Downstream new product development and upstream process innovation

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Abstract

In assembly industries, when input prices are low, downstream firms can easily introduce new products. Because the introduction of new products increases the demand for inputs, upstream firms promote cost-reducing research and development (R&D). We consider both downstream R&D aimed at introducing new products and upstream R&D aimed at cost reduction. We show that if the upstream R&D is efficient (inefficient), the new products introduced downstream become strategic complements (substitutes). Furthermore, because the introduction of new downstream products decreases input prices through upstream R&D, it has a positive endogenous spillover effect on rival downstream firms.

Keywords: New product introduction; Cost-reducing R&D; Upstream input supplier; R&D efficiency; Endogenous spillover

JEL classification: L13; D43; O31

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1 Introduction

In vertical production relationships, research and development (R&D) by the upstream supplier plays a key role. When markets expand due to the introduction or development of new products by downstream firms, the use of inputs increases, and cost-reducing R&D by the upstream firms becomes even more important (Fontana and Guerzoni 2008).[1] In assembly industries, if new products are introduced downstream, they must be capable of promoting upstream investment to reduce the cost of inputs to production. For example, in the automobile industry, batteries are a key input for electric vehicles (EVs). Statharas et al. (2019) find that high prices for batteries can raise the purchase price of EVs considerably. Further, they empirically show that a higher battery cost reduces the market share of battery EVs. To expand EV production, it has been emphasized that a reduction in the price of the batteries is highly important (Japan Small and Medium Enterprise Management Consultant Association (J-SMECA), Hiroshima prefecture branch 2012).

In this paper, we consider the role of upstream cost-reducing R&D when downstream producers introduce new products. We build a model comprising an innovative upstream input supplier and two downstream final good producers that develop new products in a case where all final goods are differentiated. Under this vertical structure, we consider a four-stage game: In the first stage, each downstream firm decides whether to introduce a new product. In the second stage, the upstream firm makes cost-reducing investments. In the third stage, the upstream firm decides on its input price. In the final stage, the downstream firms compete *à la* Cournot.[2]

We show that if all downstream firms introduce new products, the upstream cost-

[1]When market size is large, process innovation becomes a more important innovative activity for firms. In fact, according to Fontana and Guerzoni (2008), in many sectors, the majority of firms involved in process R&D operate in medium–large or large markets.

[2]In addition, we consider Bertrand competition in Section 5.

reducing investment level is maximized and, at the same time, the cheapest input price is achieved. When a downstream firm introduces a new product, the demand for inputs rapidly increases. For example, if all downstream firms introduce new products, input demands double. This rapid expansion of downstream input demand increases the incentives for investment by the upstream firm. As investments in cost-reducing input production rise, upstream production costs fall and hence the input price falls. Therefore, the input price falls to its lowest level or the cheapest price when all downstream firms invest.

Further, we show that a downstream firm's strategic behavior against its rivals in introducing a new product mainly depends on upstream production efficiency. The production efficiency of the upstream supplier that makes an effort to reduce costs is equivalent to the efficiency of its R&D. Suppose that a downstream firm introduces a new product. This behavior increases demand for inputs, so the upstream firm actively invests to reduce its production cost and thus the input price falls. Because the price-cost margin of the downstream firms widens as the input price falls, and results in a scenario similar to a market expansion, the incentive of the rival firm to introduce a new product becomes stronger. Introducing a new product results in the downstream firm stealing some of its rival's market share, which weakens the rival's incentives to introduce a new product. At this time, if the efficiency of the upstream R&D is high, the input price drops rapidly because of the introduction of the new product. Then, this fall of the input price facilitates the rival competing and introducing a new product. Thus, the race to introduce a new product begins to be characterized by strategic complements. Conversely, if the upstream R&D efficiency is low, the effect of stealing the rival's market share is dominant and thus, the race to introduce a new product is characterized by strategic substitutes.

Our model makes two contributions to the literature. The first contribution is to partially extend the framework of d'Aspremont and Jacquemin (1988). d'Aspremont and

Jacquemin (1988) treat R&D spillovers as exogenous, such that one firm's cost-reducing R&D affects the rival firm's marginal cost exogenously. Conversely, in our model, the effect of one firm's product R&D on the rival firm's marginal cost is endogenous. We believe that our model complements the study of d'Aspremont and Jacquemin (1988).

The second contribution of our study concerns the relation between the input price and downstream investment. When the upstream agent determines its price after observing the downstream investment decision, the upstream agent extracts the downstream R&D benefit by setting a higher input price. When decision-making follows this timing, the existing studies consider that the input price becomes higher if downstream firms invest more. It is well known that this behavior by the upstream agent impedes downstream investment (e.g., Banerjee and Lin 2003, Gilbert and Cvsa 2003, Haucap and Wey 2004). However, in our model, despite the fact that we follow the same timing for decision-making, we argue that the more the downstream firms invest, the lower is the input price. Hence, if all downstream firms invest, the input price falls to its the lowest possible level. This occurs because the introduction of the new downstream product rapidly expands the demand for inputs; in response, the upstream supplier increases its R&D into reducing input costs, which leads to the input price falling. Our result indicates the importance of upstream cost-reducing R&D in the vertical production structure.

This paper is related to two strands in the literature. One strand comprises studies on the introduction of new products in an oligopoly (Basak and Mukherjee 2018)[3] and the other comprises studies that focus on upstream innovation in vertically related markets (e.g.,

[3]Dawid et al. (2010) considered new product development in a duopoly setting. However, their model has no upstream sector, and the R&D types differ between the two firms: one firm engages in a project involving new product development and the other firm engages in cost-reducing R&D. Although Dobson and Waterson (1996) and Grossman (2007) considered a similar scenario to ours, in which firms choose their number of differentiated goods, there is no upstream market in these models.

Chen and Sappington 2010, Hu et al. 2020, Macho-Stadler et al. 2021, Pinopoulos 2020, Stefanadis 1997). Basak and Mukherjee (2018) considered the introduction of a new product in a unionized duopoly. They showed that, among many different settings, the strategic substitute equilibrium, such that "one firm only introduces a new product," appears,[4] whereas the strategic complementary equilibrium appears if and only if labor unions are firm specific (i.e., decentralized labor unions) and product differentiation is *asymmetric*. Our model shows that when an upstream supplier engages in cost-reducing investment, both the strategic complements equilibrium and the strategic substitutes equilibrium appear. This is in sharp contrast to the results of Basak and Mukherjee (2018).

Although some researchers have focused on upstream process innovation, their models and purposes differ substantially from ours. Chen and Sappington (2010) considered the effects of vertical integration and separation on upstream innovation. Under a general demand system, Macho-Stadler et al. (2021) examined the relationship between firms' decisions on organizational structures and upstream R&D. Hu et al. (2020) considered upstream R&D, but their purpose was to examine the relationship between upstream cost-reducing investments and cross-holdings among downstream firms. Pinopoulos (2020) analyzed types of input pricing behaviors, for example, two-part tariffs, by an upstream firm that engages in cost-reducing R&D. Stefanadis (1997) investigated R&D competition between two upstream suppliers, and examined the possibility that exclusive supply contracts with a downstream firm discourage upstream innovation.

The remainder of this paper is structured as follows. In Sections 2 and 3, we establish the basic model and present the analysis of the model, respectively. In Section 4, we perform

[4]This case is the same as the scenario in which there is one multi-product firm and one single-product firm. Hence, the strategic substitute type of equilibrium in our new product introduction model includes that scenario. Inomata (2018) and Kawasaki et al. (2014) also considered the coexistence of multi-product and single-product firms.

a welfare analysis and in Section 5, we examine downstream price competition. Finally, in Section 6, we draw conclusions.

2 Model

We consider a vertically related market with an upstream firm (U) and two symmetric downstream firms (Di, $i = 1, 2$). Di uses one unit of input to produce one unit of the final product, and it competes in Cournot fashion.[5] For simplicity, we omit other production costs for Di. U decides the input price w and makes a take-it-or-leave-it offer. For example, in the United States, the Robinson–Patman Act is enforced, so U charges a uniform input price for Di.[6]

U engages in R&D to reduce the constant marginal cost $c \in (0, 1)$. To create demand by introducing a new product, Di chooses whether to conduct R&D by paying a fixed cost f (> 0). Let Di's existing product be $q_{e,i}$ and its new product be $q_{n,i}$. When $D1$ and $D2$ introduce new products, the inverse demand is:[7]

$$\begin{aligned} p_{e,i} &= 1 - q_{e,i} - \gamma(q_{n,i} + q_{e,j} + q_{n,j}), \\ p_{n,i} &= 1 - q_{n,i} - \gamma(q_{e,i} + q_{e,j} + q_{n,j}), \end{aligned} \tag{1}$$

where $p_{e,i}$ ($p_{e,j}$) is the price of the existing product Di (Dj) and $p_{n,i}$ ($p_{n,j}$) is the price of the new product Di (Dj), $i \neq j$ and $i, j = 1, 2$. The parameter γ ($0 \leq \gamma < 1$) measures the degree of product substitutability among final products. Final products are independent if $\gamma = 0$, whereas they are homogeneous if $\gamma = 1$.

[5]Our main results do not alter in Bertrand competition. For more details, see Section 5.

[6]Even if U conducts price discrimination, our results do not alter.

[7]The other possible setting is that the existing and new products are differentiated. The formula in such a case is $p_{e,i} = 1 - (q_{e,i} + q_{e,j}) - \gamma(q_{n,i} + q_{n,j})$, $i, j = 1, 2;\ i \neq j$. However, as our main results do not alter, we use a simpler form (1).

The gross profit of Di is:

$$\pi_{Di}(q_{e,i}, q_{n,i}) \equiv (p_{e,i} - w)q_{e,i} + (p_{n,i} - w)q_{n,i}. \tag{2}$$

If Di innovates, its profit is $\pi_{Di}(q_{e,i}, q_{n,i}) - f$; otherwise, its profit is $\pi_{Di}(q_{e,i}, 0)$.

The profit of U is:

$$\pi_U \equiv (w - (c - x))Q - kx^2, \tag{3}$$

where x is the investment level and kx^2 is the R&D cost. k (> 0) denotes the R&D efficiency. Q is the demand for inputs. $Q = \sum_i q_{e,i}$ if no one innovates. $Q = \sum_i q_{e,i} + q_{n,j}$ if only Dj innovates. $Q = \sum_i q_{e,i} + \sum_i q_{n,i}$ if everyone innovates.

We consider the following four-stage game. In the first stage, $D1$ and $D2$ independently and simultaneously choose whether to conduct R&D by paying the fixed cost (I) or choosing not to pay (N). In the second stage, U decides the investment level. In the third stage, U charges the input price. Finally, downstream firms compete *à la* Cournot.

This timing structure corresponds to the difficulty involved in R&D. Generally, product development requires a sunk cost, such as a long-term contract with researchers, and it takes a long time. Hence, downstream R&D occurs at the first stage. We assume that production of a prototype and repeated safety tests are not required, so the second stage is upstream R&D. The downstream firm can flexibly adjust its production and thus the quantity of final products is decided in the final stage. The solution concept is the subgame perfect Nash equilibrium.

3 Results

Depending on the downstream investment decisions, four regimes can arise: II, IN, NI, and NN. Because downstream firms are symmetric, IN and NI are the same. We call II the *all-product-developers regime*, IN and NI the *mixed regime*, and NN the *no-one-*

invests regime. Using (1)–(3), we obtain the equilibrium outcomes for each regime. The derivations of the equilibrium outcomes in each regime are reported in Appendix B.

The no-one-invests regime: $\boldsymbol{NN}$. Each Di produces only the existing product; hence, $q_{n,i} = 0$. Thus, we obtain the following equilibrium outcomes:

$$\begin{aligned} &w^{NN} = \frac{(1+c)(\gamma+2)k-1}{2(\gamma+2)k-1}; \quad x^{NN} = \frac{1-c}{2(\gamma+2)k-1}; \quad \pi_U^{NN} = \frac{(1-c)^2 k}{2(\gamma+2)k-1}, \\ &q_{e,i}^{NN} = \frac{(1-c)k}{2(\gamma+2)k-1}; \quad \pi_{Di}^{NN} = \left(q_{e,i}^{NN}\right)^2 \quad \text{for } i = 1, 2. \end{aligned} \tag{4}$$

Mixed regime: $\boldsymbol{IN}$ or $\boldsymbol{NI}$. When Di invests and Dj does not, Di produces both existing and new products but Dj only produces its existing product. Therefore, $q_{e,i} > 0$, $q_{n,i} > 0$, $q_{e,j} > 0$, and $q_{n,j} = 0$ ($i, j = 1, 2$ and $i \neq j$). From this, we obtain the following:

$$\begin{aligned} &x^{IN} = x^{NI} = \frac{(1-c)(3-\gamma)}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}, \\ &w^{IN} = \frac{2(1+c)(2+2\gamma-\gamma^2)k-(3-\gamma)}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}; \quad \pi_U^{IN} = \frac{(1-c)^2(3-\gamma)k}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}, \\ &q_{e,1}^{IN} = q_{n,1}^{IN} = \frac{(1-c)(2-\gamma)k}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}; \quad q_{e,2}^{IN} = \frac{2(1-c)k}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}, \\ &\pi_{D1}^{IN} = \frac{2(1-c)^2(2-\gamma)^2(1+\gamma)k^2}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2}; \quad \pi_{D2}^{IN} = \left(q_{e,2}^{IN}\right)^2. \end{aligned} \tag{5}$$

Note that $w^{IN} = w^{NI}$, $q_{e,2}^{NI} = q_{n,2}^{NI} = q_{e,1}^{IN} = q_{n,1}^{IN}$, $q_{e,2}^{IN} = q_{e,1}^{NI}$, $\pi_{D2}^{NI} = \pi_{D1}^{IN}$, and $\pi_{D1}^{NI} = \pi_{D2}^{IN}$.

All-product-developers regime: $\boldsymbol{II}$. Because our differentiated products are produced in this regime, we obtain the following equilibrium outcomes:

$$\begin{aligned} &w^{II} = \frac{(1+c)(2\gamma+1)k-1}{(4\gamma+2)k-1}; \quad x^{II} = \frac{1-c}{(4\gamma+2)k-1}; \quad \pi_U^{II} = \frac{(1-c)^2 k}{(4\gamma+2)k-1}, \\ &q_{e,i}^{II} = q_{n,i}^{II} = \frac{(1-c)k}{2\left[(4\gamma+2)k-1\right]}; \quad \pi_{Di}^{II} = \frac{(1-c)^2(\gamma+1)k^2}{2\left[(4\gamma+2)k-1\right]^2} \quad \text{for } i = 1, 2. \end{aligned} \tag{6}$$

To ensure that U has a positive marginal cost after investment, we need Assumption 1.

Assumption 1. $k > k_0 \equiv \dfrac{1}{2c(1+2\gamma)}$.

We establish Proposition 1 from Equations (4)–(6).

Proposition 1. *(i) The equilibrium level of the R&D investment of U is largest in the all-product-developers regime, intermediate size in the mixed regime, and smallest in the no-one-invests regime. More precisely,* $x^{II} > x^{IN} = x^{NI} > x^{NN}$. *(ii) In all regimes, a higher R&D efficiency of U increases its investment level. Higher product substitutability decreases the level of R&D investment of U. More precisely,* $\partial x^r/\partial k < 0$ *and* $\partial x^r/\partial \gamma < 0$, *where* $r = II, IN, NI, NN$.

Proof. See Appendix A.

Proposition 1 yields Proposition 2.

Proposition 2. *(i) The equilibrium level of the input price is highest in the no-one-invests regime, intermediate size in the mixed regime, and lowest in the all-product-developers regime. Formally,* $w^{NN} > w^{IN} = w^{NI} > w^{II}$. *(ii) In all regimes, a higher R&D efficiency of U lowers the input price. Higher product substitutability raises the input price. More precisely,* $\partial w^r/\partial k > 0$ *and* $\partial w^r/\partial \gamma > 0$, *where* $r = II, IN, NI, NN$.

Proof. See Appendix A.

The logic behind part (i) of Proposition 1 is as follows. When U engages in cost-reducing investment, it will invest a large amount of funds if it can sell a large amount of its input. Innovation by Di increases the number of product varieties, so the demand for input also expands. If $D1$ and $D2$ innovate, the input demand is the largest among all the regimes. Therefore, investment will also be at its highest level in this situation. Conversely, when no one innovates, the level of investment is the lowest possible among all the regimes. If only Di innovates, investment rises to an intermediate level.

Part (ii) is intuitive. The first result is natural. Although a larger γ makes competition more intense, in our model, it reduces the size of the downstream market. The latter effect is dominant, which causes the input demand to shrink. This impedes upstream investment.

Proposition 1 immediately yields Proposition 2. Because a higher investment level corresponds to a lower input price, we obtain the ranking of the input price. Part (ii) is natural, in that the effects of γ are similar to those in part (ii) of Proposition 1.

To derive the equilibrium of the game, we introduce two threshold functions Φ_I and Φ_N, as follows: $\Phi_I \equiv \pi_{D1}^{IN} - \pi_{D1}^{NN} = \pi_{D2}^{NI} - \pi_{D2}^{NN}$ and $\Phi_N \equiv \pi_{D1}^{II} - \pi_{D1}^{NI} = \pi_{D2}^{II} - \pi_{D2}^{IN}$.[8] These thresholds are related to the gain or loss that results from the deviation from equilibrium regimes NN and II, respectively.

Φ_I and Φ_N are given by:

$$\Phi_N(k,\gamma) \equiv \frac{(1-c)^2(1-\gamma)k^2 \left[\begin{array}{l} 1+4\gamma-\gamma^2 + 16(2+6\gamma+6\gamma^2+2\gamma^3-\gamma^4)k^2 \\ -\, 8(2+4\gamma+3\gamma^2-\gamma^3)k \end{array} \right]}{2[(4\gamma+2)k-1]^2[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} > 0 \quad \text{and}$$

$$\Phi_I(k,\gamma) \equiv \frac{(1-c)^2(1-\gamma)k^2[8(8+8\gamma-4\gamma^4)k^2 - 8(2+2\gamma+\gamma^2-\gamma^3)k - 2\gamma^2+5\gamma-1]}{[2(\gamma+2)k-1]^2[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} > 0.$$

Di innovates if $f < \min\{\Phi_I(k,\gamma), \Phi_N(k,\gamma)\}$ and does not innovate if $f > \max\{\Phi_I(k,\gamma), \Phi_N(k,\gamma)\}$. Hence, the mixed regime, $IN\&NI$, can appear if $\Phi_N(k,\gamma) < \Phi_I(k,\gamma)$; and the complementary equilibrium, $NN\&II$, can appear if $\Phi_I(k,\gamma) < \Phi_N(k,\gamma)$. These arguments yield Proposition 3.

Proposition 3.

1. Suppose that $k \in (k_0,\, 1/(4\gamma))$. *Then,* $\Phi_I(k,\gamma) < \Phi_N(k,\gamma)$. *(i) If* $f < \Phi_I(k,\gamma)$, II *appears; (ii) if* $f > \Phi_N(k,\gamma)$, NN *appears; and (iii) if* $\Phi_I(k,\gamma) \le f \le \Phi_N(k,\gamma)$, $NN\&II$ *can appear.*

2. Suppose that $k > 1/(4\gamma)$ *or* $1/(4\gamma) \le k_0$. *Then,* $\Phi_N(k,\gamma) < \Phi_I(k,\gamma)$. *(i) If* $f < \Phi_N(k,\gamma)$, II *appears; (ii) if* $f > \Phi_I(k,\gamma)$, NN *appears; and (iii) if* $\Phi_N(k,\gamma) \le f \le \Phi_I(k,\gamma)$, $IN\&NI$ *can appear.*

[8] Chowdhury (2005) defined Φ_I as a *nonstrategic benefit* of R&D and Φ_N as a *strategic benefit* of R&D.

Proof. See Appendix A.

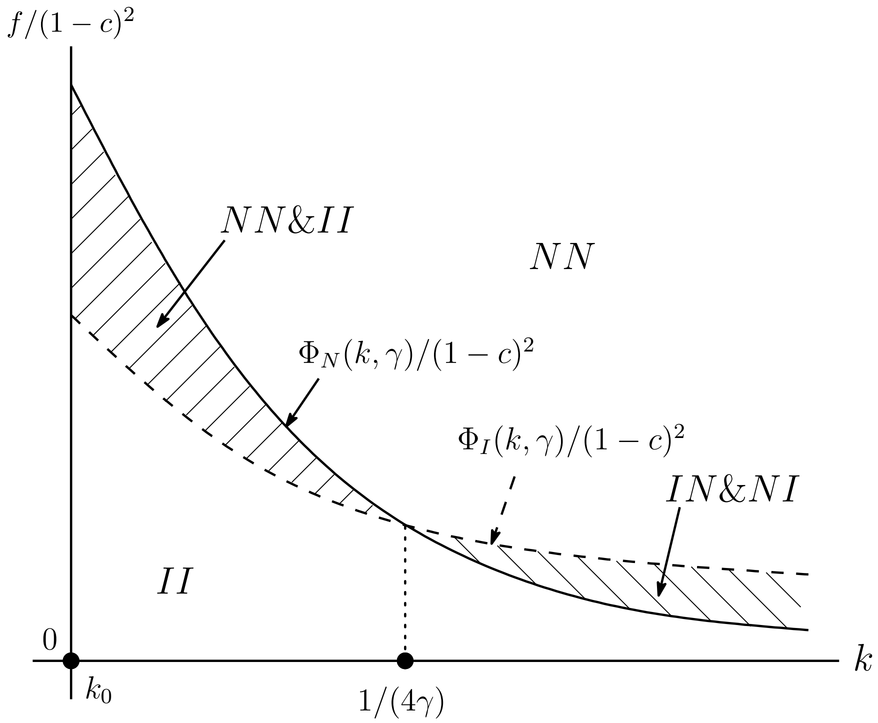

Figure 1: Equilibrium of the new product introduction game in $(k, \frac{f}{(1-c)^2})$-space ($k_0 < 1/(4\gamma)$)

When the fixed cost f is small (large) because I (N) is the dominant strategy, II (NN) appears. If f is an intermediate size, Di's strategy depends on the upstream R&D efficiency k: (i) if k is small, $NN\&II$ can appear; and (ii) if k is large, $IN\&NI$ can appear (see Figure 1).

The intuition behind Proposition 3 is as follows: (i) When k is small, upstream R&D is efficient. In this case, if Di deviates from NN, because the range of the fall in the input price is larger, the downstream production costs fall significantly. However, this prompts production by the rival firm, and creates more intense competition in the existing product market. Hence, even if the deviation increases the number of products in the market, because the benefits of R&D can be lost, Di does not deviate from NN. Furthermore, if Di deviates from II, its market share halves and the input price jumps. Hence, Di does not deviate from II.

Basak and Mukherjee (2018) found that, in a unionized duopoly, the emergence of the complementary equilibrium requires *asymmetric product differentiation* and *decentralized unions*. Conversely, we show that the complementary equilibrium appears even if there is no asymmetry in product differentiation. This implies that upstream R&D plays an important role in downstream innovation and, therefore, adds new insights to the literature.

(ii) When k is large or γ is large (i.e., $k > 1/(4\gamma)$ or $1/(4\gamma) \leq k_0$), because upstream R&D is inefficient or the downstream market is competitive, the effects of upstream investment on the input price weaken, and the effect of horizontal competition (γ) intensifies. When γ becomes large (closes to 1), competition in the downstream market intensifies. As a result, the firm's incentives to raise the costs of its rival to weaken the intensive competition becomes stronger, leading to the equilibrium of strategic substitutes appearing. If Di deviates from II, its product market halves and the input price rises. However, the input price is relatively high because of the larger k (Proposition 2). Thus, the production cost is also high, and the profit from the new product market becomes relatively small; that is, the R&D benefit is small. As a deviation from II raises the input price, it also increases the rival's cost. This results in a lessening of the intensive competition in the existing product market. Because the R&D benefit is small and competition in the existing product market declines, Di has an incentive to choose N if the rival chooses I. The deviation from NN increases the sales of the products and lowers the input price. Then, although the R&D benefit is small, the input price is high because k is large. Hence, a fall in production cost as a result of a decrease in the input price becomes a more attractive option. Di chooses I when the rival chooses N.

Our model makes two contributions to the literature. The first contribution is to endogenize the R&D spillover in d'Aspremont and Jacquemin (1988). In their model, even though one firm's cost-reducing R&D investment decreases its rival's marginal cost, the "R&D spillover" is an exogenous variable, and it tends to be considered as a horizontal technology transmission among innovative firms. More specifically, in d'Aspremont and Jacquemin (1988), if $A > 0$ is defined as a constant marginal cost of R&D firms, the cost function of firm i is defined as $C_i(q_i, x_i, x_j) = [A - x_i - \beta x_j]q_i,\ i, j = 1, 2,\ i \neq j$ with $0 < \beta < 1$. The term related to the R&D spillover is "βx_j." Thus, firm i can gain a β-ratio of the results of the rival firm's R&D investment without any payments. The best response function is a strategic substitute (complement) if the spillover rate β is small (large).[9]

By contrast, in our model, there is no R&D spillover β or its related term βx_j. In our model, although one firm's product R&D reduces its rival's marginal cost, the reduction is caused by a reduction in the input price due to the introduction of a new final good. That is, through the input price w, the upstream cost-reducing investment is vertically transmitted to the downstream product R&D, and the degree of the spillover depends on the level of upstream R&D efficiency k. As shown in Proposition 3, if k is small, upstream R&D investment is efficient and the degree of vertical spillovers (involving upstream cost-reducing R&D leading to an input price reduction) is large. Then, in the downstream R&D game, the equilibrium of strategic complements can appear. Conversely, if k is large and the degree of upstream R&D spillovers is small, the equilibrium in downstream R&D game can become one of strategic substitutes. Hence, it can be considered that we partially extend the model of d'Aspremont and Jacquemin (1988).

Our second contribution concerns the relation between the input price and downstream investment. In our model, the upstream input supplier decides on its price after observing the downstream investment decision. Nevertheless, when all downstream firms invest, the

[9]Henriques (1990) showed that in the d'Aspremont and Jacquemin (1988) model, each firm's best response to the R&D level is a strategic complement (substitute) if $\beta > 1/2$ ($\beta < 1/2$).

input price is at its lowest among all the regimes (see Proposition 2).

When the input supplier decides its price after observing the downstream investment decision, it is well known that it will raise the input price to appropriate the benefits of the downstream investment (Banerjee and Lin 2003, Gilbert and Cvsa 2003). Banerjee and Lin (2003) emphasized such hold-up behavior by the upstream supplier; they showed that if the input price is decided after downstream investment, because the upstream input supplier raises its price and appropriates the downstream investment benefit, the input price becomes higher, which impedes downstream investment activities.[10] Furthermore, the existing research has emphasized that the opportunistic behavior by upstream appears even if the upstream agent is a labor union (i.e., a wage setter) (Haucap and Wey 2004). In the argument of Banerjee and Lin (2003), if the input supplier decides its price after observing downstream investment behavior, because a higher input price occurs, the larger the size of the downstream investment, the higher is the input price faced by the downstream firm. By contrast, we find the opposite results.

This difference is based on the following two points: (i) the upstream input supplier's cost-reducing investment activity, and (ii) the downstream product R&D. When a downstream firm invests, the demand for inputs rapidly (discontinuously) increases. Furthermore, when all downstream firms invest, demands for inputs doubles. The rapid increase in the input demand through the downstream product R&D raises the investment incentives of the upstream supplier higher. As shown in Proposition 1, upstream investment depends on downstream investment, that is, the size of input demand. In addition, the upstream cost-reducing investment level determines the input price level. Because the cost-reducing investment level is maximized if all downstream firms invest, the input price is at its lowest

[10]Banerjee and Lin (2003) showed that a fixed-price contract (i.e., a contract under which the input price is decided first in the game) that uses the input price resolves this hold-up problem. Conversely, Takauchi and Mizuno (2019) demonstrated that a fixed-price contract can harm upstream and downstream firms.

among all the regimes when all downstream firms invest. Our result implies that upstream R&D is very influential in vertical structures and thus offers new insights in the literature on innovation and vertical production chains.

4 Welfare analysis

Even if a downstream firm innovates, it will not capture all the surplus that it generates. Therefore, if the innovation cost f is large, downstream firms cease their attempts to introduce new products, and underinvestment (i.e., welfare loss) occurs. In this section, we provide the conditions that can cause underinvestment.

To avoid unnecessary algebraic complexity and to facilitate our welfare analysis, we set a lower limiting value for k. Therefore, we make Assumption 2.

Assumption 2. $k \geq \dfrac{1}{2}$.

First, we discuss underinvestment in terms of consumer surplus. We define consumer surplus and gross total surplus (excluding the downstream R&D cost f) as follows:

$$CS = \frac{q_{e,1}^2 + q_{e,2}^2 + q_{n,1}^2 + q_{n,2}^2}{2} + \gamma\left[q_{e,1}(q_{e,2} + q_{n,1} + q_{n,2}) + q_{e,2}(q_{n,1} + q_{n,2}) + q_{n,1}q_{n,2}\right]$$
$$- p_{e,1}q_{e,1} - p_{n,1}q_{n,1} - p_{e,2}q_{e,2} - p_{n,2}q_{n,2},$$

and $TS = CS + \pi_U + \sum_i \pi_{Di}$.

We have the following equilibrium surpluses. In the all-product-developers regime, the equilibrium surplus is:

$$CS^{II} = \frac{(1-c)^2(3\gamma+1)k^2}{2[(4\gamma+2)k-1]^2}; \quad TS^{II} = \frac{(1-c)^2 k[(13\gamma+7)k-2]}{2[(4\gamma+2)k-1]^2}.$$

In the mixed regime, the equilibrium surplus is:

$$CS^{IN} = \frac{(1-c)^2(\gamma^3 - 7\gamma^2 + 8\gamma + 6)k^2}{[\gamma + (-4\gamma^2 + 8\gamma + 8)k - 3]^2},$$
$$TS^{IN} = \frac{(1-c)^2 k[(7\gamma^3 - 33\gamma^2 + 24\gamma + 42)k - (\gamma - 3)^2]}{[\gamma + (-4\gamma^2 + 8\gamma + 8)k - 3]^2}.$$

Note that $CS^{IN} = CS^{NI}$ and $TS^{IN} = TS^{NI}$.

In the no-one-invests regime, the equilibrium surplus is:

$$CS^{NN} = \frac{(1-c)^2(\gamma+1)k^2}{[1-2(\gamma+2)k]^2}; \quad TS^{NN} = \frac{(1-c)^2 k[(3\gamma+7)k - 1]}{[1-2(\gamma+2)k]^2}.$$

By comparing the consumer surpluses, we obtain Result 1.

Result 1. *(i) Assume that "II" appears if the equilibrium regime is II or NN. Then, from the viewpoint of consumer surplus, underinvestment in downstream occurs if $f > \Phi_N$.*
(ii) Assume that "NN" appears if the equilibrium regime is II or NN. Then, from the viewpoint of consumer surplus, underinvestment in downstream occurs if $f > \min\{\Phi_N, \Phi_I\}$.

Proof. See Appendix A.

Consumers always welcome an increase in the variety of goods that they consume. Moreover, from Proposition 3, equilibrium regime II is not realized when the R&D cost f is large. Therefore, the consumer surplus is not maximized if f is large.

Next, we discuss underinvestment in terms of total surplus. We will not present the algebraic proof, focusing on intuitive discussions. For the precise conditions of underinvestment, see the Online Appendix.

We assume that $k > \max\{1/2, k_0\}$. To consider the best regime that maximizes the total surplus, we define the gross benefits of an increase in the number of downstream firms conducting R&D: $\Psi_{21}^{TS} \equiv TS^{II} - TS^{IN} = TS^{II} - TS^{NI}$, $\Psi_{10}^{TS} \equiv TS^{IN} - TS^{NN} = TS^{NI} - TS^{NN}$, and $\Psi_{20}^{TS} \equiv (TS^{II} - TS^{NN})/2$. More precisely, a rise in the number of downstream firms conducting R&D increases the total surplus if the following conditions are satisfied: $\Psi_{21}^{TS} > f$, $\Psi_{10}^{TS} > f$, or $\Psi_{20}^{TS} > f$.

No downstream firm conducts R&D if the R&D cost is large: $f > \max\{\Phi_I, \Phi_N\}$. Then, underinvestment in terms of the total surplus occurs when the gross benefits Ψ_{10}^{TS} and Ψ_{20}^{TS} are larger than $\max\{\Phi_I, \Phi_N\}$. Calculating the threshold ranking between Φ_I, Φ_N, Ψ_{10}^{TS}, and Ψ_{20}^{TS}, we can show that $\max\{\Psi_{10}^{TS}, \Psi_{20}^{TS}\} > \max\{\Phi_I, \Phi_N\}$. Hence, if $\max\{\Psi_{10}^{TS}, \Psi_{20}^{TS}\} > f > \max\{\Phi_I, \Phi_N\}$, the downstream firms do not conduct R&D, which results in underinvestment.

To confirm this result, we use a numerical example to illustrate the total surplus benefit from downstream R&D, which yields Figures 2 and 3. In these figures, the horizontal axis represents the fixed cost f of introducing a new product. For Figure 2 (or Figure 3), we assume that $c = 2/5$, $k = 1$, and $\gamma = 1/5$ (or 2/5, $k = 5$, and $\gamma = 1/5$). Each figure has three lines. The bold gray, dashed black, and solid black lines represent $TS^{II} - 2f$, $TS^{IN} - f (= TS^{NI} - f)$, and TS^{NN}, respectively. Each figure has four vertical dotted lines: Φ_I, Φ_N, Ψ_{10}^{TS}, and Ψ_{20}^{TS}. From Proposition 3, Φ_I and Φ_N determine the investment decision for downstream firms. Additionally, at $f = \Psi_{10}^{TS}$ and $f = \Psi_{20}^{TS}$, $TS^{IN} - f = TS^{NI} - f = TS^{NN}$ and $TS^{II} - 2f = TS^{NN}$, respectively. In Figures 2 and 3, we can confirm that underinvestment occurs if f takes an intermediate value.

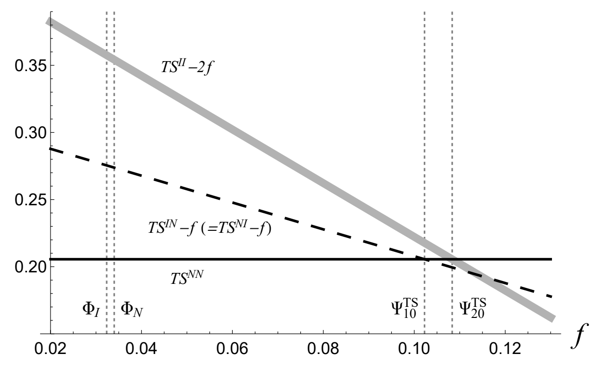

Figure 2: Total welfare comparison: $\boldsymbol{k = 1}$

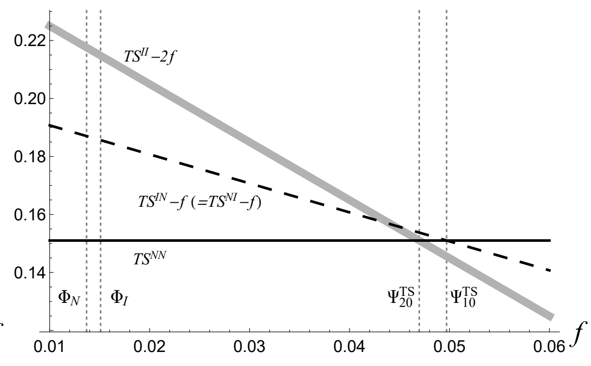

Figure 3: Total welfare comparison: $\boldsymbol{k = 5}$

Note: In both Figures 2 and 3, $c = 2/5$ and $\gamma = 1/5$.

We explain the intuition behind the result for underinvestment in Figures 2 and 3. First, we consider a case with a small f. At $f = 0$, downstream firms always invest. Additionally, the total surplus increases with the number of investing downstream firms because downstream investment increases final demand. From the continuity of the total surplus function, both $D1$ and $D2$ invest, and this investment decision is socially optimal if f is small.

Second, we consider a case with a large f. When no downstream firm invests, the total surplus is independent of f; when at least one downstream firm invests, the total surplus decreases with f. Hence, we obtain two threshold values: Ψ_{10}^{TS} and Ψ_{20}^{TS}. From Proposition 3, no downstream firm invests if f is large and, therefore, the equilibrium number of investing downstream firms is socially optimal.

Finally, we consider a case in which f takes an intermediate value. In our model, $\max\{\Phi_I, \Phi_N\} < \max\{\Psi_{10}^{TS}, \Psi_{20}^{TS}\}$. Hence, for any $\max\{\Phi_I, \Phi_N\} < f < \max\{\Psi_{10}^{TS}, \Psi_{20}^{TS}\}$, no downstream firm invests even though it is socially desirable for both downstream firms to invest. In the case with $\Phi_N < f < \Phi_I$ in Figure 3, the equilibrium investment decisions are $II\&NN$. Hence, whether the underinvestment of downstream firms occurs depends on the manner in which the equilibrium is refined. If NN (or II) is realized under $\Phi_N < f < \Phi_I$, downstream investment is insufficient (or socially optimal).

5 Downstream price competition

In this section, we discuss the case in which downstream firms compete on price to show the robustness of the main result in the previous sections. In differentiated Bertrand competition, the demand function depends on whether downstream firms introduce a new product.

No-one-invests regime: *NN*. In this regime, the demand function is:

$$q_{e,i} = \frac{(1-\gamma) - p_{e,i} + \gamma p_{e,j}}{1-\gamma^2}; \quad q_{e,j} = \frac{(1-\gamma) - p_{e,j} + \gamma p_{e,i}}{1-\gamma^2}, \quad i \neq j.$$

Mixed regime: *IN* or *NI*. As only Di introduces a new product, the demand function is:

$$\begin{aligned} q_{e,i} &= \frac{(1-\gamma) - (\gamma+1)p_{e,i} + \gamma(p_{n,i} + p_{e,j})}{(1-\gamma)(2\gamma+1)}, \\ q_{e,j} &= \frac{(1-\gamma) - (\gamma+1)p_{e,j} + \gamma(p_{e,i} + p_{n,i})}{(1-\gamma)(2\gamma+1)}, \\ q_{n,i} &= \frac{(1-\gamma) - (\gamma+1)p_{n,i} + \gamma(p_{e,i} + p_{e,j})}{(1-\gamma)(2\gamma+1)}. \end{aligned}$$

All-product-developers regime: *II*. In this regime, the demand function is:

$$\begin{aligned} q_{e,i} &= \frac{(1-\gamma) - (2\gamma+1)p_{e,i} + \gamma(p_{n,i} + p_{n,j} + p_{e,j})}{(1-\gamma)(3\gamma+1)}, \\ q_{n,i} &= \frac{(1-\gamma) - (2\gamma+1)p_{n,i} + \gamma(p_{n,j} + p_{e,i} + p_{e,j})}{(1-\gamma)(3\gamma+1)}. \end{aligned}$$

The timing of the game is similar to that of the Cournot competition case. By applying the same procedure as those in the previous setting, we obtain each regime's equilibrium outcomes, which are shown in Appendix C. Using their equilibrium outcomes, we can derive the best response of downstream firms.

Suppose that Dj introduces a new product. If $f < \hat{\Phi}_I$, Di introduces a new product; otherwise, it does not, and

$$\hat{\Phi}_I \equiv (1-c)^2(\gamma-1)k^2 \left[\frac{\gamma+1}{(2(\gamma-2)(\gamma+1)k+1)^2} - \frac{2(2\gamma+1)(3\gamma+2)^2}{(\gamma(\gamma+5)+4(2\gamma+1)((\gamma-2)\gamma-2)k+3)^2}\right].$$

Suppose that Dj does not introduce a new product. If $f < \hat{\Phi}_N$, Di introduces a new product; otherwise, it does not, and

$$\hat{\Phi}_N \equiv \frac{1}{2}(1-c)^2(\gamma-1)(\gamma+1)k^2 \left[\frac{8(\gamma+1)^2(2\gamma+1)}{(\gamma(\gamma+5)+4(2\gamma+1)((\gamma-2)\gamma-2)k+3)^2} - \frac{3\gamma+1}{(\gamma-2(3\gamma+1)k+1)^2}\right].$$

These two thresholds, $\hat{\Phi}_I$ and $\hat{\Phi}_N$, have similar properties to those in the Cournot competition case; that is, even if the downstream market has a different form of competition, we obtain similar results.[11] The logic behind the results in the Bertrand case is very similar to that of Proposition 3. Hence, we conclude that the complementary equilibrium appears without asymmetry in product differentiation and verify that Proposition 3 has a certain robustness.

6 Conclusion

In vertically related markets such as assembly industries, the key input price is important. This is because the price of a key input greatly influences the final good production. If the key input is priced cheaply because the downstream production cost is low, then downstream firms can easily introduce a new product. Cost-reducing R&D investment in upstream firms has an important role. Using an upstream monopoly and downstream duopoly model, we consider both upstream cost-reducing R&D and the introduction of a new product downstream. We find that upstream R&D efficiency (or production efficiency) determines the downstream behavior in terms of introducing a new product. If the upstream firm is efficient, the newly introduced downstream product is a strategic complement. However, if the upstream firm is less efficient, the downstream product introduction is a strategic substitute.

In our model, there is no R&D spillover among firms. However, upstream R&D affects the downstream strategic behavior by affecting the downstream firm's marginal cost through the input price. Hence, it can be said that our model partially extends d'Aspremont and Jacquemin's (1988) framework, which treats a horizontal R&D spillover among firms as exogenous.

[11] The figure illustrating the equilibrium in the case of differentiated Bertrand competition is almost identical to that in the Cournot case and, therefore, we omit it.

Although we considered a simple vertical structure in this paper, our analysis is limited to a domestic or single region's market. In future research, it would be interesting to consider what equilibrium patterns arise when downstream firms have two options, not only production for domestic consumers, but also the introduction of a new product for foreign consumers. In addition, although it is outside the scope of our paper to consider the effect of vertical integration/separation on upstream and downstream innovation, we expect that innovation activities would be influenced. The reason is that when upstream and downstream firms are integrated, their organizational forms alter. We leave the examination of this topic to future research.

Acknowledgements

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Appendix A. Proofs

Proof of Proposition 1. (i) $x^{II} - x^{IN} = \frac{2k(1-c)(1-\gamma)}{L_N} > 0$ and $x^{IN} - x^{NN} = \frac{2k(1-c)(1-\gamma)(2-\gamma)}{L_I} > 0$, where $L_N \equiv [(4\gamma+2)k-1][4(2+2\gamma-\gamma^2)k-(3-\gamma)]$ and $L_I \equiv [2(\gamma+2)k-1][4(2+2\gamma-\gamma^2)k-(3-\gamma)]$. (ii) The partial derivative of x^r with respect to k yields $\partial x^{II}/\partial k = -\frac{2(1-c)(2\gamma+1)}{[(4\gamma+2)k-1]^2} < 0$, $\partial x^{IN}/\partial k = -\frac{4(1-c)(3-\gamma)(2+2\gamma-\gamma^2)}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} < 0$, and $\partial x^{NN}/\partial k = -\frac{2(1-c)(\gamma+2)}{[2(\gamma+2)k-1]^2} < 0$. The partial derivative of x^r with respect to γ yields $\partial x^{II}/\partial \gamma = -\frac{4(1-c)k}{[(4\gamma+2)k-1]^2} < 0$, $\partial x^{IN}/\partial \gamma = -\frac{4(1-c)(\gamma^2-6\gamma+8)k}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} < 0$, and $\partial x^{NN}/\partial \gamma = -\frac{2(1-c)k}{[2(\gamma+2)k-1]^2} < 0$. □

Proof of Proposition 2. (i) $w^{NN} - w^{IN} = \frac{k(1-c)(2-\gamma)(1-\gamma)}{L_I} > 0$ and $w^{IN} - w^{II} = \frac{k(1-c)(1-\gamma)}{L_N} > 0$. (ii) The partial derivative of w^r with respect to k yields $\partial w^{II}/\partial k = \frac{(1-c)(2\gamma+1)}{[(4\gamma+2)k-1]^2} > 0$, $\partial w^{IN}/\partial k = \frac{2(1-c)(3-\gamma)(2+2\gamma-\gamma^2)}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} > 0$, and $\partial w^{NN}/\partial k = \frac{(1-c)(\gamma+2)}{[2(\gamma+2)k-1]^2} > 0$. The partial derivative of w^r with respect to γ yields $\partial w^{II}/\partial\gamma = \frac{2(1-c)k}{[(4\gamma+2)k-1]^2} > 0$, $\partial w^{IN}/\partial\gamma = \frac{2(1-c)k(4-\gamma)(2-\gamma)}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} > 0$, and $\partial w^{NN}/\partial\gamma = \frac{(1-c)k}{[2(\gamma+2)k-1]^2} > 0$. □

Proof of Proposition 3. By comparing Φ_N with Φ_I, we obtain:

$$\Phi_N - \Phi_I = \frac{(1-c)^2(1-\gamma)^2k^2(1-4\gamma k)\ g(k,\gamma)}{2[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[\gamma+\left(-4\gamma^2+8\gamma+8\right)k-3\right]^2}$$

and $g(k,\gamma) \equiv 16(3\gamma^4+5\gamma^3+16\gamma^2+22\gamma+8)k^3-12(5\gamma^3+5\gamma^2+8\gamma+6)k^2+24\gamma^2k-3\gamma+3$.

We show that $g(k,\gamma) > 0$ and sign$\{\Phi_N - \Phi_I\}$ depend only on $1-4\gamma k$. To prove that $g(k,\gamma) > 0$, it is sufficient to show that $g(k,\gamma)$ has its minimum value at $k = k_0$ and $c = 1$, and that this value is positive.

First, we show that $g(k,\gamma)$ is an increasing function of k; that is, $g(k,\gamma)$ is smallest at $k = k_0$. The first derivative $g(k,\gamma)$ with respect to k is $\partial g(k,\gamma)/\partial k = 24[2(3\gamma^4 + 5\gamma^3 + 16\gamma^2 + 22\gamma + 8)k^2 - (5\gamma^3 + 5\gamma^2 + 8\gamma + 6)k + \gamma^2]$. $\partial g(k,\gamma)/\partial k$ is a quadratic function of k and the coefficient of k^2 is positive. Hence, by solving $\partial g(k,\gamma)/\partial k > 0$ for k, we obtain $k < k_1$ and $k > k_2$, where k_1 and k_2 are roots of $g(k,\gamma) = 0$ for k and $k_1 < k_2$.

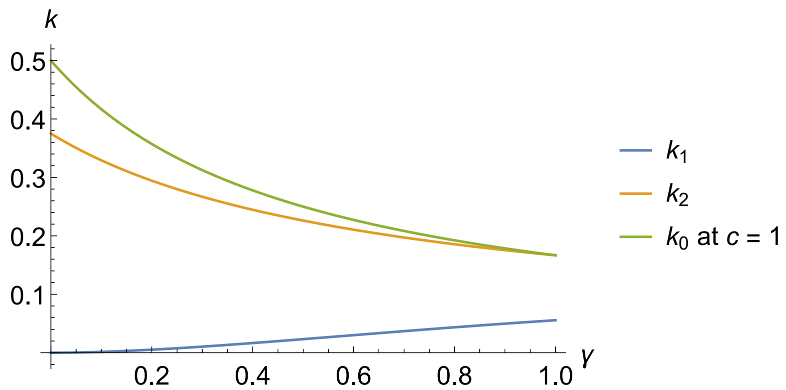

Figure 4: k_0 at $c = 1$ and the two roots of $g(k,\gamma) = 0$

As $k_0 = 1/[2c(2\gamma+1)]$ decreases with c, k_0 has its minimum value at $c = 1$. We illustrate

k_1, k_2, and k_0 at $c=1$ in Figure 4. Using numerical calculation, we find that for $\gamma \in [0,1]$, the unique root of $k_2 - k_0|_{c=1} = 0$ is $\gamma = 1$. Hence, $\partial g(k,\gamma)/\partial k > 0$ for any $k > k_0$. Therefore, $g(k,\gamma)$ has its minimum value at $k = k_0$.

Second, we show $\partial g(k_0,\gamma)/\partial c < 0$. The derivation yields:

$$\frac{\partial g(k_0,\gamma)}{\partial c} = \frac{\partial g(k_0,\gamma)}{\partial k} \times \frac{\partial k_0}{\partial c} = \frac{\partial g(k_0,\gamma)}{\partial k}\left[\frac{-1}{2c^2(2\gamma+1)}\right] < 0.$$

The last inequality is satisfied because $\partial g(k,\gamma)/\partial k > 0$. Hence, $g(k_0,\gamma)$ is a decreasing function for c and it has its minimum value when $c = 1$.

From the above discussion, $g(k_0,\gamma)$ has the following minimum value at $k = k_0$ and $c=1$: $g(k_0,\gamma)\big|_{c=1} = \frac{(1-\gamma)^2(\gamma+1)}{(2\gamma+1)^3} > 0$. Because $g(k_0,\gamma)\big|_{c=1}$ is positive, $\forall k > k_0$, $g(k,\gamma) > 0$. This result implies that $\operatorname{sign}\{\Phi_N - \Phi_I\}$ depends only on "$1 - 4\gamma k$." Hence, $\Phi_N > \Phi_I$ iff $k < 1/(4\gamma)$. □

Proof of Result 1. Case (i). From Proposition 2, the equilibrium regime is either "$IN\&NI$" or "NN" if $f > \Phi_N$; and the equilibrium regime is "II" if $f < \phi_N$. Hence, to prove the first part, we need to show that $CS^{II} > CS^{IN} (= CS^{NI}) > CS^{NN}$. This is because underinvestment occurs only if $f > \Phi_N$.

Case (ii). Applying a similar discussion to that in case (i), we find that underinvestment occurs only if $f > \min\{\Phi_N, \Phi_I\}$, where the equilibrium regime is also either "$IN\&NI$" or "NN." Hence, in both cases, if we show that $CS^{II} > CS^{IN} > CS^{NN}$, the proof is complete.

First, we consider $\operatorname{sign}\{CS^{II} - CS^{IN}\}$:

$$CS^{II} - CS^{IN} = \frac{(1-c)^2(1-\gamma)k^2\ \psi_1^{CS}}{2[(4\gamma+2)k-1)]^2\left[\gamma + (-4\gamma^2+8\gamma+8)\,k - 3\right]^2},$$

where $\psi_1^{CS} \equiv 8(2+10\gamma+9\gamma^2-4\gamma^3-2\gamma^4)k^2 - 8\gamma(2-\gamma^2)k - \gamma^2 + 2\gamma - 3$.

$\operatorname{sign}\{CS^{II} - CS^{IN}\}$ depends only on ψ_1^{CS}. Because ψ_1^{CS} is a quadratic function of k and the coefficient of k^2 is positive, $\psi_1^{CS} = 0$ has two roots: k_1^{CS} and k_2^{CS}. By solving

$\psi_1^{CS} > 0$ for k, we obtain $k < k_1^{CS}$ or $k > k_2^{CS}$, where:

$$k_1^{CS} \equiv \frac{2\gamma(2-\gamma^2)-\sqrt{6\gamma^4-40\gamma^3+34\gamma^2+52\gamma+12}}{4(2+10\gamma+9\gamma^2-4\gamma^3-2\gamma^4)}; \quad k_2^{CS} \equiv \frac{2\gamma(2-\gamma^2)+\sqrt{6\gamma^4-40\gamma^3+34\gamma^2+52\gamma+12}}{4(2+10\gamma+9\gamma^2-4\gamma^3-2\gamma^4)}.$$

We compare k_2^{CS} with k_0. We consider the case where $c = 1$. Using numerical calculation, we find that $\forall\gamma \in [0, 1)$, $k_0|_{c=1} > k_2^{CS}$. Because k_0 takes the minimum value at $c = 1$, $k_0 > k_2^{CS} > k_1^{CS}$ for any $c > 0$. Hence, $CS^{II} - CS^{IN} > 0$.

Next, we consider $CS^{IN} - CS^{NN}$ and apply a proof similar to that above:

$$CS^{IN} - CS^{NN} = -\frac{(c-1)^2(\gamma-1)k^2\ \psi_2^{CS}}{(1-2(\gamma+2)k)^2\left[\gamma + \left(-4\gamma^2+8\gamma+8\right)k - 3\right]^2},$$

where: $\psi_2^{CS} \equiv 4(3\gamma^4 - 6\gamma^3 - 6\gamma^2 + 16\gamma + 8)k^2 - 4\gamma(\gamma^2 - 2\gamma + 2)k - 3 + 2\gamma$.

sign$\{CS^{IN} - CS^{NN}\}$ depends only on ψ_2^{CS}. Because the coefficient of k^2 in ψ_2^{CS} is positive, by solving $\psi_2^{CS} > 0$ for k, we obtain $k < k_3^{CS}$ or $k > k_4^{CS}$, where:

$$k_3^{CS} \equiv \frac{\gamma(\gamma^2-2\gamma+2)-(2-\gamma)\sqrt{\gamma^4-6\gamma^3+\gamma^2+14\gamma+6}}{2(3\gamma^4-6\gamma^3-6\gamma^2+16\gamma+8)}; \quad k_4^{CS} \equiv \frac{\gamma(\gamma^2-2\gamma+2)+(2-\gamma)\sqrt{\gamma^4-6\gamma^3+\gamma^2+14\gamma+6}}{2(3\gamma^4-6\gamma^3-6\gamma^2+16\gamma+8)}.$$

We show that $k_0 > k_4^{CS}(> k_3^{CS})$. At $c = 1$, using numerical calculation, we find that, $\forall\gamma \in [0, 1)$, $k_0|_{c=1} > k_4^{CS}$. Because k_0 has its minimum value at $c = 1$, for any $c > 0$, $k_0 > k_4^{CS} > k_3^{CS}$ holds. Therefore, $CS^{IN} - CS^{NN} > 0$. $\square$

Appendix B. Derivation of Equilibrium Outcomes

This appendix derives the equilibrium outcomes in each regime.

No-one-invests regime: *NN*. First, we derive the equilibrium quantity of the final product. Because each firm decides the quantity of the final product to maximize profit, the first-order condition (FOC) is:

$$\frac{\partial \pi_{Di}}{\partial q_{e,i}} = 1 - w - 2q_{e,i} - \gamma q_{e,j} = 0.$$

Consequently, we obtain $q_{e,i} = \frac{1}{2}(1 - w - q_{e,j})$. From this reaction function, we obtain:

$$q_{e,i} = q_{e,j} = \frac{1-w}{2+\gamma}. \tag{7}$$

Substituting eq. (7) into the upstream firm's profit function, we obtain:

$$\pi_U = \frac{2(1-w)(w-(c-x))}{r+2} - kx^2.$$

Because the upstream firm decides w and x to maximize its profit, the FOCs are:

$$\frac{\partial \pi_U}{\partial w} = \frac{2(1-2w+(c-x))}{\gamma+2} = 0,$$
$$\frac{\partial \pi_U}{\partial x} = \frac{2(1-w)}{2+\gamma} - 2kx = 0.$$

Thus, we obtain the equilibrium outcomes listed in the paper.

Mixed regime: *IN* or *NI*. Without loss of generality, we assume that only D_i invests. First, we derive the equilibrium quantity of final products. Because each firm decides the quantity of final products to maximize profit, the FOCs are:

$$\frac{\partial \pi_i}{\partial q_{e,i}} = 1 - w - 2q_{e,i} - 2\gamma q_{n,i} - \gamma q_{e,j} = 0,$$
$$\frac{\partial \pi_i}{\partial q_{n,i}} = 1 - w - 2q_{n,i} - 2\gamma q_{e,i} - \gamma q_{e,j} = 0,$$
$$\frac{\partial \pi_j}{\partial q_{e,j}} = 1 - w - 2q_{e,j} - \gamma(q_{e,i} + q_{n,i}) = 0.$$

Consequently, we obtain:

$$q_{e,i} = q_{n,i} = \frac{1}{2(1+\gamma)}(1 - w - q_{e,j}),$$
$$q_{e,j} = \frac{1}{2}(1 - w - \gamma(q_{e,i} + q_{n,i})).$$

From the above reaction functions, we obtain:

$$q_{e,i} = q_{n,i} = \frac{(1-w)(2-\gamma)}{2(2+2\gamma-\gamma^2)}; \quad q_{e,j} = \frac{1-w}{2+2\gamma-\gamma^2}. \tag{8}$$

Substituting eq. (8) into the upstream firm's profit function, we obtain:

$$\pi_U = \frac{(1-w)(2-\gamma)(w-(c-x))}{2+2\gamma-\gamma^2} - kx^2.$$

Because the upstream firm decides w and x to maximize its profit, the FOCs are:

$$\frac{\partial \pi_U}{\partial w} = \frac{(3-\gamma)(-2w+1+(c-x))}{2+2\gamma-\gamma^2} = 0,$$
$$\frac{\partial \pi_u}{\partial x} = \frac{(1-w)(3-\gamma)}{2+2\gamma-\gamma^2} - 2kx = 0.$$

Thus, we obtain the equilibrium outcomes listed in the paper.

All-product-developers regime: *II*. First, we derive the equilibrium quantity of the final product. Because each firm decides the quantity of final products to maximize profit, the FOCs are:

$$\frac{\partial \pi_{Di}}{\partial q_{e,i}} = 1 - w - 2q_{e,i} - 2\gamma q_{n,i} - (q_{e,j} + q_{n,j})\gamma = 0,$$
$$\frac{\partial \pi_{Di}}{\partial q_{n,i}} = 1 - w - 2q_{n,i} - 2\gamma q_{e,i} - (q_{e,j} + q_{n,j})\gamma = 0.$$

Consequently, we obtain $q_{e,i} = q_{n,i} = \dfrac{1-w-(q_{e,j}+q_{n,j})\gamma}{2(1+\gamma)}$. From this, we obtain:

$$q_{e,i} = q_{n,i} = q_{e,j} = q_{n,j} = \frac{1-w}{2(2\gamma+1)}. \tag{9}$$

Substituting eq. (9) into the upstream firm's profit function, we obtain:

$$\pi_U = \frac{2(1-w)(w-(c-x))}{2\gamma+1} - kx^2.$$

Because the upstream firm decides w and x to maximize its profit, the FOCs are:

$$\frac{\partial \pi_U}{\partial w} = \frac{2(-2w+1+(c-x))}{2\gamma+1} = 0,$$
$$\frac{\partial \pi_U}{\partial x} = \frac{(2(1-w))}{(1+2\gamma)} - 2kx = 0.$$

Thus, we obtain the equilibrium outcomes listed in the paper.

Appendix C. SPNE outcomes in downstream Bertrand.

To identify Bertrand rivalry, we attach "ˆ" to the variables of the equilibrium solutions.

No-one-invests regime: *NN*

$\hat{w}^{NN} = \frac{1-(c+1)(2-\gamma)(\gamma+1)k}{1-2(2-\gamma)(\gamma+1)k}$, $\hat{x}^{NN} = \frac{1-c}{2(2-\gamma)(\gamma+1)k-1}$, $\hat{\pi}_U^{NN} = \frac{(1-c)^2k}{2(2-\gamma)(\gamma+1)k-1}$,

$\hat{p}_{e,i}^{NN} = \frac{1-(\gamma+1)k(c-2\gamma+3)}{2(\gamma^2-\gamma-2)k+1}$, and $\hat{\pi}_{Di}^{NN} = \frac{(1-c)^2(1-\gamma)(\gamma+1)k^2}{(2(\gamma^2-\gamma-2)k+1)^2}$.

Mixed regime: *IN* or *NI*

$\hat{w}^{IN} = \frac{2(c+1)(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}{4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}$,

$\hat{x}^{IN} = \frac{(1-c)(\gamma(\gamma+5)+3)}{4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}$, $\hat{\pi}_U^{IN} = \frac{(1-c)^2(\gamma(\gamma+5)+3)k}{4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}$,

$\hat{p}_{e,i}^{IN} = \frac{-(2\gamma+1)k(c(\gamma+1)(\gamma+2)+5(1-\gamma)\gamma+6)+\gamma(\gamma+5)+3}{\gamma(\gamma+5)-4(2\gamma+1)((2-\gamma)\gamma+2)k+3}$, $\hat{p}_{n,i}^{IN} = \frac{-(2\gamma+1)k(c(\gamma+1)(\gamma+2)+5(1-\gamma)\gamma+6)+\gamma(\gamma+5)+3}{\gamma(\gamma+5)-4(2\gamma+1)((2-\gamma)\gamma+2)k+3}$,

$\hat{p}_{e,j}^{IN} = \frac{-2(2\gamma+1)k(2\gamma c+c+2(1-\gamma)\gamma+3)+\gamma(\gamma+5)+3}{\gamma(\gamma+5)-4(2\gamma+1)((2-\gamma)\gamma+2)k+3}$, $\hat{\pi}_{Di}^{IN} = \frac{2(1-c)^2(1-\gamma)(2\gamma+1)(3\gamma+2)^2k^2}{(4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3))^2}$,

and $\hat{\pi}_{Dj}^{IN} = \frac{4(1-c)^2(1-\gamma)(\gamma+1)^3(2\gamma+1)k^2}{(4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3))^2}$.

All-product-developers regime: *II*

$\hat{w}^{II} = \frac{(c+1)(3\gamma+1)k-(\gamma+1)}{(6\gamma+2)k-(\gamma+1)}$, $\hat{x}^{II} = \frac{(1-c)(\gamma+1)}{(6\gamma+2)k-(\gamma+1)}$, $\hat{\pi}_U^{II} = \frac{(1-c)^2(\gamma+1)k}{(6\gamma+2)k-(\gamma+1)}$, $\hat{q}_{e,i}^{II} = \frac{(3\gamma+1)k(\gamma c+c-\gamma+3)-2(\gamma+1)}{4(3\gamma+1)k-2(\gamma+1)}$,

and $\hat{\pi}_{Di}^{II} = \frac{(1-c)^2(1-\gamma)(\gamma+1)(3\gamma+1)k^2}{2[2(3\gamma+1)k-(\gamma+1)]^2}$.

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