Exhaustible Resources, Welfare, and Technological Progress in the Stochastically Growing Open Economy

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Summary in One Slide

Will shortages of natural resources constrain economic growth?

- Yes, the amount of natural resources on earth fixed!

...perhaps no? Two apologies:

1. Resource-saving technological progress.
2. Trade. Import from abroad.

Drawbacks of these:

1. Technological progress...
   - may not come. Arrival rate stochastic.
   - is not necessarily resource-saving.
2. Import from abroad...
   - is possible at the country level.
   - is impossible at the global level.

Goal: Construct a stochastic, open, endogenous growth model that can handle these considerations.
World Crude Oil Production, 1900-2010

More oil has been discovered than has been used.

All the oil will be gone in 61 years... *Source: Weil (2013, p.465).*
Basket = Aluminum, coal, copper, lead, iron ore, and zinc. Deflated by CPI. *Source:* Jones (2016HM, p.31). Basically price ↘ (not ↗?!)!
Resource scarcity $\implies$ Price $\uparrow$...but price $\downarrow$!? Source: Weil (2013, p.481).
Literature Review I

- Dasgupt and Heal (1974RES)
  Essential resource today ⇒ Discovery (Tech progress) ⇒ Inessential resource ”tomorrow” ⇒ But discovery date random/stochastic.

- Solow (1978AmEcon)
  CES production f ⇒ ”Resources vs Capital/Labor” ⇒ Resources much less ”important” ⇒ Resource scarcity = Not big problem.

- Pindyck (1984RES)
  Stochastic resource dynamics. Higher resource uncertainty σ ⇒ Ambiguous effects on extraction rates ν (σ ↑ ⇒ ν?).

- Vita (2007EcoEcon)
  Extends the human-capital-based endogenous growth model of Lucas (1988JME) + substitutability b/w exhaustible resources and ”secondary materials” (recycled) ⇒ Affects growth during transitional dynamics.
Cheviakov and Hartwick (2009EcoEcon)
Extends Solow (1956QJE) + Exhaustible resources. Depreciation rate of physical capital $\delta \uparrow \Rightarrow$ Destroy an economy...but can be saved by tech progress. $\Rightarrow$ Sustained growth.

Aghion and Howitt (2009, Ch.16)
AK model with exhaustible resources $\Rightarrow$ Zero growth. But the creative destruction ("Schumpeterian") model with exhaustible resources $\Rightarrow$ Sustained growth.

Romer (2012, Sect 1.8)
Solow model with exhaustible resources $\Rightarrow$ Zero growth. But tech progress $\Rightarrow$ Undo resource scarcity. $\Rightarrow$ Sustained growth.

(Optimistic?) Consensus?
Anyway ”Tech progress $\Rightarrow$ Undo resource scarcity. $\Rightarrow$ Sustained growth?”
1. Introduction
   - Summary
   - Some Data
   - Literature Review

2. The Model
   - Capital Accumulation, Resource, and Household
   - Stochastic Technology and Resource Uncertainty
   - Stochastic Optimization

3. Welfare Analysis
   - Prelude
   - Welfare and Brownian Uncertainty
   - Welfare and Poisson Uncertainty

4. Concluding Remarks
Model Features

- Like Vita (2007EcoEcon), will use the Lucas (1988JME) model.


- Endogenous growth featuring human capital.

- Stochastic technological progress following Bucci et al. (2011JEZN), Hiraguchi (2013JEZN), and Hiraguchi (2014MacroDyn).

- Stochastic resource dynamics following Pindyck (1980JPE) and Pindyck (1984RES).

- The world economy consisting of "small" \( J \) countries (indexed by \( j = 1, \ldots, J \)). Or \( J \to \infty \). Can use \( \varphi_j \) of the global resource stock \( \bar{S} \).

⇒ Shut down the possibility of importing resources (from "abroad" ...).
No leisure. Work or learn. $u(t)$ control variable. Time allocation matters!
Common (Cobb-Douglas) Production Technology

\[
Y_i(t) = F(A_j(t), L_j(t), K_j(t), H_j(t), S_j(t)) \\
\text{Output} = (A_j(t)L_j(t))^{\alpha} K_j(t)^{\beta} (u_j(t)H_j(t))^{\gamma} (\vartheta_j \bar{S}(t))^{1-\alpha-\beta-\gamma} \tag{1}
\]

where

\( A_j(t) = \text{Technology} \)

\( K_j(t) = \text{Physical capital stock} \)

\( H_j(t) = \text{Human capital stock} \)

\( S_j(t) = \text{Exhaustible resource stock} = \vartheta_j \bar{S}. \text{ So } \sum_{j=1}^{J} \vartheta_j = 1. \)

Note: Labor \( L_j(t) = 1 \) in what follows (for simplicity).
World Resource Sharing

Too simple to be true. Will comment on this assumption later.
Physical and Human Capital Accumulation

Goods mkt clearing condition implies (\forall j)

\[ Y_j(t) = \underbrace{C_j(t)}_{\text{Consumption}} + \underbrace{l_j(t)}_{\text{Investment}} \]  (2)

Thus, physical capital accumulation in country j is

\[ dK_j(t) = Y_j(t)dt - C_j(t)dt - \delta_k K_j(t)dt \]  (3)

Human capital accumulation in country j:

\[ dH_j(t) = b(1 - u_j(t))H_j(t)dt - \delta_h H_j(t)dt \]  (4)

- \( b > 0 \) = efficiency of human capital accumulation.
- \( \delta_i (i = k, h) \) = depreciation rate of capital.
Stochastic Technology: A Geometric Brownian Motion (Wiener) Process and Many Poisson Jump Processes

Following Hiraguchi (2014MacroDyn):

\[
dA_j(t) = \mu A_j(t)\,dt + \sigma_a A_j(t)\,dz_{ja} + \sum_{n=1}^{N} \beta_n A_j(t)\,dq_{jn} \quad (5)
\]

where

- \( \mu > 0 \) = rate of technological progress
- \( \sigma_a > 0 \) = diffusion coefficient of technology
- \( z_{ja} \): Brownian motion process s.th. \( z_{t+\Delta} - z_t \sim N(0, \Delta) \) for any \( \Delta \).
- \( dz_t = \lim_{\Delta \searrow 0} (z_{t+\Delta} - z_t) \) with moments \( \mathcal{E}(dz_t) = 0 \) and \( \mathcal{V}(dz_t) = dt \).
- \( q_{jn} \): Poisson jump process with arrival rate \( \lambda_n \) and a jump of size \( \beta_n \).
In principle, fluctuates around the deterministic path, with infrequent...
Stochastic Resource Dynamics

Extend Pindyck (1980JPE) and Pindyck (1984RES) by including jumps:

\[
dS_j(t) = -\nu S_j(t)\,dt + \sigma_s S_j(t)\,dz_s + \sum_{m=1}^{M} \beta_m S_j(t)\,dq_{jm}
\]

(6)

where

- \(\nu > 0\) = depletion/extraction rate
- \(\sigma_s > 0\) = diffusion coefficient of resource
- \(z_{js}\): Brownian motion process s.th. \(z_{t+\Delta} - z_t \sim N(0, \Delta)\) for any \(\Delta\).
- \(q_{jm}\): Poisson jump process with arrival rate \(\lambda_m\) and jumps of size \(\beta_m\).
- \(\eta\): Correlation coefficient of \(dz_{ja}\) and \(dz_{js}\), i.e. \((dz_{ja})(dz_{js}) = \eta dt\)! Key assumption to think about resource-technology nexus.
Standard CRRA Utility

Preferences of a representative household in country $j$ are:

$$E \int_{0}^{\infty} e^{-\rho t} \frac{C_j(t)^{1-\phi} - 1}{1 - \phi} dt$$

- $E = \text{Expectation operator with respect to the information set available to the representative household}$
- $\rho > 0 = \text{subjective discount rate}$
- $\phi > 0 = \text{index of risk aversion}$

In sum, the optimization problem is to
Stochastic Optimization in Continuous Time: Summary

Maximize

\[ E \int_{0}^{\infty} e^{-\rho t} \frac{C_j(t)^{1-\phi} - 1}{1 - \phi} dt \]

subject to

\[ dK_j(t) = Y_j(t)dt - C_j(t)dt - \delta_k K_j(t)dt \]

\[ dH_j(t) = b(1 - u_j(t))H_j(t)dt - \delta_h H_j(t)dt \]

\[ dA_j(t) = \mu A_j(t)dt + \sigma_a A_j(t)dz_{ja} + \sum_{n=1}^{\infty} \beta_n A_j(t)dq_{jn} \]

\[ dS_j(t) = -\nu S_j(t)dt + \sigma_s S_j(t)dz_{js} + \sum_{m=1}^{\infty} \beta_m S_j(t)dq_{jm} \]
HJB Partial Differential Equation: Recursively Represented

- Lagrangian $\mathcal{L}$ or Hamiltonian $\mathcal{H} \Rightarrow$ Cannot be used here.
- Let $V_j(K_j, A_j, H_j, S_j)$ denote value function (indirect utility function).
- Write down the Hamilton-Jabobi-Bellman (HJB) equation:

$$
\rho V_j(K_j, A_j, H_j, S_j) = \max_{C_j, u_j} \left( \frac{C_j(t)^{1-\phi} - 1}{1 - \phi} + \frac{E(\cdots)}{dt} \right) \\
\text{"Ito-JumpTerms"}
$$

(8)

- Figure out the closed-form representation of the value function $V_j(K_j, A_j, H_j, S_j)$ that satisfies (8)!
- Unfortunately yet no algorithm (since Merton (1975RES!)) ⇒ Must use the "guess and verify" method, i.e. no "method" in effect...
Waiting for the ”Divine Revelation”

- Can prove: There exists no closed-form solution (as usual, see Wälde (2011JEDC) survey, among others). Stochastic growth models ⇒ Analytical solution extremely rate (”diamond”).

Only two options;

- Give up. Numerical simulation such as the value function iteration, perturbation method of Schmitt-Grohé and Uribe (2004JEDC), projection method (Xu, 2017JEDC), finite-difference method, etc. to ”numerically” solve.

- Parameter restriction (Xie, 1991JPE; Xie, 1994JET; Rebelo and Xie, 1999JME; Smith, 2007BEJM; Bucci et al., 2011JEZN; Marsiglio and La Torre, 2012EconModel; Hiraguchi, 2013JEZN) is the last stand! Rarely works, but worth trying!

- If $\phi = \beta$ (risk aversion = physical capital share),
The "Divine Revelation" Realized - Transparent

then the closed-form solution is

$$V_j(K_j, A_j, H_j, S_j) = X K_j^{1-\beta} + Y_j A_j^\alpha H_j^\gamma S_j^{1-\alpha-\beta-\gamma} + \mathbb{Z}$$

where

$$X = X(\rho, \beta, \delta_k)$$

$$Y_j = Y_j(\rho, \alpha, \mu, b, \delta_h, \nu, \gamma, \sigma_a, \sigma_s, \eta, \beta_n, \beta_m, \lambda_n, \lambda_m, \psi_j)$$

$$Z = Z(\rho, \beta)$$

Moreover (two control variables also explicit)

$$C_j/K_j = C_j/K_j(\rho, \beta, \delta_k)$$

$$u_j = u_j(\rho, \alpha, \mu, b, \delta_h, \nu, \gamma, \sigma_a, \sigma_s, \eta, \beta_n, \beta_m, \lambda_n, \lambda_m)$$
Before Welfare Analysis: Comments on This Assumption

- No resource $S$ exchange among countries. Admittedly unrealistic.
- More realistic: resource exchange among countries.
- Trade-off b/w realistic assumption and existence of analytical solution.
- All attempts so far have failed. Any way to make realistic assumption, while preserving analytical solution?...Work in progress on this point.
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4 Concluding Remarks
Following Turnovsky (1997, 2000) and Tsuboi (2018JEZN), the value function $V_j(K_j, A_j, H_j, S_j) = \text{Measure of welfare.}$

- **Numerical simulation unnecessary.** Just analytically check $\partial V_j/\partial x > 0 (< 0)$ for key parameters $x$.

- **Time flies!** Let me illustrate with MATLAB.
Higher Correlation $\eta$ Improves Welfare $V_j$ ($\partial V_j / \partial \eta > 0$)
Why Higher Correlation Improves Welfare ($\partial V_j/\partial \eta > 0$)?

Remember:

$$u_j = u_j(\rho, \alpha, \mu, b, \delta_h, \nu, \gamma, \sigma_a, \sigma_s, \eta, \beta_n, \beta_m, \lambda_n, \lambda_m)$$

Thus, $\eta \uparrow \Rightarrow u_j \downarrow \Rightarrow 1 - u_j \uparrow \Rightarrow H_j(t) \uparrow \Rightarrow V(.,.,.H_j,.) \uparrow$
Larger technology shock $\sigma_a$ and Welfare $V_j$: Correlation

$\eta = 1$ - $\eta = 0$ - $\eta = -1$
Larger resource shock $\sigma_s$ and Welfare $V_j$: Correlation
Higher Arrival Rate $\lambda_n$ of Technology and Welfare $V_j$

\[ \beta = 1\% \quad \beta = 0 \quad \beta = -1\% \]
Higher Arrival Rate $\lambda_m$ of Resource and Welfare $V_j$
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Concluding Remarks

Q: "Will shortages of natural resources constrain economic growth?"

Construct the open Uzawa-Lucas model with technological progress and resource dynamics driven by stochastic processes. Analytically show that

- Higher correlation $\eta$ improves welfare $V_j$.
- Higher uncertainty deteriorates welfare when technological progress is not resource-saving ($\eta = 0$ or $\eta < 0$).
- Higher uncertainty improves welfare when technological progress is resource-saving ($\eta > 0$), as long as $\sigma$ is small enough.
- Higher arrival rate of technology $\lambda_n$ improves welfare if its jump size is positive ($\beta_n > 0$).

Policy Implications: Tech progress is welcome, but not enough to undo resource scarcity. Only resource-saving tech progress can undo resource scarcity (at the global level) and improve welfare. A: "Probably NO!"
The End

My Heartfelt Thanks for Your Attention!


References II


References V


