

# Multiple equilibria and symmetry-breaking in a dynamic game of trade and transboundary pollution control

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## Abstract

This paper studies a dynamic two-country game in which trade in manufactured goods interacts with transboundary pollution and public abatement. Pollution lowers agricultural productivity, changes relative prices, and feeds back into manufacturing demand and emissions. Countries choose Markov abatement policies subject to a non-negativity constraint on the pollution stock. The resulting linear–quadratic structure admits a complete regime characterization by the set of active abaters. We show that multiple locally stable steady states can arise even when countries are ex ante symmetric. In particular, two symmetric positive-stock branches may coexist, and the lower-pollution branch delivers higher welfare. Clean steady states can also be symmetric or asymmetric, with welfare differences driven entirely by the allocation of abatement costs. Trade liberalization affects not only emissions but also equilibrium selection: it tends to expand the region supporting the high-pollution upper branch, shrink the welfare-superior lower branch, and make some clean-boundary equilibria harder to sustain.

**JEL classification:** C73; F18; H41; Q56.