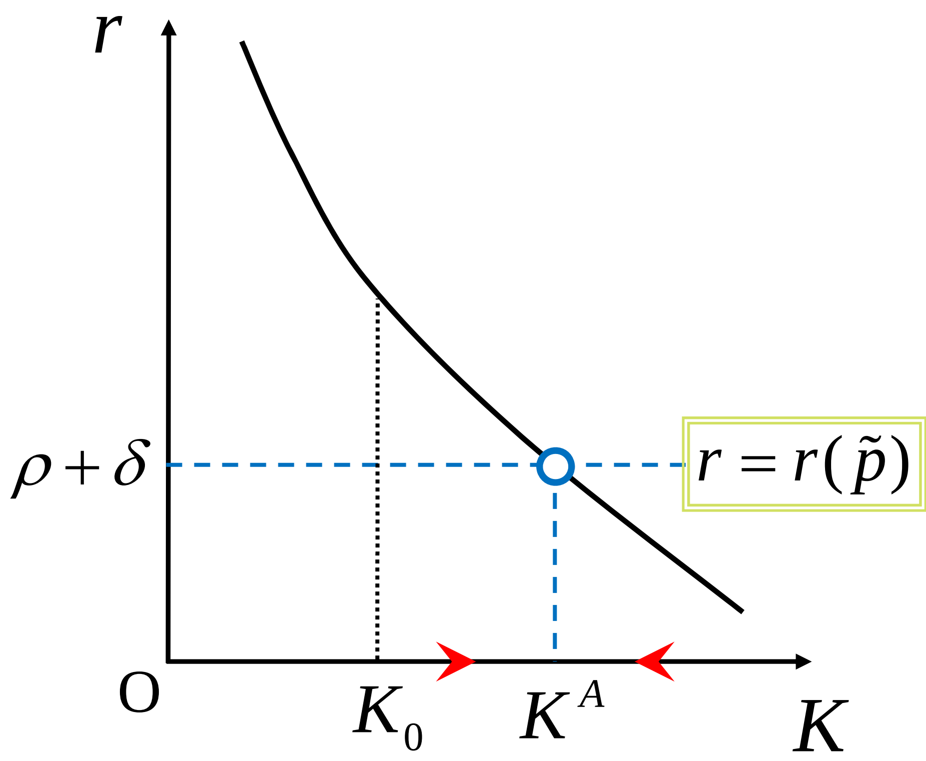
the marginal utility of the consumable capital = the shadow value of capital

Monotonic relation between K and r

Generally, there is a negative relation between capital accumulation and the rental on capital: the rental rate decreases when capital stock increases.

This yields the uniqueness of an autarkic steady state and its saddle-point stability.

$$K\uparrow \;\Rightarrow\; \begin{cases} Y_1\downarrow \\ C_1\uparrow \end{cases} \;\Rightarrow\; p\uparrow \;\Rightarrow\; r(p)\downarrow$$



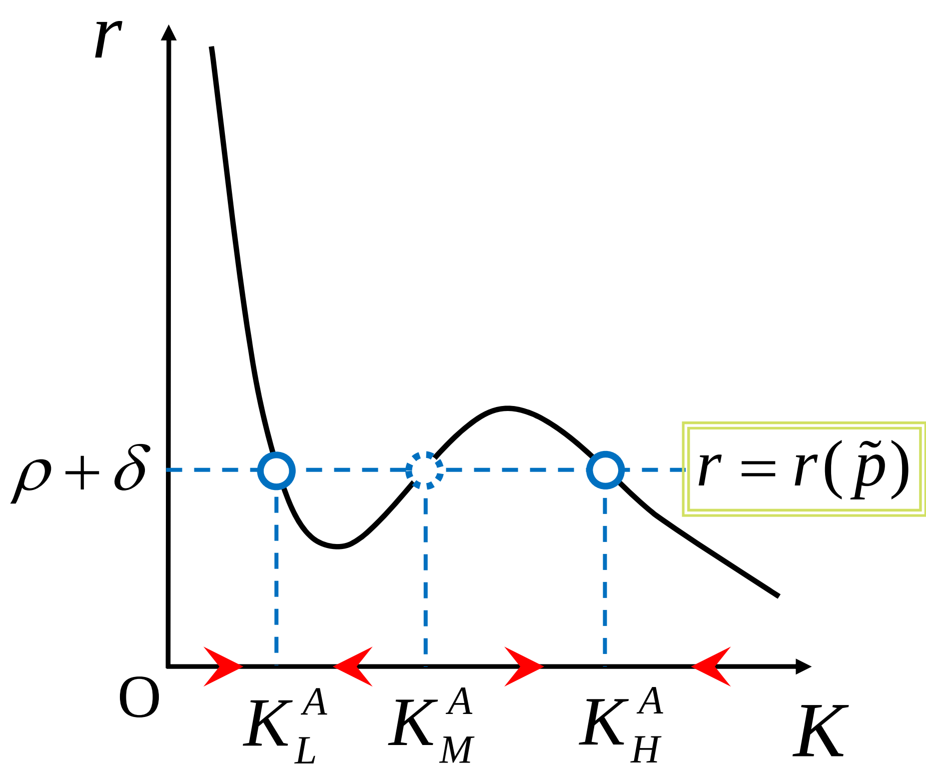
Non-monotonic relation between K and r

Suppose that labor-intensive good 1 is inferior.

Then, the more capital countries accumulate, the less labor-intensive good is demanded, and hence the more capital is needed for producing goods.

So, there is a possibility of a non-monotonic relation between the capital stock and the return on capital, and the model will exhibit rich dynamic properties.

$$K\uparrow \;\Rightarrow\; \begin{cases} Y_1 \downarrow \\ C_1 \downarrow\downarrow \end{cases} \;\Rightarrow\; p\downarrow \;\Rightarrow\; r(p)\uparrow$$



The steady state Rybczynski line

At any steady state in Autarky,

$Y_1 = r'(\tilde{p})K + w'(\tilde{p})L = C_1,$

$Y_2 = C_2 + \delta K$ hold.

So,

$\tilde{a}_{1w}Y_1 + \tilde{a}_{2w}Y_2 = L,$

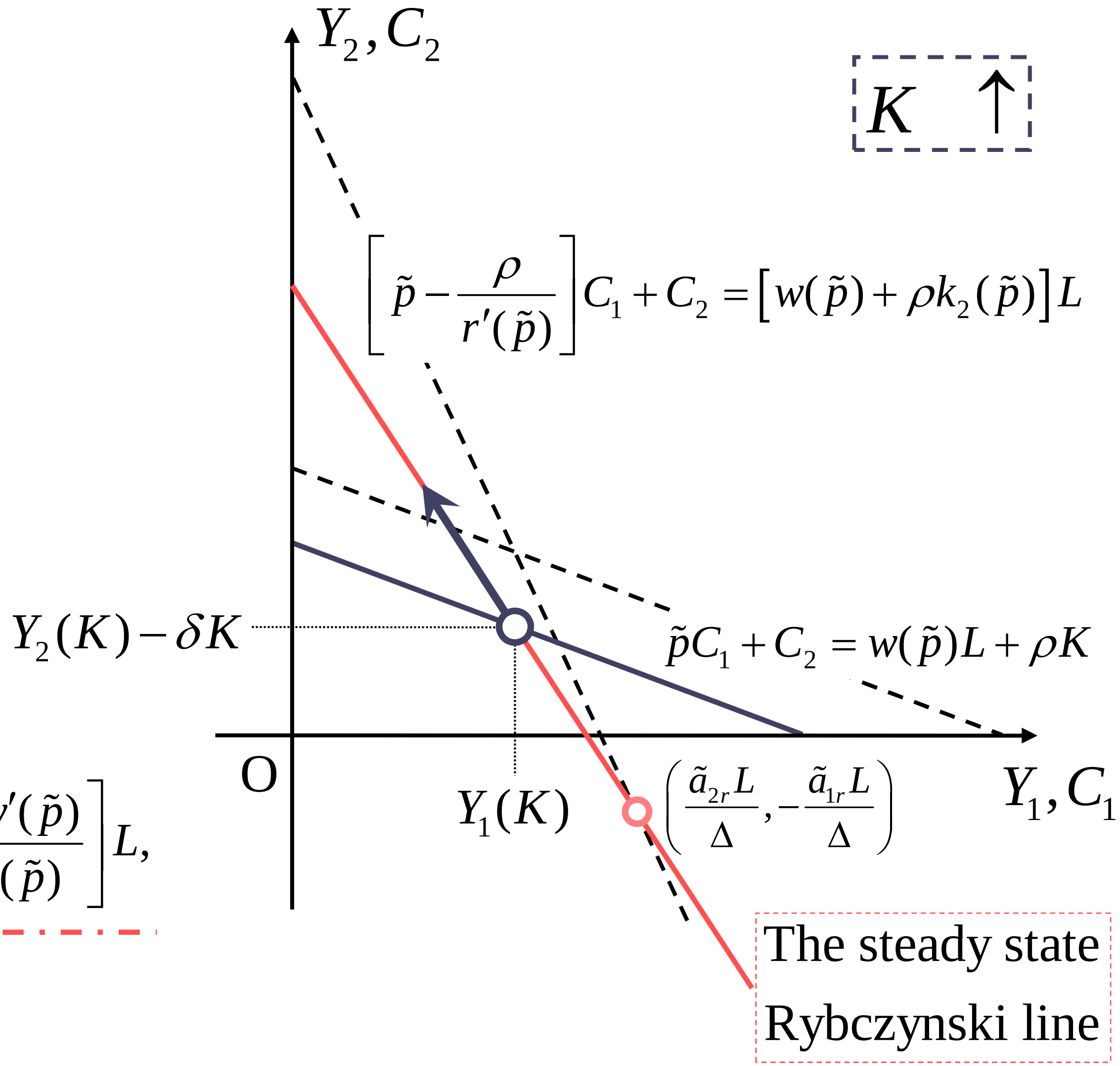
$\tilde{p}Y_1 + Y_2 = w(\tilde{p})L + r(\tilde{p})K$

$\left(\tilde{a}_{iw} \equiv a_{iw}(w(\tilde{p}), r(\tilde{p})),\ i = 1, 2\right)$

yield

$$\left[\frac{\tilde{a}_{1w}}{\tilde{a}_{2w}} + \frac{\delta}{r'(\tilde{p})}\right]C_1 + C_2 = \left[\frac{1}{\tilde{a}_{2w}} + \frac{\delta w'(\tilde{p})}{r'(\tilde{p})}\right]L,$$

$$\tilde{p}C_1 + C_2 = w(\tilde{p})L + \left[r(\tilde{p}) - \delta\right]K.$$



Lemma 1

Let K be the steady state capital stock.

Then, the outputs of two goods at the steady state,

$(Y_1(K), Y_2(K)),$ are derived from the intersection

between the steady state Rybczynski line,

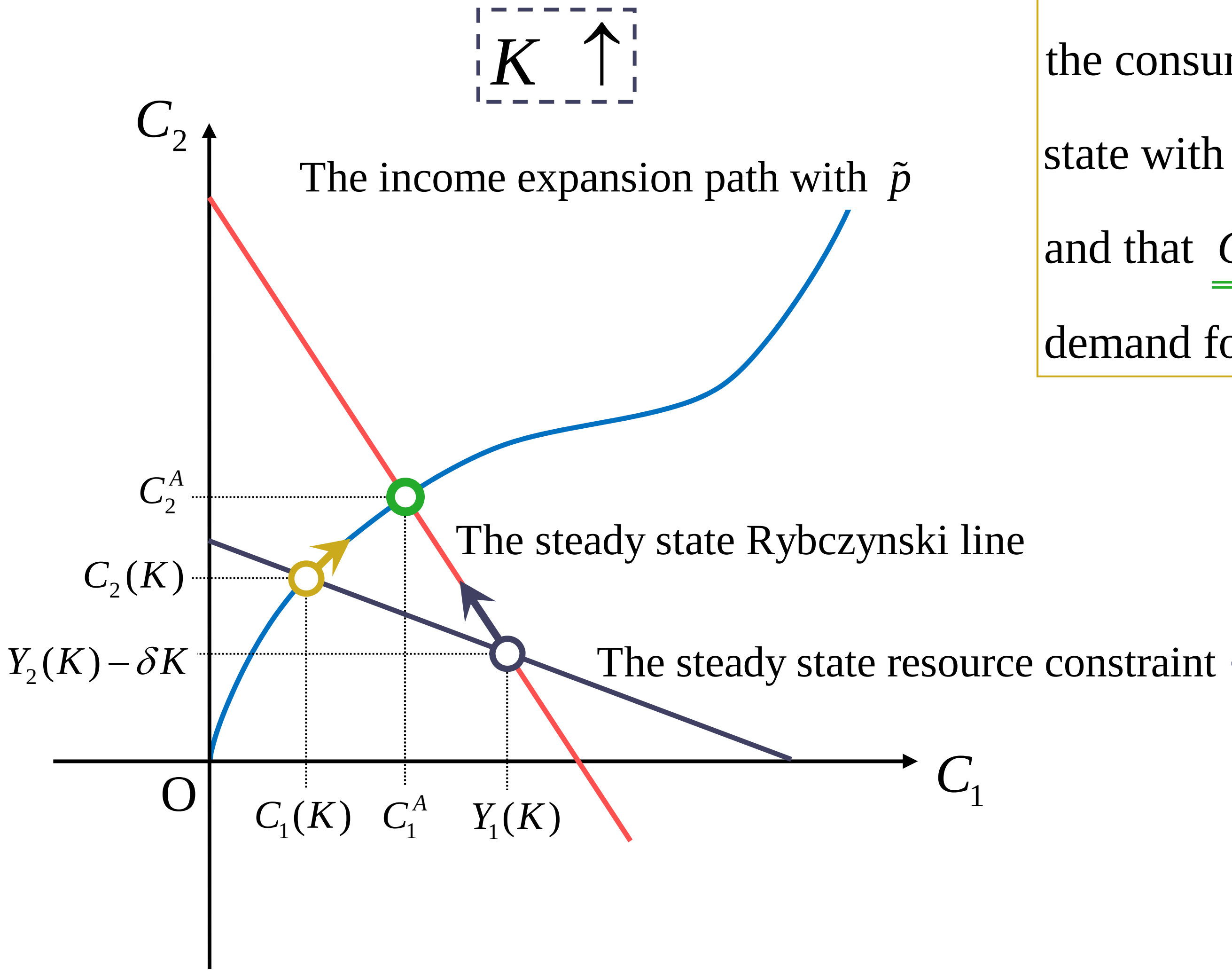
$$\left[\tilde{p} - \frac{\rho}{r'(\tilde{p})}\right] C_1 + C_2 = \left[w(\tilde{p}) + \rho k_2(\tilde{p})\right] L,$$

and the steady state resource constraint,

$$\tilde{p} C_1 + C_2 = w(\tilde{p}) L + \rho K,$$

as $(Y_1(K), Y_2(K)) = (C_1, C_2 + \delta K).$

The autarkic steady state



Notice that $\left(C_1(K), C_2(K)\right)$ denotes the consumption bundles at a steady state with capital stock K, and that $C_1(K) - Y_1(K)$ is an excess demand for good 1.

The budget constraint for households with capital stock K and investment δK

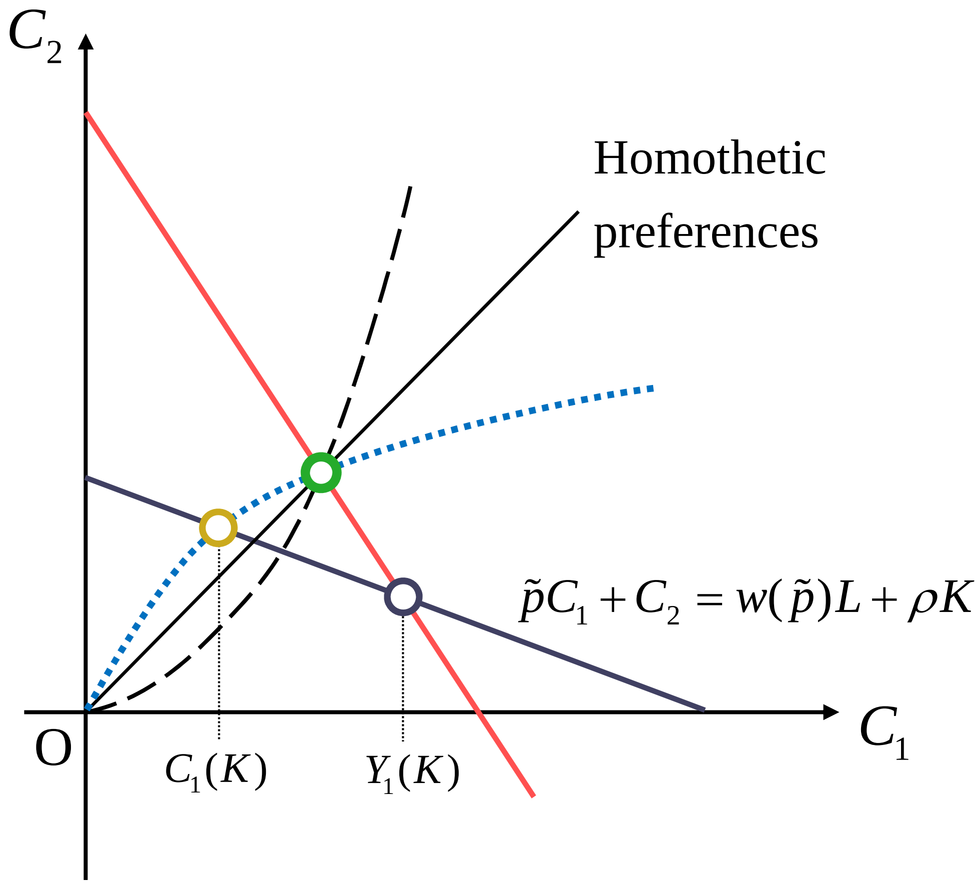
Proposition 1

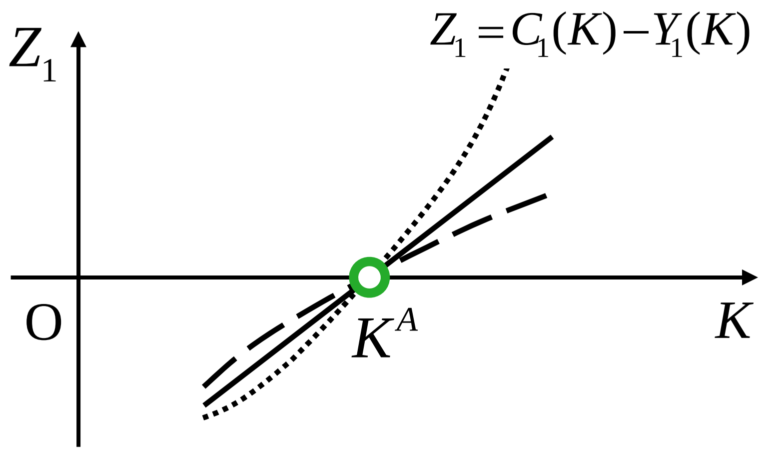
An intersection between the steady state Rybczynski line and the income expansion path with the steady state price of good 1 corresponds to an autarkic steady state:

$$K^A = \frac{C_1^A - w'(\tilde{p})L}{r'(\tilde{p})} \quad \text{and} \quad \lambda^A = u_2(C_1^A, C_2^A).$$

Therefore, it uniquely exists as long as labor intensive good 1 is normal and preferences exhibit neither a satiation level nor a minimum subsistence level.

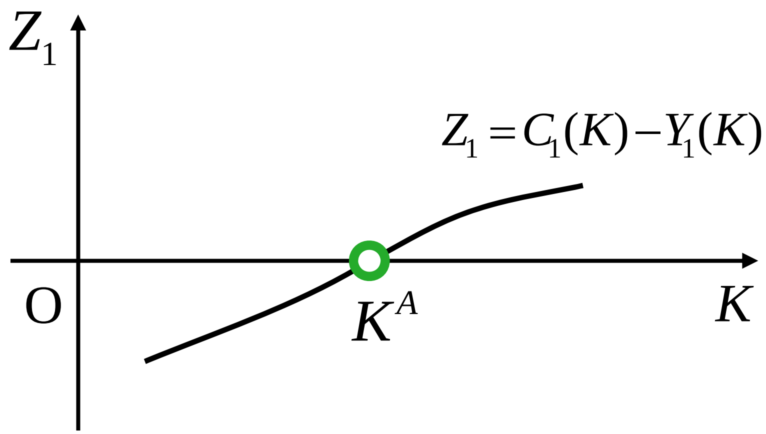
The excess demand for good 1





Lemma 2

With normality in consumption, the steady state in autarky is a saddle point.

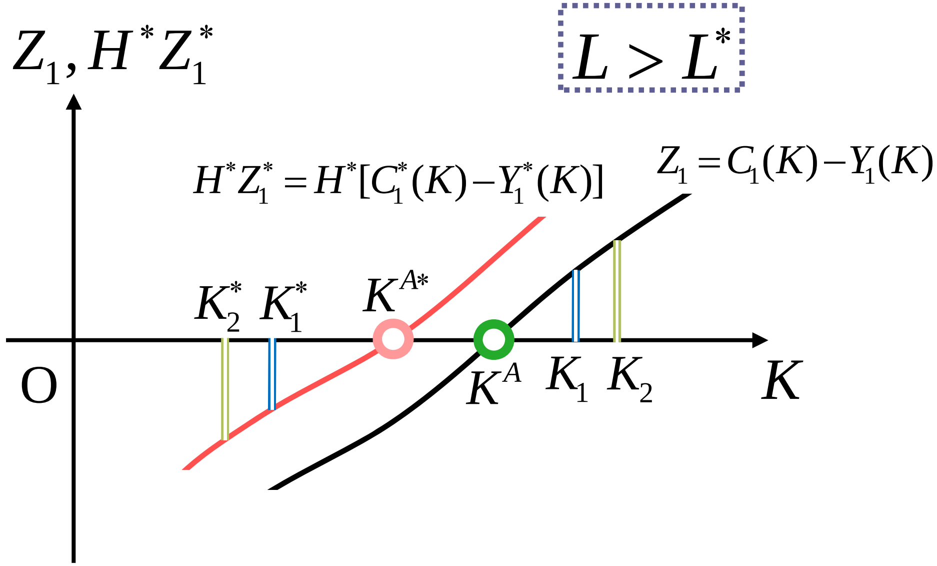


Intuition: Lemma 2 can be interpreted as indicating that if an increase in the capital stock above the autarkic steady state creates an excess demand for the labor intensive good, then there will be an increase in the price and a decrease in the rental on capital in the economy.

The foreign country

Assumption:

The home and foreign countries have identical utility functions, u, identical technologies, $a_i, i = 1, 2$, $\rho = \rho^*$, and $\delta = \delta^*$.



A steady state equilibrium with trade is a pair (K, K^*) such that

$Z_1(K) + H^* Z_1^*(K^*) = 0$ with $K \in \left[k_1(\tilde{p})L, k_2(\tilde{p})L\right]$ and $K^* \in \left[k_1(\tilde{p})L^*, k_2(\tilde{p})L^*\right]$,

where H^* is the number of households in the foreign country and $Z_1^*(.) = Z_1(.)$ iff $L^* = L$.

Lemma 3

With normality in consumption, all the free trade steady states are saddle points.

(The intuition for this result is the same as for Lemma 2.)

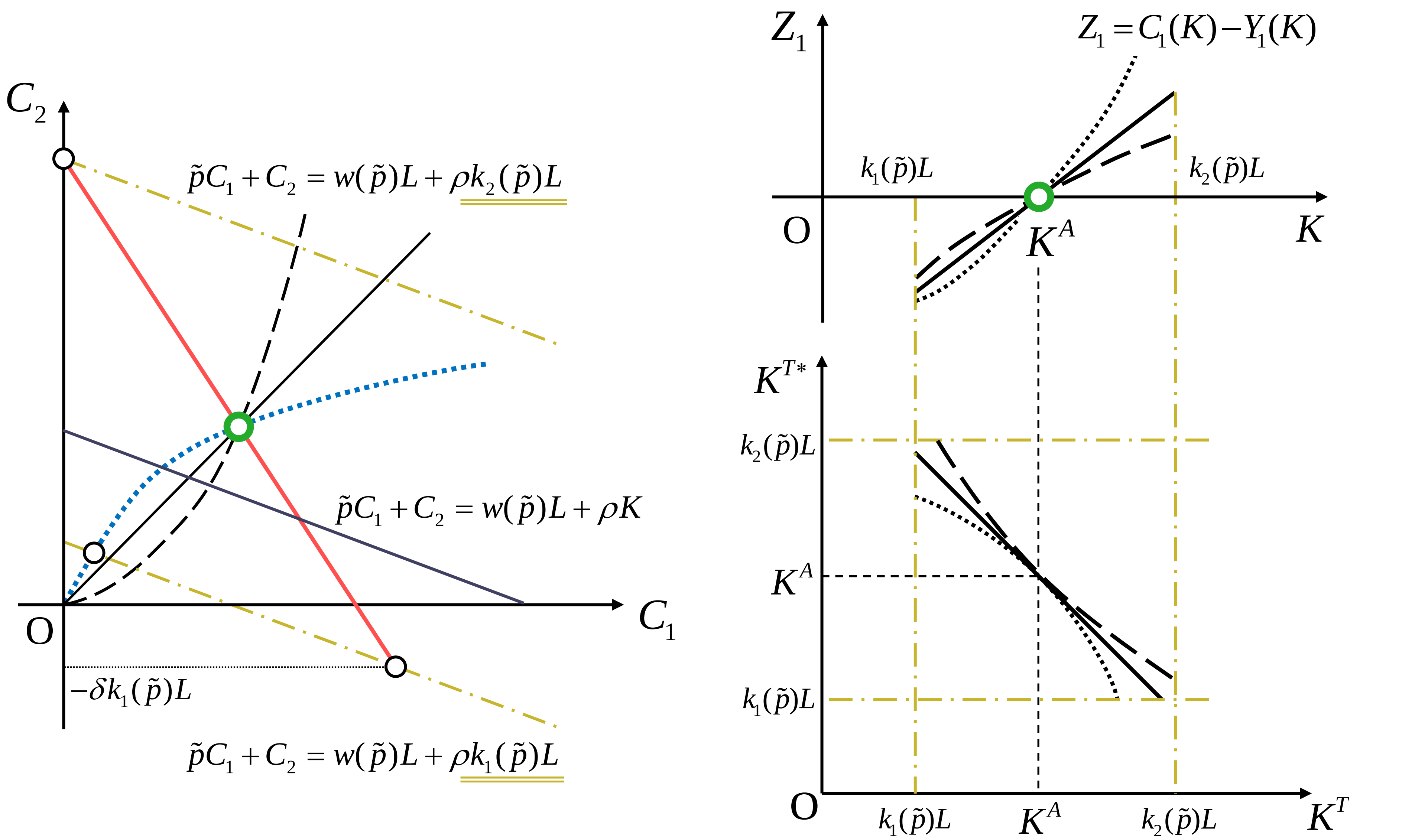
Steady state values of λ and λ^*

Letting (K^T, K^{T*}) be one of the steady state free trade pairs, the values of λ and λ^* at the steady state are given by

$$\lambda^T = u_2(C_1(K^T), C_2(K^T))$$

and $\lambda^{T*} = u_2(C_1^*(K^{T*}), C_2^*(K^{T*}))$.

The locus of (K^T, K^{T*}) with $H^*=1$ and $L=L^*$



Remark 1

For given technologies, preferences, and a labor endowment in each country, we can draw, for each country,

(i) the steady state Rybczynski line,

(ii) the income expansion path with $\tilde{p}$,

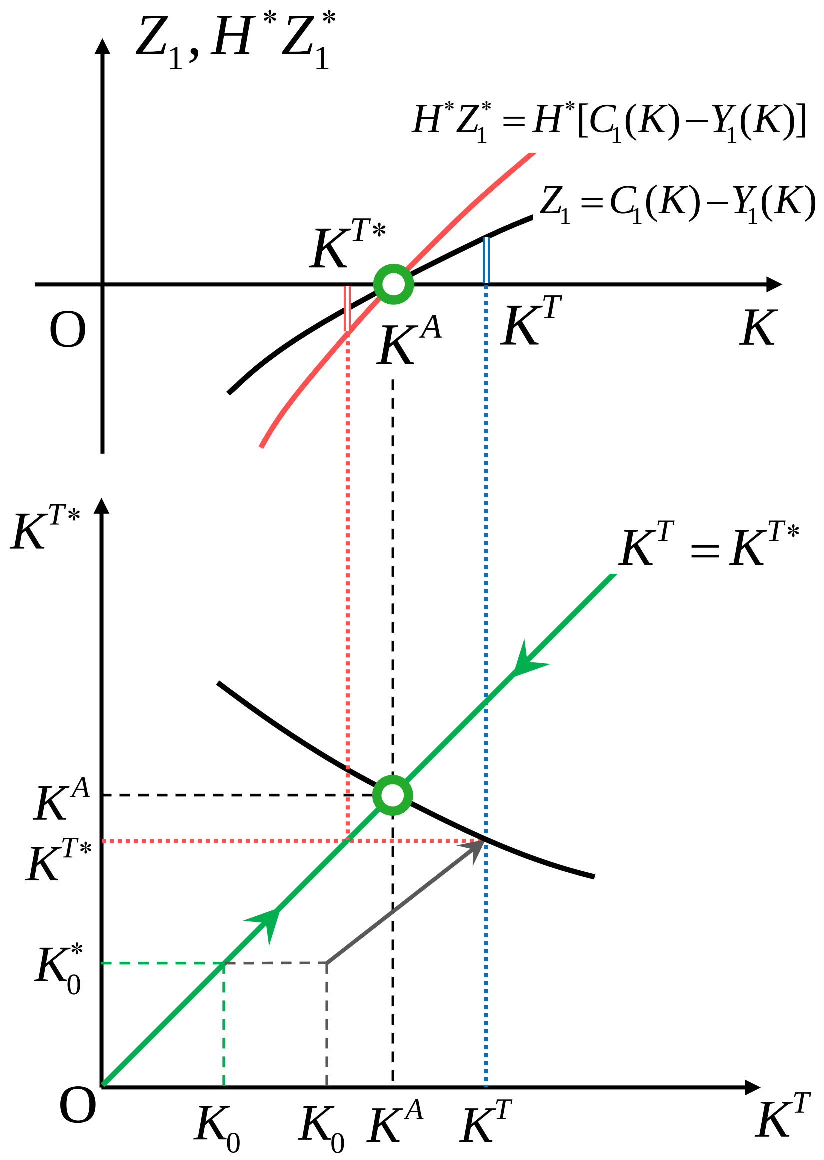
and (iii) the steady state resource constraints at the highest and the lowest capital stocks consistent with incomplete specialization.

They yield the steady state excess demand function for each country, from which we can precisely derive the locus of (K^T, K^{T*}).

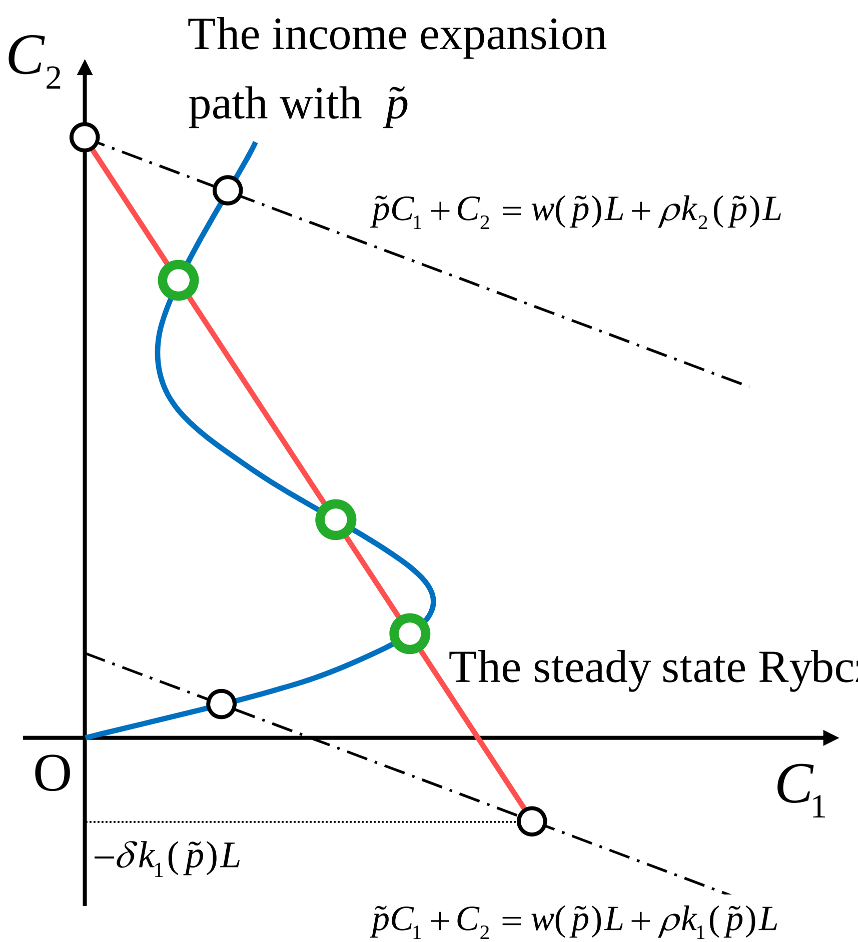
Proposition 2 (Heckscher-Ohlin theorem)

Let goods be normal and $L = L^*$.

Then, the initially capital abundant country remains capital abundant along the dynamic general equilibrium path to the steady state, and the capital abundant country exports the capital intensive good at the steady state.



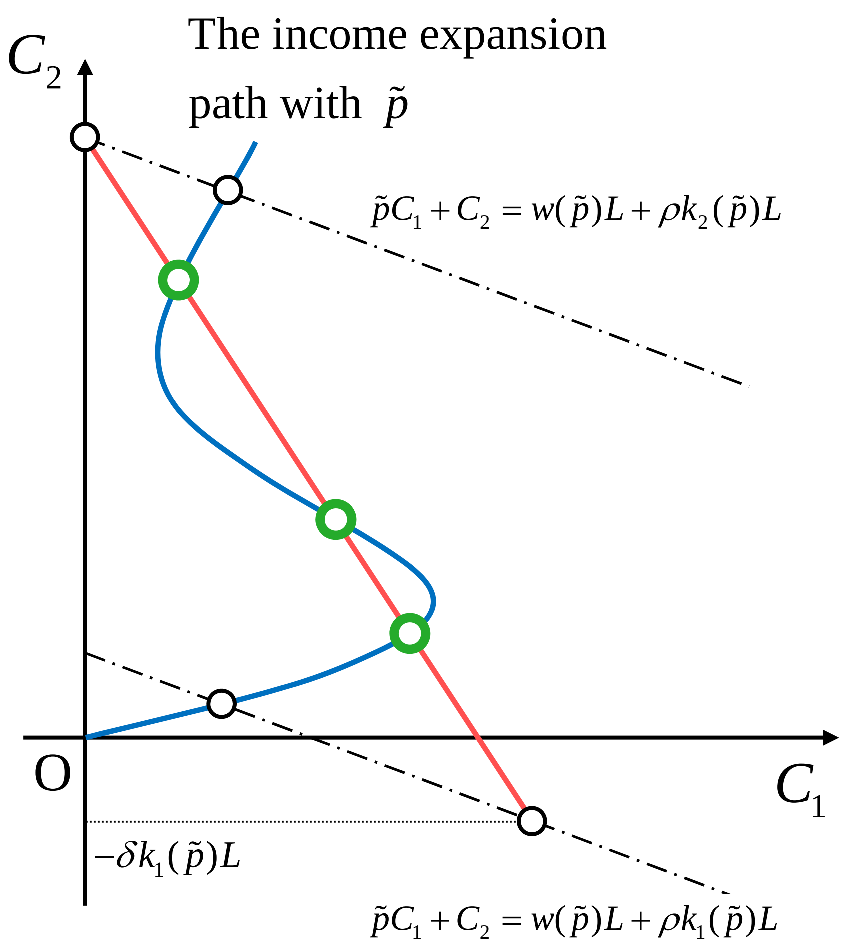
An example with inferior goods

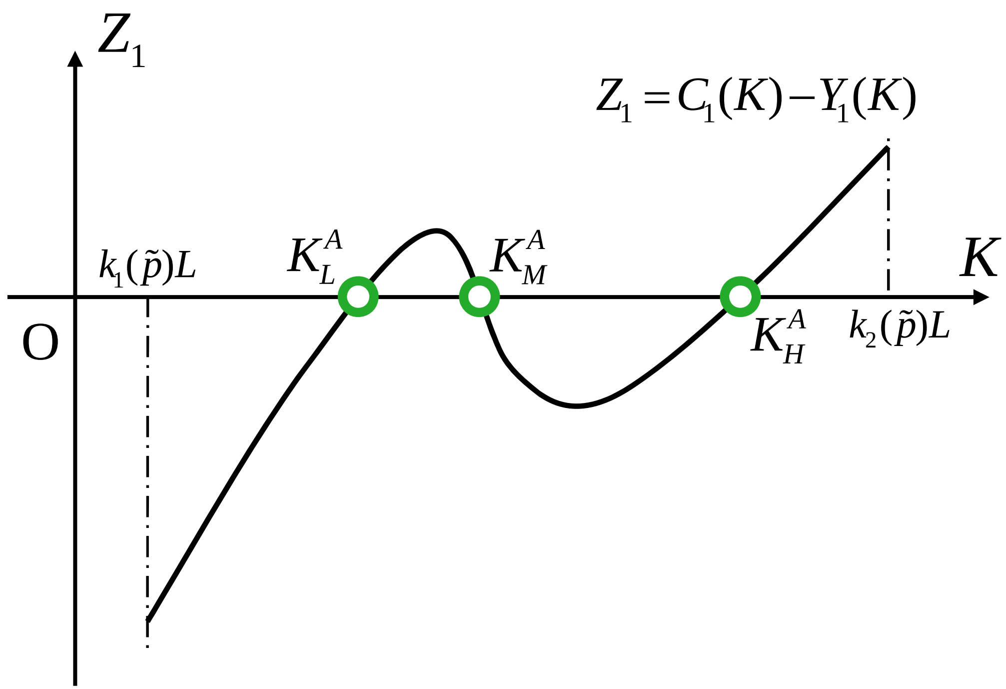


A multiplicity of steady states in autarky is possible when labor intensive good 1 is inferior at some range of income and $\tilde{a}_{2w}$, the labor input coefficient in capital intensive sector 2, is sufficiently small.

The steady state Rybczynski line: $\left[\frac{\tilde{a}_{1w}}{\tilde{a}_{2w}} + \frac{\delta}{r'(\tilde{p})}\right] C_1 + C_2 = \left[\frac{1}{\tilde{a}_{2w}} + \frac{\delta w'(\tilde{p})}{r'(\tilde{p})}\right] L$

The non-monotonic excess demand



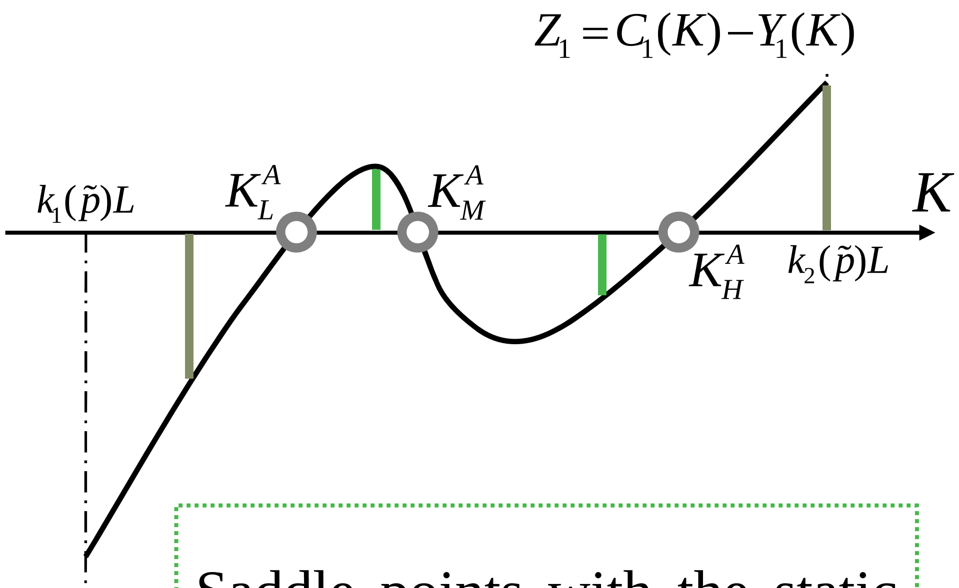


Proposition 3 (Stability condition)

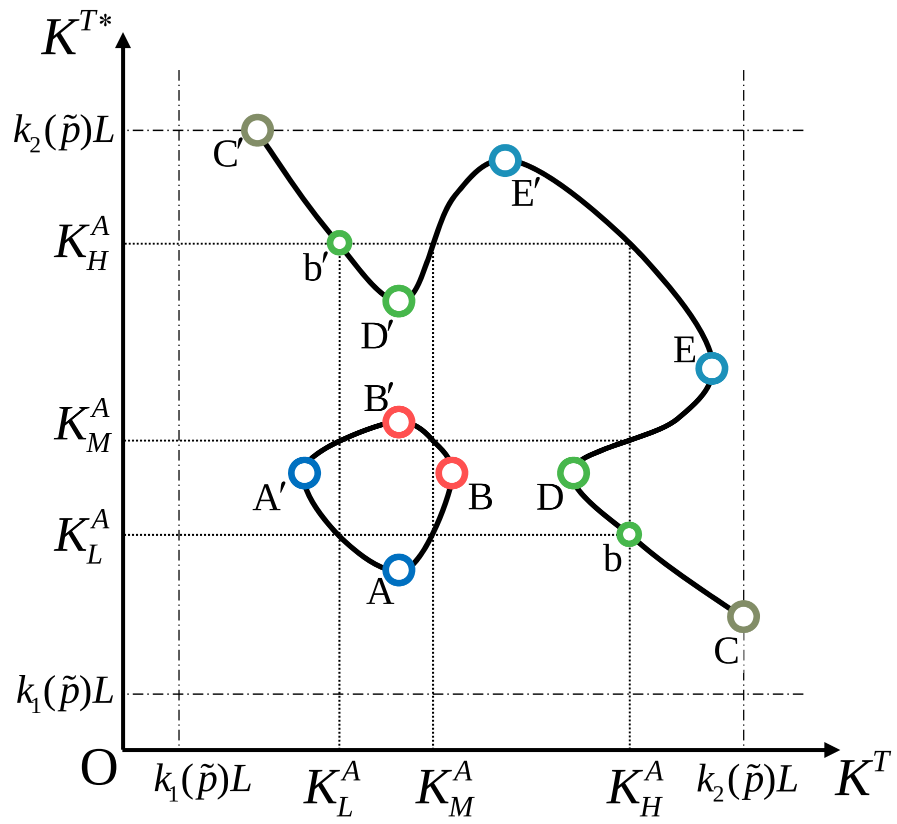
If the steady state demand function in each country is upward sloping at the value of capital stock in a free trade steady state, then the steady state is a saddle point.

If it is downward and the discount factor in each country is the same, then the steady state is unstable.

The locus of (K^T, K^{T*}) with $H^* = 1$ and $L = L^*$



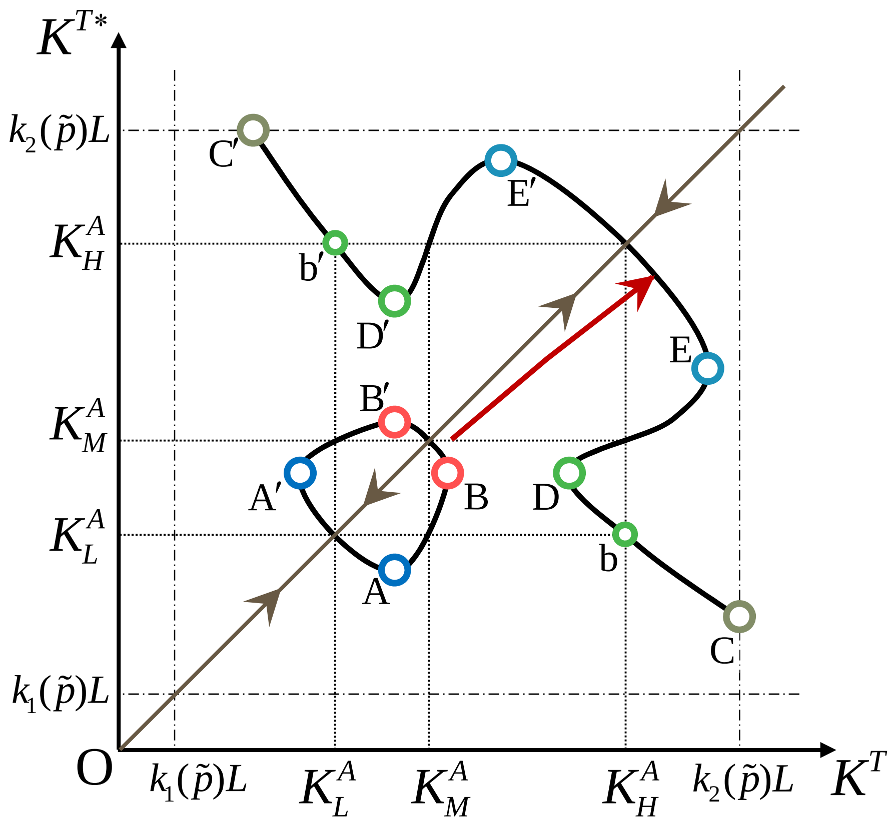
Saddle points with the static H-O theorem being violated



Remark 2

Under the symmetry on countries' fundamentals except for their initial capital stocks, the capital abundant country remains capital abundant along the trajectory to the steady state.

However, it is possible that the trade pattern varies along the path due to inferiority in consumption.



Concluding remarks

We have shown that main results in dynamic H-O models (with non-homothetic preferences) can be derived and/or examined from some diagrams which represent the basic functions in static models such as the Rybczynski line, an income expansion path, and an excess demand function.

For given technologies, preferences, and a labor endowment in each country, we have derived the diagrams and shown that they can clarify not only the existence and the multiplicity of steady states in autarky and under free trade, but also their stabilities and the static and the dynamic Heckscher-Ohlin theorems.