A Factor Endowment Theory of International Trade under Imperfect Competition and Increasing Returns*

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Abstract

Constructing a two-good (a competitive and monopolized goods), two-primary factor (capital and labor) and two-country model of international trade where the monopolized sector is subject to increasing returns to scale, we establish an oligopolistic version of the Heckscher-Ohlin Theorem.

1 Introduction

The determination of trade pattern is a central topic in trade theory. The Heckscher-Ohlin Theorem has been one of the fundamental theorems. Since the Theorem is based on the assumptions of perfect competition and constant returns to scale, it is quite natural to ask whether the Theorem, or its modified version, is still true even under conditions of imperfect competition and increasing returns.

The answers that have been so far proposed are affirmative. Lahiri and Ono (1995) and Shimomura (1998) considered two-country models in which there is an oligopolistic and increasing-returns-to-scale sector in each country and proved their oligopolistic versions of the Heckscher-Ohlin Theorem. However, both of them assumed free entry in any production sector. Thus, it has been an open question whether the factor endowment theory of international trade survives imperfect competition and increasing returns when entry into the increasing-returns-to-scale sector is assumed to be impossible and the industry is monopolized by a fixed number of firms.[1]

The purpose of this paper is to prove that we can still give an affirmative answer to the above question even in the case of prohibited entry into the increasing-returns-to-scale sector. More specifically, constructing a two-country by two-factor by two-sector general equilibrium model of international trade in which one sector is monopolized under conditions of increasing returns to scale while the other sector is perfectly competitive under conditions of constant returns to scale, we establish an oligopolistic version of the Heckscher-Ohlin Theorem.

The Lahiri-Ono-Shimomura result and the main result in this paper together imply that the traditional factor endowment theory of international trade survives imperfect competition and increasing returns.

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[1]In what follows, we normalize the fixed number to be unity.

Let us relate this paper to the literature on general equilibrium models of trade under imperfect competition. It was Melvin and Warne (1973) who established the basic two-factor and two-good general equilibrium framework with monopolized sectors. Then, Markusen (1981) provided the now-classical trade-pattern proposition in the framework. Assuming that constant returns to scale prevail in both monopolized and competitive sectors and that preferences in each country are described by a CES utility function with the elasticity of consumption substitution being greater than one, Markusen proved that in the free-trade Cournot-Nash equilibrium a large country imports the monopolized good.

On the other hand, Kemp and Shimomura (2002) assumed that the monopolized sector is subject to increasing returns to scale and showed that the large country exports the increasing-returns good. The difference between Markusen and Kemp and Shimomura come from the fact that identical agents in each country behave cooperatively.[2]

Wong (1995) derived an oligopolistic Heckscher-Ohlin Theorem under the condition that the production possibility curves of trading countries intersect at the production point which would be realized in an autarkic equilibrium in one of the countries.

Those contributions to the determination of trade pattern under imperfect competition leave a couple of problems unexplored.

First, the above Markusen trade-pattern proposition does not cover the case in which factor endowment ratios differ between the trading countries, say the home and foreign countries. While Wong considers the difference, he did that only under the aforementioned specific condition concerning the relative positions of the two production possibility curves. Naturally one may want to make clear the relationship between the pattern of international trade and the international distribution of factor endowments when there is an *arbitrary* difference in factor endowment ratios between the home and foreign countries.

Second, except for Kemp and Shimomura (2002), the pattern of international trade was studied only for the case in which constant returns prevail everywhere. On the other hand, the present paper derives an oligopolistic Heckscher-Ohlin theorem under the assumption that the monopolized sector in each country is subject to increasing returns to scale.

Let us outline our main results. First, we establish a fundamental proposition concerning the relationship between the home and foreign Cournot-Nash equilibrium outputs and the international distribution of factor endowments. Let $(\bar{K}, \bar{L})$ and (K^*, L^*) be pairs of home and foreign capital and labor endowments and (Y^N, Y^{*N}) be a pair of Cournot-Nash equilibrium pair of home and foreign outputs in monopolized sectors. The fundamental proposition asserts that there exists a positive value, say γ, such that if the foreign country is more labor (resp. capital)-abundant country in a modified sense such that

$$(K^* - \bar{K}) < (resp. >) \gamma (L^* - \bar{L})$$

then $Y^N < Y^{*N}$ if the monopolized good is labor-intensive (resp. capital-intensive) than the competitive good. The main difference between the standard Heckscher-Ohlin model and the present model is in that while γ is equal to $\bar{K}/\bar{L}$ in the former model, it is equal to the capital-labor ratio in the competitive sector.

After we prove the fundamental proposition, we proceed to establish our oligopolistic Heckscher-Ohlin Theorem. Assume that the income effects on the demand for the monopolized good is sufficiently weak. Then, the fundamental proposition implies the oligopolistic Heckscher-Ohlin Theorem as follows: See Figure 1, where the slope of $mE\bar{m}$ (resp. $lE\bar{l}$) evaluated at E is the capital/labor ratio of the labor- (resp. capital-) intensive good. Suppose that the monopolized good is capital-intensive. Then if the foreign country is labor-(resp.

[2]See Kemp and Long (1992) and Kemp and Shimomura (1995).

capital-) abundant in the (modified) sense such that its factor endowment point is below (resp. above) $mE\bar{m}$

The organization of this paper is as follows. Section 2 sets up the model. Section 3 proves the fundamental proposition. Section 4 establishes the oligopolistic version of the Heckscher-Ohlin Theorem. Section 5 discusses the relationship between our theorem and Markusen's which has been the most important contribution in this field. Section 6 makes a couple of concluding remarks.

2 The Model

The model is a standard general equilibrium system which is often used in trade theory. The trading world consists of two countries, say the home and foreign countries. There are two tradable goods, the monopolized and competitive goods, and two primary factors of production, labor L and capital K.

2.1 The assumptions concerning technologies, factor endowments and preferences

2.1.1 production technologies

The production technology of the monopolized good is described by a homothetic production function

$$Y_1 = F(f^1(K_1, L_1)), \tag{1}$$

where $F(\cdot)$ is twice-differentiable, monotonely increasing and strictly convex in f^1, $F(0) = 0$ and $f^1(\cdot)$ is twice-differentiable, monotonely increasing, quasi-concave and linearly homogeneous in K_1 and L_1. Thus, increasing returns to scale prevail in the monopolized sector. On the other hand, the production technology of the competitive good is described by a linearly homogeneous production function

$$Y_2 = f^2(K_2, L_2), \tag{2}$$

where $f^2(\cdot)$ is twice-differentiable, monotonely increasing and quasi-concave in K_2 and L_2. Thus, the competitive sector is subject to constant returns to scale.

There is no technological difference between the home and foreign countries.

2.1.2 factor endowments

Factor endowments are assumed to be internationally immobile (but domestically mobile), exogenously given and constant in each country. The full employment conditions in the home country are

$$\bar{L} = L_1 + L_2 \tag{3}$$
$$\bar{K} = K_1 + K_2, \tag{4}$$

where $\bar{K}$ and $\bar{L}$ are the home capital and labor endowments.

2.1.3 Preferences

There is no difference in preferences between the home and foreign countries in the sense that the home community utility function,

$$u = U(C_1, C_2),$$

where C_1 (resp. C_2) is the home consumption of the monopolized (resp. competitive) good, has the same shape as the foreign community utility function[3]

$$u^* = U(C_1^*, C_2^*)$$

The function $U(\cdot)$ is assumed to have all standard properties as a utility function. Denoting by p and I (resp. I^*) the price of the monopolized good in terms of the competitive good and the home (resp. foreign) national income. The Marshallian demand functions of the monopoly good in the home and foreign countries are

$$C_1 = D(p, I) \text{ and } C_1^* = D(p, I^*). \tag{5}$$

The population of each country is normalized to be one, and, for simplicity, in what follows, we use the word "monopolist" in the meaning of the representative oligopolistic firm in each country.

2.2 The general equilibrium model

Under free trade and for a given pair of the home and foreign monopolized outputs (Y_1, Y_1^*), the world market-clearing condition for the monopolized good is

$$D(p, I) + D(p, I^*) = Y_1 + Y_1^*, \tag{6}$$

where I and I^* are

$$I \equiv pY_1 + Y_2 \text{ and } I^* \equiv pY_1^* + Y_2^*, \tag{7}$$

Solving (6) for p, we derive the inverse demand function

$$p = \Gamma(Y_1 + Y_1^*, I, I^*). \tag{8}$$

Using the inverse demand function, we can write the objective function of the home monopolist as

$$\Pi \equiv \Gamma(Y_1 + Y_1^*, I, I^*)Y_1 - (wL_1 + rK_1), \tag{9}$$

where r and w are the rental and wage rates. She seeks to maximize (9), under her subjective assumption that the foreign output level, the factor prices and the home and foreign incomes are unaffected by her choice of Y_1[4].

[3] In what follows we attach an asterisk (*) to the variables belonging to the foreign country

[4] The assumption is standard in the literature. See Melvin and Warne (1973) and Markusen (1981).

We assume that there exists an interior solution[5] that maximizes (9), the first-order condition is

$$\Gamma_y(Y_1+Y_1^*,I,I^*)Y_1+\Gamma(Y_1+Y_1^*,I,I^*)-\frac{\partial}{\partial Y_1}(wL_1+rK_1)=0, \tag{10}$$

where Γ_y denotes the partial derivative of Γ with respect to $(Y_1+Y_1^*)$. Similarly, we can obtain the first-order condition for the foreign monopolist.

$$\Gamma_y(Y_1+Y_1^*,I,I^*)Y_1^*+\Gamma(Y_1+Y_1^*,I,I^*)-\frac{\partial}{\partial Y_1^*}(w^*L_1^*+r^*K_1^*)=0. \tag{11}$$

Due to the assumption that the home and foreign monopolist thinks that the effect of her choice of output on factor prices is unaffected, the home and foreign production points, (Y_1,Y_2) and (Y_1^*,Y_2^*), must be both on the production possibility curves of the respective countries and the slopes of the curves at the points are equal to the ratios of marginal costs of the two goods. That is, defining the home production possibility curve as

$$\begin{aligned} G(\phi(Y_1),\bar{L},\bar{K}) &\equiv \max_{K_i,L_i,\ i=1,2} f^2(K_2,L_2) \\ \text{subject to } \phi(Y_1) &\leq f^1(K_1,L_1) \\ \bar{L} &\geq L_1+L_2 \\ \bar{K} &\geq K_1+K_2, \end{aligned} \tag{12}$$

we see that

$$\begin{aligned} \frac{\partial}{\partial Y_1}G(\phi(Y_1),\bar{L},\bar{K}) &= \frac{\partial G}{\partial\phi(Y_1)}\phi'(Y_1) \\ &= -\frac{(\text{the marginal cost of the monopolized good})}{(\text{the marginal cost of the competitive good})}. \end{aligned} \tag{13}$$

Since the production of the competitive good is subject to constant returns to scale and we assume that the competitive good serves as the numeraire, the denominator of the right-hand side of (13) is unity. Therefore, the first-order condition (10) can be rewritten to

$$\begin{aligned} \Psi(Y_1,Y_1^*,I,I^*,\bar{L},\bar{K}) &\equiv \Gamma_y(Y_1+Y_1^*,I,I^*)Y_1+\Gamma(Y_1+Y_1^*,I,I^*) \\ &\quad +G_\phi(\phi(Y_1),\bar{L},\bar{K})\phi'(Y_1) \\ &= 0, \end{aligned} \tag{14}$$

where

$$G_\phi(\phi(Y_1),\bar{L},\bar{K}) \equiv \frac{\partial G}{\partial\phi(Y_1)}.$$

[5] Y_1 is interior if $0<Y_1<\bar{Y}_1\equiv F(f^1(\bar{K},\bar{L}))$, the maximum output of good 1.

Making a parallel argument, we obtain the first-order condition concerning the foreign monopolist

$$\begin{aligned}\Psi^*(Y_1, Y_1^*, I, I^*, L^*, K^*) &\equiv \Gamma_y(Y_1 + Y_1^*, I, I^*)Y_1^* + \Gamma(Y_1 + Y_1^*, I, I^*) \\ &\quad + G_\phi(\phi(Y_1^*), L^*, K^*)\phi'(Y_1^*) \\ &= 0. \end{aligned} \tag{15}$$

Considering the definitions of the home and foreign national incomes and production possibility curves, we can write the home and foreign national incomes as the functions of the home and foreign outputs and factor endowments, $\Omega \equiv (Y_1, Y_1^*, \bar{L}, \bar{K}, L^*, K^*)$ [6]

$$I(\Omega), I^*(\Omega), \tag{16}$$

The substitution of (16) into (14) and (15) yields

$$\begin{aligned}\tilde{\Psi}(\Omega) &\equiv MR(\Omega) + G_\phi(\phi(Y_1), \bar{L}, \bar{K})\phi'(Y_1) \\ &= 0 \qquad (17) \\ \tilde{\Psi}^*(\Omega) &\equiv MR^*(\Omega) + G_\phi(\phi(Y_1^*), L^*, K^*)\phi'(Y_1^*) \\ &= 0 \qquad (18)\end{aligned}$$

where

$$MR(\Omega) \equiv \Gamma_y(Y_1 + Y_1^*, I(\Omega), I^*(\Omega))Y_1 + \Gamma(Y_1 + Y_1^*, I(\Omega), I^*(\Omega)) \tag{19}$$

$$MR^*(\Omega) \equiv \Gamma_y(Y_1 + Y_1^*, I(\Omega), I^*(\Omega))Y_1^* + \Gamma(Y_1 + Y_1^*, I(\Omega), I^*(\Omega)) \tag{20}$$

(17) and (18) determine the Cournot-Nash equilibrium home and foreign outputs of the monopolized goods, Y_1^N and Y_1^{*N}, for given home and foreign factor endowments, $\bar{L}, \bar{K}, L^*$, and K^*.

[6] Making use of the production possibility curve, the home and foreign national incomes are

$$I = pY_1 + G(\phi(Y_1), \bar{K}, \bar{L}), \;\; I^* = pY_1^* + G(\phi(Y_1^*), K^*, L^*)$$

The substitution of these equalities into (8) yields

$$p = \Gamma(Y_1 + Y_1^*, pY_1 + G(\phi(Y_1), \bar{K}, \bar{L}), pY_1^* + G(\phi(Y_1^*), K^*, L^*)$$

Solving this equation for p for a given Ω and denote the solution by $p(\Omega)$. We then define

$$I(\Omega) \equiv p(\Omega)Y_1 + G(\phi(Y_1), \bar{K}, \bar{L}), I^*(\Omega) \equiv p(\Omega)Y_1^* + G(\phi(Y_1^*), K^*, L^*).$$

Finally we note that the standard competitive general equilibrium theory ensures us that under some mild conditions on utility functions $p(\Omega)$ exists uniquely. Though it has been customary to assume that any monopolist takes national income and factor prices as given, imposing restriction is open to a critique that it is unrealistic. Based on this view, Tawada and Okawa (1995) examined the role of income effects in models including Markusen's.

In what follows, we make the following assumptions mainly concerning the existence and stability of the Cournot-Nash equilibrium positive output pair (Y_1^N, Y_1^{*N}).

Assumption 1: There is a neighborhood of a given home factor endowment pair $(\bar{K},\ \bar{L})$, say $\Theta(\bar{K},\ \bar{L})$, such that for any foreign factor endowment pair (K^*, L^*) in $\Theta(\bar{K},\ \bar{L})$ there exists a unique Cournot-Nash equilibrium pair (Y_1^N, Y_1^{*N}) for which production in each country is incompletely specialized.

Assumption 2: The partial derivatives of $\tilde{\Psi}(Y_1, Y_1^*, \bar{L}, \bar{K}, L^*, K^*)$ and $\tilde{\Psi}^*(Y_1, Y_1^*, \bar{L}, \bar{K}, L^*, K^*)$ with respect to Y_1 and Y_1^*. respectively, i.e., $\partial\tilde{\Psi}/\partial Y_1$ and $\partial\tilde{\Psi}^*/\partial Y_1^*$, are both negative at the equilibrium point (Y_1^N, Y_1^{*N}).

Assumption 3: The locus (17) is steeper than the locus (18) on the $(Y_1,\ Y_1^*)$-plane where Y_1 is measured horizontally at the Cournot-Nash equilibrium pair.

$$\left| \left. \frac{\partial\tilde{\Psi}/\partial Y_1}{\partial\tilde{\Psi}/\partial Y_1^*} \right|_{(Y_1^N, Y_1^{*N})} \right| > \left| \left. \frac{\partial\tilde{\Psi}^*/\partial Y_1}{\partial\tilde{\Psi}^*/\partial Y_1^*} \right|_{(Y_1^N, Y_1^{*N})} \right| \tag{21}$$

For brevity, we subsequently focus only on the case in which the loci (17) and (18) are negatively sloped. As far as Assumptions 1-3 hold, we can obtain the same main results in this paper even if the loci are positively sloped.

3 The Fundamental Proposition

Let us first consider the relationship between the international distribution of factor endowments $(\bar{L}, \bar{K}, L^*, K^*)$ and the Cournot-Nash equilibrium output pair (Y_1^N, Y_1^{*N}).

To do so, let us consider the following system of equations that is slightly different from the system (17) and (18):

$$MR(\Omega) + G_\phi(\phi(Y_1), \bar{L}, \bar{K})\phi'(Y_1) = 0 \tag{22}$$
$$MR^*(\Omega) + G_\phi(\phi(Y_1^*), \bar{L}, \bar{K})\phi'(Y_1^*) = 0. \tag{23}$$

Note that (22) is the same as (17). Considering the definitions of the home and foreign marginal revenues $MR(\Omega)$ and $MR^*(\Omega)$ in (19) and (20), we see

$$MR(y, y, \bar{L}, \bar{K}, L^*, K^*) \equiv MR^*(y, y, \bar{L}, \bar{K}, L^*, K^*) \text{ in } y.$$

Therefore, for any solution y_0 to the following equation,

$$MR(y_0, y_0, \bar{L}, \bar{K}, L^*, K^*) + G_\phi(\phi(y_0), \bar{L}, \bar{K})\phi'(y_0) = 0,$$

the pair $(Y_1, Y_1^*) = (y_0, y_0)$ satisfies both (22) and (23). It follows that the loci of (22) and (23) have to intersect with each other on the 45^o line, as is depicted in Figure 2.

Now let us compare the loci, (23) and (18). If the international difference in factor endowments is not very large, the relative position of the two loci depends on the sign of

$$-G_\phi(\phi(y_0), L^*, K^*) - [-G_\phi(\phi(y_0), \bar{L}, \bar{K})] \tag{24}$$

For example, if this sign (24) is positive, we see that

$$\begin{aligned} 0 &= MR^*(y_0, y_0, \bar{L}, \bar{K}, L^*, K^*) + G_\phi(\phi(y_0), \bar{L}, \bar{K})\phi'(y_0) \\ &> MR^*(y_0, y_0, \bar{L}, \bar{K}, L^*, K^*) + G_\phi(\phi(y_0), L^*, K^*)\phi'(y_0). \end{aligned}$$

It follows from Assumptions 2 and 3 that there is a positive ε such that, by changing Y_1 from y_0 to $(y_0 - \varepsilon)$,

$$MR^*(y_0 - \varepsilon, y_0, \bar{L}, \bar{K}, L^*, K^*) + G_\phi(\phi(y_0), L^*, K^*)\phi'(y_0) = 0$$

That is, the locus (18) is located below the locus (23), like the line AE_2B in Figure 2. Making a parallel argument, we see that the locus (18) is located above the locus (23), if (24) is negative. Therefore, either E_1 or E_2 is the Cournot-Nash equilibrium point, i.e., the solution to the system of equations (17) and (18). Inspecting the two intersections E_1 and E_2, we have the lemma.

Lemma 1. *$Y_1^N >$ (resp. $<$)Y_1^{*N}, if*

$$-G_\phi(\phi(y_0), L^*, K^*) - [-G_\phi(\phi(y_0), \bar{L}, \bar{K})] > \text{(resp. } <)0.$$

Now, let us examine what determines the sign of (24). If the absolute values of $\Delta L \equiv L^* - \bar{L}$ and $\Delta K \equiv K^* - \bar{K}$ are not large, we have

$$\begin{aligned} & sign\Big\{ - G_\phi(\phi(y_0), L^*, K^*) - [-G_\phi(\phi(y_0), \bar{L}, \bar{K})]\Big\} \\ = \;& -sign\Big\{G_{\phi L}(\phi(y_0), \bar{L}, \bar{K})\Delta L + G_{\phi K}(\phi(y_0), \bar{L}, \bar{K})\Delta K\Big\} \\ = \;& -sign\Big\{G_{\phi K}\Delta L\Big(\frac{G_{\phi L}}{G_{\phi K}} + \frac{\Delta K}{\Delta L}\Big)\Big\}. \end{aligned} \tag{25}$$

Moreover, by making some calculations, we can derive[7]

$$sign[G_{\phi K}] = sign\Big[\frac{K_1}{L_1} - \frac{K_2}{L_2}\Big] \text{ and } \frac{G_{\phi L}}{G_{\phi K}} = -\frac{K_2}{L_2},$$

That is, $G_{\phi K}$ is positive (resp. negative) if the monopolized good is more capital-intensive (resp. labor-intensive) than the competitive good, and $-\frac{G_{\phi L}}{G_{\phi K}}$ is equal to the factor intensity of the competitive good. Therefore, we can rewrite (25) as

$$\begin{aligned} & sign\Big\{Y_1^N - Y_1^{*N}\Big\} \\ = \;& sign\Big\{ - G_\phi(\phi(y_0), L^*, K^*) - [-G_\phi(\phi(y_0), \bar{L}, \bar{K})]\Big\} \\ = \;& sign\Big\{\Big(\frac{K_1}{L_1} - \frac{K_2}{L_2}\Big)\Delta L\Big(\frac{K_2}{L_2} - \frac{\Delta K}{\Delta L}\Big)\Big\}, \end{aligned} \tag{26}$$

[7]For these properties of the production possibility curve, see Long (1982).

which ensures us that the following proposition is established.

Proposition. *See Figure 1. There is a border line in a neighborhood of the home factor endowment point $(\bar{K}, \bar{L})$ such that if the foreign country is labor- (resp. capital-) abundant in the sense that the foreign factor endowment point (K^*, L^*) is below (resp. above) the border line and the monopolized good is labor- (resp. capital-) intensive, then $Y_1^{*N} > Y_1^N$. If the foreign country is labor- (resp. capital-) abundant but the monopolized good is capital- (resp. labor-) intensive, then $Y_1^{*N} < Y_1^N$. The slope of the border line at E is equal to the capital/labor ratio of the competitive good.*

Proof. See Figure 3A, where we assume that the monopolized good is labor-intensive, $r' < 0$. Thus, we can depict the border line like $lE\bar{l}$ the slope of which, K_2/L_2, is larger than $\bar{K}/\bar{L}$.

Let us focus on the area above the border line. First, take any point in the area $SE\bar{l}$. There, we see that $\Delta L \equiv L^* - \bar{L} > 0$ and $\Delta K/\Delta L > K_2/L_2$. It follows from (26) that

$$Y_1^N - Y_1^{*N} > 0 \tag{27}$$

Next, take any point in the area KES, where $\Delta K/\Delta L < 0$ and $\Delta L < 0$. It follows from (26) that $Y_1^N > Y_1^{*N}$. Finally, take any point in the area lEO, where $\Delta L < 0$ and $\Delta K/\Delta L < 0 < K_2/L_2$. Again, $Y_1^N - Y_1^{*N} > 0$.

A parallel argument can be made to prove that $Y_1^N < Y_1^{*N}$ below the border line. Similarly, we can prove the proposition in the case of the capital-intensive monopolized good, $K_1/L_1 - K_2/L_2 > 0$. See Figure 3B. Note that, as is shown in the figure, the border line is $mE\bar{m}$ in that case. ■

Remark 1. *y_0 depends on (K^*, L^*) and $(\bar{K}, \bar{L})$. Therefore, considering that K_1/L_2 in (26) depends on y_0, i.e., $K_2/L_2 \equiv k_2(y_0)$, one may naturally wonder whether $k_2(y_0)$ and $\Delta K/\Delta L$ are independent with each other. To make clear that they are independent, let us consider the symmetric system*

$$\begin{aligned} MR(Y_1, Y_1^*, \bar{L}, \bar{K}, \bar{L}, \bar{K}) - MC(Y_1, \bar{L}, \bar{K}) &= 0 \\ MR^*(Y_1, Y_1^*, \bar{L}, \bar{K}, \bar{L}, \bar{K}) - MC(Y_1^*, \bar{L}, \bar{K}) &= 0. \end{aligned}$$

The solution $(Y_1, Y_1^*) = (\tilde{y}_0, \tilde{y}_0)$ in the 45^o-line and $\tilde{y}_0$ is apparently independent of (K^*, L^*). Therefore, we can choose (K^*, L^*) in a neighborhood of $(\bar{K}, \bar{L})$ whichever $k_2(y_0) > \Delta K/\Delta L$ or $k_2(y_0) < \Delta K/\Delta L$ holds. Since by making $|\Delta K|$ and $|\Delta L|$ small while keeping $\Delta K/\Delta L$ constant, we can make $|\tilde{y}_0 - y_0|$ small as well. Hence, we can choose (K^*, L^*) in a neighborhood of $(\bar{K}, \bar{L})$ whichever $k_2(y_0) > \Delta K/\Delta L$ or $k_2(y_0) < \Delta K/\Delta L$ holds.

4 The Oligopolistic Heckscher-Ohlin Theorem

Now, let us derive the main result in this paper. Consider the "virtual" equilibrium point (y_0, y_0) in Figure 2. We have the "virtual" world market-clearing condition for the monopolized good.

$$D(p_0, p_0 y_0 + G(\phi(y_0), \bar{K}, \bar{L})) + D(p_0, p_0 y_0 + G(\phi(y_0), K^*, L^*)) = y_0 + y_0,$$

where p_0 denotes the "virtual" equilibrium price. Apparently, for some I, there is no international trade in the "virtual" equilibrium:

$$D(p_0, I) = y_0,$$

and we denote I which satisfies this equation by I_0. Then, as we did in the previous section, let us change the foreign factor endowments from $(\bar{K}, \bar{L})$ to the real pair (K^*, L^*). As a result, not only the equilibrium output pair changes from (y_0, y_0) to (Y_1^N, Y_1^{*N}), but also the equilibrium price and home and foreign incomes also change. Thus, when $|\Delta K|$ and $|\Delta L|$ are small, the differences between the "virtual" and real equilibrium values approximately satisfy

$$\begin{aligned} & \Delta D + \Delta D^* \\ = & \Big(\frac{\partial D}{\partial p}\Big|_{p_0, I_0} \times \Delta p + \frac{\partial D}{\partial I}\Big|_{p_0, I_0} \times \Delta I \Big) \\ & + \Big(\frac{\partial D}{\partial p}\Big|_{p_0, I_0} \times \Delta p + \frac{\partial D}{\partial I}\Big|_{p_0, I_0} \times \Delta I^* \Big) \\ = & \Delta Y_1 + \Delta Y_1^*, \end{aligned} \tag{28}$$

where Δ represents the deviation between the Cournot-Nash and "virtual" variables, e.g., $\Delta D \equiv D^N - D(p_0, I_0)$, $\Delta p \equiv p^N - p_0$, and $\Delta I \equiv I^N - I_0$. Assume that the community utility function is such that the income effects on the home and foreign Marshallian demands for the monopolized good are weak enough to satisfy

$$sign[\Delta D] = sign\Big[\frac{\partial D}{\partial p}\Big|_{p_0, I_0} \times \Delta p \Big] = sign[\Delta D^*]. \tag{29}$$

We are ready for proving the oligopolistic Heckscher-Ohlin Theorem.

Theorem (The Oligopolistic Heckscher-Ohlin Theorem). *Suppose that Assumptions 1-3 are satisfied and that the income effects are sufficiently small so that (29) holds. The country which is capital- (resp. labor-) abundant in the sense stated in Proposition exports the capital- (resp. labor-) intensive good.*

Proof. First of all, note that, as is clear from Figure 2,

$$sign[\Delta Y_1] = -sign[\Delta Y_1^*] \tag{30}$$

Suppose that $\Delta Y_1 (\equiv Y_1^N - y_0) > 0$. It follows from (30) that $\Delta Y_1^* (\equiv Y_1^{*N} - y_0) < 0$. If $\Delta D > 0$, (29) implies that $\Delta D^* > 0$. Therefore, letting D^N and D^{*N} be the consumption at the Cournot-Nash equilibrium, we have

$$\begin{aligned} \Delta D^* - \Delta Y_1^* &= (+) - (-) \\ &= (D^{*N} - y_0) - (Y_1^{*N} - y_0) \\ &= D^{*N} - Y_1^{*N} \\ &= Y_1^N - D^N > 0. \end{aligned}$$

If $\Delta D < 0$,

$$\begin{aligned}\Delta D - \Delta Y_1 &= (-) - (+) \\ &= (D^N - y_0) - (Y_1^N - y_0) \\ &= D^N - Y_1^N < 0.\end{aligned}$$

Therefore, whether ΔD is positive or negative, we have

$$Y_1^N - D^N > 0, \text{ if } \Delta Y_1 > 0.$$

A parallel argument can be made for the case $\Delta Y_1 = Y_1^N - y_0 < 0$ and we obtain

$$Y_1^N - D^N < 0, \text{ if } \Delta Y_1 < 0$$

It follows that

$$\begin{aligned}sign[\Delta Y_1 - \Delta Y_1^*] &= sign[Y_1^N - Y_1^{*N}] \\ &= sign[Y_1^N - D^N]\end{aligned}$$

Therefore, Proposition implies the Theorem. ■

Remark 2. *An example of the utility function that implies the demand function with the zero income effect on the demand for the monopolized good is a quasi-linear function*

$$u(C_1, C_2) \equiv V(C_1) + aC_2,$$

where $V(C_1)$ is an increasing and strictly concave function and a is a positive constant. In this case the fundamental proposition directly implies the oligopolistic Heckscher-Ohlin Theorem.[8]

5 A Discussion

As mentioned in introduction, it is fair to say that Markusen (1981) is the first to reveal trade patterns in an oligopolistic setting although he did not allow for any difference in countries' relative factor endowments. According to him, the large country imports the monopolized good irrespective of the factor intensity ranking between goods. But what figures 3 and 4 indicate is that the large country exports the monopolized good as long as both countries' factor endowment ratio is equal and preferences have sufficiently small

[8] In the present specified case, we can construct an example in which the Cournot-Nash equilibrium uniquely exists with the second-order condition for profit maximization satisfied. A proof for it is available from the authors on request.

income effects. Thus, at first sight, one may conjecture that there exists an inconsistency between our trade-pattern theorem and Markusen's. The principal purpose of this section is to point out the incorrectness of such a conjecture by comparing two results.

First of all, we briefly review Markusen's proof of trade-pattern theorem.[9] It consists of two steps. See figure 5 in which two countries' production possibility frontiers are depicted. Suppose that the home country is small while the foreign is large and that the home production point under free trade is given by A. As a first step, he proves that the foreign counterpart must be on segment BC. And as a second step, he shows that the ray from the origin which represents the equilibrium consumption ratio of both countries must be in the cone EOF due to the market-clearing condition. As a result, the small (home) country exports the monopolized good (good 1) whereas the large (foreign) country the competitive good (good 2). Therefore, in each step, constant returns and homotheticity of preferences play a significant role.

Then, what if we relax one of these restrictions keeping the identical relative factor endowments? First, we replace the assumption of constant returns with that of increasing returns. In this case, it is impossible to exclude the foreign production point located between CD. And figures 3 and 4 tell us that the foreign country produces more monopolized good than the home country regardless of factor intensity ranking between goods. Hence, even with homotheticity of preferences maintained, only the introduction of increasing returns can subvert Markusen's trade-pattern theorem.

On the other hand, we examine what will take place when we replace homothetic preferences with quasi-linear ones with constant returns maintained. Note that the foreign production point must be in BC due to constant returns. In this case, because the demand function of good 1 becomes a function of the price only, namely income effects are zero, the market-clearing condition under free trade is given by

$$\begin{aligned} & \tilde{D}(p) + \tilde{D}(p) = Y_1^N + Y_1^{*N} \\ \Rightarrow \quad & Y_1^N - \tilde{D}(p) = \tilde{D}(p) - Y_1^{*N}, \end{aligned}$$

where $\tilde{D}(p)$ denotes the demand function derived from a quasi-linear utility function. This equation implies that the consumption point of both countries must be on the vertical line from the middle point between Y_1^N and Y_1^{*N} as in figure 6. Thus, in this case, the large (foreign) country exports the monopolized good (good 1) while the small (home) country the competitive good (good 2). Again, Markusen's theorem breaks down. It goes without saying that his result is not valid when we introduce both increasing returns and quasi-linearity of preferences. This outcome is summarized in table 1.

From these observations, our theorem maintains consistency with Markusen's. What seems inconsistency between two results stems from the difference of basic assumptions about returns to scale and specification of preferences. Accordingly, it is quite natural to obtain different conclusions because we replace Markusen's assumptions.

6 Concluding Remarks

The factor endowment theory of trade is a core theory in the traditional international economics. The main result in this paper means that the theory still has some validity even if we take into account market imperfection and increasing returns. Let us make a couple of remarks about our main result.

[9] Any interested reader is referred to the original paper of Markusen (1981).

First, the unambiguous result on trade pattern comes from our assumption of weak income effects. One may naturally wonder whether or not we can obtain a similar result when income effects are not weak. Fujiwara and Shimomura (2002) show that a modified version of the oligopolistic Heckscher-Ohlin Theorem can be derived even if the community utility function is homothetic so that income effects are significant.

Second, our main result proposes a new hypothesis concerning the determination of trade pattern which is empirically testable at least in principle. Roughly speaking, if for a positive value γ, $K^* - \bar{K}$ is smaller (resp. larger) than $\gamma(L^* - \bar{L})$, the foreign (resp. home) country exports (resp. imports) the labor-(resp. capital-) intensive good. If the volumes of factors of production are measurable, we can accept or reject this statement based on statistical data, which is one of our research agenda in future.

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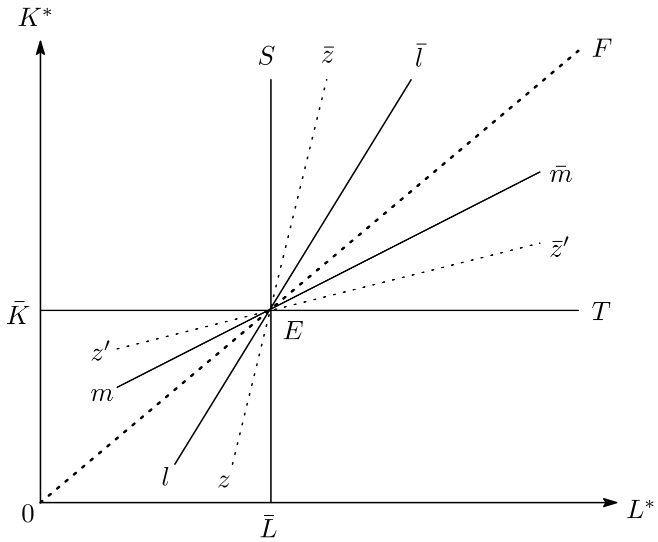

Figure 1:

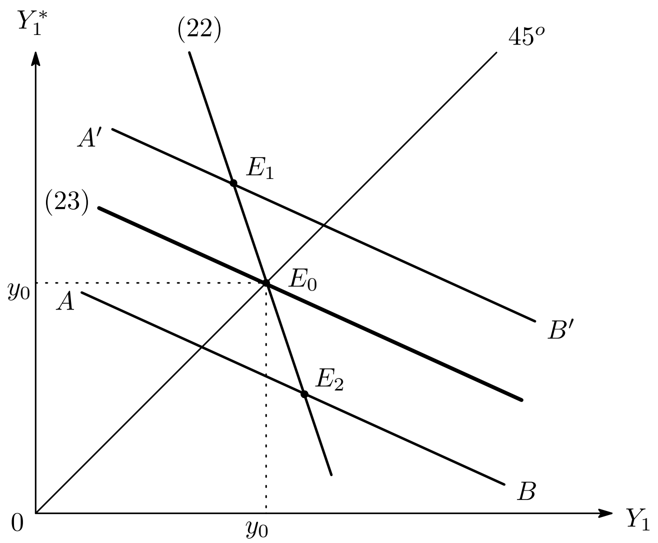

Figure 2: AB (resp. $A'B'$) is the locus of (18) if (24) is positive (resp. negative).

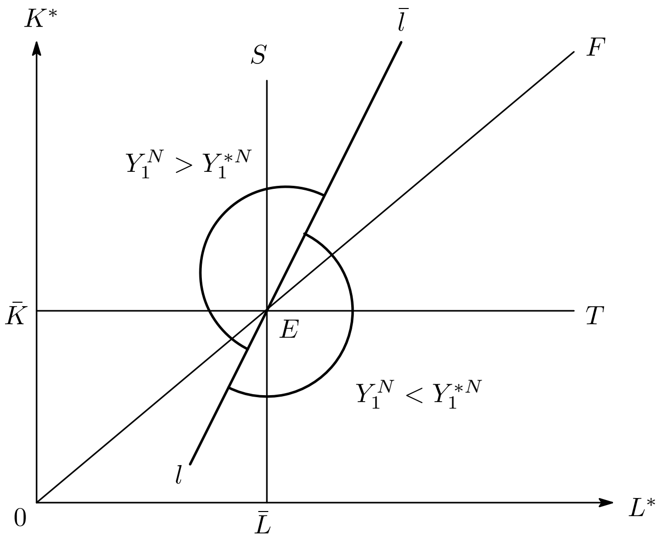

Figure 3: The monopolized good is labor-intensive.

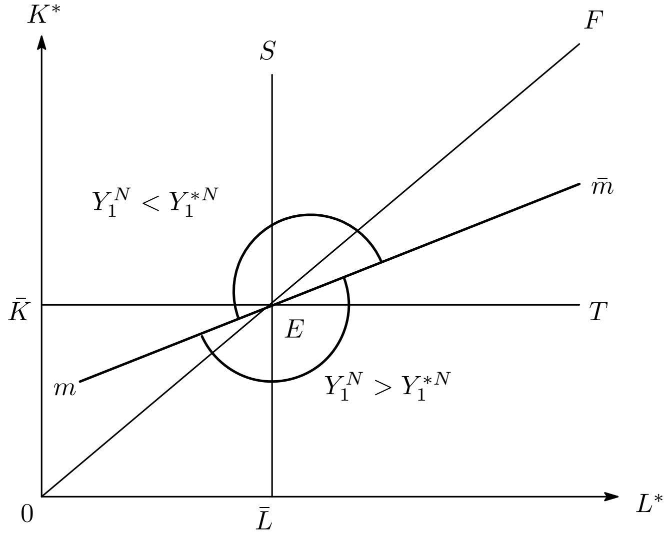

Figure 4: The monopolized good is capital-intensive.

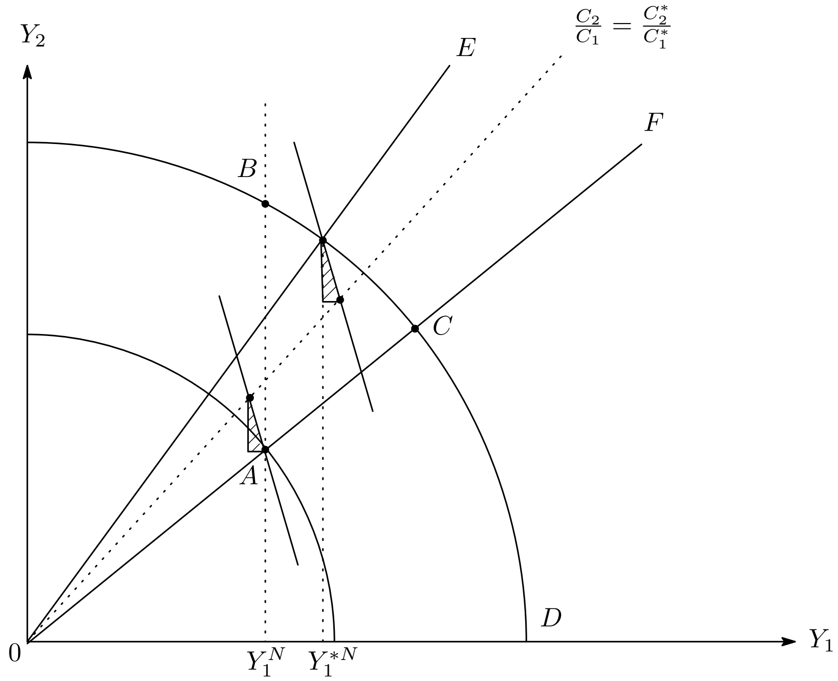

Figure 5: Markusen's Trade-Pattern Theorem

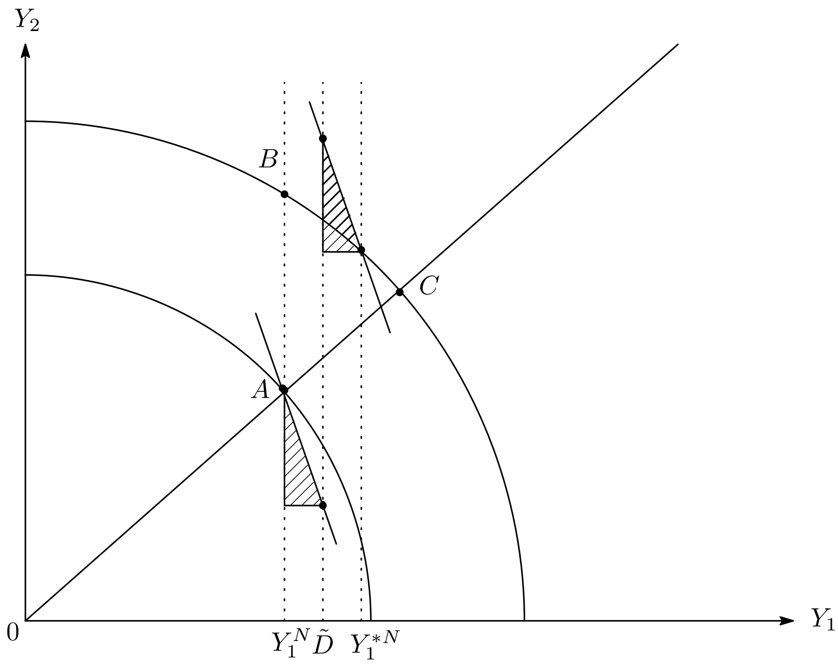

Figure 6: Trade Pattern with Quasi-Linear Preferences

	constant returns	increasing returns
Markusen (1981): homothetic preferences	imports	exports
Fujiwara and Shimomura: quasi-linear preferences	exports	exports

Table 1. Each cell represents whether the large country exports or imports the monopolized good.