

# Quantifying the Optimal Mix of Tariffs and Linear Income Taxes

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## Abstract

Donald Trump and his supporter, Marc Andreessen, advocate using tariffs as a major revenue source of the U.S. government instead of income taxes. To assess this idea, this paper quantifies the optimal mix of import tariffs and linear income taxes. I develop a two-country general equilibrium model where heterogeneous households elastically supply labor, the two countries exchange goods as in the Armington model, and the Home government finances non-rivalrous and non-excludable public goods with ad valorem tariffs and linear income taxes. A unique equilibrium exists. Calibrating the model for the U.S. and the rest of the world as of 2017, the optimal U.S. tax rates are a 25-percent tariff and a 20-percent income tax. At the optimum, tariffs account for only 3.3 percent of government revenue.

Keywords: Income Taxes, Tariffs, Quantitative Trade Models

JEL classification: F13, H21, H24, H41

## 1 Introduction

Referring to Figure 1, a Trump supporter and billionaire, Marc Andreessen, tweeted

This is a really remarkable chart of tariffs as % of total federal revenue. The Second Industrial Revolution, perhaps the most fertile era for technology development and deployment in human history, was 1870-1914. ([Andreessen, 2024](#))

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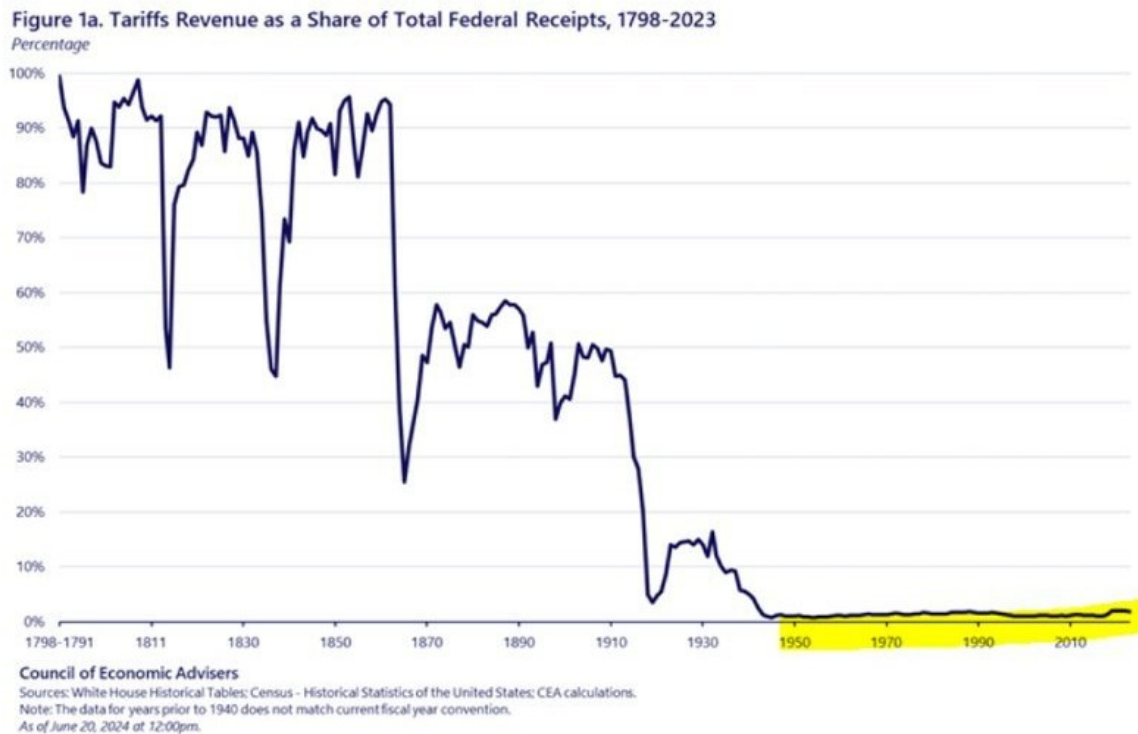


Figure 1: The Tariff Share in the U.S. Government’s Revenue

Quoting Andreessen’s tweet above, Donald Trump said

The Tariffs, and Tariffs alone, created this vast wealth for our Country. Then we switched over to Income Tax. We were never so wealthy as during this period. Tariffs will pay off our debt and, MAKE AMERICA WEALTHY AGAIN! (Trump, 2025)

With nostalgia for the era when tariffs accounted for a bulk of government revenue, these two men assert that the U.S. government should replace income taxes with tariffs for a primary source of revenue. Are they right? What are the optimal rates of import tariffs and income taxes?

To answer these questions, we develop a two-country general equilibrium model. In the model, heterogeneous households elastically supply labor, the two countries trade goods with each other as in the Armington trade model, and the Home government imposes ad valorem tariffs and linear income taxes to finance non-rivalrous and non-excludable public goods. Income taxes distort labor supply decisions, and tariffs distort consumption behavior. But, at the same time, these two taxes are valuable tax bases to provide public goods.

For such a model, we show that a unique equilibrium exists. Our primary focus is on

welfare as a function of the two kinds of taxes. In the case of a small open economy which treats the Foreign wage as exogenous, we can derive the first-order conditions of welfare with respect to the tariff and income tax rates. Assuming an interior solution, the optimal income tax rate is a non-monotonic function of a given tariff rate. However, for small tariffs (tariffs smaller than 25 percent for the trade elasticity of 4), the optimal income tax is a decreasing function of tariff rates. Therefore, for sufficiently small tariff rates, tariffs and income taxes are substitutable tax bases.

In order to quantify the optimal two dimensional vector of the tariff and the income tax, we numerically compute the case of a large open economy case where both Home and Foreign factor prices are endogenous. We calibrate the model for the United States and the rest of the world in 2017. Computing equilibria for many grids of tariffs and income taxes, we find that the tax rates maximizing the U.S. welfare is a 25-percent tariff and a 20-percent income tax. Although this optimal tariff is much higher than the actual U.S. tariff in 2017 (and 2026), the tariff revenue associated with these optimal taxes accounts for only 3.3 percent of government revenue. This number is much smaller than the tariff share in government revenue in the 19th century, which Andreessen and Trump admired.

Even if we cannot rationalize Andreessen and Trump's idea in the current era, it might be possible to justify William McKinley's very high tariffs in the late 19th century because parameter values are different between then and the current era. Among many, the striking difference between these two eras is non-tariff trade costs; transportation costs significantly declined over time. Guided by this idea, we compute the optimal pairs of tariffs and income taxes for different iceberg non-tariff trade costs. Interestingly, the optimal tariff is decreasing, and the optimal income tax is increasing in the iceberg non-tariff trade costs. Therefore, the welfare maximizing tariff was lower and the welfare maximizing income tax was higher in the 19th-century U.S. than in the current U.S. This is at odds with the actual history; tariffs decreased and income taxes increased over time.

This paper relates to literature on (i) (optimal) tariffs, (ii) (optimal) income taxes, and (iii) the intersection of these two. (i) In the literature on tariffs, [Johnson \(1950\)](#) derives a classic formula for optimal tariffs analyzing a stylized two-country model. [Costinot and Rodriguez-Clare \(2014\)](#) are a then-comprehensive review of quantitative trade models including numerical computation of optimal tariffs in a many-country trade model. [Demidova et al. \(2024\)](#) formulate a small open economy as a limit of a wide range of quantitative trade models and derive an optimal tariff formula for such an economy.<sup>1</sup>

(ii) Research on optimal income taxes has long history in theoretical public economics. There are two approaches in this regard. One is the Ramsay approach which assumes the

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<sup>1</sup>[Gros \(1987\)](#), [Costinot et al. \(2015\)](#), and [Costinot et al. \(2020\)](#) derive optimal tariff formulae for the Krugman, Dornbusch-Fischer-Samuelson, and Melitz models, respectively. [Alvarez and Lucas \(2007\)](#) numerically illustrate optimal tariffs for a small open economy case in the Eaton-Kortum model.

government's perfect information on income or wealth levels of households.<sup>2</sup> The other is the Mirrlees approach which emphasizes asymmetric information; the government has limited information on households incomes (Mirrlees, 1971). The current paper belongs to the former. Notable contributions are Sheshinski (1972) who theoretically derives optimal linear income taxes and Saez (2001) who derives optimal linear and nonlinear income taxes using sufficient statistics.

(iii) There is a niche research area on the intersection of tariffs and income taxes. To the best of my knowledge, no existing paper quantifies the optimal mix of tariffs and income taxes, whereas this paper does. There is research on a model that includes both tariffs and income taxes. Kocherlakota (2025) extends Johnson (1950)'s classic model to derive an optimal tariff formula in the presence of distortionary income taxes. Lashkaripour (2020) uses a multi-sector Eaton-Kortum model (Caliendo and Parro, 2015) to consider how tariffs (or, more generally, trade taxes) can replace income taxes for government revenue. These two papers assume the lump-sum transfer of tax revenue to households, whereas the current paper assumes that taxes are used to finance public goods.

## 2 Model

We consider a static general equilibrium model. There are two countries: Home and Foreign. The Home government levies taxes; the Foreign government does not. (Or, putting differently, we assume that there is no government in Foreign.)

### 2.1 Preferences, Consumption, and Labor Supply

In Home, individuals are heterogeneous (only) in "ability"  $\omega$ . Individual  $\omega$  in Home has the following utility function

$$U_H(\omega) = C_H(\omega)^\beta S_H^{1-\beta} - \frac{l_H(\omega)^{1+\nu}}{1+\nu}, \quad (1)$$

where  $C_H(\omega)$  is individual  $\omega$ 's consumption of the private good,  $S_H$  is consumption of the (non-exclusive and non-rivalrous) public good of anyone in Home,  $\beta \in [0, 1]$  is the Cobb-Douglas parameter governing the importance of the consumption good relative to the public good,  $l_H(\omega)$  is individual  $\omega$ 's work hours,  $\nu > 0$  is the parameter governing the labor supply elasticity.

Note that here the public good is as defined in a standard textbook in public economics; non-exclusive and non-rivalrous. This means that, an individual's consumption of the

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<sup>2</sup>The literature refer to Ramsey (1927) although his paper is about commodity taxes.

public good does not affect another individual's consumption of it. And no one can prevent any individual from consuming the public good within Home.

The consumption of the private good  $C_H(\omega)$  is, in turn, defined by

$$C_H(\omega) = \left( C_{H,H}(\omega)^{\frac{\sigma-1}{\sigma}} + C_{F,H}(\omega)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (2)$$

where  $C_{H,H}(\omega)$  is individual  $\omega$ 's consumption of the private good shipped from Home itself,  $C_{F,H}(\omega)$  is individual  $\omega$ 's consumption of the private good shipped from Foreign to Home, and  $\sigma > 1$  is the elasticity of substitution between the Home and Foreign private goods.

The budget constraint is

$$p_H C_{H,H}(\omega) + \tilde{\tau}_t d p_F C_{F,H}(\omega) \leq \tilde{\tau}_i \omega w_H l(\omega), \quad (3)$$

where  $p_H$  is the f.o.b. price of private goods produced in Home,  $p_F$  is the f.o.b. price of private goods produced in Foreign,  $\tilde{\tau}_t = 1 + \tau_t$  is the tariff factor in the Foreign good prices Home consumers face ( $\tau_t$  is the tariff rate),  $d$  is the symmetric iceberg trade costs between Home and Foreign,  $\tilde{\tau}_i = 1 - \tau_i$  is the after-income-tax portion of individual  $\omega$ 's labor income ( $\tau_i$  is the linear labor income tax Home imposes on its residents),  $w_H$  is the wage rate for effective labor in Home.

The price index for private goods in Home (which is dual to (2)) is

$$P_H = \left( p_H^{1-\sigma} + (\tilde{\tau}_t d p_F)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \quad (4)$$

Using this, individual  $\omega$ 's consumption of private goods is equalized to her real income

$$C_H(\omega) = \frac{\tilde{\tau}_i \omega w_H l_H(\omega)}{P_H}. \quad (5)$$

Plugging this into (1), the indirect utility given work hours  $l_H(\omega)$  is

$$\left( \frac{\tilde{\tau}_i \omega w_H l_H(\omega)}{P_H} \right)^\beta S_H^{1-\beta} - \frac{l_H(\omega)^{1+\nu}}{1+\nu}. \quad (6)$$

Take the first order condition of this indirect utility with respect to work hours  $l(\omega)$ . Then we have

$$l_H(\omega) = \left( \frac{\tilde{\tau}_i \omega w_H}{P_H} \right)^{\frac{\beta}{\nu+1-\beta}} \beta^{\frac{1}{\nu+1-\beta}} S_H^{\frac{1-\beta}{\nu+1-\beta}}. \quad (7)$$

Plugging this into (1), we have the indirect utility of individual  $\omega$

$$\left( \frac{\tilde{\tau}_i \omega w_H}{P_H} \right)^{\frac{\beta(1+\nu)}{\nu+1-\beta}} S_H^{\frac{(1-\beta)(\nu+1)}{\nu+1-\beta}} \left( \beta^{\frac{\beta}{\nu+1-\beta}} - \frac{1}{1+\nu} \beta^{\frac{1+\nu}{\nu+1-\beta}} \right). \quad (8)$$

Let  $F(\cdot)$  be the probability distribution function of abilities  $\omega$ . I call the expected utility integrating over potential abilities  $\omega$  as the ex ante expected utility ("ex ante" refers to the situation before abilities are realized). The ex ante expected utility is

$$\left( \frac{\tilde{\tau}_i w_H}{P_H} \right)^{\frac{\beta(1+\nu)}{\nu+1-\beta}} S_H^{\frac{(1-\beta)(\nu+1)}{\nu+1-\beta}} \left( \beta^{\frac{\beta}{\nu+1-\beta}} - \frac{1}{1+\nu} \beta^{\frac{1+\nu}{\nu+1-\beta}} \right) \int \omega^{\frac{\beta(1+\nu)}{\nu+1-\beta}} dF(\omega). \quad (9)$$

Since this is the welfare measure I focus on, I use the welfare and the ex ante expected utility interchangeably.

Plugging (7) into (5), individual  $\omega$ 's consumption of private goods is

$$C_H(\omega) = \left( \frac{\tilde{\tau}_i \omega w_H}{P_H} \right)^{\frac{\nu+1}{\nu+1-\beta}} \beta^{\frac{1}{\nu+1-\beta}} S_H^{\frac{1-\beta}{\nu+1-\beta}}. \quad (10)$$

Let  $L_H > 0$  be the population in Home. Then the aggregate consumption of private goods,  $C_H$ , in Home is

$$\begin{aligned} C_H &= L_H \int C_H(\omega) dF(\omega) \\ &= L_H \left( \frac{\tilde{\tau}_i w_H}{P_H} \right)^{\frac{\nu+1}{\nu+1-\beta}} \beta^{\frac{1}{\nu+1-\beta}} S_H^{\frac{1-\beta}{\nu+1-\beta}} \int \omega^{\frac{\nu+1}{\nu+1-\beta}} dF(\omega). \end{aligned} \quad (11)$$

## 2.2 Trade, the Government's Budget Constraint, and Production

Assume that the government transforms private goods to public goods in a one-for-one fashion. Home's aggregate expenditure (including both individuals' and the government's),  $X_H$ , is

$$X_H = P_H(C_H + S_H). \quad (12)$$

Home's expenditure share on Foreign goods,  $\pi_{F,H}$  is

$$\pi_{F,H} = \frac{(\tilde{\tau}_t d p_F)^{1-\sigma}}{p_H^{1-\sigma} + (\tilde{\tau}_t d p_F)^{1-\sigma}}. \quad (13)$$

The trade value from Foreign to Home including tariff payments,  $X_{F,H}$ , is

$$X_{F,H} = \pi_{F,H} X_H = \frac{(\tilde{\tau}_t dp_F)^{1-\sigma}}{p_H^{1-\sigma} + (\tilde{\tau}_t dp_F)^{1-\sigma}} P_H (C_H + S_H). \quad (14)$$

Then the tariff revenues in Home are

$$\frac{\tau_t}{\tilde{\tau}_t} X_{F,H} = \frac{\tau_t}{\tilde{\tau}_t} \pi_{F,H} X_H = \frac{\tau_t}{\tilde{\tau}_t} \frac{(\tilde{\tau}_t dp_F)^{1-\sigma}}{p_H^{1-\sigma} + (\tilde{\tau}_t dp_F)^{1-\sigma}} P_H (C_H + S_H). \quad (15)$$

And Home's revenues from labor income taxes are

$$L_H \int \tau_i \omega w_H l(\omega) dF(\omega). \quad (16)$$

Then the Home government's budget constraint is

$$P_H S_H = L_H \int \tau_i \omega w_H l(\omega) dF(\omega) + \frac{\tau_t}{\tilde{\tau}_t} X_{F,H}, \quad (17)$$

where the left-hand side is the government spending, the first term on the right-hand side is the income tax revenues, and the second term on the right-hand side is the tariff revenues.

Individuals are homogeneous in Foreign. Any individual has the utility function

$$U_F = C_F = \left( C_{H,F}^{\frac{\sigma-1}{\sigma}} + C_{F,F}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (18)$$

and the budget constraint

$$p_H dC_{H,F} + p_F C_{F,F} \leq w_F. \quad (19)$$

Then the indirect utility is

$$\frac{w_F}{P_F}, \quad (20)$$

where  $P_F$  is the ideal price index

$$P_F = \left( (dp_H)^{1-\sigma} + p_F^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \quad (21)$$

We move on to production. Let  $i = H, F$  indexes Home and Foreign. There is a representative firm in each country, which exhibits constant returns to scale. The profit of the representative firm in country  $i$  is

$$\pi_i = p_i y_i - w_i l_i, \quad (22)$$

where  $y_i$  is the production level of the representative firm in  $i$ , and  $l_i$  is the amount of labor the firm hires. The production function in  $i$  is linear

$$y_i = a_i l_i, \quad (23)$$

where  $a_i > 0$  is the labor productivity. We assume perfect competition and the representative firm makes zero profit. Therefore,

$$p_i = \frac{w_i}{a_i}. \quad (24)$$

Home's trade balance condition is

$$P_H(C_H + S_H)\pi_{F,H}\frac{1}{\tilde{\tau}_t} = w_F L_F \pi_{H,F}, \quad (25)$$

where the left-hand side is Home's import value (net of tariffs), and the right-hand side is Home's export value.

### 2.3 Rewriting Income Tax Revenues

We choose Home's labor as a numeraire; that is,  $w_H = 1$ . Recall that the Home government's revenues from labor income taxes are  $L_H \int \tau_i \omega w_H l(\omega) dF(\omega)$ . We rewrite the integral.

$$\begin{aligned} & \int \tau_i \omega w_H l(\omega) dF(\omega) \\ &= \tau_i \left( \frac{\tilde{\tau}_i}{P_H} \right)^{\frac{\beta}{v+1-\beta}} \beta^{\frac{1}{v+1-\beta}} S_H^{\frac{1-\beta}{v+1-\beta}} \int \omega^{\frac{v+1}{v+1-\beta}} dF(\omega). \end{aligned} \quad (26)$$

### 2.4 Equilibrium

Given  $\tau_t$  ( $\tilde{\tau}_t$ ) and  $\tau_i$  ( $\tilde{\tau}_i$ ), choosing Home's labor as a numeraire  $w_H = 1$ , an equilibrium is a tuple of  $w_F$ ,  $\pi_{H,F}$ ,  $\pi_{F,H}$ ,  $P_H$ ,  $C_H$ , and  $S_H$  such that

$$w_F L_F \pi_{H,F} = P_H(C_H + S_H)\pi_{F,H}\frac{1}{\tilde{\tau}_t}, \quad (27)$$

$$\pi_{H,F} = \frac{(d(1/a_H))^{1-\sigma}}{(d(1/a_H))^{1-\sigma} + (w_F/a_F)^{1-\sigma}}, \quad (28)$$

$$\pi_{F,H} = \frac{(\tilde{\tau}_t d(w_F/a_F))^{1-\sigma}}{(1/a_H)^{1-\sigma} + (\tilde{\tau}_t d(w_F/a_F))^{1-\sigma}}, \quad (29)$$

$$P_H = \left( (1/a_H)^{1-\sigma} + ((\tilde{\tau}_t dw_F)/a_F)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \quad (30)$$

$$C_H = L_H \left( \frac{\tilde{\tau}_i}{P_H} \right)^{\frac{\nu+1}{\nu+1-\beta}} \beta^{\frac{1}{\nu+1-\beta}} S_H^{\frac{1-\beta}{\nu+1-\beta}} \int \omega^{\frac{\nu+1}{\nu+1-\beta}} dF(\omega), \quad (31)$$

$$P_H S_H = L_H \tau_i \left( \frac{\tilde{\tau}_i}{P_H} \right)^{\frac{\beta}{\nu+1-\beta}} \beta^{\frac{1}{\nu+1-\beta}} S_H^{\frac{1-\beta}{\nu+1-\beta}} \int \omega^{\frac{\nu+1}{\nu+1-\beta}} dF(\omega) + \frac{\tau_t}{\tilde{\tau}_i} (\pi_{F,H} P_H (C_H + S_H)). \quad (32)$$

This is a system of 6 equations with 6 unknowns.

(31) and (32) can be further condensed, and (32) is rewritten as

$$S_H = \left( \frac{\left( \frac{\tau_i}{\tilde{\tau}_i} + \frac{\tau_t}{\tilde{\tau}_i} \pi_{FH} \right) L_H \left( \frac{\tilde{\tau}_i}{P_H} \right)^{k_0} \beta^{\frac{1}{\nu+1-\beta}} \Omega_0}{1 - \frac{\tau_t}{\tilde{\tau}_i} \pi_{FH}} \right)^{\frac{\nu+1-\beta}{\nu}}, \quad (33)$$

where

$$k_0 = \frac{\nu+1}{\nu+1-\beta} \quad (34)$$

and

$$\Omega_0 = \int \omega^{\frac{\nu+1}{\nu+1-\beta}} dF(\omega). \quad (35)$$

## 2.5 Existence and Uniqueness

For this (large) open economy, a unique equilibrium exists for either  $\tau_i > 0$  or  $\tau_t > 0$ . See Appendix A for the proof.

## 2.6 Pareto Distribution

In the remainder of this paper, we assume that individual abilities  $\omega$  follow the Pareto distribution with a minimum value  $\omega_{\min} > 0$  and a shape parameter  $\alpha$ . The distribution function is

$$F(\omega) = 1 - \left( \frac{\omega_{\min}}{\omega} \right)^\alpha. \quad (36)$$

So far we have two integrals over  $F(\omega)$ :  $\int \omega^{\beta k_0} dF(\omega)$  in (9) and  $\int \omega^{k_0} dF(\omega)$  in various equations. Let  $\Omega_1$  denote the latter integral. Since  $0 < \beta k_0 < k_0$ , in order to make these two integrals finite, we need

$$k_0 = \frac{\nu+1}{\nu+1-\beta} < \alpha. \quad (37)$$

We assume so in the following. Then we have

$$\Omega_0 = \frac{\alpha}{\alpha - k_0} \omega_{\min}^{k_0}, \quad \Omega_1 = \frac{\alpha}{\alpha - \beta k_0} \omega_{\min}^{\beta k_0}. \quad (38)$$

Note that none of these depends on endogenous objects of the equilibrium system.

## 2.7 Small Open Economy

Fixing the Foreign wage  $w_F$ , we can treat the Home economy as a small open economy. Assuming an interior solution, for such a case, we can derive the first-order conditions of welfare with respect to a tariff and an income tax. These are a necessary but not sufficient condition for the optimal taxes. The first-order conditions are

$$\tau_i(\tau_t) = \frac{1 - \beta - J(\tau_t)}{2 - \beta} \quad (39)$$

and

$$-(2 - \beta) \frac{P_H(\tau_t)'}{P_H(\tau_t)} + (1 - \beta) \frac{J'(\tau_t)}{\tau_i + J(\tau_t)} + (1 - \beta) \frac{H'(\tau_t)}{1 - H(\tau_t)} = 0, \quad (40)$$

where  $J$  and  $H$  are defined by

$$J(\tau_t) = \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH}(\tau_t) \cdot P_H(\tau_t) \quad (41)$$

and

$$H = \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH}(\tau_t), \quad (42)$$

and all of  $J$ ,  $H$ , and  $P_H$  are regarded as functions of the tariff rate  $\tau_t$ . Therefore, all derivatives in (40) are with respect to  $\tau_t$ . (39) implies that for a given tariff level, the optimal income tax is derived. Note that even for the small open economy, the first-order conditions only implicitly characterize the optimal tariff by (40).

In general,  $J$  is not monotonic in  $\tau_t$ . However, for a specific range of  $\tau_t$ , it is monotonic. To see this, we compute the log derivative of  $J$

$$\frac{J'(\tau_t)}{J(\tau_t)} = \frac{(1/\tau_t) + 1 - \sigma + \sigma \pi_{F,H}(\tau_t)}{1 + \tau_t}. \quad (43)$$

Therefore, we have

$$J'(\tau_t) > 0 \Leftrightarrow \frac{1}{\tau_t} + 1 - \sigma + \sigma \pi_{F,H}(\tau_t) > 0. \quad (44)$$

Since  $\pi_{F,H}(\tau_t) > 0$  for any finite  $\tau_t > 0$ ,  $\pi_{F,H}(\tau_t) > 0$ . Therefore, a sufficient condition for

the right-hand side of  $\Leftrightarrow$  in (44) is

$$\tau_t < \frac{1}{\sigma - 1}. \quad (45)$$

For example, later we use a value of  $\sigma = 5$  in quantification. In this case, if the tariff rate is less than 25 percent, the optimal income tax is decreasing in the (given) tariff rate. For a tariff rate small enough, tariffs and income taxes serve as substitutes to raise a tax base.

### 3 Calibration

We calibrate the model for the United States and the rest of the world in 2017. In the remainder of this paper, we consider the large open economy equilibrium defined in Subsection 2.4. To compute such an equilibrium, we need to calibrate various parameters. Home is the U.S., and Foreign is the rest of the world. We externally calibrate  $\nu = 1$  so that the Frisch elasticity is one. We set  $\sigma = 5$  following Broda and Weinstein (2006). We normalize the Home productivity  $a_H$  and the Home population  $L_H$  to one. The former (productivity) is just arbitrary, but the latter (population) must be one in order to avoid mechanically large welfare because of non-rivalrous public goods shared by population greater than one.

All other parameters are internally calibrated. Since the U.S. population  $L_H$  is normalized to one, the rest-of-the-world population  $L_F$  is calibrated to be the actual rest-of-the-world population relative to the U.S. population. The population data are from the World Bank. The baseline (factual) tariff rate of the United States,  $\tau_t$ , is computed as the tariff revenue divided by the import value. The tariff revenues (custom duties) are from the BEA NIPA, and the import value is from the USA Trade Online. Then, given the trade elasticity  $\sigma$  and the tariff rate  $\tau_t$ , the Head-Ries index for the iceberg non-tariff trade cost (Head and Ries, 2001) is

$$d = \left( \frac{X_{F,F} X_{H,H}}{X_{H,F} X_{F,H}} \tilde{\tau}_t^{1-\sigma} \right)^{\frac{1}{2(\sigma-1)}}, \quad (46)$$

where  $X_{i,j}$  is the trade value from  $i$  to  $j$ , where  $i$  and  $j$  are  $H$  or  $F$ .

Note that  $w_F$  is the nominal wage rate per person in Foreign, but  $w_H$  is the nominal wage rate per hour in Home. Home labor is the numeraire:  $w_H = 1$ . I calibrate the baseline  $w_F$  by the rest-of-the-world nominal GDP per capita relative to the U.S. nominal GDP per work hour.<sup>3</sup> Using (29), we have

$$\frac{\pi_{F,H}}{\pi_{H,H}} = (\tilde{\tau}_t d w_F)^{1-\sigma} \left( \frac{a_F}{a_H} \right)^{\sigma-1}. \quad (47)$$

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<sup>3</sup>I use the nominal GDP instead of the nominal labor income because what I call labor is the unique production factor in the model.

Rearranging this and using  $a_H = 1$ , we recover  $a_F$

$$a_F = \left( \frac{X_{F,H}}{X_{H,H}} \right)^{\frac{1}{\sigma-1}} \tilde{\tau}_t d w_F. \quad (48)$$

Now only the remaining parameter is the lowest value of abilities  $\omega_{\min}$ . To recover  $\omega_{\min}$ , we first express the aggregate U.S. labor income as a function of  $\omega_{\min}$ ,  $\bar{w}_H(\omega_{\min})$ ,

$$\begin{aligned} \bar{w}_H(\omega_{\min}) &= L_H \int_{\omega_{\min}}^{\infty} \omega w_H l_H(\omega) F(\omega) \\ &= L_H w_H^{\frac{\nu+1}{\nu+1-\beta}} \tilde{\tau}_i^{\frac{\beta}{\nu+1-\beta}} P_H^{-\frac{\beta}{\nu+1-\beta}} \beta^{\frac{1}{\nu+1-\beta}} \left[ \frac{\left( \frac{\tau_i}{\tilde{\tau}_i} + \frac{\tau_t}{\tilde{\tau}_t} \pi_{F,H} \right) L_H \left( \frac{\tilde{\tau}_i}{P_H} \right)^{k_0} \beta^{\frac{1}{\nu+1-\beta}}}{1 - \frac{\tau_t}{\tilde{\tau}_t} \pi_{F,H}} \right]^{\frac{1-\beta}{\nu}} \left( \frac{\alpha}{\alpha - k_0} \omega_{\min}^{k_0} \right)^{1 + \frac{1-\beta}{\nu}}. \end{aligned} \quad (49)$$

Note that given all parameters we have calibrated so far, only the unknown parameter is  $\omega_{\min}$  in this expression. The model counterpart to the nominal GDPs in Home and Foreign are  $\bar{w}_H(\omega_{\min})$  and  $w_F L_F$ . We equate the nominal GDP ratios of the model and the data

$$\frac{\bar{w}_H(\omega_{\min})}{w_F L_F} = \frac{\text{U.S. GDP}}{\text{ROW GDP}}, \quad (50)$$

where U.S. GDP and ROW GDP denote the U.S. nominal GDP and the rest-of-the-world nominal GDP in the World Bank data. We solve this for  $\omega_{\min}$  to back it out

$$\omega_{\min} = \left( \frac{\text{U.S. GDP}}{\text{ROW GDP}} \cdot \frac{w_F L_F}{\zeta} \right)^{\frac{\nu}{k_0(\nu+1-\beta)}}, \quad (51)$$

where

$$\zeta = \tilde{\tau}_i^{\frac{\beta}{\nu+1-\beta}} P_H^{-\frac{\beta}{\nu+1-\beta}} \beta^{\frac{1}{\nu+1-\beta}} \left[ \frac{\left( \frac{\tau_i}{\tilde{\tau}_i} + \frac{\tau_t}{\tilde{\tau}_t} \pi_{F,H} \right) \left( \frac{\tilde{\tau}_i}{P_H} \right)^{k_0} \beta^{\frac{1}{\nu+1-\beta}}}{1 - \frac{\tau_t}{\tilde{\tau}_t} \pi_{F,H}} \right]^{\frac{1-\beta}{\nu}} \left( \frac{\alpha}{\alpha - k_0} \right)^{\frac{\nu+1-\beta}{\nu}}. \quad (52)$$

Now the model is equipped with all parameters to carry out counterfactual experiments.

## 4 Optimal Taxes

We compute the optimal mix of ad valorem tariffs and linear income taxes using the calibrated model. In Subsection 4.1, we compute the U.S. welfare for various tariffs, given particular values of income taxes. Likewise, in Subsection 4.2, we plot the U.S. welfare against income taxes, given tariff levels. Subsection 4.3 gives the main result; we plot the

U.S. welfare against both tariffs and income taxes and find the optimal mix. Subsection 4.4 plots optimal tariffs and income taxes against iceberg non-tariff trade costs  $d$ .

## 4.1 Optimal Tariffs for Given Income Taxes

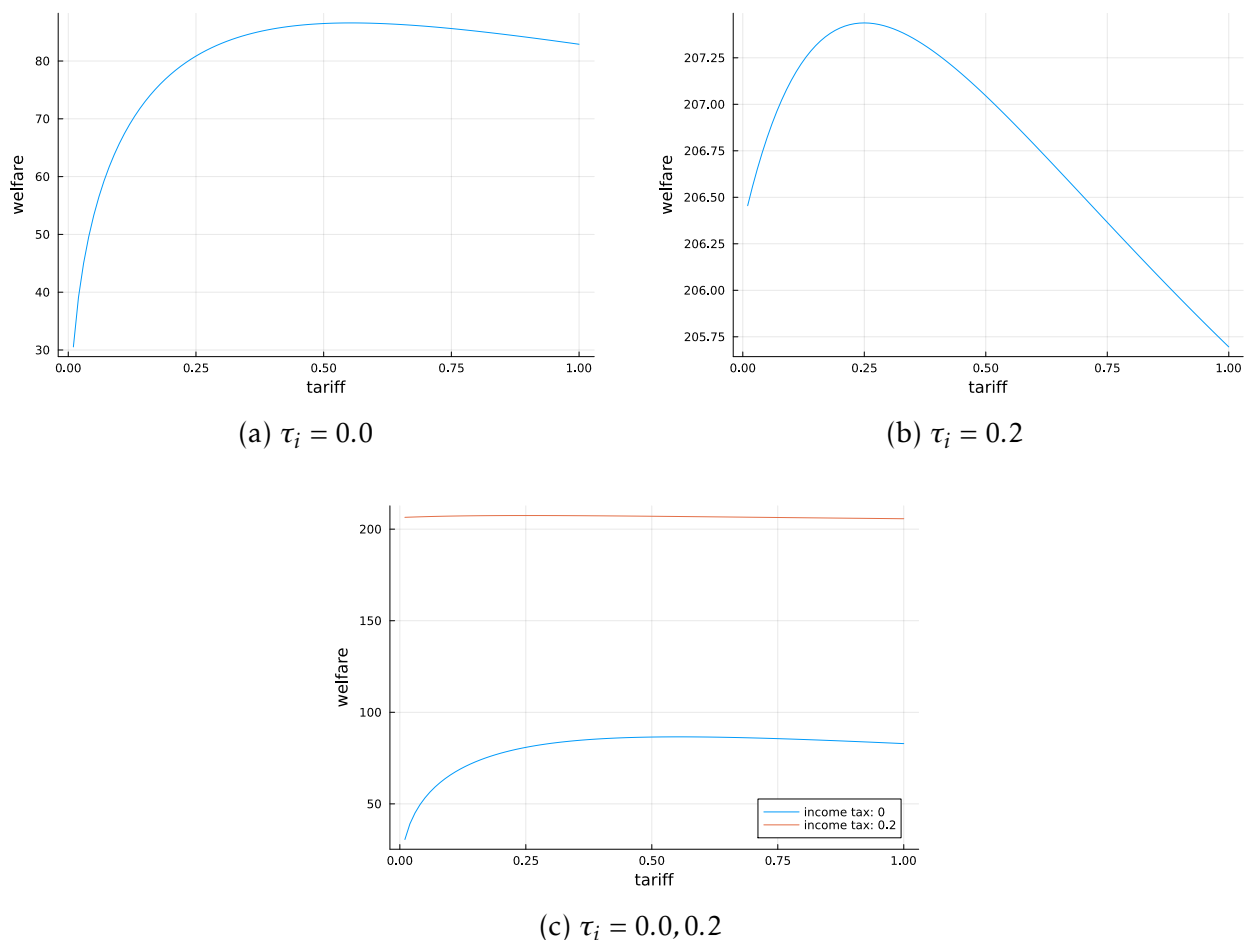


Figure 2: Welfare against Tariffs under Income Taxes of 0 and 20 Percent

We plot the U.S. welfare against various tariff rates. To do so, we fix the income tax rates at 0 and 20 percent. Figure 2a plots the U.S. welfare against tariff rates under a 0-percent income tax. If the government imposes no income tax as Andreessen and Trump dream of, the optimal tariff is 55 percent. When tariffs are only the source of government revenue, such a high tariff rate is optimal. Figure 2b is a similar plot under a 20-percent income tax. In this case, a 25-percent tariff is optimal. Since the trade elasticity is  $\sigma - 1 = 4$ , this number coincides with the value from the classic optimal tariff formula  $1/(\sigma - 1)$ .

Figure 2c puts Figures 2a and 2b together into one. Although different tariffs achieve different welfare levels for given income tax levels, actually, the welfare difference caused by income taxes is far greater than the welfare difference driven by tariffs in these cases.

## 4.2 Optimal Income Taxes for Given Tariffs

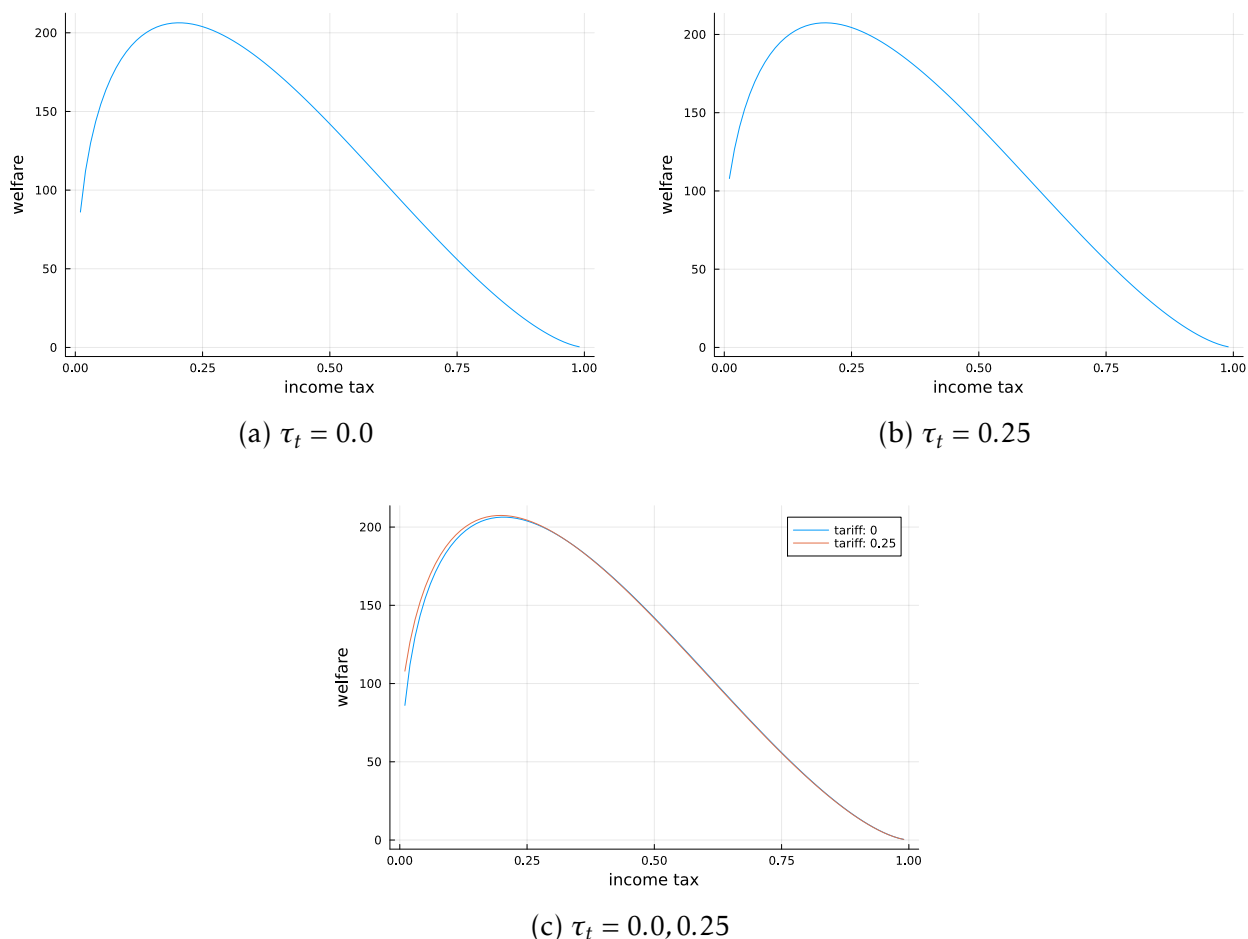


Figure 3: Welfare against Income Taxes under Tariffs of 0 and 25 Percent

We observe how welfare changes as income taxes vary under given tariff levels. Figure 3a plots the U.S. welfare against income tax rates under no tariff. The optimal income tax rate is 20 percent. Figure 3b shows a similar plot under a 25-percent tariff. Again, the optimal income tax is 20 percent.<sup>4</sup> Indeed, we put Figures 3a and 3b together into Figure 3c, and the two graphs are very similar. Therefore, income tax rates are more impactful on the U.S. welfare than do tariff rates.

## 4.3 Optimal Mix of Tariffs and Income Taxes

We compute the optimal mix of tariffs and income taxes. For this purpose, we numerically compute equilibria for many pairs of tariffs and income taxes. Figure 4 is a 3D plot of the

<sup>4</sup>We compute these equilibria with a grid of taxes of a step size of 0.01, so the optimal income taxes for these two situations can be numerically different if we use a finer grid.

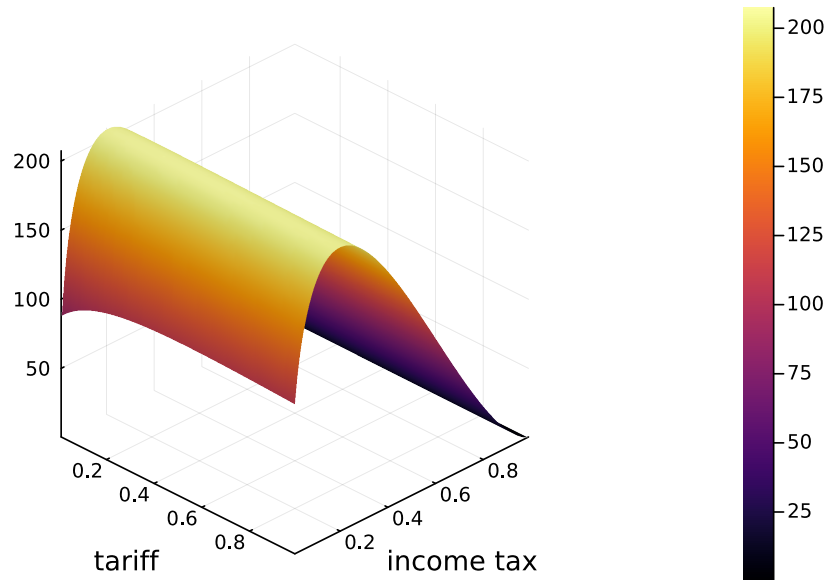


Figure 4: Welfare against Tariffs and Income Taxes

U.S. welfare against both tariffs and income taxes. The optimal mix is a 25-percent tariff and a 20-percent income tax. If the U.S. government imposes such taxes, tariffs account for only 3.3 percent of government revenue. Admittedly the 25-percent tariff is much higher than the actual U.S. tariff, the optimal tariff share in government revenue is much smaller than the one in the 19th century which Andreessen and Trump admire.

#### 4.4 Non-tariff Trade Costs and Optimal Taxes

One of many differences between the 19th century and the current era is international transportation costs. Because of technological and institutional progress, it is much easier to ship goods from one country to another now than in the 19th century. Do higher trade costs back in the 19th century rationalize a high-tariff, low-income-tax policy of William McKinley and other protectionists? To answer this question, we compute optimal mixes of tariffs and income taxes for various levels of non-tariff trade costs  $d$ .

Figure 5 plot optimal combinations of taxes against iceberg non-tariff trade costs. The optimal tariff is decreasing and the optimal income tax is increasing in non-tariff trade costs. Therefore, if the primary difference between the 19th century and the current period is transportation costs, the 19th century U.S. should have set lower tariffs and higher income taxes than the current U.S. This is at odds with the actual history. The U.S. tariff decreased and the U.S. income tax increased over time. Introducing a mechanism to justify the actual policy change over time into the model is out of the scope of this paper but an

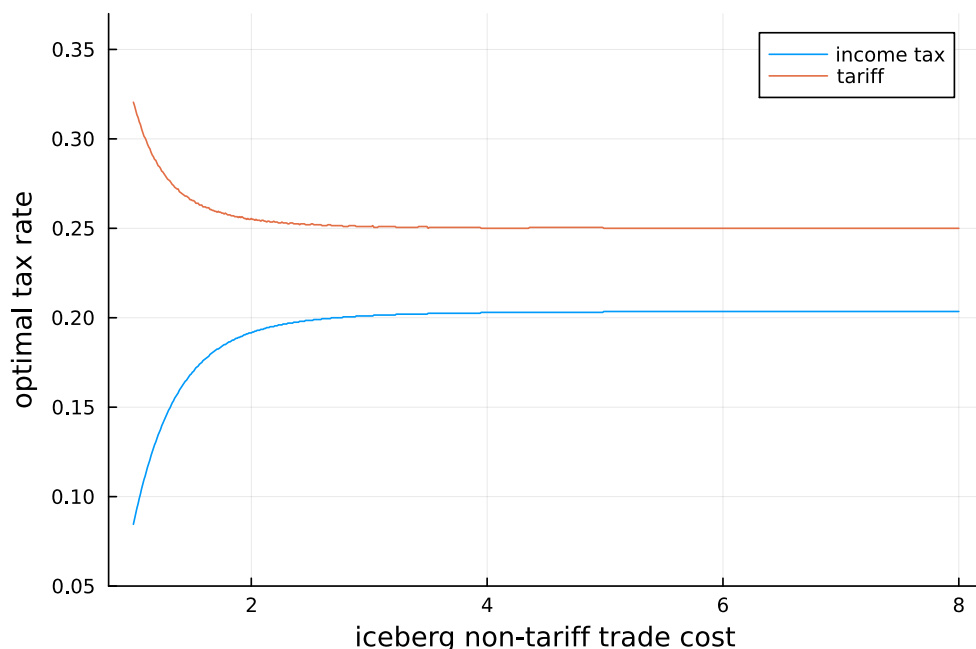


Figure 5: Optimal Mixes of Tariffs and Income Taxes against Non-tariff Trade Costs

interesting venue for future research.

## 5 Conclusion

We start with the bald statement of Marc Andreessen and Donald Trump; the U.S. government should replace income taxes with tariffs for a primary source of its revenue. To scrutinize their idea, we develop a general equilibrium model of international trade and public good provision. Then we calibrate the model for the United States and the rest of the world in 2017.

With the calibrated model in hand, we find that income taxes affect welfare more than tariffs do. The optimal mix is a 25-percent tariff and a 20-percent income tax. The optimal tariff is decreasing and the optimal income tax is increasing in iceberg trade costs.

Although tariffs' distortion in consumer behavior, income taxes' distortion in labor supply, and the terms-of-trade effect driven by tariffs are all taken into account in our model, the model misses some important consequences of tax policies. Higher income taxes might disincentivize entrepreneurship, firm creation, and innovation. Tariffs might hamper investment. One needs a more complex framework to integrate these mechanisms, and we leave it for future research.

## A Existence and Uniqueness

The equilibrium defined in Subsection 2.4 exists. And this equilibrium is unique. As we plug all of (28), (29), (30), (31), and (32) into (27), we can condense the equilibrium conditions into only one equation, Home's trade balance, with only one endogenous variable, the Foreign wage  $w_F$ .

Recall that the left-hand side on (27) is Home's export value, and the right-hand side is Home's import value. Viewing both of these exports and imports as functions of  $w_F$ , if these two take the same value only once for a single positive  $w_F$ , the unique equilibrium exists. We will show this.

First we inspect the export  $Ex(w_F) = w_F L_F \pi_{H,F}$ . Plugging (28) into this, we have

$$\begin{aligned} Ex(w_F) &= w_F L_F \frac{(d(1/a_H))^{1-\sigma}}{(d(1/a_H))^{1-\sigma} + (w_F/a_F)^{1-\sigma}} \\ &= \frac{Aw_F}{B + Cw_F^{1-\sigma}}, \end{aligned} \quad (53)$$

where  $A = L_F(d(1/a_H))^{1-\sigma} > 0$ ,  $B = (d(1/a_H))^{1-\sigma} > 0$ , and  $C = (1/a_F)^{1-\sigma} > 0$  are constants in this context. Taking the derivative of this,

$$Ex'(w_F) = \frac{A(B + Cw_F^{1-\sigma}) + AC(\sigma - 1)w_F^{1-\sigma}}{(B + Cw_F^{1-\sigma})^2} > 0. \quad (54)$$

Moreover, we can confirm

$$\lim_{w_F \rightarrow 0} Ex(w_F) = 0, \quad (55)$$

$$\lim_{w_F \rightarrow \infty} Ex(w_F) = \infty. \quad (56)$$

Therefore, the export  $Ex(w_F)$  is continuous and strictly increasing. As  $w_F \rightarrow 0$ , it tends to 0; as  $w_F \rightarrow \infty$ , it tends to  $\infty$ .

We move on to Home's imports

$$Im(w_F) = P_H(C_H + S_H)\pi_{F,H} \frac{1}{\bar{\tau}_t}. \quad (57)$$

We proceed in two steps. In the first step, we rewrite  $Im(w_F)$  as a function of Home's expenditure share on Foreign goods  $\pi_{F,H}$  instead of  $w_F$ . In so doing, we confirm that  $w_F$  affects  $Im$  only through  $\pi_{F,H}$ . Then, in the second step, we identify the behavior of  $Im$  ultimately as a function of  $w_F$  through  $\pi_{F,H}$ .

The government's budget constraint (32) and the aggregate consumption of private

goods (31) imply

$$\left(1 - \frac{\tau_t}{\tilde{\tau}_t} \pi_{F,H}\right) S_H = \left(\frac{\tau_i}{\tilde{\tau}_i} + \frac{\tau_t}{\tilde{\tau}_t} \pi_{F,H}\right) C_H. \quad (58)$$

Define  $r_i$  and  $s_t$  by  $r_i = \frac{\tau_i}{\tilde{\tau}_i}$  and  $s_t = \frac{\tau_t}{\tilde{\tau}_t}$ . Then we have

$$S_H = B(\pi_{F,H}) C_H, \quad (59)$$

where  $B(\pi_{F,H}) = \frac{r_i + s_t \pi_{F,H}}{1 - s_t \pi_{F,H}}$ . Plugging this into (31), we have

$$C_H = C_0 P_H^{-\frac{\nu+1}{\nu}} B(\pi_{F,H})^{\frac{1-\beta}{\nu}}, \quad (60)$$

where

$$C_0 = L_H^{\frac{\nu+1-\beta}{\nu}} \tilde{\tau}_i^{\frac{\nu+1}{\nu}} \beta^{\frac{1}{\nu}} \Omega_0^{\frac{\nu+1-\beta}{\nu}}, \quad (61)$$

and

$$\Omega_0 = \int \omega^{\frac{\nu+1}{\nu+1-\beta}} dF(\omega). \quad (62)$$

And we have

$$S_H = C_0 P_H^{-\frac{\nu+1}{\nu}} B(\pi_{F,H})^{\frac{1-\beta+\nu}{\nu}}. \quad (63)$$

Therefore,

$$C_H + S_H = C_0 P_H^{-\frac{\nu+1}{\nu}} B(\pi_{F,H})^{\frac{1-\beta}{\nu}} (1 + B(\pi_{F,H})). \quad (64)$$

Plugging (64) into (57) yields

$$Im = C_0 P_H^{-\frac{1}{\nu}} B(\pi_{F,H})^{\frac{1-\beta}{\nu}} (1 + B(\pi_{F,H})) \pi_{F,H} \frac{1}{\tilde{\tau}_t}. \quad (65)$$

The Home price index can be rewritten using  $\pi_{F,H}$

$$P_H = \frac{1}{a_H} (1 - \pi_{F,H})^{\frac{1}{\sigma-1}}. \quad (66)$$

And we have

$$1 + B(\pi_{F,H}) = \frac{1}{\tilde{\tau}_i (1 - s_t \pi_{F,H})}. \quad (67)$$

Plugging these two into (65), we have

$$Im = C_0 \left( \frac{1}{a_H} (1 - \pi_{F,H})^{\frac{1}{\sigma-1}} \right)^{-\frac{1}{\nu}} \left( \frac{r_i + s_t \pi_{F,H}}{1 - s_t \pi_{F,H}} \right)^{\frac{1-\beta}{\nu}} \left( \frac{1}{\tilde{\tau}_i (1 - s_t \pi_{F,H})} \right) \pi_{F,H} \frac{1}{\tilde{\tau}_t}. \quad (68)$$

Notite that  $\pi_{F,H}$  is the only endogenous variable on which  $Im$  depends. In this sense,

$w_F$  affects  $Im$  only through  $\pi_{F,H}$ . I write  $Im(\pi_{F,H})$  to emphasize  $Im$  as a function of  $\pi_{F,H}$ . Let  $C_1 = C_0 \cdot \left(\frac{1}{a_H}\right)^{-\frac{1}{\nu}} \frac{1}{\tilde{\tau}_t} \frac{1}{\tilde{\tau}_t}$ . Then  $Im(\pi_{F,H}) = C_1(1 - \pi_{F,H})^{-\frac{1}{\nu(\sigma-1)}} \left(\frac{r_i + s_t \pi_{F,H}}{1 - s_t \pi_{F,H}}\right)^{\frac{1-\beta}{\nu}} \left(\frac{1}{1 - s_t \pi_{F,H}}\right) \pi_{F,H}$ . Taking logs of both sides,

$$\log Im(\pi_{F,H}) = \log C_1 - \frac{1}{\nu(\sigma-1)} \log(1 - \pi_{F,H}) + \frac{1-\beta}{\nu} \log(r_i + s_t \pi_{F,H}) - \frac{1-\beta+\nu}{\nu} \log(1 - s_t \pi_{F,H}) + \log \pi_{F,H}. \quad (69)$$

We confirm that the last four terms are all strictly increasing functions of  $\pi_{F,H}$ . Since a log is a monotone transformation,  $Im(\pi_{F,H})$  is also a strictly increasing function. We can confirm that  $Im(\pi_{F,H}) \rightarrow \infty$  as  $\pi_{F,H} \rightarrow 1$ , and  $Im(\pi_{F,H}) \rightarrow 0$  as  $\pi_{F,H} \rightarrow 0$ . Ultimately we want to relate  $Im$  to  $w_F$ , instead of  $\pi_{F,H}$ . Only the endogenous variable in (29) is  $w_F$ , so we view  $\pi_{F,H}$  as a function of  $w_F$ . We can write  $\pi_{F,H} = \frac{Dw_F^{1-\sigma}}{E + Dw_F^{1-\sigma}}$ , where  $D = \left(\frac{\tilde{\tau}_t d}{a_F}\right)^{1-\sigma}$ , and  $E = \left(\frac{1}{a_H}\right)^{1-\sigma}$ . Then  $\pi'_{F,H} = -\frac{(\sigma-1)DEw_F^{-\sigma}}{(E + Dw_F^{1-\sigma})^2} < 0$ , so  $\pi_{F,H}$  is strictly decreasing in  $w_F$ . Moreover, we can confirm that  $\pi_{F,H} \rightarrow 1$  as  $w_F \rightarrow 0$  and that  $\pi_{F,H} \rightarrow 0$  as  $w_F \rightarrow \infty$ . Therefore, viewing  $Im$  as a function of  $w_F$ ,  $Im$  is strictly decreasing in  $w_F$ ,  $Im \rightarrow \infty$  as  $w_F \rightarrow 0$ , and  $Im \rightarrow 0$  as  $w_F \rightarrow \infty$ .

Therefore, the two functions  $Ex(w_F)$  and  $Im(w_F)$  intersect only once.

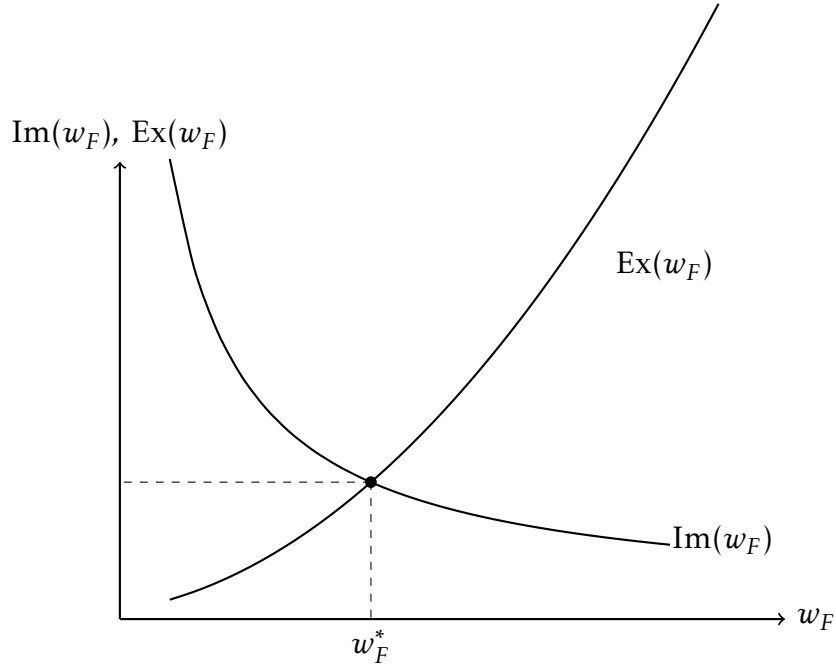


Figure 6: The import and export schedules intersect at a unique equilibrium foreign wage  $w_F^*$ .

## B Details on the Small Open Economy

So far the factor prices and the price indices of private goods in Home and Foreign are all endogenously determined. Following the language of international trade theory, we call this general case as a "large" open economy. In order to obtain an analytical expression of welfare as a function of a tariff and a labor income tax, we consider Home as a small open economy. Home is small enough so that any behavior in Home does not affect Foreign prices. That is, we fix the foreign wage  $w_F$ . In the system of general equilibrium, (27) determines  $w_F$ . Therefore we drop this equation in the remainder of this subsection (Small Open Economy). Given the fixed  $w_F$ , the trade shares  $\pi_{F,H}$ ,  $\pi_{H,F}$ , and  $P_H$  are determined by (29), (28), (30). Therefore only remaining endogenous variables are  $C_H$  and  $S_H$ , which are determined by (31) and (32), respectively.

Plugging (31) into (32) and solving for  $S_H$ , we have

$$S_H = \left[ \frac{\left( \tau_i + \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH} P_H \right) L_H \left( \frac{\tilde{\tau}_i}{P_H} \right)^{\frac{\nu+1}{\nu+1-\beta}} \beta^{\frac{1}{\nu+1-\beta}} \Omega_0}{\left( 1 - \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH} \right) P_H} \right]^{\frac{\nu+1-\beta}{\nu}}. \quad (70)$$

Define a function  $J$  by

$$J(\tau_t) = \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH}(\tau_t) \cdot P_H(\tau_t). \quad (71)$$

Then

$$S_H \propto (\tau_i + J)^{\frac{\nu+1-\beta}{\nu}}. \quad (72)$$

Plugging (70) into (31), we have

$$C_H = L_H \left( \frac{\tilde{\tau}_i}{P_H} \right)^{\frac{\nu+1}{\nu+1-\beta}} \beta^{\frac{1}{\nu+1-\beta}} \Omega_0 \left[ \frac{\left( \tau_i + \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH} P_H \right) L_H \left( \frac{\tilde{\tau}_i}{P_H} \right)^{\frac{\nu+1}{\nu+1-\beta}} \beta^{\frac{1}{\nu+1-\beta}} \Omega_0}{\left( 1 - \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH} \right) P_H} \right]^{\frac{1-\beta}{\nu}} \quad (73)$$

Since the labor in Home is the numeraire, that is,  $w_H = 1$ , the ex ante expected utility (9) is

$$\left( \frac{\tilde{\tau}_i}{P_H} \right)^{\frac{\beta(1+\nu)}{\nu+1-\beta}} S_H^{\frac{(1-\beta)(\nu+1)}{\nu+1-\beta}} \left( \beta^{\frac{\beta}{\nu+1-\beta}} - \frac{1}{1+\nu} \beta^{\frac{1+\nu}{\nu+1-\beta}} \right) \int \omega^{\frac{\beta(1+\nu)}{\nu+1-\beta}} dF(\omega). \quad (74)$$

Let  $W_S$  be the ex ante expected utility (welfare) in this small open economy. Then

$$\log W_S = \frac{\beta(1+\nu)}{\nu+1-\beta} \log \left( \frac{\tilde{\tau}_i}{P_H} \right) + \frac{(1-\beta)(\nu+1)}{\nu+1-\beta} \log S_H + \log \Omega_1, \quad (75)$$

where

$$\log \Omega_1 = \log \left( \beta^{\frac{\beta}{\nu+1-\beta}} - \frac{1}{1+\nu} \beta^{\frac{1+\nu}{\nu+1-\beta}} \right) + \int \omega^{\frac{\beta(1+\nu)}{\nu+1-\beta}} dF(\omega) \quad (76)$$

is a constant irrelevant to tax rates. To compute  $\log W_S$ , we need an expression for  $\log S_H$

$$\begin{aligned} \log S_H &= \frac{\nu+1-\beta}{\nu} \\ &\cdot \left[ \log \left( \tau_i + \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH} P_H \right) + \log L_H + \frac{\nu+1}{\nu+1-\beta} \log \left( \frac{\tilde{\tau}_i}{P_H} \right) + \frac{1}{\nu+1-\beta} \log \beta + \log \Omega_0 - \log \left( 1 - \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH} \right) - \log P_H \right]. \end{aligned} \quad (77)$$

Let  $\log \xi = \log L_H + \frac{1}{\nu+1-\beta} \log \beta + \log \Omega_0$ , which is a constant irrelevant to tax rates. In (75), only the first two terms on the right-hand side are related to tax rates. Therefore we pick up the first two terms and denote them as  $\log \tilde{W}_S$ . Then we have

$$\begin{aligned} &\log \tilde{W}_S \\ &= \frac{\nu+1}{\nu} \log(\tilde{\tau}_i) - \frac{(2-\beta)(\nu+1)}{\nu} \log P_H + \frac{(1-\beta)(\nu+1)}{\nu} \log \left( \tau_i + \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH} P_H \right) \\ &\quad - \frac{(1-\beta)(\nu+1)}{\nu} \log \left( 1 - \frac{\tau_t}{\tilde{\tau}_t} \pi_{FH} \right) + \frac{(1-\beta)(\nu+1)}{\nu} \log \xi \\ &= \frac{\nu+1}{\nu} \log(\tilde{\tau}_i) - \frac{(2-\beta)(\nu+1)}{\nu} \log P_H(\tau_t) + \frac{(1-\beta)(\nu+1)}{\nu} \log(\tau_i + J(\tau_t)) \\ &\quad - \frac{(1-\beta)(\nu+1)}{\nu} \log(1 - H(\tau_t)) + \frac{(1-\beta)(\nu+1)}{\nu} \log \xi. \end{aligned} \quad (78)$$

Dividing (78) by  $(\nu+1)/\nu$ , we have

$$\log \check{W}_S = \log \tilde{\tau}_i - (2-\beta) \log P_H + (1-\beta) \log(\tau_i + J) - (1-\beta) \log(1-H) + \tilde{\xi}, \quad (79)$$

where  $\tilde{\xi} = (1-\beta) \log \xi$  is a constant.

The first derivative with respect to  $\tau_i$  is

$$\frac{\partial \log \check{W}_S}{\partial \tau_i} = -\frac{1}{\tilde{\tau}_i} + (1-\beta) \frac{1}{\tau_i + J}. \quad (80)$$

Equating this with zero yields the optimal income tax given a tariff (through  $J$ )

$$\tau_i(\tau_t) = \frac{1-\beta-J}{2-\beta}. \quad (81)$$

The first derivative with respect to  $\tau_t$  is

$$\frac{\partial \log \check{W}_S}{\partial \tau_t} = -(2 - \beta) \frac{P'_H}{P_H} + (1 - \beta) \frac{J'}{\tau_i + J} + (1 - \beta) \frac{H'}{1 - H}. \quad (82)$$

The second derivatives (including the cross derivative) are

$$f_{ii} = \frac{\partial^2 \log \check{W}_S}{\partial \tau_i^2} = -\frac{1}{(1 - \tau_i)^2} - (1 - \beta) \frac{1}{(\tau_i + J)^2}, \quad (83)$$

$$f_{it} = \frac{\partial^2 \log \check{W}_S}{\partial \tau_t \partial \tau_i} = -(1 - \beta) \frac{J'}{(\tau_i + J)^2}, \quad (84)$$

$$f_{tt} = \frac{\partial^2 \log \check{W}_S}{\partial \tau_t^2} = -(2 - \beta) \frac{P''_H P_H - (P'_H)^2}{P_H^2} + (1 - \beta) \frac{J'' J - (J')^2}{(\tau_i + J)^2} + (1 - \beta) \frac{H''(1 - H) + (H')^2}{(1 - H)^2}. \quad (85)$$

We cannot determine the sign of the determinant of the Hessian globally. If

$$f_{tt} - \frac{f_{it}^2}{f_{ii}} < 0 \quad (86)$$

at the two taxes satisfying the first-order conditions, these taxes are at least a local maximizer of welfare.

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