Exhaustible Resources, Welfare, and Technological Progress in the Stochastically Growing Open Economy

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Summary in One Slide

Will shortages of natural resources constrain economic growth?

- Yes, the amount of natural resources on earth fixed!

...perhaps no? Two apologies:

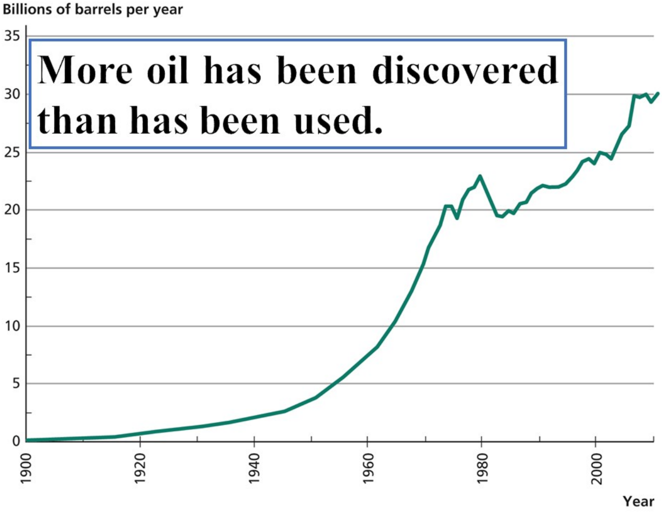
1. Resource-saving technological progress.
2. Trade. Import from abroad.

Drawbacks of these:

1. Technological progress...
   - may not come. Arrival rate stochastic.
   - is not necessarily resource-saving.
2. Import from abroad...
   - is possible at the country level.
   - is impossible at the global level.

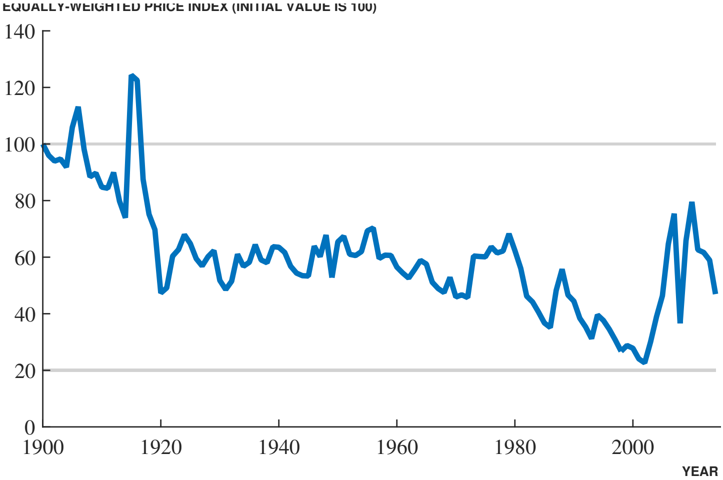
Goal: Construct a stochastic, open, endogenous growth model that can handle these considerations.

World Crude Oil Production, 1900-2010



All the oil will be gone in 61 years...*Source*: Weil (2013, p.465).

The Real Price of Industrial Commodities



Basket = Aluminum, coal, copper, lead, iron ore, and zinc. Deflated by CPI. *Source:* Jones (2016HM, p.31). Basically price ↘ (not ↗?)!

Real Price of Oil, 1861-2010

Resource scarcity ⇒ Price ↗...but price ↘!? *Source*: Weil (2013, p.481).

Literature Review I

- Dasgupt and Heal (1974RES)

Essential resource today ⇒ Discovery (Tech progress) ⇒ Inessential resource "tomorrow" ⇒ But discovery date random/stochastic.

- Solow (1978AmEcon)

CES production f ⇒ "Resources vs Capital/Labor" ⇒ Resources much less "important" ⇒ Resource scarcity = Not big problem.

- Pindyck (1984RES)

Stochastic resource dynamics. Higher resource uncertainty $\sigma \Rightarrow$ Ambiguous effects on extraction rates ν ($\sigma \uparrow \Rightarrow \nu$?)!

- Vita (2007EcoEcon)

Extends the human-capital-based endogenous growth model of Lucas (1988JME) + substitutability b/w exhaustible resources and "secondary materials" (recycled) ⇒ Affects growth during transitional dynamics.

Literature Review II

- Cheviakov and Hartwick (2009EcoEcon)

Extends Solow (1956QJE) + Exhaustible resources. Depreciation rate of physical capital $\delta \uparrow \Rightarrow$ Destroy an economy...but can be saved by tech progress. $\Rightarrow$ Sustained growth.

- Aghion and Howitt (2009, Ch.16)

AK model with exhaustible resources $\Rightarrow$ Zero growth. But the creative destruction ("Schumpeterian") model with exhaustible resources $\Rightarrow$ Sustained growth.

- Romer (2012, Sect 1.8)

Solow model with exhaustible resources $\Rightarrow$ Zero growth. But tech progress $\Rightarrow$ Undo resource scarcity. $\Rightarrow$ Sustained growth.

- (Optimistic?) Consensus?

Anyway "Tech progress $\Rightarrow$ Undo resource scarcity. $\Rightarrow$ Sustained growth?"

Model Features

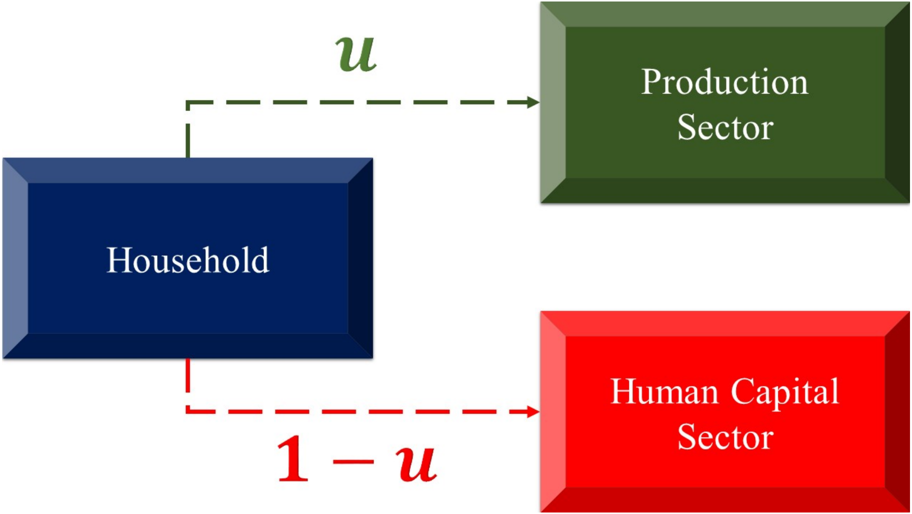
- Like Vita (2007EcoEcon), will use the Lucas (1988JME) model.

*More precisely: Uzawa (1965IER) - Lucas (1988JME) model.

- Endogenous growth featuring human capital.
- Stochastic technological progress following Bucci et al. (2011JEZN), Hiraguchi (2013JEZN), and Hiraguchi (2014MacroDyn).
- Stochastic resource dynamics following Pindyck (1980JPE) and Pindyck (1984RES).
- The world economy consisting of "small" J countries (indexed by $j = 1, ..., J$). Or $J \to \infty$. Can use ϑ_j of the global resource stock $\bar{S}$.

$\Rightarrow$ Shut down the possibility of importing resources (from "abroad"...).

Uzawa (1965IER) - Lucas (1988JME) Two-Sector Model



No leisure. Work or learn. $u(t)$ control variable. Time allocation matters!

Common (Cobb-Douglas) Production Technology

$$
\begin{aligned}
\underbrace{Y_i(t)}_{Output} &= F(A_j(t), L_j(t), K_j(t), H_j(t), S_j(t)) \\
&= (A_j(t)L_j(t))^{\alpha} K_j(t)^{\beta} (u_j(t)H_j(t))^{\gamma} (\underbrace{\vartheta_j}_{Share} \bar{S}(t))^{1-\alpha-\beta-\gamma}
\end{aligned} \quad (1)
$$

where

$A_j(t)$ = Technology

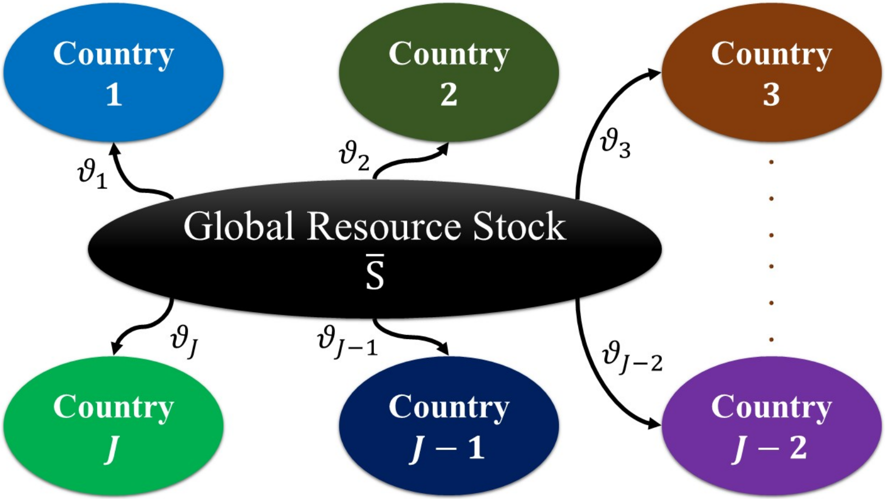
$K_j(t)$ = Physical capital stock

$H_j(t)$ = Human capital stock

$S_j(t)$ = Exhaustible resource stock $= \vartheta_j \bar{S}$. So $\sum_{j=1}^{J} \vartheta_j = 1$.

Note: Labor $L_j(t) = 1$ in what follows (for simplicity).

World Resource Sharing



Too simple to be true. Will comment on this assumption later.

Physical and Human Capital Accumulation

Goods mkt clearing condition implies ($\forall j$)

$$Y_j(t) = \underbrace{C_j(t)}_{Consumption} + \underbrace{I_j(t)}_{Investment} \tag{2}$$

Thus, physical capital accumulation in country j is

$$dK_j(t) = Y_j(t)dt - C_j(t)dt - \delta_k K_j(t)dt \tag{3}$$

Human capital accumulation in country j:

$$dH_j(t) = b(\underbrace{1 - u_j(t)}_{Learning})H_j(t)dt - \delta_h H_j(t)dt \tag{4}$$

- $b > 0$ = efficiency of human capital accumulation.
- $\delta_i (i = k, h)$ = depreciation rate of capital.

Stochastic Technology: A Geometric Brownian Motion (Wiener) Process and Many Poisson Jump Processes

Following Hiraguchi (2014MacroDyn):

$$dA_j(t) = \underbrace{\mu A_j(t)}_{Drift}\, dt + \underbrace{\sigma_a A_j(t)}_{Diffusion}\, dz_{ja} + \underbrace{\sum_{n=1}^{N} \beta_n A_j(t) dq_{jn}}_{Jump} \tag{5}$$

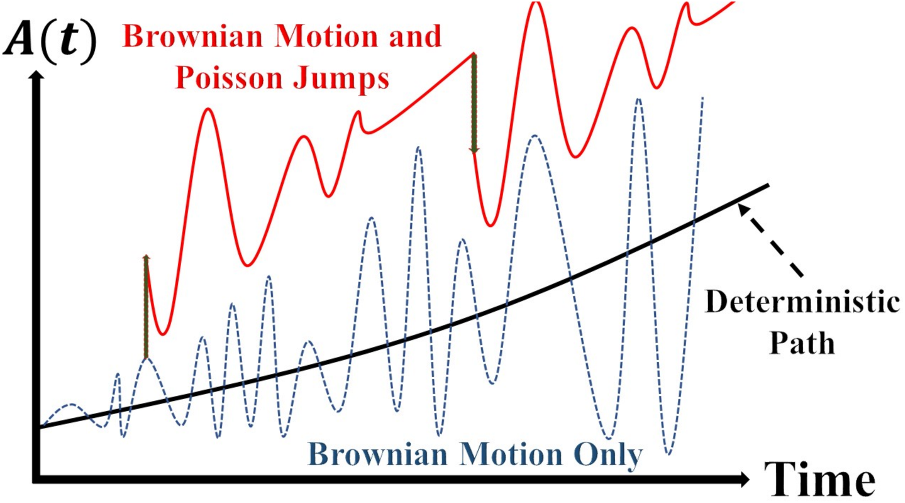
where

- $\mu > 0$ = rate of technological progress
- $\sigma_a > 0$ = diffusion coefficient of technology
- z_{ja}: Brownian motion process s.th. $z_{t+\Delta} - z_t \sim N(0, \Delta)$ for *any* Δ.

$dz_t = \lim_{\Delta \searrow 0}(z_{t+\Delta} - z_t)$ with moments $\mathcal{E}(dz_t) = 0$ and $\mathcal{V}(dz_t) = dt$.

- q_{jn}: Poisson jump process with arrival rate λ_n and a jump of size β_n.

Stochastic Processes: Illustration by Hand



In principle, fluctuates around the deterministic path, with infrequent

Stochastic Resource Dynamics

Extend Pindyck (1980JPE) and Pindyck (1984RES) by including jumps:

$$dS_j(t) = \underbrace{-\nu S_j(t)}_{Drift} dt + \underbrace{\sigma_s S_j(t)}_{Diffusion} dz_{js} + \underbrace{\sum_{m=1}^{M} \beta_m S_j(t) dq_{jm}}_{Jump} \tag{6}$$

where

- $\nu > 0$ = depletion/extraction rate
- $\sigma_s > 0$ = diffusion coefficient of resource
- z_{js}: Brownian motion process s.th. $z_{t+\Delta} - z_t \sim N(0, \Delta)$ for *any* Δ.
- q_{jm}: Poisson jump process with arrival rate λ_m and jumps of size β_m.
- η: Correlation coefficient of dz_{ja} and dz_{js}, i.e. $(dz_{ja})(dz_{js}) = \eta dt$! Key assumption to think about resource-technology nexus.

Standard CRRA Utility

Preferences of a representative household in country j are:

$$E \int_0^\infty e^{-\rho t} \frac{C_j(t)^{1-\phi} - 1}{1-\phi} dt \tag{7}$$

- E = Expectation operator with respect to the information set available to the representative household
- $\rho > 0$ = subjective discount rate
- $\phi > 0$ = index of risk aversion

In sum, the optimization problem is to

Stochastic Optimization in Continuous Time: Summary

Maximize

$$E \int_0^\infty e^{-\rho t} \frac{C_j(t)^{1-\phi} - 1}{1-\phi} dt$$

subject to

$$dK_j(t) = Y_j(t)dt - C_j(t)dt - \delta_k K_j(t)dt$$

$$dH_j(t) = b(1 - u_j(t))H_j(t)dt - \delta_h H_j(t)dt$$

$$dA_j(t) = \mu A_j(t)dt + \sigma_a A_j(t)dz_{ja} + \sum_{n=1}^{N} \beta_n A_j(t)dq_{jn}$$

$$dS_j(t) = -\nu S_j(t)dt + \sigma_s S_j(t)dz_{js} + \sum_{m=1}^{M} \beta_m S_j(t)dq_{jm}$$

HJB Partial Differential Equation: Recursively Represented

- Lagrangian $\mathcal{L}$ or Hamiltonian $\mathcal{H}$ $\Rightarrow$ Cannot be used here.
- Let $V_j(K_j, A_j, H_j, S_j)$ denote value function (indirect utility function).
- Write down the Hamilton-Jabobi-Bellman (HJB) equation:

$$\rho V_j(K_j, A_j, H_j, S_j) = max_{C_j, u_j} \left(\frac{C_j(t)^{1-\phi} - 1}{1-\phi} + \underbrace{\frac{E(...)}{dt}}_{''Ito-JumpTerms''} \right) \tag{8}$$

- Figure out the closed-form representation of the value function $V_j(K_j, A_j, H_j, S_j)$ that satisfies (8)!
- Unfortunately yet no algorithm (since Merton (1975RES!)) $\Rightarrow$ Must use the "guess and verify" method, i.e. no "method" in effect...

Waiting for the "Divine Revelation"

- Can prove: There exists no closed-form solution (as usual, see Wälde (2011JEDC) survey, among others). Stochastic growth models $\Rightarrow$ Analytical solution extremely rate ("diamond").

Only two options;

- Give up. Numerical simulation such as the value function iteration, perturbation method of Schmitt-Grohé and Uribe (2004JEDC), projection method (Xu, 2017JEDC), finite-difference method, etc. to "numerically" solve.
- Parameter restriction (Xie, 1991JPE; Xie, 1994JET; Rebelo and Xie, 1999JME; Smith, 2007BEJM; Bucci et al., 2011JEZN; Marsiglio and La Torre, 2012EconModel; Hiraguchi, 2013JEZN) is the last stand! Rarely works, but worth trying!

- If $\phi = \beta$ (risk aversion = physical capital share),

The "Divine Revelation" Realized - Transparent

then the closed-form solution is

$$V_j(K_j, A_j, H_j, S_j) = \mathbb{X} K_j^{1-\beta} + \mathbb{Y}_j A_j^{\alpha} H_j^{\gamma} S_j^{1-\alpha-\beta-\gamma} + \mathbb{Z}$$

where

$$\mathbb{X} = \mathbb{X}(\rho, \beta, \delta_k)$$

$$\mathbb{Y}_j = \mathbb{Y}_j(\rho, \alpha, \mu, b, \delta_h, \nu, \gamma, \sigma_a, \sigma_s, \eta, \beta_n, \beta_m, \lambda_n, \lambda_m, \vartheta_j)$$

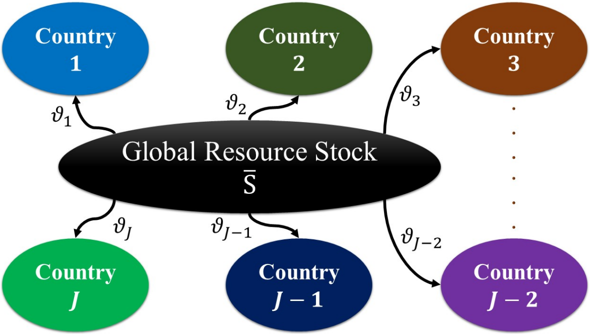
$$\mathbb{Z} = \mathbb{Z}(\rho, \beta)$$

Moreover (two control variables also explicit)

$$C_j/K_j = C_j/K_j(\rho, \beta, \delta_k)$$

$$u_j = u_j(\rho, \alpha, \mu, b, \delta_h, \nu, \gamma, \sigma_a, \sigma_s, \eta, \beta_n, \beta_m, \lambda_n, \lambda_m)$$

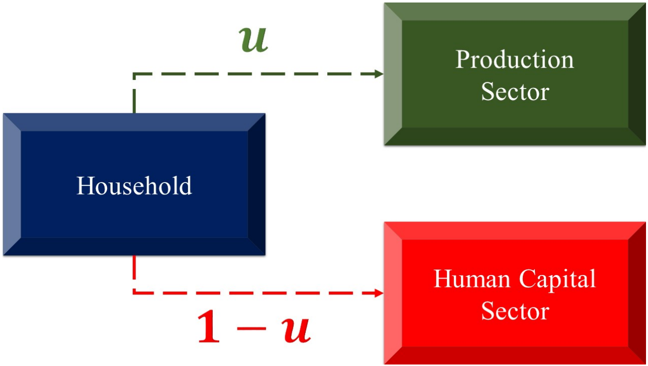
Before Welfare Analysis: Comments on This Assumption



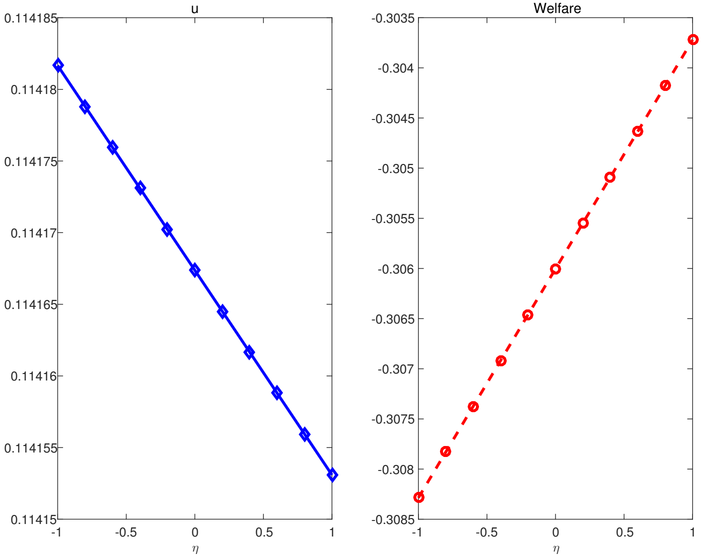
- No resource S exchange among countries. Admittedly unrealistic.
- More realistic: resource exchange among countries.
- Trade-off b/w realistic assumption and existence of analytical solution.
- All attempts so far have failed. Any way to make realistic assumption, while preserving analytical solution?...Work in progress on this point.

Welfare Analysis: Prelude

- Following Turnovsky (1997, 2000) and Tsuboi (2018JEZN), the value function $V_j(K_j, A_j, H_j, S_j)$ = Measure of welfare.
- Numerical simulation unnecessary. Just analytically check $\partial V_j/\partial x > 0(< 0)$ for key parameters x.
- Time flies! Let me illustrate with MATLAB.



Higher Correlation η Improves Welfare V_j ($\partial V_j/\partial \eta > 0$)

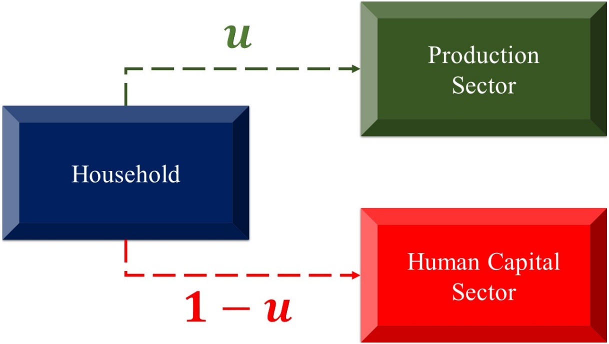


Why Higher Correlation Improves Welfare ($\partial V_j / \partial \eta > 0$)?

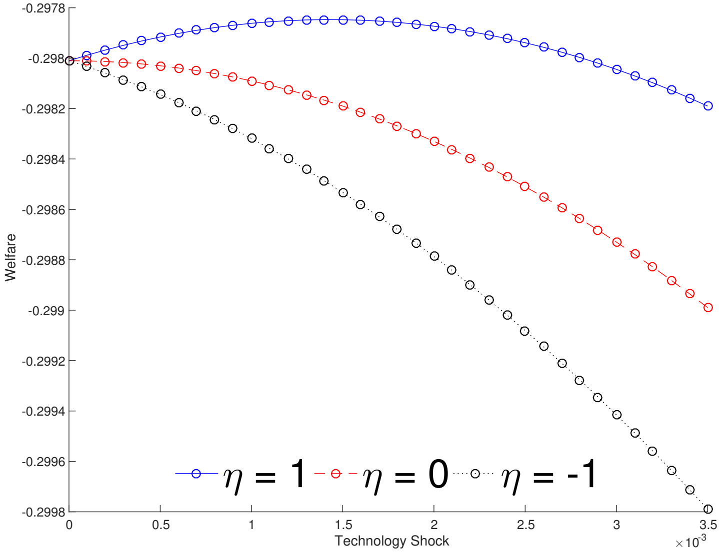
Remember:

$$u_j = u_j(\rho, \alpha, \mu, b, \delta_h, \nu, \gamma, \sigma_a, \sigma_s, \underbrace{\eta}_{(-)}, \beta_n, \beta_m, \lambda_n, \lambda_m)$$

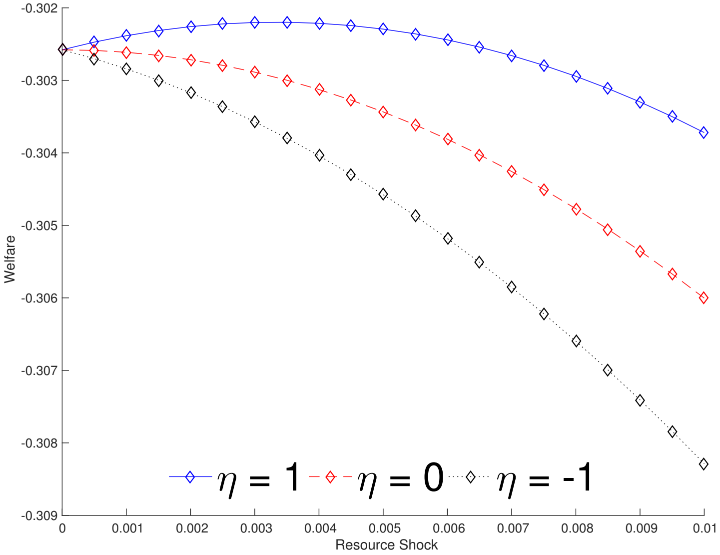
Thus, $\eta \uparrow \Rightarrow u_j \downarrow \Rightarrow 1 - u_j \uparrow \Rightarrow H_j(t) \uparrow \Rightarrow V(., ., H_j, .) \uparrow$



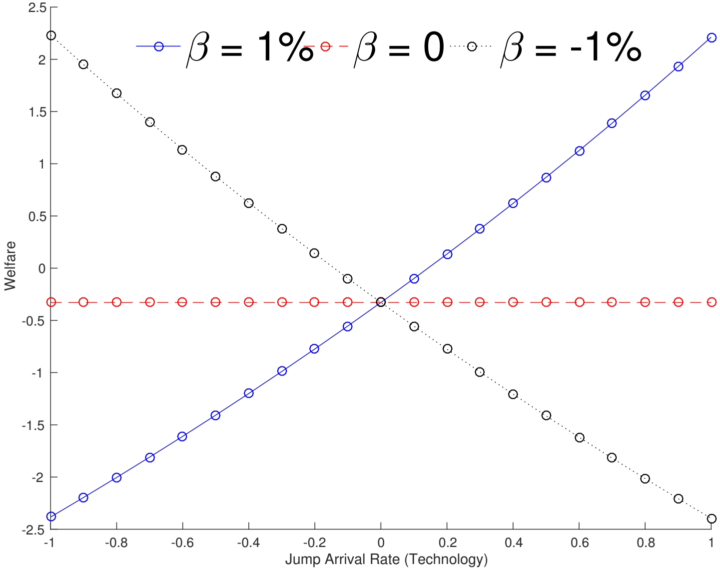
Larger technology shock σ_a and Welfare V_j: Correlation



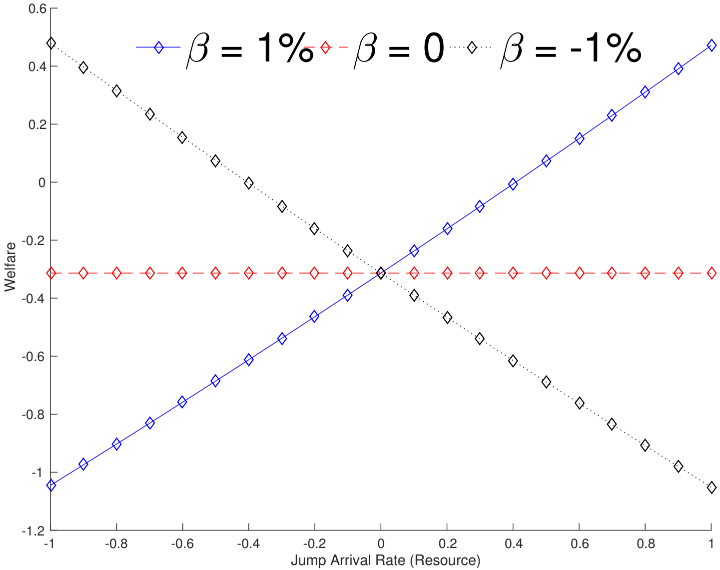
Larger resource shock σ_s and Welfare V_j: Correlation



Higher Arrival Rate λ_n of Technology and Welfare V_j



Higher Arrival Rate λ_m of Resource and Welfare V_j



Concluding Remarks

Q: "Will shortages of natural resources constrain economic growth?"

Construct the open Uzawa-Lucas model with technological progress and resource dynamics driven by stochastic processes. Analytically show that

- Higher correlation η improves welfare V_j.
- Higher uncertainty deteriorates welfare when technological progress is not resource-saving ($\eta = 0$ or $\eta < 0$).
- Higher uncertainty improves welfare when technological progress is resource-saving ($\eta > 0$), as long as σ is small enough.
- Higher arrival rate of technology λ_n improves welfare if its jump size is positive ($\beta_n > 0$).

Policy Implications: Tech progress is welcome, but not enough to undo resource scarcity. Only resource-saving tech progress can undo resource scarcity (at the global level) and improve welfare. A: "Probably NO!"

The End

My Heartfelt Thanks for Your Attention!

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