Can a Lump-Sum Transfer Make Everyone Enjoy the Gains from Free Trade?

Yasukazu Ichino*
Faculty of Economics, Konan University

January 2010

Abstract

I examine lump-sum transfer rules to redistribute the gains from free trade. When individuals anticipate that free trade policy will come with a lump-sum transfer, they are going to change their behaviors under autarky in order to get larger transfers. In spite of this falsification, can a lump-sum transfer still make everyone enjoy the gains from free trade? I found a condition under which a lump-sum transfer can make it, but in general it is difficult to achieve Pareto gains from free trade.

Key Words: Gains from trade, lump-sum transfer, redistribution
JEL Classification Numbers: F11, H23

*Address: 8-9-1 Okamoto, Higashinada-ku Kobe 658-8501, Japan.
E-mail: yichino@center.konan-u.ac.jp

1 Introduction

It is well known that free trade creates winners and losers. However, with an appropriate lump-sum transfer, Pareto improvement is achieved as the country moves from autarky to free trade. For example, see Kemp (1962), and Grandmont and McFadden (1972). Essentially, the idea of Pareto-improving lump-sum transfer rules is to let what each individual consumed under autarky affordable under free trade, too.

Later, Dixit and Norman (1986) pointed out practical difficulties of implementing such lump-sum transfer rules. In implementing lump-sum transfer to make everyone better off after free trade, the government has to figure out not only who are taxed and who are subsidized, but also how much is taxed or subsidized for each individual. To do so, the government needs to collect a lot of information about the characteristics of individuals. Moreover, when asked by the government, each individual may not reveal his characteristics that the government is going to use to calculate the amount of transfer. As Dixit and Norman (1986) writes, individuals have an incentive to manipulate their behavior "so as to mislead the planner about these characteristics and secure a larger net transfer."

How do they manipulate their behavior? Does that manipulation matter? Wong (1997) studies these questions in a general setting. His answer to the questions is, it may not matter very much: he argues that a lump-sum transfer rule that allows each individual to consume under free trade what they used to consume under autarky can achieve Pareto gains from free trade, in spite that consumers manipulate their behavior. However, partly because he worked on a very general setting, Wong did not analyze whether and how consumers' manipulation of their behavior affects the autarkic equilibrium price, and how the change in the autarkic equilibrium price will affect the performance of the lump-sum transfer.

In this paper, I work on a simpler, more specific setting than Wong: two goods, quasi-linear utility. By doing so, I can analyze whether and how consumers' manipulation of their behavior influences the autarkic equilibrium price. Then, I ask the questions Wong asked: How do consumers manipulate their behavior? Does that manipulation matter? Contrary to Wong's, my answer is, it does matter quite. I found that it is in general difficult to redistribute the gains from free trade to everyone as a country moving from autarky to free trade, if the government cannot observe individuals' preferences and if individuals anticipate a lump-sum transfer under free trade.

The reason is as follows. Knowing that the lump-sum transfer is based on how much they buy and sell under autarky, individuals distort their consumption under autarky, in order to increase the compensation they receive from the government or to decrease the lump-sum tax they have to pay to the government. This results in a drop of the autarkic equilibrium price, which is likely to make the lump-sum transfer ineffective.

2 Model

We consider an exchange economy with two goods, good x and good y, where good y is a numeraire. There are H individuals: $h = 1, 2, ...H$. Individual h's excess demand for two goods are respectively denoted by x_h and y_h. Individual h's endowment of good x is denoted by ω_h and that of good y is denoted by $\bar{y}_h$.

In this model, we have two periods: in period 0, the economy is under autarky; in period 1, the economy is under free trade.

We assume that the utility function is quasi-linear. Individual h's utility maximization problem in period 0 is given by

$$\max_{x_h, y_h} u_h(x_h + \omega_h) + y_h + \bar{y}_h$$
$$\text{s.t. } px_h + y_h \leq 0,$$

where $u_h(\cdot)$ is individual h's subutility function, with $u_h'(\cdot) > 0$ and $u_h''(\cdot) < 0$, and p is the price of good x.

Solving the first-order conditions gives the excess demand functions $x_h(\omega_h, p) = u_h'^{-1}(p) - \omega_h$ and $y_h(\omega_h, p) = -px_h(\omega_h, p)$. From the excess demand function, the autarkic equilibrium price p^a is defined by $\sum_h x_h(\omega_h, p^a) = 0$.

Let $x_h^a \equiv x_h(\omega_h, p^a)$ denote the excess demand (or, we call it "transaction") of individual h in the autarkic equilibrium. Then, we can define the indirect utility of individual h in the autarkic equilibrium: $U_h^a \equiv U_h(x_h^a, p^a) \equiv u_h(x_h^a + \omega_h) - p^a x_h^a + \bar{y}_h$.

In period 1, when the country allows free trade, the price of good x is equal to the world price p^* (we assume the economy is small). Then, transaction and indirect utility of individual h in the free-trade equilibrium are respectively denoted by $x_h^* \equiv x_h(\omega_h, p^*)$ and $U_h^* \equiv U_h(x_h^*, p^*) \equiv u_h(x_h^* + \omega_h) - p^* x_h^* + \bar{y}_h$. We assume that $p^* < p^a$. Namely, the world price of good x is smaller than the autarky price.

2.1 Gains from trade falls unequally

In aggregate, there are gains from free trade:

$$\begin{aligned}\sum_h U_h^* - \sum_h U_h^a &= \sum_h [U_h(x_h^*, p^*) - U_h(x_h^a, p^*)] + \sum_h [U_h(x_h^a, p^*) - U_h(x_h^a, p^a)] \\ &= \sum_h [U_h(x_h^*, p^*) - U_h(x_h^a, p^*)] + (p^a - p^*) \sum_h x_h^a > 0\end{aligned}$$

since the first term is positive (given p^*, the utility maximizing consumption is x_h^*) and the second term is zero because $\sum_h x_h^a = 0$.

However, the gains from trade do not fall equally to everyone. The buyers of good x gain from free trade:

$$\begin{aligned}U_h^* - U_h^a &= [U_h(x_h^*, p^*) - U_h(x_h^a, p^*)] + [U_h(x_h^a, p^*) - U_h(x_h^a, p^a)] \\ &= [U_h(x_h^*, p^*) - U_h(x_h^a, p^*)] + (p^a - p^*) x_h^a > 0,\end{aligned}$$

provided $x_h^a > 0$ (i.e., individual h is a buyer under autarky). On the other hand, the sellers of good x lose from free trade:

$$\begin{aligned}U_h^* - U_h^a &= [U_h(x_h^*, p^*) - U_h(x_h^*, p^a)] + [U_h(x_h^*, p^a) - U_h(x_h^a, p^a)] \\ &= (p^a - p^*) x_h^* + [U_h(x_h^*, p^a) - U_h(x_h^a, p^a)] < 0,\end{aligned}$$

provided that $x_h^* < 0$ (i.e., individual h is a seller under free trade).

3 Traditional lump-sum transfer

Let the government give some lump-sum transfer in period 1 to make sure that everyone can enjoy the gains from free trade. Suppose that the government gives a lump-sum transfer $z_h^a = -(p^a - p^*) x_h^a$ to individual h in period 1 (note that $z_h^a > 0$ is a subsidy and $z_h^a < 0$ is a tax). Then, the budget constraint of individual h in period 1 becomes $p^* x_h + y_h \leq z_h^a$. It is easy to show that this transfer makes (x_h^a, y_h^a) affordable under p^*:

$$p^* x_h + y_h \leq z_h^a \Leftrightarrow p^* x_h + y_h \leq -(p^a - p^*) x_h^a \Leftrightarrow p^* x_h + y_h \leq p^* x_h^a + y_h^a.$$

Therefore, with this transfer, everyone gets better off by free trade. That is, $U_h^* + z_h^a > U_h^a$ for all h. In addition, with this lump-sum transfer, the government budget is balanced since $\sum_h z_h^a = -(p^a - p^*) \sum_h x_h = 0$.

3.1 Consumption Falsification

The discussion above that the free trade policy with the lump-sum transfer z_h^a makes everyone better off, however, depends on an implicit assumption that the government can figure out what z_h^a's are. For the government to calculate z_h^a, it has to know each individual's utility function and endowments. But this is not likely the case. Without knowing preferences and endowments of each individual, the government is not able to calculate x_h^a, and thus cannot figure out z_h^a. Then, the only way for the government to find the amount of lump-sum transfer for individual h is to *observe* the transaction made by individual h in period 0 (under autarky). However, if each individual knows that the government is going to observe his transaction in period 0 to determine the amount of transfer for him, then he may want to change his transaction from x_h^a in order to get larger transfer in period 1 (under free trade). We call this "transaction falsification."

Given this, the question we want to ask in this paper is as follows: how does each individual falsify his transaction in period 0, and how does it affect the equilibrium price in period 0? If the individuals falsify their transaction in period 0, does this mean that Pareto gains from free trade cannot be achieved?

To examine these questions, let us now clarify our setting. As we have mentioned, in the following analysis the government knows neither endowments nor preferences of each individual; the government just observes transaction of each individual in period 0 (under autarky). In period 0, all individuals know that free trade will be allowed in period 1. They also know that the lump-sum transfer $z_h = -(p - p^*)\, x_h$ will be given in period 1, where x_h is the observed transaction of individual h in period 0 and p is the prevailing price in period 0. We refer this form of transfer, $z_h = -(p - p^*)\, x_h$, as "the traditional lump-sum transfer." Notice that the traditional transfer $z_h = -(p - p^*)\, x_h$ has the same spirit as $z_h^a = -(p^a - p^*)\, x_h^a$: z_h makes what individual h consumed in period 0 (under autarky) affordable in period 1 (under free trade). Also, note that if there were no transaction falsification, the traditional lump-sum transfer would be identical to z_h^a in equilibrium. We are going to investigate whether this traditional lump-sum transfer can redistribute the gains from free trade so as to achieve Pareto improvement, even when there is transaction falsification.

Now, the utility maximization problem of individual h should be written as follows:

$$\max_{x_h, y_h, x_h^1, y_h^1} u_h(x_h + \omega_h) + y_h + \bar{y}_h + \rho_h \left[u_h(x_h^1 + \omega_h) + y_h^1 + \bar{y}_h \right]$$

$$\text{s.t. } px_h + y_h \leq 0, \text{ and } p^* x_h^1 + y_h^1 \leq -(p - p^*)\, x_h$$

where x_h and y_h are period-0 transaction of good x and y; x_h^1 and y_h^1 are period-1 transaction of good x and y, respectively, and ρ_h is a discount factor for individual h, $0 \leq \rho_h \leq 1$ for all h. Solving the first-order conditions, we have $x_h^1 = x_h^*$ and $x_h = u_h'^{-1}(p + \rho_h (p - p^*)) - \omega_h$. Here, the period-1 transaction is the same as the one we saw in Section 2. But the period-0 transaction is now different: anticipating that the transfer will be given in period 1, individual h sees the cost of consuming one unit of good x in period 0 not p but $p + \rho_h (p - p^*)$.

Now, let p^0 denote the equilibrium price in period 0 under transaction falsification.

It is determined by $\sum_h u_h'^{-1}\left(p^0 + \rho_h\left(p^0 - p^*\right)\right) = \sum_h \omega_h$. How is p^0 different from p^a? That is, how does transaction falsification affect the equilibrium autarky price? We found the following result.

Lemma 1 $p^* < p^0 < p^a$.

Proof. First, compare the individual demand function under non-falsification $u_h'^{-1}(p)$ and under falsification $u_h'^{-1}(p + \rho_h(p - p^*))$. For any given p, $u_h'^{-1}(p) > u_h'^{-1}(p + \rho_h(p - p^*)) \Leftrightarrow p - p^* > 0$, and $u_h'^{-1}(p) = u_h'^{-1}(p + \rho_h(p - p^*)) \Leftrightarrow p - p^* = 0$. Since this is true for all h, $\sum_h u_h'^{-1}(p) > \sum_h u_h'^{-1}(p + \rho_h(p - p^*)) \Leftrightarrow p - p^* > 0$, and $\sum_h u_h'^{-1}(p) = \sum_h u_h'^{-1}(p + \rho_h(p - p^*)) \Leftrightarrow p - p^* = 0$. Graphically, the demand curve of non-falsification is steeper than that of falsification, and they intersect at p^* (see Figure 1). In other words, at those quantities less than the quantity such that $\sum_h u_h'^{-1}(p) = \sum_h u_h'^{-1}(p + \rho_h(p - p^*))$, the demand curve of non-falsification is above that of falsification. The equilibrium non-falsification price p^a is determined by $\sum_h u_h'^{-1}(p^a) = \sum_h \omega_h$ and by assumption $p^* < p^a$. Thus, the intersection of the aggregate falsification demand curve $\sum_h u_h'^{-1}(p + \rho_h(p - p^*))$ and the aggregate supply $\sum_h \omega_h$, at which p^0 is determined, is below the intersection of $\sum_h u_h'^{-1}(p)$ and $\sum_h \omega_h$. Therefore, $p^* < p^0 < p^a$. ■

With transaction falsification, the equilibrium price in period 0 falls. Intuitively, this is because sellers try to sell more in period 0 in order to get more subsidy in period 1, and buyers try to buy less in order to lower tax they have to pay.

Having found the period-0 equilibrium price under transaction falsification, now we can define the period-0 equilibrium transaction as $x_h^0 \equiv u_h'^{-1}\left(p^0 + \rho_h\left(p^0 - p^*\right)\right) - \omega_h$. Is x_h^0 larger or smaller than x_h^a? Namely, when anticipating the transfer of period 1, is individual h going to consume more or less in period 0? There are two opposing effects. To get larger transfer of period 1, an individual wants to buy less (or sell more). This effect tends to make x_h^0 less than x_h^a. On the other hand, since good x is now cheaper ($p^0 < p^a$), an individual wants to buy more (or sell less). This effect tends to make x_h^0 more than x_h^a. Which effect is dominating depends on ρ_h. In fact, if everyone has the same discount factor, these two effects just cancel each other, and the falsified transaction x_h^0 is just equal to the non-falsified transaction x_h^a for all h.

Lemma 2 $x_h^0 = x_h^a$ *for all* $h \Leftrightarrow \rho_h = \rho$ *for all* h.

Proof. ($\Leftarrow$) Suppose that $x_h^0 < x_h^a$ for some h. Then, $p^0 + \rho_h \left(p^0 - p^*\right) > p^a$. Since $\sum_h x_h^0 = \sum_h x_h^a$, there must be another agent h' such that $x_{h'}^0 > x_{h'}^a$. Thus $p^0 + \rho_{h'} \left(p^0 - p^*\right) < p^a$. However, since $\rho_h = \rho_{h'}$, this is a contradiction.

($\Rightarrow$) $x_h^0 = x_h^a$ for all h implies $p^0 + \rho_h \left(p^0 - p^*\right) = p^a$ for all h. Then, ρ_h's must be the same for all h. ■

Now, let's consider the case where ρ_h's are different. For those individuals with large ρ_h, it holds that $p^0 + \rho_h \left(p^0 - p^*\right) > p^a$, and thus $x_h^0 < x_h^a$. This means that for those individuals with large ρ_h, securing larger transfer in period 0 is more important than utilizing the cheaper price in period 0. On the other hand, for those individuals with small ρ_h, it holds that $p^0 + \rho_h \left(p^0 - p^*\right) < p^a$, and thus $x_h^0 > x_h^a$.

3.2 Can everyone gain from free trade?

We are now ready to answer our main question. Even when the individuals falsify their transaction in period 0 in order to affect the transfer they receive in period 1, can the traditional lump-sum transfer, which guarantees the period-0 consumption bundle affordable in period 1, still make everyone enjoy the gains from free trade? To see this, we are going to compare $U_h^0 + \rho_h \left(U_h^* + z_h\right)$ with $U_h^a + \rho_h U_h^a$, where $U_h^0 \equiv U_h \left(x_h^0, p^0\right) \equiv u_h(x_h^0 + \omega_h) - p^0 x_h^0 + \bar{y}_h$, and $z_h = -\left(p^0 - p^*\right) x_h^0$ (hereafter, z_h is to denote the traditional lump-sum transfer *in equilibrium* when there is transaction falsification). That is, we are comparing the welfare when free trade policy comes in period 1 with the traditional lump-sum transfer under transaction falsification, with the welfare when the economy stays in autarky in period 1.

First, we consider the special case where all individuals have the same discount factor.

3.2.1 When $\rho_h = \rho$ for all h

Proposition 1 *If $\rho_h = \rho$ for all h, then $U_h^0 + \rho_h (U_h^* + z_h) = U_h^a + \rho_h U_h^*$ for all h.*

Proof.

$$\begin{aligned} & [U_h^0 + \rho_h (U_h^* + z_h)] - [U_h^a + \rho_h U_h^*] = (U_h^0 + \rho z_h) - U_h^a \\ = \; & U_h(x_h^0, p^0 + \rho \left(p^0 - p^*\right)) - U_h(x_h^a, p^a) = 0. \end{aligned}$$

The last equality comes from Lemma 2. ■

Therefore, when $\rho_h = \rho$, the traditional lump-sum transfer is completely neutralized. That is, even though the free trade policy in period 1 is accompanied by the lump-sum transfer, the transaction falsification in period 0 makes the resulting intertemporal welfare just equal to the one with free trade policy not accompanied by any transfers.

Because of Proposition 1, when $\rho_h = \rho$ for all h, comparing $U_h^0 + \rho_h (U_h^* + z_h)$ with $U_h^a + \rho_h U_h^a$ is reduced to comparing U_h^* with U_h^a. Hence, we readily have the following corollary.

Corollary 1 *If $\rho_h = \rho$ for all h, then $U_h^0 + \rho_h (U_h^* + z_h) > U_h^a + \rho_h U_h^a$ for h such that $x_h^a \geq 0$, and $U_h^0 + \rho_h (U_h^* + z_h) < U_h^a + \rho_h U_h^a$ for h such that $x_h^* \leq 0$.*

From free trade, the buyers of good x still gain and the sellers of good x still lose. When $\rho_h = \rho$ for all h, due to transaction falsification, the traditional lump-sum transfer $z_h = -\left(p^0 - p^*\right) x_h^0$ fails to redistribute the gains from free trade to everyone.

3.2.2 Different ρ_h

Next, we consider the cases where ρ_h's are different.

Let us examine buyers first. As we confirm below, in spite of transaction falsification, buyers of good x are made better off by free trade with the lump-sum transfer $z_h = -\left(p^0 - p^*\right) x_h^0$.

Proposition 2 *$U_h^0 + \rho_h (U_h^* + z_h) > U_h^a + \rho_h U_h^a$ for h such that $x_h^a \geq 0$.*

Proof.

$$
\begin{aligned}
& \left[U_h^0+\rho_h\left(U_h^*+z_h\right)\right]-\left[U_h^a+\rho_h U_h^a\right]=\left[\left(U_h^0+\rho_h z_h\right)-U_h^a\right]+\rho_h\left(U_h^*-U_h^a\right) \\
= \; & U_h\left(x_h^0, p^0+\rho_h\left(p^0-p^*\right)\right)-U_h\left(x_h^a, p^0+\rho_h\left(p^0-p^*\right)\right)+U_h\left(x_h^a, p^0+\rho_h\left(p^0-p^*\right)\right)-U_h\left(x_h^a, p^a\right) \\
& +\rho_h\left[U_h\left(x_h^*, p^*\right)-U_h\left(x_h^a, p^*\right)+U_h\left(x_h^a, p^*\right)-U_h\left(x_h^a, p^a\right)\right] \\
= \; & U_h\left(x_h^0, p^0+\rho_h\left(p^0-p^*\right)\right)-U_h\left(x_h^a, p^0+\rho_h\left(p^0-p^*\right)\right)+\rho_h\left[U_h\left(x_h^*, p^*\right)-U_h\left(x_h^a, p^*\right)\right] \\
& +\left[p^a-\left(p^0+\rho_h\left(p^0-p^*\right)\right)\right] x_h^a+\rho_h\left(p^a-p^*\right) x_h^a \\
= \; & U_h\left(x_h^0, p^0+\rho_h\left(p^0-p^*\right)\right)-U_h\left(x_h^a, p^0+\rho_h\left(p^0-p^*\right)\right)+\rho_h\left[U_h\left(x_h^*, p^*\right)-U_h\left(x_h^a, p^*\right)\right] \\
& +\left(1+\rho_h\right)\left(p^a-p^0\right) x_h^a \\
> \; & 0.
\end{aligned}
$$

■

How about sellers? With the lump-sum transfer, can sellers of good x be made better off by free trade? As we show in Proposition 3, due to transaction falsification, some sellers may be made worse off by free trade even though they receive the lump-sum transfer.

Proposition 3 *For h such that $x_h^* \leq 0$, if ρ_h is small enough such that $p^a \geq p^0 + \rho_h\left(p^0-p^*\right)$, then $U_h^0+\rho_h\left(U_h^*+z_h\right)<U_h^a+\rho_h U_h^a$.*

Proof.

$$
\begin{aligned}
& \left[U_h^0+\rho_h\left(U_h^*+z_h\right)\right]-\left[U_h^a+\rho_h U_h^a\right]=\left[\left(U_h^0+\rho_h z_h\right)-U_h^a\right]+\rho_h\left(U_h^*-U_h^a\right) \\
= \; & \left[U_h\left(x_h^0, p^0+\rho_h\left(p^0-p^*\right)\right)-U_h\left(x_h^0, p^a\right)\right]+\left[U_h\left(x_h^0, p^a\right)-U_h\left(x_h^a, p^a\right)\right] \\
& +\rho_h\left[U_h\left(x_h^*, p^*\right)-U_h\left(x_h^*, p^a\right)+U_h\left(x_h^*, p^a\right)-U_h\left(x_h^a, p^a\right)\right] \\
= \; & \left[p^a-\left(p^0+\rho_h\left(p^0-p^*\right)\right)\right] x_h^0+\left[U_h\left(x_h^0, p^a\right)-U_h\left(x_h^a, p^a\right)\right] \\
& +\rho_h\left(p^a-p^*\right) x_h^*+\rho_h\left[U_h\left(x_h^*, p^a\right)-U_h\left(x_h^a, p^a\right)\right] \\
< \; & 0.
\end{aligned}
$$

Note that $x_h^0 \leq 0$ for those h such that $x_h^* \leq 0$ since $p^0 > p^*$. ■

Intuitively, this proposition is explained as follows. When a seller of good x has small ρ_h, because the cheaper-price effect outweighs the securing-larger-transfer effect, he sells less in period 0 under transaction falsification than under non-falsification. Thus, the lump-sum transfer he receives in period 1 will be smaller. Furthermore, the fall in period-0 price due to transaction falsification makes his period-0 welfare smaller (because he is a seller of good x). As a result, when there is transaction falsification, he is made worse off by free trade policy in period 1 even though the lump-sum transfer is given.

Proposition 3 gives us a *necessary* condition under which the lump-sum transfer can make everyone better off: the traditional lump-sum transfer successfully redistributes the gains from trade to everyone *only if* all sellers of good x have large discount factor so that $p^0 + \rho_h \left(p^0 - p^*\right) > p^a$: with all sellers having large ρ_h's, they sell more in period 0 so as to secure large amount of lump-sum transfer in period 1. As a result they can be enough compensated under free trade. In Appendix A, we present an example to demonstrate that there is actually a case where everyone is made better off.

3.3 Discussion

So, after all, how do we evaluate the performance of the traditional lump-sum transfer rule $z_h = -\left(p^0 - p^*\right) x_h^0$? Can it successfully redistribute the gains from free trade to everyone, in spite of transaction falsification? Our answer is, yes, it can, but not always. Whether this lump-sum transfer can make everyone better off depends on the distribution of individuals' characteristics. Specifically, it can make everyone better off only when all of the sellers of good x (i.e., those who have larger endowment of good x) care more about the future than the buyers do.

How is this likely to happen? Casually speaking, there seems no special reason to believe that the individuals having larger endowment of good x is more patient than those having smaller endowment. Then, the situation under which this transfer rule makes everyone better off is not specially likely to happen. Moreover, with the assumption that the government does not know the preferences of each individual, it is natural to consider that the government does not know the discount factor of each individual either. So, the government will not be able to identify the situation where the lump-sum transfer rule works fine. Overall, our conclusion is that the traditional lump-sum transfer rule $z_h = -\left(p^0 - p^*\right) x_h^0$ does not perform very well when there is transaction falsification.

4 Any linear transfer rules

Thus far, we have examined a particular form of lump-sum transfer, $z_h = -\left(p^0 - p^*\right) x_h^0$, because it is a transfer rule that guarantees the period-0 consumption bundle affordable in period 1. We have shown that this traditional transfer rule does not perform well to redistribute the gains from free trade to everyone.

A reason why this transfer rule does not work well is that the amount of transfer is affected by the autarkic price drop from p^a to p^0 due to transaction falsification. To avoid this, we may want to look for other rules of lump-sum transfer. For example, how about $z_h = -\left(p^a - p^*\right) x_h$? With this rule, the amount of transfer is not directly affected by the autarkic price change. To analyze such a transfer rule, in this section, we consider a more general class of lump-sum transfer rules. Specifically, we examine a class of transfer rules that are budget balancing and linear in x_h.

A budget-balancing, linear lump-sum transfer rule is denoted by $z\left(x_h\right) = s x_h$, where s can be a constant, or it can be a function of the autarkic price p, as in the case of the traditional transfer rule. So, when necessary, we write $s\left(p\right)$. Are there any linear transfer rules that outperform the traditional transfer rule $z_h = -\left(p^0 - p^*\right) x_h^0$? This is the question we ask in this section. As we will see below, the answer to this question is essentially no. Basically, any linear transfer rules have the similar characteristics to the traditional lump-sum transfer rule we have studied in the last section.

With a general linear lump-sum transfer, the utility maximization problem is written as follows:

$$\max_{x_h, y_h, x_h^1, y_h^1} u_h(x_h + \omega_h) + y_h + \bar{y}_h + \rho_h \left[u_h(x_h^1 + \omega_h) + y_h^1 + \bar{y}_h \right]$$

$$\text{s.t. } px_h + y_h \leq 0, \text{ and } p^* x_h^1 + y_h^1 \leq s x_h$$

This setting is exactly the same as the one we had in Section 3.1, except $-(p - p^*) x_h$ is replaced by $s x_h$. Solving the first-order conditions, we derive $x_h^1 = x_h^*$ and $x_h = u_h'^{-1}(p - \rho_h s) - \omega_h$. The equilibrium autarky price p^0 is determined by $\sum_h u_h'^{-1}\left(p^0 - \rho_h s\left(p^0\right)\right) = \sum_h \omega_h$. We let x_h^0 denote the equilibrium transaction in period 0: $x_h^0 = u_h'^{-1}\left(p^0 - \rho_h s\left(p^0\right)\right) - \omega_h$. Then, in equilibrium, the lump-sum transfer is $z\left(x_h^0\right) = s\left(p^0\right) x_h^0$.

4.1 When $\rho_h = \rho$ for all h

Now, we can show that the same lemma holds as in the case of the traditional transfer.

Lemma 3 *Let $z\left(x_h^0\right) = s x_h^0$. $x_h^0 = x_h^a$ for all $h \Leftrightarrow \rho_h = \rho$ for all h.*

Proof. ($\Leftarrow$) Suppose that $x_h^0 < x_h^a$ for some h. Then, $p^0 - \rho_h s\left(p^0\right) > p^a$. Since $\sum_h x_h^0 = \sum_h x_h^a$, there must be another agent h' such that $x_{h'}^0 > x_{h'}^a$. Thus $p^0 - \rho_{h'} s\left(p^0\right) < p^a$. However, since $\rho_h = \rho_{h'}$, this is a contradiction.

($\Rightarrow$) $x_h^0 = x_h^a$ for all h implies $p^0 - \rho_h s\left(p^0\right) = p^a$ for all h. Then, it must be that ρ_h are the same for all h. ■

Using this lemma, again, we can show that any linear transfer rules are neutralized when $\rho_h = \rho$ for all h.

Proposition 4 *Let $z\left(x_h^0\right) = s x_h^0$. If $\rho_h = \rho$ for all h, then $U_h^0 + \rho_h\left(U_h^* + z_h\right) = U_h^a + \rho_h U_h^*$ for all h.*

Proof.

$$\begin{aligned}
& U_h^0 + \rho_h \left(U_h^* + z\left(x_h^0\right)\right) - \left(U_h^a + \rho_h U_h^*\right) \\
= & \left(U_h^0 + \rho z\left(x_h^0\right)\right) - U_h^a \\
= & U_h\left(x_h^0, p^0 - \rho_h s\right) - U_h\left(x_h^a, p^a\right) = 0.
\end{aligned}$$

■

Therefore, any linear and budget-balancing transfer rules cannot make everyone better off when everyone has the same discount factor.

4.2 When ρ_h's are different

So, again, to see if linear transfers work well, we have to look at the cases where ρ_h's are different. Before doing this, let us take a closer look at how the characteristics of the linear lump-sum transfer rules $z\left(x_h^0\right) = s\left(p^0\right) x_h^0$ affect the equilibrium autarkic price p^0. Lemma 4 shows that as long as the transfer is positive for sellers (subsidy for sellers) and negative for buyers (tax on buyers), i.e., $s\left(p^0\right) \leq 0$, the equilibrium autarkic price falls due to transaction falsification. Then, Lemma 5 shows that as long as the size of transfer is not too large, the equilibrium autarkic price p^0 is higher than the world price.

Lemma 4 *$p^0 \leq p^a \Leftrightarrow s\left(p^0\right) \leq 0$.*

Proof. ($\Leftarrow$) Suppose $p^0 > p^a$. $s\left(p^0\right) \leq 0$ implies $p^0 - \rho_h s\left(p^0\right) > p^0$ for all h. Then, $p^0 - \rho_h s\left(p^0\right) > p^0 > p^a$ for all h. This implies $x_h^0 < x_h^a$ for all h. But this is a contradiction since $\sum_h x_h^0 = \sum_h x_h^a$.

($\Rightarrow$) Suppose $p^0 \leq p^a$. $s\left(p^0\right) > 0$ implies $p^0 > p^0 - \rho_h s\left(p^0\right)$. Then, $p^a \geq p^0 > p^0 - \rho_h s\left(p^0\right)$ for all h. This implies $x_h^a < x_h^0$ for all h, but this is a contradiction. ■

Lemma 5 *If $0 \leq -s\left(p^0\right) \leq p^a - p^*$, then $p^* \leq p^0 \leq p^a$.*

Proof. Note that $-s\left(p^0\right) \geq 0$ implies $p^0 \leq p^a$ from Lemma 4. Given that $p^a - p^* \geq -s\left(p^0\right)$, we have $p^0 - p^* \geq \left(p^0 - s\left(p^0\right)\right) - p^a \geq \left(p^0 - \bar{\rho}_h s\left(p^0\right)\right) - p^a$, where $\bar{\rho}_h = \max_h \rho_h$.

Since $p^0 - \bar{\rho}_h s\left(p^0\right) \geq p^a$ (otherwise, $x_h^0 > x_h^a$ for all h, which cannot happen), it holds that $p^0 \geq p^*$. ■

Having these lemmas, we can show the following: the results we have derived in the case of the traditional transfer (Section 3) is generalized to the case of any linear transfer rules, provided that the transfer is positive (subsidy) for sellers and negative (tax) for buyers, and its size is not too large.

Proposition 5 *Let $z\left(x_h^0\right) = s x_h^0$ and suppose that $0 \leq -s\left(p^0\right) \leq p^a - p^*$. (1) For h such that $x_h^a > 0$, $U_h^0 + \rho_h\left(U_h^* + z_h\right) > U_h^a + \rho_h U_h^a$. (2) For h such that $x_h^* \leq 0$, if $p^a \geq p^0 - \rho_h s$ (i.e., if ρ_h is relatively small), then $U_h^0 + \rho_h\left(U_h^* + z_h\right) < U_h^a + \rho_h U_h^a$.*

Proof. (1) For those individuals such that $x_h^a > 0$,

$$\begin{aligned}
& \left[U_h^0 + \rho_h\left(U_h^* + z_h\right)\right] - \left[U_h^a + \rho_h U_h^a\right] \\
= & \left[U_h\left(x_h^0, p^0 - \rho_h s\right) - U_h\left(x_h^a, p^0 - \rho_h s\right)\right] + \left[U_h\left(x_h^a, p^0 - \rho_h s\right) - U_h\left(x_h^a, p^a\right)\right] \\
& + \rho_h\left[U_h\left(x_h^*, p^*\right) - U_h\left(x_h^a, p^*\right) + U_h\left(x_h^a, p^*\right) - U_h\left(x_h^a, p^a\right)\right] \\
= & \left[U_h\left(x_h^0, p^0 - \rho_h s\right) - U_h\left(x_h^a, p^0 - \rho_h s\right)\right] + \rho_h\left[U_h\left(x_h^*, p^*\right) - U_h\left(x_h^a, p^*\right)\right] \\
& + \left[\left(p^a - p^0\right) + \rho_h\left(\left(p^a - p^*\right) + s\right)\right] x_h^a \\
> & \; 0
\end{aligned}$$

since $p^a - p^* \geq -s$ by assumption and $p^a \geq p^0$ by Lemma 4.

(2) For those individuals such that $x_h^* \leq 0$,

$$\begin{aligned}
& \left[U_h^0 + \rho_h\left(U_h^* + z_h\right)\right] - \left[U_h^a + \rho_h U_h^a\right] \\
= & \left[U_h\left(x_h^0, p^0 - \rho_h s\right) - U_h\left(x_h^0, p^a\right)\right] + \left[U_h\left(x_h^0, p^a\right) - U_h\left(x_h^a, p^a\right)\right] \\
& + \rho_h\left[U_h\left(x_h^*, p^*\right) - U_h\left(x_h^*, p^a\right) + U_h\left(x_h^*, p^a\right) - U_h\left(x_h^a, p^a\right)\right] \\
= & \left[p^a - \left(p^0 - \rho_h s\right)\right] x_h^0 + \left[U_h\left(x_h^0, p^a\right) - U_h\left(x_h^a, p^a\right)\right] \\
& + \rho_h\left(p^a - p^*\right) x_h^* + \rho_h\left[U_h\left(x_h^*, p^a\right) - U_h\left(x_h^a, p^a\right)\right] \\
< & \; 0
\end{aligned}$$

since $p^a > p^0 - \rho_h s$, and since $x_h^0 \leq x_h^* < 0$ because of $p^0 \geq p^*$, which is from Lemma 5. ■

Thus, the discussion we had in Section 3.3 applies to any linear transfer rules such that $0 \leq -s\left(p^0\right) \leq p^a - p^*$. That is, these linear transfer rules do not perform well in making everyone enjoy the gains from free trade. It does so only when all sellers of good x are more patient than the buyers of good x.

In Appendix B, we consider the other linear transfer rules, such that $0 < p^a - p^* < -s\left(p^0\right)$, and that $s\left(p^0\right) > 0$. For those lump-sum transfer rules, too, whether they can make everyone better off depends on the distribution of individuals' discount factors. Therefore, practically speaking, those transfer rules, either, do not perform well in making everyone enjoy the gains from free trade.

5 Concluding remarks

In this paper, we examined lump-sum transfer rules to redistribute the gains from free trade to everyone. Specifically, we considered a transfer rule that makes what each individual consumed under autarky affordable under free trade. When individuals anticipate that free trade policy will come with this lump-sum transfer, they are going to falsify their transaction under autarky in order to get larger transfer. In spite of this transaction falsification, can this lump-sum transfer successfully redistribute the gains from free trade to everyone? Our answer is, yes, it can, but not always. Whether this lump-sum transfer can make everyone better off depends on the distribution of individuals' characteristics. More specifically, it can make everyone better off only when all of the sellers of good x (i.e., those who have larger endowment of good x) care more about the future than the buyers do. The similar result will hold for any linear, budget-balancing transfer rules.

Thus, if we want to examine more generally how well lump-sum transfer rules work, we have to consider *nonlinear* transfer rules. By using a technique of principal-agent models, we can look for a welfare-maximizing lump-sum transfer rule that makes everyone better off and that is incentive compatible. This will be a next topic of my research.

Appendix

A. An example that everyone is made better

Here we provide an example that everyone is made better off by free trade policy with the traditional lump-sum transfer $z_h = -\left(p^0 - p^*\right) x_h^0$. Let the subutility function $u_h(\omega_h + x_h)$ be quadratic: $u_h(\omega_h + x_h) = \frac{\beta_h}{\gamma_h}(\omega_h + x_h) - \frac{1}{2\gamma_h}(\omega_h + x_h)^2$. Suppose that there are two individuals. Let $\rho_1 = 0.05$, $\rho_2 = 1$, $\beta_1 = \beta_2 = 1$, $\gamma_1 = 2$, $\gamma_2 = 1$, $\omega_1 = 0.249$, $\omega_2 = 0.751$, and $p^* = \frac{1}{4}$.

We can calculate the equilibrium prices as follows:

$p^a = \frac{(\beta_1 + \beta_2) - (\omega_1 + \omega_2)}{(\gamma_1 + \gamma_2)} = \frac{1}{3}$; and

$p^0 = \frac{(\beta_1 + \beta_2) - (\omega_1 + \omega_2) + (\gamma_1\rho_1 + \gamma_2\rho_2)\, p^*}{(\gamma_1 + \gamma_2) + (\gamma_1\rho_1 + \gamma_2\rho_2)} = 0.310\,98.$

The equilibrium consumptions of good x are as follows:

$x_1^a + \omega_1 = \frac{1}{3}$;

$x_2^a + \omega_2 = \frac{2}{3}$;

$x_1^* + \omega_1 = \frac{1}{2}$;

$x_2^* + \omega_2 = \frac{3}{4}$;

$x_1^0 + \omega_1 = 0.37195$; and

$x_2^0 + \omega_2 = 0.62805$.

The indirect utilities are:

$U_1^a = 0.11078 + \bar{y}_1$;

$U_2^a = 0.47256 + \bar{y}_2$;

$U_1^* = 0.12475 + \bar{y}_1$;

$U_2^* = 0.469 + \bar{y}_2$;

$U_1^0 + \rho_1 z_1^2 = 0.11278 + \bar{y}_1$; and

$U_2^0 + \rho_2 z_2^2 = 0.47656 + \bar{y}_2$.

In this case everyone is made better off since:

$\left(U_1^0 + \rho_1\left(U_1^* + z_1\right)\right) - \left(U_1^a + \rho_1 U_1^a\right) = 0.00270 > 0$; and

$\left(U_2^0 + \rho_2\left(U_2^* + z_2\right)\right) - \left(U_2^a + \rho_2 U_2^a\right) = 0.00044 > 0$.

B. General linear transfer rules: $z_h = s\left(p^0\right) x_h^0$

B-1. When $0 < p^a - p^* < -s\left(p^0\right)$

Since $s\left(p^0\right) < 0$, in this case it holds that $p^a > p^0$ (See, Lemma 4). For buyers,

$$\begin{aligned} & \left[U_h^0 + \rho_h\left(U_h^* + z_h\right)\right] - \left[U_h^a + \rho_h U_h^a\right] \\ = & \left[U_h\left(x_h^0, p^0 - \rho_h s\right) - U_h\left(x_h^a, p^0 - \rho_h s\right)\right] + \rho_h\left[U_h\left(x_h^*, p^*\right) - U_h\left(x_h^a, p^*\right)\right] \\ & + \left[\rho_h\left(p^a - p^*\right) - \left(\left(p^0 - \rho_h s\right) - p^a\right)\right] x_h^a. \end{aligned}$$

When $x_h^a \geq 0$, a sufficient condition for this to be positive is $\rho_h\left(p^a - p^*\right) \geq \left(p^0 - \rho_h s\right) - p^a$. This condition is satisfied when ρ_h is not too large.

For sellers,

$$\begin{aligned} & \left[U_h^0 + \rho_h\left(U_h^* + z_h\right)\right] - \left[U_h^a + \rho_h U_h^a\right] \\ = & \left[p^a - \left(p^0 - \rho_h s\right)\right] x_h^0 + \left[U_h\left(x_h^0, p^a\right) - U_h\left(x_h^a, p^a\right)\right] \\ & + \rho_h\left(p^a - p^*\right) x_h^* + \rho_h\left[U_h\left(x_h^*, p^a\right) - U_h\left(x_h^a, p^a\right)\right]. \end{aligned}$$

When $x_h^* \leq 0$, a sufficient condition for this to be negative is $\left(p^a - \left(p^0 - \rho_h s\right)\right) x_h^0 \leq 0$. This is, $p^a \geq \left(p^0 - \rho_h s\right)$ and $x_h^0 \leq 0$, or $p^a \leq \left(p^0 - \rho_h s\right)$ and $x_h^0 \geq 0$. However, the latter one, $p^a \leq \left(p^0 - \rho_h s\right)$ and $x_h^0 \geq 0$, can't happen since $p^a \leq \left(p^0 - \rho_h s\right) \Leftrightarrow x_h^a \geq x_h^0$ and since $0 > x_h^a$. Therefore, a sufficient condition for this to be negative is $p^a \geq \left(p^0 - \rho_h s\right)$ and $x_h^0 \leq 0$. In terms of ρ_h, this is that ρ_h is small enough to satisfy $p^a \geq \left(p^0 - \rho_h s\right)$, but not too small to violate $x_h^0 \leq 0$.

B-2. When $s\left(p^0\right) > 0$

In this case $p^a < p^0$ by Lemma 4. For buyers,

$$\begin{aligned} & \left[U_h^0 + \rho_h\left(U_h^* + z_h\right)\right] - \left[U_h^a + \rho_h U_h^a\right] \\ = & \left[U_h\left(x_h^0, p^0 - \rho_h s\right) - U_h\left(x_h^a, p^0 - \rho_h s\right)\right] + \rho_h\left[U_h\left(x_h^*, p^*\right) - U_h\left(x_h^a, p^*\right)\right] \\ & + \left[\rho_h\left(p^a - p^*\right) + \left(p^a - \left(p^0 - \rho_h s\right)\right)\right] x_h^a. \end{aligned}$$

When $x_h^a \geq 0$, a sufficient condition for this to be positive is $p^a \geq \left(p^0 - \rho_h s\right)$. This condition is satisfied when ρ_h is large enough.

For sellers,

$$\begin{aligned}
& \left[U_h^0 + \rho_h \left(U_h^* + z_h\right)\right] - \left[U_h^a + \rho_h U_h^a\right] \\
= \; & \left[p^a - \left(p^0 - \rho_h s\right)\right] x_h^0 + \left[U_h\left(x_h^0, p^a\right) - U_h\left(x_h^a, p^a\right)\right] \\
& + \rho_h \left(p^a - p^*\right) x_h^* + \rho_h \left[U_h\left(x_h^*, p^a\right) - U_h\left(x_h^a, p^a\right)\right].
\end{aligned}$$

When $x_h^* \leq 0$, a sufficient condition for this to be negative is $\left(p^a - \left(p^0 - \rho_h s\right)\right) x_h^0 \leq 0$. This is $p^a \geq \left(p^0 - \rho_h s\right)$ and $x_h^0 \leq 0$, or $p^a \leq \left(p^0 - \rho_h s\right)$ and $x_h^0 \geq 0$. However, again, the latter one, $p^a \leq \left(p^0 - \rho_h s\right)$ and $x_h^0 \geq 0$, can't happen since $p^a \leq \left(p^0 - \rho_h s\right) \Leftrightarrow x_h^a \geq x_h^0$ and since $0 > x_h^a$. Therefore, a sufficient condition for this to be negative is $p^a \geq \left(p^0 - \rho_h s\right)$ and $x_h^0 \leq 0$. In terms of ρ_h, this is that ρ_h is large enough to satisfy $p^a \geq \left(p^0 - \rho_h s\right)$, but not too large to violate $x_h^0 \leq 0$.

References

[1] Che, Jiahua and Giovanni Facchini (2007), "Dual Track Reforms: with and without Losers," *Journal of Public Economics*, 91, pp. 2291-2306.

[2] Dixit, Avinash and Victor Norman (1986), "Gains from Trade without Lump-Sum Compensation," *Journal of International Economics*, 21, pp. 111-122.

[3] Feenstra, Robert C. and Tracy R. Lewis (1991), "Distributing the Gains from Trade with Incomplete information," *Economics and Politics* 3, pp. 21-39.

[4] Grandmont, J. M., and D. McFadden (1972), "A Technical Note on Classical Gains from Trade," *Journal of International Economics*, 2, pp. 109-125.

[5] Kemp, Murray C. (1962) "The Gain from International Trade," *Economic Journal*, 72, pp. 803-819.

[6] Wong, Kar-yiu (1995), *International Trade in Goods and Factor Mobility*, Cambridge, Mass.: MIT Press.

[7] Wong, Kar-yiu (1997), "Gains from Trade with Lump-Sum Compensation." *Japanese Economic Review,* 48, pp. 132–146.

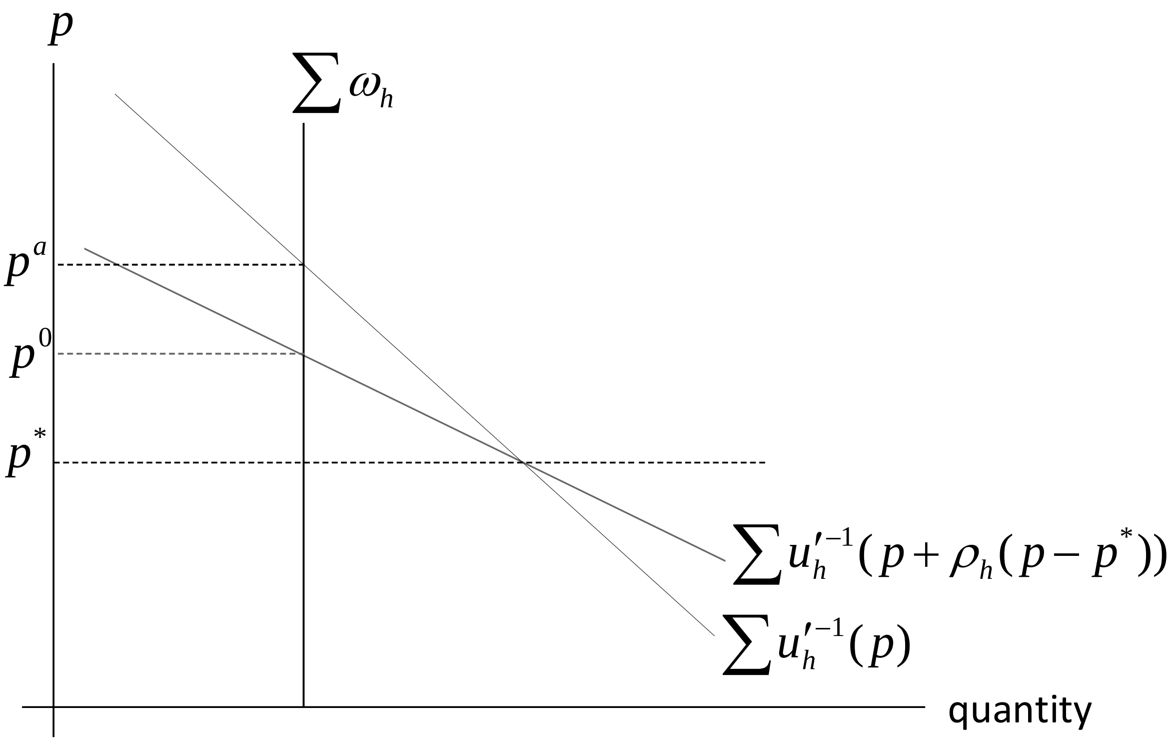

Figure 1: Lemma 1: the equilibrium price in Period 0 (under autarky)