FDI Subsidies in a General Oligopolistic Equilibrium Model

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Abstract

We employ a general oligopolistic equilibrium model to analyze the welfare effect of subsidies to fixed costs of FDI. Specifically, we construct the model that exporting and FDI industries co-exist in an economy. In addition, firms in the exporting industries produce goods under oligopolistic competition while a firm in the FDI industry produces goods under monopoly. Under this situation, we consider the welfare impact of the subsidies under different financing sources such as a labor income tax and a consumption tax. The results indicate that a small subsidy financed by consumption taxes may improve welfare. The reason is that small subsidies financed by labor income taxes do not affect the wage, and thus do not alter any other economic variables. On the other hand, small subsidies financed by consumption taxes influence the demand and supply condition, which subsequently decreases the wage. This reduction in the wage can improve welfare.

Keywords: FDI; subsidy; welfare analysis; general oligopolistic equilibrium

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1 Introduction

According to a report by UNCTAD (2022), the amount of FDI in 2021 was approximately 1.6 trillion dollars, which indicates a 64% increase compared to the previous year. FDI, which decreased during the COVID-19 pandemic, is now showing a tendency to increase again. Inducing multinational enterprises (MNEs) can generally bring direct benefits to host countries by lowering the prices of goods they supply through the transfer of production, which saves transportation costs across borders, and labor costs in the host country (when the host country has low wages). Moreover, foreign capital and job creation brought by MNEs are evaluated as benefits for the host country's interests. The policymakers have thus been open to the foreign affiliates of MNEs. Countries that hold positive views towards FDI are attempting to attract FDI through incentive measures, such as tax breaks, employment promotion subsidies, and construction of industrial facilities. In fact, these measures affect the MNE's behavior.

A concrete example can be seen in the provision of subsidies to the semiconductor industry. In Japan, the "5G Promotion Act," which includes subsidies for the semiconductor industry, was implemented in 2022. As a result, Taiwan Semiconductor Manufacturing Company (TSMC) began constructing a plant in Kumamoto, subsidized with 476 billion yen, while Micron Technology announced the construction of a plant in Hiroshima in 2022. In 2023, Samsung Electronics announced its plan to establish a factory in Yokohama, with an estimated subsidy amount ranging between 10 and 15 billion yen. In the United States, the "CHIPS Act," which incorporates subsidies for the semiconductor industry, was proposed in 2020 and passed in 2022. In response to this expectation, TSMC announced its expansion into Arizona in 2020, and Samsung Electronics revealed its plan to build a plant in Texas in 2021. (The amount of subsidies for these companies is currently under adjustment.)

Therefore, government subsidies for semiconductor manufacturers are regarded as an important factor in investment decision-making.[1] However, it cannot be immediately asserted that policies aimed at attracting FDI necessarily enhance the welfare of the host country.[2] In the end, it is

[1]Host countries' policies on FDI by overseas firms are debatable. For instance, Wells *et al.* (2001) demonstrate that a elimination of tax incentives for foreign firms does not have a correlation with the inflow of FDI into host countries. On the other hand, Lim (2008) reveals that the establishment of a investment promotion agency (IPA) in a host country leads to an increase in FDI inflows.

[2]The positive effect of FDI on the host country's economy is not clear, and further discussion is necessary. For example, Alfaro *et al.* (2004) demonstrate that FDI contributes to the host country's economic growth when the financial market in the host country is developed. On the other hand, Borensztein *et al.* (1998) and Carkovic and Levine (2005) reveal that FDI does not have an impact on the host country's economic growth.

necessary to compare the benefits brought by the presence of MNEs attracted by subsidies with the fiscal costs incurred to provide such subsidies.

In developed countries, taxes used for economic policies such as subsidies for FDI are often financed through labor income taxes. This is because governments in developed countries can collect the tax income easily. Tanzi and Zee (2000) point out that labor income taxes are generally less distortionary than other taxes. However, even in developed countries, there are cases where governments struggle to collect income taxes. According to OECD Revenue Statistics, labor income taxes account for a very small percentage of the total tax revenue in Chile and Colombia (approximately 10.8% for Chile and 6.7% for Colombia) in 2021. The main source of tax revenue in these two countries is consumption tax, accounting for about 50% of total tax revenue. Regarding less developed countries such as China, Indonesia, Thailand, and Vietnam, the proportion of labor income tax to total tax revenue is low, at 5.6%, 10.8%, 11.3%, and 8.0% respectively. On the other hand, the proportion of consumption tax to total tax revenue in these countries is high, with figures of 41.6%, 42.6%, 57.1%, and 42.6% respectively. Thus, the consumption tax can be used for economic policies such as FDI subsidies.

The above discussion raises a question of how the different financing sources of subsidies towards FDI affect welfare. More specifically, the question arises as to how the welfare effects of providing FDI subsidies for "fixed costs" to attract the foreign monopoly firm such as TSMC would vary depending on the financing source of the FDI subsidies.

To answer this question, we utilize a general oligopolistic equilibrium (GOLE) model developed by a series of papers by J. Peter Neary.[3] The GOLE model aims to incorporate general equilibrium into oligopoly models. In the construction process of the model, it assumes that firms have significant market power within the industry or sector they operate in but are small economic players in the overall economy. Therefore, the GOLE model enables the analysis of the crucially important effects of market power in a global context. This aspect describes the real economy. Over the past four decades, there has been a significant increase in the proportion of pure profits in gross value added, accompanied by a substantial decline in the share of labor. The results of Autor *et al.* (2017), Autor *et al.* (2020), and De Loecker *et al.* (2020) suggest that this trend may be driven by a rise in industry concentration. Consequently, the modeling of strategic interactions among large firms has become increasingly important.[4]

[3]See Neary (2003a), Neary (2003b), Neary (2007), Eckel and Neary (2010), Neary (2010), and Neary (2016).

[4]The literature using a GOLE model has been expanding. See Bastos and Kreickemeier (2009), Egger and Etzel

We construct a model assuming two symmetric countries (Home and Foreign) with the presence of export industries, Home FDI industries, and Foreign FDI industries using the fundamental GOLE model developed by Neary (2016). In each export industries, Home and Foreign firms produce goods under oligopolistic competition. In each Home FDI industry, only the Home monopolistic firm produces goods, and in each Foreign FDI industry, only the Foreign monopolistic firm produces goods. To produce goods, we assume the labor coefficients of the export industries and the FDI industries are different. Furthermore, we assume the labor coefficients of the Home and Foreign FDI firms are same. We consider the situation where the Home government provides small subsidies for fixed costs for the Foreign monopolistic FDI firm. In addition, the Foreign government provides those for the Home monopolistic FDI firm symmetrically. The results show that the small subsidies financed by consumption taxes can improve welfare.

As for the welfare impact of FDI subsidies financed by labor income taxes, the result indicates that such subsidies do not affect welfare because labor income taxes do not create distortions in consumption, and subsidies do not affect the production level of firms. Thus, the wage and prices of goods remain unchanged, and subsidies have no effect on welfare.

Regarding the welfare effect of FDI subsidies financed by consumption taxes, the result suggests that such subsidies can improve welfare. Similar to the labor income tax case, subsidies do not affect the production level of firms directly, but through consumption taxes, it distorts demand and supply. This negative shock affects the labor market, resulting in a decrease in the wage when the government provides a small subsidy. The wage reduction leads to lower producer prices. Assuming the labor coefficient of the exporting industries are smaller than that of Home and Foreign FDI industries, consumer prices (producer prices plus consumption taxes) of exporting industries increase, while those of Home and Foreign FDI industries decrease under the assumption that the labor coefficient of the export industries are smaller than that of FDI industries.

Welfare tends to improve when the consumer prices of exporting industries are low and those of FDI industries are high. Specifically, welfare tends to increase when trade costs are small, fixed costs of FDI are at an intermediate level, and the exporting industries' labor coefficient is large enough. When trade costs are small, consumer prices of exporting industries are small while those of FDI industries are high. When fixed costs of FDI are at the intermediate level, it confirms that the wage, outputs, and prices are positive. In addition, the size of the negative shock on

(2012), Fujiwara (2017), and Beladi and Chakrabarti (2019).

the producer prices in the both industries is not so different with the intermediate level of the fixed costs. When the exporting industries' labor coefficient is large enough, the response to wage changes becomes larger. This leads the result that a negative shock induced by wage reduction on producer prices of exporting industries is significant large. In addition, this large negative shock results in the small increase in the consumer prices of the exporting industries. Thus, welfare tends to increase under these conditions.

This paper contributes to a vast literature on the welfare effects of FDI subsidies, providing the first attempt (to the best of my knowledge) at applying a framework with GOLE to the policy issue of FDI. The model has the advantage of allowing for a precise description of monopolistic FDI firms behavior with subsidies. Comparing welfare effects of fixed cost subsidies financed by labor income taxes with consumption taxes is a natural question to investigate with this model, yet it is an under-explored issue.

This paper is closely related to Fujiwara (2017), Chor (2009), and Han *et al.* (2023). Fujiwara (2017) analyzes the welfare effects of trade liberalization and FDI liberalization in an economy where both export and FDI industries coexist, using the GOLE model. The result shows that FDI liberalization always improves welfare. Different from this paper, he assumes firms in both industries produce under oligopolistic competition. In addition, he does not analyze the effects of subsidies. Chor (2009) constructs the model with heterogeneous firms based on Helpman *et al.* (2004) to assess the welfare effects of FDI subsidies for both fixed and variable costs financed by lump-sum labor income taxes. He shows that a small FDI subsidy always improves welfare because when the amount of subsidy is small, the benefits derived from the selection effect (moving from export to FDI) outweigh the fiscal costs. Han *et al.* (2023) extends the model presented by Chor (2009). They compare the welfare effects of variable costs subsidies for FDI financed by labor income taxes with consumption taxes and corporate income taxes. They reveal that subsidies financed by labor income taxes has always better impacts on welfare than both consumption and corporate income taxes. This result stems from the existence of distortion by both taxes because the distortion has the additional negative effects on demand and supply. In contrast to their papers, we assume monopolistic FDI firms and compare the welfare effects of fixed costs FDI subsidies financed by labor income taxes and consumption taxes. In addition, we analyze the welfare effects under general equilibrium model.[5]

[5]See Haaland and Wooton (1999), Fumagalli (2003), Skaksen (2005), and Pennings (2005) for other literature on the welfare effect of FDI subsidies.

The remaining parts of the paper are as follows. Section 2 describes the basic model. Section 3 examines the welfare effect of subsidies for fixed costs financed by labor income taxes. Section 4 analyzes the welfare effect of subsidies for fixed costs financed by consumption taxes. Section 5 concludes.

2 Basic Model

2.1 Demand

Our model is based on Neary (2016). There are two identical countries (Home and Foreign) that consist of duopolistic and monopoly industries in a unit interval $[0, 1]$. The representative consumer in Home has a following continuum-quadratic preference:

$$U[q^H(z)] = \int_0^1 \left\{ aq^H(z) - \frac{\left[q^H(z)\right]^2}{2} \right\} dz.$$

The representative consumer maximizes the utility subject to the budget constraint:

$$\int_0^1 p^H(z)q^H(z)dz \leq Y$$

where Y is consumer income. Solving the utility maximization problem gives the first order condition as follow:

$$a - q^H(z) = \lambda p^H(z)$$

where λ is the Lagrangean multiplier and the marginal utility of income. Following Neary (2016), firms are assumed to have market power in their product markets, but they have little influence on the whole economy. Thus, we set $\lambda = 1$. The demand function becomes:

$$q^H(z) = a - p^H(z)$$

Substituting the demand function into the utility function yields the indirect utility function:

$$V^H = \frac{a - \sigma_2^H}{2} \text{ where } \sigma_2^H = \int_0^1 \left[p^H(z)\right]^2 dz.$$

This expression is the convenient way to evaluate consumer welfare because the indirect utility only depends on the second moment of prices.

2.2 Supply

The producers maximize their profits. There is a set of exporting industries $X \in [2z^*, 1]$ in the economy. Firms in exporting industries compete under oligopoly. In addition, the economy consists of a set of Home FDI industries $I_H \in [0, z^*]$ and a set of Foreign FDI industries $I_F \in [z^*, 2z^*]$. In these FDI industries, the firms compete under monopoly. In each industry, firms produce homogeneous goods. With assumptions of market segmentation and the oligopoly in the exporting industries, the inverse demand functions for exporting goods in Home and Foreign are:

$$p_X^H = a - q_X^{HH} - q_X^{FH}$$
$$p_X^F = a - q_X^{FF} - q_X^{HF}$$

where q^{HH} is Home firm's good for Home, q^{FH} is Foreign firm's good for Home, q^{FF} is Foreign firm's good for Foreign, and q^{HF} is Home firm's good for Foreign.

In FDI industries, Home and Foreign firms produce for both their own countries and the other countries under monopoly in each country. The inverse demand functions in Home become:

$$p_{I_H}^{HH} = a - q_{I_H}^{HH}, \quad p_{I_H}^{FH} = a - q_{I_H}^{FH}.$$

The inverse demand functions in Foreign become:

$$p_{I_F}^{FF} = a - q_{I_F}^{FF}, \quad p_{I_F}^{HF} = a - q_{I_F}^{HF}.$$

Regarding the production technology, constant marginal labor input for exporting industries is α_X and it for Home and Foreign FDI industries is α_I. Firms in the exporting industries incur specific trade costs, τ, to provide goods the other countries. Firms in the FDI industries have to pay the fixed unit of labor, f, for constructing an additional plant. We further assume that fixed costs of FDI have to be payed by labor in a source country. Taking these assumptions into account, profit functions of Home exporting and FDI firms become:

$$\pi_X^H = p_X^H q_X^{HH} + p_X^F q_X^{HF} - w\alpha_X(q_X^{HH} + q_X^{HF}) - \tau q_X^{HF} \tag{1}$$

$$\pi_I^H = p_I^{HH} q_I^{HH} + p_I^{HF} q_I^{HF} - w\alpha_I(q_I^{HH} + q_I^{HF}) - wf, \tag{2}$$

where w is the wage in Home, which is same level as in Foreign due to symmetric assumption. Naturally, the foreign firms' profit is determined analogously. In the exporting industries, the firms produce goods under a Cournot competition. First order conditions of a Home exporting firm become:

$$a - w\alpha_X - 2q_X^{HH} - q_X^{FH} = 0, \quad a - w\alpha_X - 2q_X^{HF} - q_X^{FF} - \tau = 0.$$

The symmetric assumption for two countries leads relationships, $q_X^{HH} = q_X^{FF}$ and $q_X^{HF} = q_X^{FH}$. Given these conditions, the Cournot equilibrium outputs become:

$$q_X^{HH} = \frac{a - w\alpha_X + \tau}{3}, \quad q_X^{HF} = \frac{a - w\alpha_X - 2\tau}{3}. \tag{3}$$

A firm in a Home FDI industry produces goods under monopoly in each market, thus the monopoly equilibrium outputs from first order conditions become:

$$q_I^{HH} = \frac{a - w\alpha_I}{2}, \quad q_I^{HF} = \frac{a - w\alpha_I}{2}. \tag{4}$$

Again, due to the assumption of the identical countries, the outputs of a Foreign FDI firm become: $q_I^{HH} = q_I^{FF}$ and $q_I^{HF} = q_I^{FH}$.

2.3 Labor market equilibrium

In the GOLE model, the wage is endogenously determined. To close the model, we need the one further condition. In each country, labor supplies L units of inelastic labor supply. This labor supply must be equal to labor demand. The labor market clearing condition of Home is:

$$\begin{aligned}
L &= \int_0^{z^*} \alpha_I q_I^{HH} dI_H + \int_{z^*}^{2z^*} (\alpha_I q_I^{FH} + f) dI_F + \int_{2z^*}^{1} \alpha_X (q_X^{HH} + q_X^{HF}) dX \\
&= \frac{2z^* a\alpha_I - 2z^*\alpha_I^2 w + 2z^* f}{2} + \frac{2(1-2z^*)a\alpha_X - (1-2z^*)\tau\alpha_X - 2(1-2z^*)\alpha_X^2 w}{3}
\end{aligned}$$

Solving the above labor market clearing condition gives the endogenous wage as follow:

$$w = \frac{[2(1-2z^*)\alpha_X + 3z^*\alpha_I]a - (1-2z^*)\alpha_X\tau - 3(L - z^* f)}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2}. \tag{5}$$

From the equilibrium wage, we can establish the following lemma:

Lemma 1. *Trade costs negatively affect the wage and fixed costs positively affect it.*

Proof. See Appendix. □

The intuition of this lemma is as follows. If the trade costs decline, the total outputs of the Home exporting firms, $\int_{2z^*}^1 (q_X^{HH} + q_X^{HF}) dX$, increase. This increase in the outputs results in the more labor demand and the higher wage in Home. Thus, the trade costs negatively affect the wage. On the other hand, if the fixed costs decrease, the outputs of Home and Foreign FDI firms for the Home consumer, $\int_0^{z^*} q_I^{HH} dI_H$ and $\int_{z^*}^{2z^*} q_I^{HF} dI_F$, do not change. However, the reduction in fixed costs decreases the labor demand directly and the wage declines in Home. Therefore, the fixed costs positively affect the wage.

2.4 General equilibrium

Here, we derive the general equilibrium outputs and prices explicitly. Substituting (5) into (3), We can yield general equilibrium outputs of a Home firm in an exporting industry as follows:

$$q_X^{HH} = \frac{-z^*\alpha_I(\alpha_X - \alpha_I)a + [(1-2z^*)\alpha_X^2 + z^*\alpha_I^2]\tau + \alpha_X(L - z^* f)}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} \tag{6}$$

$$q_X^{HF} = \frac{-z^*\alpha_I(\alpha_X - \alpha_I)a - [(1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2]\tau + \alpha_X(L - z^* f)}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2}. \tag{7}$$

Using (6) and (7), a general equilibrium price of the Home exporting industry becomes:

$$p_X^H = \frac{[2(1-2z^*)\alpha_X^2 + 2z^*\alpha_X\alpha_I + z^*\alpha_I^2]a + z^*\alpha_I^2\tau - 2\alpha_X[L - z^* f]}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2}. \tag{8}$$

We obtain the following lemma immediately.

Lemma 2. *Both trade costs and fixed costs of FDI positively affect the general equilibrium price of the Home exporting industries.*

Proof. See Appendix. □

The intuition of this lemma can be interpreted as follows. When there is the reduction in trade costs, the marginal costs of the Home exporting firm, denoted as $w\alpha_X + \tau$ in (1), decrease. Additionally, Lemma 1 suggests an additional effect resulting from the wage reduction caused by the reduction in trade costs. This effect contributes positively to the marginal costs. However, there are conflicting effects on the price, but the former effect (the decrease in marginal costs) outweighs the latter effect. Consequently, the reduction in trade costs leads to a decline in both the marginal costs and the exporting price. On the other hand, the decrease in fixed costs of FDI only affects the marginal costs negatively, due to the wage reduction explained in Lemma 1. Therefore, as the fixed costs of FDI decrease, the price of the exporting industries also decreases.

Substituting (5) into (4), we get the general equilibrium outputs of a Home FDI firms as follow:

$$q_I^{HH} = q_I^{HF} = \frac{2(1-2z^*)\alpha_X(\alpha_X - \alpha_I)a + (1-2z^*)\alpha_X\alpha_I\tau + 3\alpha_I[L - z^*f]}{2[2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2]}. \tag{9}$$

Using (9) and the symmetric assumption, general equilibrium prices of goods supplied by the Home and Foreign FDI firms in Home become:

$$p_I^{HH} = p_I^{FH} = \frac{[2(1-2z^*)\alpha_X^2 + 2(1-2z^*)\alpha_X\alpha_I + 6z^*)\alpha_I^2]a - (1-2z^*)\alpha_X\alpha_I\tau - 3\alpha_I[L - z^*f]}{2[2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2]}. \tag{10}$$

From the general equilibrium FDI prices, we yield the following lemma:

Lemma 3. *Trade costs negatively affect the general equilibrium prices of the Home and Foreign FDI firms in Home but fixed costs of the FDI firm positively affect them.*

Proof. See Appendix. □

The intuition of this lemma can be summarized as follows. When trade costs decrease, the marginal costs of the FDI firm, denoted as $w\alpha_I$, increase. This is because trade costs have a negative impact on the wage, as shown in Lemma 1. As a result, the equilibrium prices of the

FDI firms in Home increase due to the reduction in trade costs. On the other hand, it is already established that a reduction in fixed costs has a positive effect on the wage. Therefore, when the fixed costs of FDI decrease, the equilibrium prices of the FDI firms become lower.

3 Subsidies financed by labor income taxes

In this section, we show how small FDI subsidies financed by labor income taxes affect welfare in Home.

3.1 Demand and supply with subsidies financed by labor income taxes

The utility maximization problem of the Home consumer becomes:

$$\max_{\hat{q}^H(z)} \int_0^1 \left\{ a\hat{q}^H(z) - \frac{\left[\hat{q}^H(z)\right]^2}{2} \right\} dz \;\; \text{s.t.} \;\; \int_0^1 [\hat{p}^H(z)]\hat{q}^H(z)dz \leq Y - tY + s_{lit} \int_0^{z^*} wfdI_H,$$

where s_{lit} is subsidies financed by labor income taxes. In this paper, we impose the budget neutral condition. In other words, the Government revenue (tY) is equalised to total subsidies for Foreign FDI firms ($\int_{z^*}^{2z^*} wfdI_F$). The symmetric assumption leads $\int_{z^*}^{2z^*} wfdI_F = \int_0^{z^*} wfdI_H$. This implies that the equation, $-tY + s_{lit} \int_0^{z^*} wfdI_H = 0$, holds. Therefore, the budget constraint, $Y - tY + s_{lit} \int_0^{z^*} wfdI_H$, is equal to Y. This is the same budget constraint as the basic model. This new budget constraint has no effect on the consumer behavior. Thus, the demand function under labor income taxes is same as one in the basic model.

Subsidies for fixed costs of FDI change the profit function of a Home FDI firm in (2) as follows:

$$\hat{\pi}_I^H = p_I^{HH} q_I^{HH} + p_I^{HF} q_I^{HF} - w\alpha_I (q_I^{HH} + q_I^{HF}) - w(1 - s_{lit})f, \tag{11}$$

Profit functions of the Home FDI firm with the fixed costs subsidies do not change the firm behavior. Therefore, the outputs of the Home FDI firm is same as (4). Also, the profit function and outputs in exporting industries are same as (1) and (3). With these conditions, we next consider a Home government's budget condition.

3.2 Budget Neutral with subsidies financed by labor income taxes

We assume that the Home government uses all revenues from labor income taxes for subsidies for the Foreign FDI firm. Using (1) and (11), the Home government's revenue from the labor income taxes is given as:

$$tY = t(\int_{2z^*}^{1} \pi_X^H dX + \int_0^{z^*} \hat{\pi}_I^H dI_H + wL).$$

The Home government's spending for the Foreign FDI firms becomes:

$$s_{lit} \int_{z^*}^{2z^*} wf dI_F = z^* s_{lit} wf.$$

The budget neutral conditions leads the relationship of t and s_{lit} as follow:

$$\begin{aligned} t &= \frac{z^* s_{lit} wf}{\int_{2z^*}^{1} \pi_X^H dX + \int_0^{z^*} \hat{\pi}_I^H dI_H + wL} \\ &= \frac{z^* s_{lit} wf}{\int_{2z^*}^{1} \pi_X^H dX + \int_0^{z^*} \pi_I^H dI_H + wL + z^* s_{lit} wf}. \end{aligned}$$

Differentiating t in the above equation with respect to s_{lit} around $s_{lit} = 0$, we have:

$$\left.\frac{\partial t}{\partial s_{lit}}\right|_{s_{lit}=0} = \frac{z^* wf \left(\int_{2z^*}^{1} \pi_X^H dX + \int_0^{z^*} \pi_I^H dI_H + wL\right)}{\left(\int_{2z^*}^{1} \pi_X^H dX + \int_0^{z^*} \pi_I^H dI_H + wL\right)^2} = \frac{z^* wf}{\int_{2z^*}^{1} \pi_X^H dX + \int_0^{z^*} \pi_I^H dI_H + wL} > 0.$$

Naturally, this relationship means that the government has to impose heavy taxes if the government spending increases. In the next subsection, we consider welfare effects of subsidies in the Home country.

3.3 Welfare analysis with subsidies financed by labor income taxes

An indirect utility function can be explicitly rewritten as:

$$V^H = \frac{a - \int_0^{z^*} \left[p_I^{HH}(z)\right]^2 dI_H - \int_{z^*}^{2z^*} \left[p_I^{FH}(z)\right]^2 dI_F - \int_{2z^*}^{1} \left[p_X^H(z)\right]^2 dX}{2}.$$

As we discussed, the demand and supply are same as the case where subsidies do not exist. Thus, the indirect utility function does not consist of either subsidies or labor income taxes. If we differentiate the indirect utility function with respect to subsidies financed by labor income taxes around $s_{lit} = 0$, we have:

$$\left.\frac{\partial V^H}{\partial s_{lit}}\right|_{s_{lit}=0} = 0.$$

We immediately establish the following proposition.

Proposition 1. *Small subsidies financed by labor income taxes have no effects on welfare.*

The intuition of this result is straightforward. In the GOLE model, the welfare change occurs through the wage change. Under small subsidies financed by the labor income taxes, both factors do not affect the demand and supply. In the end, these stable factors which do not affect the labor market have the same wage rate whether subsidies exist or not. Therefore, small subsidies do not have any effect on welfare.

4 Subsidies financed by consumption taxes

In this section, we analyse how small subsidies financed by consumption taxes affect Home welfare. To facilitate the welfare analysis, we assume that consumption taxes are the specific form.[6] In addition, we give the upper bar any economic variables with subsidies which are different from the basic model.

4.1 Demand with subsidies financed by consumption taxes

The utility maximization problem of the Home representative consumer with consumption taxes with the budget neutral of the Home government becomes:

$$\max_{\bar{q}^H(z)} \int_0^1 \left\{ a\bar{q}^H(z) - \frac{\left[\bar{q}^H(z)\right]^2}{2} \right\} dz \quad \text{s.t.} \quad \int_0^1 [\bar{p}^H(z) + t]\bar{q}^H(z)dz \leq \bar{Y}.$$

Solving this problem, we have:

$$\bar{q}^H(z) = \bar{\lambda}[a - t - \bar{p}^H(z)].$$

[6] Consumption taxes are the normally a value-added way. However, we cannot solve the model with value-added consumption taxes. Thus, we apply specific consumption taxes.

Set $\bar{\lambda} = 1$, we have the following demand function:

$$\bar{q}^H(z) = a - t - \bar{p}^H(z).$$

Naturally, the demand decreases in consumption taxes. Substituting the demand function into utility function, we have:

$$\bar{V}^H = \frac{a - \bar{\sigma}_H^2}{2} \text{ where } \bar{\sigma}_H^2 = \int_0^1 [\bar{p}^H(z) + t]^2 dz \tag{12}$$

With consumption taxes, the indirect utility depends on the second moment of consumer prices (producer prices plus consumption taxes).

4.2 Supply with subsidies financed by consumption taxes

The producers face the inverse demand function under consumption taxes as follow:

$$\bar{p}^H(z) = a - t - \bar{q}^H(z). \tag{13}$$

As we discussed in the previous section, subsidies for FDI fixed costs change the profit function of the Home FDI firm but do not affect the profit function of the Home exporting firms. The profit functions of each mode with subsidies financed by consumption taxes become:

$$\begin{aligned}
\bar{\pi}_X^H &= \bar{p}_X^H \bar{q}_X^{HH} + \bar{p}_X^F \bar{q}_X^{HF} - \bar{w}\alpha_X[\bar{q}_X^{HH} + \bar{q}_X^{HF}] - \tau \bar{q}_X^{HF}, \\
\bar{\pi}_I^H &= \bar{p}_I^{HH} \bar{q}_I^{HH} + \bar{p}_I^{HF} \bar{q}_I^{HF} - \bar{w}\alpha_I[\bar{q}_I^{HH} + \bar{q}_I^{HF}] - (1 - s_{ct})\bar{w}f,
\end{aligned}$$

where s_{ct} is subsidies financed by consumption taxes. In the same procedure as in Section 2, the Cournot equilibrium outputs of the Home exporting firm become:

$$\bar{q}_X^{HH} = \frac{a - t - \bar{w}\alpha_X + \tau}{3}, \quad \bar{q}_X^{HF} = \frac{a - t - \bar{w}\alpha_X - 2\tau}{3}, \tag{14}$$

and the monopoly equilibrium outputs of the Home FDI firm become:

$$\bar{q}_I^{HH} = \frac{a - t - \bar{w}\alpha_I}{2}, \quad \bar{q}_I^{HF} = \frac{a - t - \bar{w}\alpha_I}{2} \tag{15}$$

The difference from the basic model is that consumption taxes distort the optimal outputs while the subsidies do not affect them directly.

4.3 Labor market equilibrium with subsidies financed by consumption taxes

In this subsection, we endogenize the wage with subsidies financed consumption taxes. The labor market clearing condition becomes:

$$\begin{aligned} L &= \int_0^{z^*} \alpha_I \bar{q}_I^{HH} dI_H + \int_{z^*}^{2z^*} (\alpha_I \bar{q}_I^{FH} + f) dI_F + \int_{2z^*}^{1} \alpha_X (\bar{q}_X^{HH} + \bar{q}_X^{HF}) dX \\ &= \frac{2z^*(a-t)\alpha_I - 2z^*\alpha_I^2 \bar{w} + 2z^* f}{2} + \frac{2(1-2z^*)(a-t)\alpha_X - (1-2z^*)\tau\alpha_X - 2(1-2z^*)\alpha_X^2 \bar{w}}{3}. \end{aligned}$$

Solving this equation yields the endogenous wage with the subsidies as follow:

$$\bar{w} = \frac{[2(1-2z^*)\alpha_X + 3z^*\alpha_I](a-t) - (1-2z^*)\alpha_X\tau - 3(L - z^* f)}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2}. \tag{16}$$

The subsidies do not affect the equilibrium wage directly but they affect the wage through consumption taxes. To ensure the positive wage, the term, $L - z^* f$, is not so large.[7]

4.4 General equilibrium with subsidies financed by consumption taxes

Substituting (16) into (14), we have the general equilibrium outputs of the Home exporting firm. They are explicitly given as:

$$\bar{q}_X^{HH} = \frac{-z^*\alpha_I(\alpha_X - \alpha_I)(a-t) + [(1-2z^*)\alpha_X^2 + z^*\alpha_I^2]\tau + \alpha_X(L - z^* f)}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} \tag{17}$$

$$\bar{q}_X^{HF} = \frac{-z^*\alpha_I(\alpha_X - \alpha_I)(a-t) - [(1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2]\tau + \alpha_X(L - z^* f)}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2}. \tag{18}$$

Using (17) and (18), a general equilibrium price of the Home exporting firm becomes:

$$\bar{p}_X^{H} = \frac{[2(1-2z^*)\alpha_X^2 + 2z^*\alpha_X\alpha_I + z^*\alpha_I^2](a-t) + z^*\alpha_I^2\tau - 2\alpha_X[L - z^* f]}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2}. \tag{19}$$

[7] We impose further assumption, $L - 2z^* f > 0$ to solve the model. This is the same assumption as Fujiwara (2017).

With subsidies financed by consumption taxes, lemma 2 still hold.

Also, substituting (16) into (15), the general equilibrium outputs of the Home FDI firm are given as:

$$\bar{q}_I^{HH} = \bar{q}_I^{HF} = \frac{2(1-2z^*)\alpha_X(\alpha_X - \alpha_I)(a-t) + (1-2z^*)\alpha_X\alpha_I\tau + 3\alpha_I[L - z^* f]}{2[2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2]}. \tag{20}$$

Using (20) and symmetric assumption, general equilibrium prices of goods supplied by the Home and Foreign FDI firms in Home are given as:

$$\begin{aligned}\bar{p}_I^{HH} &= \bar{p}_I^{FH} \\ &= \frac{[2(1-2z^*)\alpha_X^2 + 2(1-2z^*)\alpha_X\alpha_I + 6z^*)\alpha_I^2](a-t) - (1-2z^*)\alpha_X\alpha_I\tau - 3\alpha_I[L - z^* f]}{2[2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2]}.\end{aligned} \tag{21}$$

4.5 Budget Neutral with subsidies financed by consumption taxes

Same as the case of labor income taxes, the Home government uses all revenues from consumption taxes to subsidies for foreign FDI firms. The budget neutral condition of the Home government becomes:

$$t\int_0^1 \bar{q}^H(z)dz = s_{ct}\int_{z^*}^{2z^*} \bar{w} f dI_F$$

From the above equation, consumption taxes can be expressed as a function of subsidies as follow [8]:

$$t = \frac{B - \sqrt{B^2 - 4AC}}{2A}, \tag{22}$$

where

$$\begin{aligned}A &\equiv 6z^*(1-2z^*)(\alpha_X - \alpha_I)^2, \\ B &\equiv Aa + 3z^*(1-2z^*)\alpha_I(\alpha_X - \alpha_I)\tau + 3[2(1-2z^*)\alpha_X + 3z^*\alpha_I][L - (1-s_{ct})z^* f], \\ C &\equiv 3z^* s_{ct} f\left\{[2(1-2z^*)\alpha_X + 3z^*\alpha_I]a - (1-2z^*)\alpha_X\tau - 3(L - z^* f)\right\}.\end{aligned}$$

[8] See the detail for Appendix

Using (22), we investigate the effects of small subsidies on consumption taxes. Differentiating consumption taxes with respect to subsidies around $s_{ct} = 0$, we have:

$$\left.\frac{\partial t}{\partial s_{ct}}\right|_{s_{ct}=0} = \frac{\left.\frac{\partial C}{\partial s_{ct}}\right|_{s_{ct}=0}}{B|_{s_{ct}=0}}$$
$$= \frac{3z^* f \left\{[2(1-2z^*)\alpha_X + 3z^*\alpha_I]a - (1-2z^*)\alpha_X\tau - 3(L - z^* f)\right\}}{B|_{s_{ct}=0}} > 0. \quad (23)$$

This implies that the Home government has to impose the heavier taxes when subsidies increase from $s_{ct} = 0$.

4.6 Effects of small subsidies on the wage and producer prices

Before proceeding the welfare analysis, we confirm the effects of small subsidies on the wage and the producer prices. First, we reveal the effect of small subsidies the wage. Differentiating (16) with respect to the subsidies around $s_{ct} = 0$, we have:

$$\left.\frac{\partial \bar{w}}{\partial s_{ct}}\right|_{s_{ct}=0} = \frac{-[2(1-2z^*)\alpha_X + 3z^*\alpha_I]}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} \cdot \left.\frac{\partial t}{\partial s_{ct}}\right|_{s_{ct}=0} < 0.$$

We immediately obtain the following lemma.

Lemma 4. *Small FDI subsidies for fixed cost financed by the consumption taxes negatively affect the wage.*

The intuition of this lemma is straightforward. The small subsidies decrease the demand and supply through the increase in consumption taxes explained in (23). This implies that the Home firms need the less labor force. Thus, the wage becomes smaller if the subsidies increase.

Second, we investigate the effect of small subsidies on the producer price of the Home exporting firm. Differentiating (19) with respect to subsidies around $s_{ct} = 0$, we obtain:

$$\left.\frac{\partial \bar{p}_X^H}{\partial s_{ct}}\right|_{s_{ct}=0} = \frac{-[2(1-2z^*)\alpha_X^2 + 2z^*\alpha_X\alpha_I + z^*\alpha_I^2]}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} \cdot \left.\frac{\partial t}{\partial s_{ct}}\right|_{s_{ct}=0} < 0.$$

From this result, we can establish the following lemma.

Lemma 5. *Small FDI subsidies for fixed costs financed by consumption taxes negatively affect the producer price of the exporting industries in Home.*

The intuition of this lemma is as follows. When consumption taxes increase, the change in the outputs of the Home exporting firm, as shown in (17) and (18), depends on the size of the labor coefficients, α_X and α_I. However, according to (13), consumption taxes have negative effects on the prices. Considering this direct effect of consumption taxes on the prices, subsidies have negative effects on the price of the Home exporting industries due to the relationship in (23).

Finally, we show the effect of small subsidies on the prices of Home and Foreign FDI industries. Differentiating (21) with respect to subsidies around $s_{ct} = 0$, we yield:

$$\left.\frac{\partial \bar{p}_I^{HH}}{\partial s_{ct}}\right|_{s_{ct}=0} = \left.\frac{\partial \bar{p}_I^{FH}}{\partial s_{ct}}\right|_{s_{ct}=0} = \frac{-[2(1-2z^*)\alpha_X^2 + 2(1-2z^*)\alpha_X\alpha_I + 6z^*)\alpha_I^2]}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} \cdot \left.\frac{\partial t}{\partial s_{ct}}\right|_{s_{ct}=0} < 0.$$

From this result, we obtain the following lemma.

Lemma 6. *Small FDI subsidies for fixed costs financed by consumption taxes negatively affect the producer prices of the Home and Foreign FDI firms in Home.*

The intuition of this lemma is similar to Lemma 5. When subsidies increase, the change in the outputs of both the Home and Foreign FDI firms in the Home market, as shown in (20), relies on the difference between the labor coefficients. However, with the direct negative effect of consumption taxes on the prices, small subsidies have the negative effects on the prices of both the Home and Foreign FDI goods in the Home market.

4.7 Welfare analysis with subsidies financed by consumption taxes

Here, we analyze the effect of subsidies for FDI fixed costs financed by consumption taxes on welfare. The indirect utility function (12) can be rewritten as:

$$\bar{V}^H = \frac{a - \int_0^{z^*}(\bar{p}_I^{HH}+t)^2 dI_H - \int_{z^*}^{2z^*}(\bar{p}_I^{FH}+t)^2 dI_F - \int_{2z^*}^{1}(\bar{p}_X^{H}+t)^2 dX}{2},$$

where

$$\bar{p}_X^H + t = \frac{[2(1-2z^*)\alpha_X^2 + 2z^*\alpha_X\alpha_I + z^*\alpha_I^2]a - 2z^*\alpha_I(\alpha_X - \alpha_I)t + z^*\alpha_I^2\tau - 2\alpha_X[L - z^*f]}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2}, \quad (24)$$

$$\begin{aligned}
\bar{p}_I^{HH} + t &= \bar{p}_I^{FH} + t \\
&= \frac{\left\{ \begin{array}{l} [2(1-2z^*)\alpha_X^2 + 2(1-2z^*)\alpha_X\alpha_I + 6z^*)\alpha_I^2]a + 2(1-2z^*)\alpha_X(\alpha_X - \alpha_I)t \\ -(1-2z^*)\alpha_X\alpha_I\tau - 3\alpha_I[L - z^*f] \end{array} \right\}}{2[2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2]}. \quad (25)
\end{aligned}$$

The welfare effects of subsidies on welfare around $s_{ct} = 0$ are given as:

$$\begin{aligned}
\left.\frac{\partial \bar{V}^H}{\partial s_{ct}}\right|_{s_{ct}=0} &= \frac{-z^*(1-2z^*)(\alpha_I - \alpha_X)}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} \\
&\quad \cdot \left\{ 2\alpha_I(\bar{p}_X^H + t)|_{s_{ct}=0} - \alpha_X[(\bar{p}_I^{HH} + t)|_{s_{ct}=0} + (\bar{p}_I^{FH} + t)|_{s_{ct}=0}] \right\} \cdot \left.\frac{\partial t}{\partial s_{ct}}\right|_{s_{ct}=0}, \quad (26)
\end{aligned}$$

From (26), the welfare effects depend on the effects of subsidies on the consumer prices, $\bar{p}_X^H + t$, $\bar{p}_I^{HH} + t$, and $\bar{p}_I^{FH} + t$. Moreover, the effects of subsidies on welfare depend on the difference between the labor coefficients, $\alpha_X - \alpha_I$. In this paper, we focus on the case that $\alpha_X - \alpha_I < 0$ holds.[9] With the condition, $\alpha_X - \alpha_I < 0$, we obtain the following proposition of the effects of subsidies on the consumer prices of exporting and FDI industries.

Proposition 2. *The consumer price of exporting goods becomes higher while the that of FDI goods becomes lower with the small FDI subsidies financed by consumption taxes.*

Proof. Differentiating (24) and (25) with respect to s_{ct} around $s_{ct} = 0$ with $\alpha_X - \alpha_I < 0$, we have:

$$\begin{aligned}
\left.\frac{\partial(\bar{p}_X^H + t)}{\partial s_{ct}}\right|_{s_{ct}=0} &= \frac{2z^*\alpha_I(\alpha_I - \alpha_X)}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} \cdot \left.\frac{\partial t}{\partial s_{ct}}\right|_{s_{ct}=0} > 0, \\
\left.\frac{\partial(\bar{p}_I^{HH} + t)}{\partial s_{ct}}\right|_{s_{ct}=0} &= \left.\frac{\partial(\bar{p}_I^{FH} + t)}{\partial s_{ct}}\right|_{s_{ct}=0} = \frac{-2(1-2z^*)\alpha_X(\alpha_I - \alpha_X)}{2[2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2]} \cdot \left.\frac{\partial t}{\partial s_{ct}}\right|_{s_{ct}=0} < 0.
\end{aligned}$$

□

[9] See Appendix for the result with $\alpha_X - \alpha_I > 0$. With $\alpha_X - \alpha_I = 0$, the result is same as in the case that subsidies are financed by labor income taxes. This is because small subsidies do not affect the outputs in general equilibrium, (17), (18), and (20). This implies that the consumer prices, (24) and (25), are equal to (8) and (10). Thus, the subsidies do not affect welfare.

The intuition of this lemma is as follows. Subsidies provide greater advantages for FDI firms compared to exporting firms, given the condition $\alpha_X - \alpha_I < 0$. This is due to the reduction in variable costs by wage reductions. Lemma 5 and Lemma 6 show that the producer prices of both exporting and FDI goods decrease, which has a downward effect on consumer prices. However, consumption taxes have upward effects on consumer prices. Regarding consumer prices of goods in the exporting industries in the Home market, the downward effect is larger than the upward effect. Conversely, for goods supplied by Home and Foreign FDI firms in the Home market, the downward effect is smaller than the upward effect.

From the above discussion, under the condition, $\alpha_X - \alpha_I < 0$, the welfare improvement stems from the reduction in the consumer prices of exporting industries's goods. We can rewrite (26) as:

$$\left.\frac{\partial \bar{V}^H}{\partial s_{ct}}\right|_{s_{ct}=0} = \frac{-z^*(1-2z^*)(\alpha_I - \alpha_X)}{[2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2]^2} \cdot \Omega_{ct} \cdot \left.\frac{\partial t}{\partial s_{ct}}\right|_{s_{ct}=0},$$

where

$$\Omega_{ct} = 2(\alpha_I - \alpha_X)[(1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2]a + \alpha_I[(1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2]\tau - \alpha_X\alpha_I(L - z^*f).$$

With the condition, $\alpha_X - \alpha_I < 0$, and (23), equation (26) indicates that welfare improves when the consumer prices of the exporting industries are low and those of the Home and Foreign FDI industries are high. In other words, the welfare improvement occurs when the term, Ω_{ct}, is negative. Specifically, we obtain the following proposition.

Proposition 3. *With $\alpha_X - \alpha_I < 0$, small FDI subsidies for fixed costs tend to enhance the welfare when trade costs are small, fixed costs of FDI are at an intermediate level, and the labor coefficient of exporting industries is large enough.*

Proof. See Appendix. □

Around $s_{ct} = 0$, equations, (24) and (25), become equal to (8) and (10). When trade costs become smaller, we can observe that the consumer prices of goods in exporting industries become lower, while those in FDI industries become higher, as shown in Lemma 2 and Lemma 3.

When the fixed costs of FDI become smaller, all consumer prices become smaller from Lemma 2 and Lemma 3. However, the size of the drop in the consumer prices in exporting industries is smaller than that in the addition of the consumer prices in Home and Foreign FDI industries.

This implies that the fixed costs of FDI cannot be too small for having negative Ω_{ct}. On the other hand, when the fixed costs of FDI become larger, the rise in the consumer prices in exporting industries is smaller than that in the addition of the consumer prices in Home and Foreign FDI industries. Apparently, this implies that welfare change tends to be positive. However, if the the fixed costs of FDI are too large, it violates the condition of the positive wage.[10] From the above discussions, the fixed costs of FDI must be at an intermediate level.

When the labor coefficient of exporting industries is sufficiently large, the producer price of exporting firms around $s_{ct} = 0$, denoted as $\bar{p}_X^H = (a - \tau + 2\bar{w}\alpha_X)/3$, decreases significantly due to the wage reduction as shown in Lemma 4. This implies that the increase in consumer prices of exporting firms become significantly smaller. The sufficiently small increase in consumer prices of exporting industries leads to only minor negative effects on welfare. With this condition, the term, Ω_{ct}, tends to be negative and thus welfare tends increase by the subsidies.

5 Conclusion

We use the GOLE model developed by Neary (2016) to evaluate the welfare impact of host country small FDI subsidies aimed at attracting a monopolistic foreign firm. Specifically, we examine the welfare implications of small subsidies for fixed cost of FDI, while considering the differences in financing sources for these subsidies.

In our analysis, we show the small FDI subsidies that are financed through labor income taxes had no impact on welfare. As labor income taxes do not cause distortions in consumption and subsidies do not affect the production level of firms. As a result, the wage and prices of goods remain constant, and subsidies do not affect welfare. This finding is in contrast to the result of Chor (2009) that the small subsidies financed by labor income taxes always improve welfare in the host country.

On the other hand, consumption taxes have negative effects on both demand and supply. This negative shock leads to a decrease in the wage, even when the government provides small subsidies. With the assumption that the labor coefficient of the exporting industries is smaller than that of FDI industries, consumer prices for exporting industries increase while those for FDI industries decrease. Welfare improves when the consumer prices of exporting industries are low and those of

[10] See the detail for Appendix

FDI industries are high. Specifically, welfare improves by the subsidy when trade costs are small, fixed costs of FDI are at an intermediate level, and the exporting industries' labor coefficient is large enough. When trade costs are small, producer prices of exporting industries are low while those of FDI industries are high. When fixed costs of FDI are at the intermediate level, it confirms that the wage, outputs, and prices are positive, Furthermore, the negative shock on the producer prices in the both industries is not so different. When the exporting industries' labor coefficient is large enough, the response to change in the wage reduction becomes larger, resulting in a larger negative shock on producer prices of exporting firms. This implies that the difference calculated by subtraction consumer prices of exporting industries from those of FDI industries tends to become negative. This finding differs from the result of Han *et al.* (2023), suggesting that subsidies to FDI financed by consumption taxes have a greater potential to enhance welfare than those by labor income taxes do.

These new results which are different from previous literature are important for economic policy by the government. For example, if we focus on trade costs, it is considered that the trade costs between Japan and Korea are low. Therefore, if Japan provides subsidies to Samsung Electronics, it would be more effective to use consumption taxes as a source of financing. On the other hand, it is considered that trade costs between Japan and the United States are high. Therefore, if Japan provides subsidies to Micron Technology, using labor income taxes as a better financing source for subsidies. However, the actual implementation and governance of any such schemes will have to be carried out carefully, in order to determine what the appropriate financing source for FDI subsidy level should be. In addition, this paper does not consider the effect of subsidies on the supply form of firms. The phenomena that firms previously supplied overseas through exports but change their strategy using FDI due to subsidies happens in the real world. This is an important issue for future work to investigate within a GOLE model setting.

References

Alfaro, L., Chanda, A., Kalemli-Ozcan, S. and Sayek, S. (2004) Fdi and economic growth: the role of local financial markets, *Journal of International Economics*, **64**, 89–112.

Autor, D., Dorn, D., Katz, L., Patterson, C. and van Reenen, J. (2020) The fall of the labor share and the rise of superstar firms*, *The Quarterly Journal of Economics*, **135**, 645–709.

Autor, D., Dorn, D., Katz, L. F., Patterson, C. and Van Reenen, J. (2017) Concentrating on the fall of the labor share, *American Economic Review*, **107**, 180–85.

Bastos, P. and Kreickemeier, U. (2009) Unions, competition and international trade in general equilibrium, *Journal of International Economics*, **79**, 238–247.

Beladi, H. and Chakrabarti, A. (2019) Multidivisional firms, internal competition, and comparative advantage: Baye et al. meet neary, *Journal of International Economics*, **116**, 50–57.

Borensztein, E., De Gregorio, J. and Lee, J.-W. (1998) How does foreign direct investment affect economic growth?, *Journal of International Economics*, **45**, 115–135.

Carkovic, M. and Levine, R. (2005) Does foreign direct investment accelerate economic growth, *Does foreign direct investment promote development*, **195**, 220.

Chor, D. (2009) Subsidies for fdi: Implications from a model with heterogeneous firms, *Journal of International Economics*, **78**, 113–125.

De Loecker, J., Eeckhout, J. and Unger, G. (2020) The rise of market power and the macroeconomic implications*, *The Quarterly Journal of Economics*, **135**, 561–644.

Eckel, C. and Neary, J. P. (2010) Multi-product firms and flexible manufacturing in the global economy, *Review of Economic Studies*, **77**, 188–217.

Egger, H. and Etzel, D. (2012) The impact of trade on employment, welfare, and income distribution in unionized general oligopolistic equilibrium, *European Economic Review*, **56**, 1119–1135.

Fujiwara, K. (2017) Trade and FDI liberalization in a general oligopolistic equilibrium, *Japan and the World Economy*, **41**, 45–49.

Fumagalli, C. (2003) On the welfare effects of competition for foreign direct investments, *European Economic Review*, **47**, 963–983.

Haaland, J. I. and Wooton, I. (1999) International competition for multinational investment, *The Scandinavian Journal of Economics*, **101**, 631–649.

Han, W., Wang, J. and Wang, X. L. (2023) Benefits of foreign direct investment subsidies: The role of funding sources, *Pacific Economic Review*, forthcoming.

Helpman, E., Melitz, M. J. and Yeaple, S. R. (2004) Export versus fdi with heterogeneous firms, *American Economic Review*, **94**, 300–316.

Lim, S.-H. (2008) How investment promotion affects attracting foreign direct investment: Analytical argument and empirical analyses, *International Business Review*, **17**, 39–53.

Neary, J. P. (2003a) Competitive versus comparative advantage, *The World Economy*, **26**, 457–470.

Neary, J. P. (2003b) Presidential address: Globalization and market structure, *Journal of the European Economic Association*, **1**, 245–271.

Neary, J. P. (2007) Cross-border mergers as instruments of comparative advantage, *The Review of Economic Studies*, **74**, 1229–1257.

Neary, J. P. (2010) Two and a half theories of trade, *The World Economy*, **33**, 1–19.

Neary, J. P. (2016) International trade in general oligopolistic equilibrium, *Review of International Economics*, **24**, 669–698.

Pennings, E. (2005) How to maximize domestic benefits from foreign investments: the effect of irreversibility and uncertainty, *Journal of Economic Dynamics and Control*, **29**, 873–889.

Skaksen, M. R. (2005) Should governments subsidise inward foreign direct investment?, *The Scandinavian Journal of Economics*, **107**, 123–140.

Tanzi, V. and Zee, H. (2000) Tax policy for emerging markets: Developing countries, *National Tax Journal*, **53**, 299–322.

UNCTAD (2022) *World Investment Report 2022*.

Wells, L. T. J., Allen, n. J., Morisset, J. and Pirnia, N. (2001) *Using Tax Incentives to Compete for Foreign Investment*, The World Bank.

Appendix

A.1 Proof for lemma 1

Differentiating (5) with respect to τ and f, we have:

$$\frac{\partial w}{\partial \tau} = \frac{-(1-2z^*)\alpha_X}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} < 0,$$
$$\frac{\partial w}{\partial f} = \frac{3z^*}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} > 0.$$

A.2 Proof for lemma 2

Differentiating (8) with respect to τ and f, we obtain:

$$\frac{\partial p_X^H}{\partial \tau} = \frac{z^*\alpha_I^2}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} > 0,$$
$$\frac{\partial p_X^H}{\partial f} = \frac{2z^*\alpha_X}{2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2} > 0.$$

A.3 Proof for lemma 3

Differentiating (10) with respect to τ and f, we yield:

$$\frac{\partial p_I^{HH}}{\partial \tau} = \frac{\partial p_I^{FH}}{\partial \tau} = \frac{-(1-2z^*)\alpha_X\alpha_I}{2[2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2]} < 0,$$
$$\frac{\partial p_I^{HH}}{\partial f} = \frac{\partial p_I^{FH}}{\partial f} = \frac{3z^*\alpha_I}{2[2(1-2z^*)\alpha_X^2 + 3z^*\alpha_I^2]} > 0.$$

A.4 Detail for the budget neutral condition

Using (14) and (15), the government's revenue is

$$\begin{aligned} t\int_0^1 \bar{q}^H(z)dz &= t\left\{\int_0^{z^*} \bar{q}_I^{HH} dI_H + \int_{z^*}^{2z^*} \bar{q}_I^{FH} dI_F + \int_{2z^*}^1 \left(\bar{q}_X^{HH} + \bar{q}_X^{FH}\right) dX\right\} \\ &= \frac{t\left[(2-z^*)(a-t) - (1-2z^*)\tau - \{2(1-2z^*)\alpha_X + 3z^*\alpha_I\}\,\bar{w}\right]}{3}. \end{aligned} \tag{A.1}$$

The government's spending is

$$s_{ct}\int_{z^*}^{2z^*}\bar{w}f dI_F = z^* s_{ct}\bar{w}f. \tag{A.2}$$

Equating (A.1) to (A.2) gives:

$$\begin{aligned} & t\int_0^1 \bar{q}^H(z)dz = s_{ct}\int_{z^*}^{2z^*}\bar{w}f dI_F \\ & \Leftrightarrow (2-z^*)at-(2-z^*)t^2-(1-2z^*)\tau t-\{2(1-2z^*)\alpha_X+3z^*\alpha_I\}\,\bar{w}t-3z^*s_{ct}\bar{w}f=0. \end{aligned} \tag{A.3}$$

Define $\bar{w}$ as follow:

$$\begin{aligned} \bar{w} &= \frac{[2(1-2z^*)\alpha_X+3z^*\alpha_I](a-t)-(1-2z^*)\alpha_X\tau-3(L-z^*f)}{2(1-2z^*)\alpha_X^2+3z^*\alpha_I^2} \\ &\equiv \frac{w_{N1}(a-t)-w_{N2}-w_{N3}}{w_D}. \end{aligned}$$

Using this notation, (A.3) can be rewritten as:

$$\begin{aligned} & (2-z^*)w_D at-(2-z^*)w_D t^2-(1-2z^*)\tau w_D t-w_{N1}(w_{N1}a-w_{N2}-w_{N3})t+w_{N1}^2t^2 \\ & \quad -3z^*s_{ct}f(w_{N1}a-w_{N2}-w_{N3})+3z^*s_{ct}fw_{N1}t=0 \\ \Leftrightarrow & \left[(2-z^*)w_D-w_{N1}^2\right]t^2 \\ & \quad -\{(2-z^*)w_D a-(1-2z^*)\tau w_D-w_{N1}(w_{N1}a-w_{N2}-w_{N3})+3z^*s_{ct}fw_{N1}\}\,t \\ & \quad +3z^*s_{ct}f(w_{N1}a-w_{N2}-w_{N3})=0 \\ \Leftrightarrow & At^2-Bt+C=0. \end{aligned} \tag{A.4}$$

The term A, B, and C are defined as:

$$\begin{aligned} & A \equiv 6z^*(1-2z^*)(\alpha_X-\alpha_I)^2, \\ & B \equiv Aa+3z^*(1-2z^*)\alpha_I(\alpha_X-\alpha_I)\tau+3[2(1-2z^*)\alpha_X+3z^*\alpha_I][L-(1-s_{ct})z^*f], \\ & C \equiv 3z^*s_{ct}f\left\{[2(1-2z^*)\alpha_X+3z^*\alpha_I]a-(1-2z^*)\alpha_X\tau-3(L-z^*f)\right\}. \end{aligned}$$

First, we show A, B, and C are positive. A is positive obviously. As for C, we assume that the wage in (16) is positive. Thus C is also positive. Regarding B, we can rewrite B as:

$$\begin{aligned} B &= 3z^*(1-2z^*)(\alpha_X-\alpha_I)[2(\alpha_X-\alpha_I)a+\alpha_I\tau] \\ &\quad + 3[2(1-2z^*)\alpha_X+3z^*\alpha_I][L-(1-s_{ct})z^*f] \end{aligned} \tag{A.5}$$

The sign of (A.5) depends on the difference between the labor coefficients, $\alpha_X-\alpha_I$. However, B is always positive. If the difference is positive, $\alpha_X-\alpha_I>0$, we can easily confirm B is positive. If the difference is negative, $\alpha_X-\alpha_I<0$, the proof that $B>0$ is bit complex. Suppose $B\leq 0$. We can rewrite (A.5) as:

$$2(\alpha_I-\alpha_X)a \leq \alpha_I\tau - \frac{3[2(1-2z^*)\alpha_X+3z^*\alpha_I](L-z^*f+s_{ct}z^*f)}{z^*(1-z^*)(\alpha_I-\alpha_X)}. \tag{A.6}$$

From (6) and (7), the condition of the positive outputs of the exporting industries can be expressed as:

$$\begin{aligned} \bar{q}_X^{HH}+\bar{q}_X^{HF}>0 &\Leftrightarrow 2z^*\alpha_I(\alpha_I-\alpha_X)a - z^*\alpha_I^2\tau + 2\alpha_X(L-z^*f)>0. \\ &\Leftrightarrow 2(\alpha_I-\alpha_X)a > \alpha_I\tau - \frac{2\alpha_X(L-z^*f)}{z^*\alpha_I} + 2(\alpha_I-\alpha_X)t. \end{aligned} \tag{A.7}$$

Combining (A.6) and (A.7), we have:

$$\begin{aligned} &\alpha_I\tau - \frac{3[2(1-2z^*)\alpha_X+3z^*\alpha_I](L-z^*f+s_{ct}z^*f)}{z^*(1-z^*)(\alpha_I-\alpha_X)} > \alpha_I\tau - \frac{2\alpha_X(L-z^*f)}{z^*\alpha_I} + 2(\alpha_I-\alpha_X)t \\ &\Rightarrow \alpha_I\tau - \frac{3[2(1-2z^*)\alpha_X+3z^*\alpha_I](L-z^*f+s_{ct}z^*f)}{z^*(1-z^*)(\alpha_I-\alpha_X)} > \alpha_I\tau - \frac{2\alpha_X(L-z^*f)}{z^*\alpha_I} \\ &\Leftrightarrow z^*[2(1-2z^*)\alpha_X^2+6(1-2z^*)\alpha_X\alpha_I+9z^*\alpha_I^2](L-z^*f) \\ &\quad + 3z^*\alpha_I[3z^*\alpha_I^2+2(1-2z^*)\alpha_X^2]s_{ct}z^*f<0. \end{aligned}$$

This is the contradiction. Thus, with the positive out put of the exporting industries, B is always positive regardless of the sign of the difference between the labor coefficients.

Second, we consider the condition that equation (A.4) has real solutions. To have real solutions, equation (A.4) needs to satisfy the following condition: $B^2-4AC>0$. We can rewrite B and

$4AC$ as:

$$B = Aa + B_1 + w_{N1}(w_{N3} + 3s_{ct}z^*f)$$
$$4AC = 12As_{ct}z^*f(w_{N1}a - w_{N2} - w_{N3})$$

Using the above two equations, the condition, $B^2 - 4AC > 0$, is explicitly given as:

$$\begin{aligned} & B^2 - 4AC > 0 \\ & \Leftrightarrow A^2a^2 + 2Aw_{N1}(w_{N3} - 3s_{ct}z^*f)a + B_1^2 + w_{N1}^2(w_{N3} + 3s_{ct}z^*f)^2 \\ & \quad + 2B_1w_{N1}(w_{N3} + 3s_{ct}z^*f) + 12As_{ct}z^*f(w_{N2} + w_{N3}) > 0 \end{aligned} \tag{A.8}$$

If the condition, $2Aw_{N1}(w_{N3} - 3s_{ct}z^*f)a > 0$, holds, (A.8) holds because other terms are positive. We can rewrite $2Aw_{N1}(w_{N3} - 3s_{ct}z^*f)a$ as $6Aw_{N1}(L - z^*f - s_{ct}f)$. Recall that we assume $L - 2z^*f > 0$. With this condition, $6Aw_{N1}(L - z^*f - s_{ct}f) > 0$ holds due to $s_{ct} \in [0, 1]$. Thus, $B^2 - 4AC > 0$ holds and t has real solutions as follow:

$$t = \frac{B \pm \sqrt{B^2 - 4AC}}{2A} \tag{A.9}$$

Finally, we consider which real solution is appropriate. Assume the positive outputs of all industries, we need $a - t > 0$. Using $t = (B + \sqrt{B^2 - 4AC})/(2A)$ and $\sqrt{B^2 - 4AC} > Aa$ from (A.8), we can rewrite $a - t > 0$ as:

$$\begin{aligned} & a - t > 0 \\ & \Leftrightarrow 2Aa - B - \sqrt{B^2 - 4AC} > 0 \\ & \Leftrightarrow Aa - [B_1 + w_{N1}(w_{N3} + 3s_{ct}z^*f)] - \sqrt{B^2 - 4AC} > 0 \\ & \Rightarrow -[B_1 + w_{N1}(w_{N3} + 3s_{ct}z^*f)] > 0 \end{aligned}$$

This is contradiction. Therefore, if $t = (B + \sqrt{B^2 - 4AC})/(2A)$ holds, we have $a - t \leq 0$.

On the other hand, Using $t = (B - \sqrt{B^2 - 4AC})/(2A)$ and $\sqrt{B^2 - 4AC} > B_1 + w_{N1}(w_{N3} +$

$3s_{ct}z^*f)$ from (A.8), we can rewrite $a - t > 0$ as:

$$\begin{aligned}
&a - t > 0 \\
&\Leftrightarrow 2Aa - B + \sqrt{B^2 - 4AC} > 0 \\
&\Leftrightarrow Aa - [B_1 + w_{N1}(w_{N3} + 3s_{ct}z^*f)] + \sqrt{B^2 - 4AC} > 0 \\
&\Rightarrow Aa > 0
\end{aligned}$$

This always holds. Thus, if $t = (B - \sqrt{B^2 - 4AC})/(2A)$ holds, we have $a - t > 0$.

From the above discussions, we have:

$$t = \frac{B - \sqrt{B^2 - 4AC}}{2A}.$$

This function is the same equation as (22).

A.5 Proof for Proposition 3.

We show the conditions that the inequality, $\Omega_{ct} < 0$, holds. The procedure of the proof is divided into three steps. In step 1, we derive conditions of the positive wage, outputs, and prices in general equilibrium around $s_{ct} = 0$ respectively. In step 2, we derive combining conditions that all positive wages, outputs, and prices hold. In step 3, given the combining conditions in step 2, we derive conditions that welfare increases by small subsidies. In other words, we derive the conditions that $\Omega_{ct} < 0$ holds.

Step 1: Deriving each condition of the positive wage, outputs, and prices

We consider the positive wage, outputs, and prices in general equilibrium around $s_{ct} = 0$. Around $s_{ct} = 0$, the wage is equal to (5), the outputs and prices with the consumption tax in exporting industries are equal to (6), (7), and (8) and in FDI industries are equal to (9) and (10). First, we consider the condition of the positive wage. Recall $w_{N1} = 2(1 - 2z^*)\alpha_X + 3z^*\alpha_I$. From (5), the condition of the positive wage can be written as:

$$w > 0 \Leftrightarrow w_{N1}a - (1 - 2z^*)\alpha_X\tau > 3\underbrace{(L - z^*f)}_{\equiv \tilde{L}}$$

To foster the analysis, we multiply the above inequality by $2\alpha_X\alpha_I$. We can rewrite the condition as:

$$w > 0 \Leftrightarrow \underbrace{2\alpha_X\alpha_I w_{N1}a - 2(1-2z^*)\alpha_X^2\alpha_I\tau}_{\equiv w_{N4}} > 6\alpha_X\alpha_I\tilde{L}$$

Second, we consider variables in exporting industries. With $\alpha_X - \alpha_I < 0$, the condition, $q_X^{HH} > 0$, always holds. From (7) and (8), we have following conditions:

$$q_X^{HF} > 0 \Leftrightarrow 2\alpha_X\tilde{L} > 2[\underbrace{(1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2}_{\equiv X_1}]\tau - 2z^*\alpha_I(\alpha_I - \alpha_X)a$$

$$p_X^H > 0 \Leftrightarrow [\underbrace{2(1-2z^*)\alpha_X^2 + 2z^*\alpha_X\alpha_I + z^*\alpha_I^2}_{\equiv X_2}]a + z^*\alpha_I^2\tau > 2\alpha_X\tilde{L}$$

Combining these conditions and multiplying $3\alpha_I$, we have the following condition that satisfies positive outputs and prices in exporting industries:

$$\underbrace{3\alpha_I X_2 a + 3z^*\alpha_I^3\tau}_{\equiv X_3} > 6\alpha_X\alpha_I\tilde{L} > \underbrace{6\alpha_I X_1\tau - 6z^*\alpha_I^2(\alpha_I - \alpha_X)a}_{\equiv X_4}$$

Finally, we consider variables in FDI industries. From (9) and (10), we have:

$$q_I^{HH} = q_I^{HF} > 0 \Leftrightarrow 3\alpha_I\tilde{L} > 2(1-2z^*)\alpha_X(\alpha_I - \alpha_X)a - (1-2z^*)\alpha_X\alpha_I\tau$$

$$p_X^H > 0 \Leftrightarrow [\underbrace{2(1-2z^*)\alpha_X^2 + 2(1-2z^*)\alpha_X\alpha_I + 6z^*\alpha_I^2}_{\equiv I_1}]a - (1-2z^*)\alpha_X\alpha_I\tau > 3\alpha_I\tilde{L}$$

Combining these conditions and multiplying $2\alpha_X$, we obtain the following condition satisfying positive outputs and prices in FDI industries:

$$\underbrace{2\alpha_X I_1 a - 2(1-2z^*)\alpha_X^2\alpha_I\tau}_{\equiv I_2} > 6\alpha_X\alpha_I\tilde{L} > \underbrace{4(1-2z^*)\alpha_X^2(\alpha_I - \alpha_X)a - 2(1-2z^*)\alpha_X^2\alpha_I\tau}_{\equiv I_3}$$

Step 2: Deriving the combining conditions of the positive wage, outputs, and prices

We derive the overall condition that satisfies the positive wage, outputs, and prices in all industries. First, we consider the relationships of w_{N4}, X_3, and I_2. Comparing w_{N4} with X_3 and I_2, we can easily show the following relationships:

$$w_{N4} < X_3 \quad \text{and} \quad w_{N4} < I_2$$

Second, we consider the relationships of w_{N4}, X_4, and I_3. Comparing w_{N4} with X_4 and I_3, we can easily show the following relationships:

$$w_{N4} > X_4 \quad \text{and} \quad w_{N4} > I_3$$

From the above discussions, we have two conditions that ensure the positive wage, outputs, and prices in each industry as follows:

$$\begin{aligned} &X_4 < 6\alpha_X\alpha_I\tilde{L} < w_{N4} \quad \text{for exporting industries} \\ &I_3 < 6\alpha_X\alpha_I\tilde{L} < w_{N4} \quad \text{for FDI industries} \end{aligned}$$

To combine the above conditions, we need to derive the size relationship of X_4 and I_3. Comparing X_4 with I_3, we have:

$$X_4 \gtreqless I_3 \Leftrightarrow \tau \gtreqless \frac{\alpha_I - \alpha_X}{2\alpha_I}a$$

This relationship is divided into two cases below:

$$\begin{aligned} X_4 \leq I_3 \quad &\text{with} \quad 0 < \tau \leq \frac{\alpha_I - \alpha_X}{2\alpha_I}a \\ X_4 > I_3 \quad &\text{with} \quad \tau > \frac{\alpha_I - \alpha_X}{2\alpha_I}a \end{aligned}$$

Considering these two cases, we have the two conditions satisfying the positive wage, outputs, and prices in all industries as follows With $\alpha_I \geq 2\alpha_X$, we have the following two conditions:

$$X_4 \leq I_3 < 6\alpha_X\alpha_I\tilde{L} < w_{N4} \quad \text{with} \quad 0 < \tau \leq \frac{\alpha_I - \alpha_X}{2\alpha_I}a \tag{A.10}$$

$$I_3 < X_4 < 6\alpha_X\alpha_I\tilde{L} < w_{N4} \quad \text{with} \quad \tau > \frac{\alpha_I - \alpha_X}{2\alpha_I}a \tag{A.11}$$

Step 3: Deriving the condition of welfare improvement by subsidies

Recall that welfare improves when $\Omega_{ct} < 0$ holds. The condition, $\Omega_{ct} < 0$, can be written as:

$$\begin{aligned}\Omega_{ct} < 0 &\Leftrightarrow 2(\alpha_I - \alpha_X)[(1-2z^*)a_X^2 + 2z^* a_I^2]a + \alpha_I[(1-2z^*)a_X^2 + 2z^* a_I^2]\tau < \alpha_X\alpha_I(L - z^* f) \\ &\Leftrightarrow 2(\alpha_I - \alpha_X)X_1 a + \alpha_I X_1 \tau < \alpha_X \alpha_I \tilde{L} \\ &\Leftrightarrow \underbrace{12(\alpha_I - \alpha_X)X_1 a + 6\alpha_I X_1 \tau}_{\equiv \omega_1} < 6\alpha_X\alpha_I\tilde{L}\end{aligned}$$

First, we consider the size relationship of ω_1, X_4, and I_3. Comparing ω_1 with X_4 and I_3, we have:

$$\omega_1 > X_4 \quad \text{and} \quad \omega_1 > I_3$$

Proof. Suppose $\omega_1 \leq X_4$ and $\omega_1 \leq I_3$. Then, we have:

$$\begin{aligned}\omega_1 \leq X_4 &\Leftrightarrow 12(\alpha_I - \alpha_X)X_1 a + 6\alpha_I X_1\tau \leq 6\alpha_I X_1 \tau - 6z^*\alpha_I^2(\alpha_I - \alpha_X)a \\ &\Leftrightarrow 6(\alpha_I - \alpha_X)(2X_1 + z^*\alpha_I^2)a \leq 0\end{aligned}$$

and

$$\begin{aligned}\omega_1 \leq I_3 &\Leftrightarrow 12(\alpha_I - \alpha_X)X_1 a + 6\alpha_I X_1 \tau \leq 4(1-2z^*)(\alpha_I - \alpha_X)\alpha_X^2 a - 2(1-2z^*)\alpha_X^2\alpha_I\tau \\ &\Leftrightarrow 8(\alpha_I - \alpha_X)[(1-z^*)\alpha_X^2 + 3z^*\alpha_I^2]a + 2\alpha_I[3X_1 + (1-2z^*)\alpha_X^2]\tau \leq 0\end{aligned}$$

These are contradiction. Thus, we have: $\omega_1 > X_4$ and $\omega_1 > I_3$. $\square$

From these inequalities and (A.10) and (A.11), welfare increase may occur if the following relationships hold.

$$X_4 \leq I_3 < \omega_1 < 6\alpha_X\alpha_I\tilde{L} < w_{N4} \quad \text{with} \quad 0 < \tau \leq \frac{\alpha_I - \alpha_X}{2\alpha_I}a \tag{A.12}$$

$$I_3 < X_4 < \omega_1 < 6\alpha_X\alpha_I\tilde{L} < w_{N4} \quad \text{with} \quad \tau > \frac{\alpha_I - \alpha_X}{2\alpha_I}a \tag{A.13}$$

Second, we derive conditions that welfare increases by the subsidy. Specifically, we need two conditions that (A.12) and (A.13) hold. The conditions are

$$\omega_1 < 6\alpha_X\alpha_I\tilde{L} < w_{N4} \quad \text{and} \quad \omega_1 < w_{N4}$$

We require the intermediate level of $\tilde{L}$ for the condition that the first inequality, $\omega_1 < 6\alpha_X\alpha_I\tilde{L} < w_{N4}$, holds. Recall $\tilde{L} = L - z^*f$. In words, we need the intermediate level of fixed costs of FDI, f, to have the increase in welfare ($\omega_1 < 6\alpha_X\alpha_I$) and the positive wage ($6\alpha_X\alpha_I\tilde{L} < w_{N4}$) hold.

We next consider the condition that the second inequality, $\omega_1 < w_{N4}$, holds. Recall $w_{N1} = 2(1-2z^*)\alpha_X + 3z^*\alpha_I$ and $X_1 = (1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2$. The inequality, $\omega_1 < w_{N4}$, can be rewritten as:

$$\begin{aligned}\omega_1 < w_{N4} &\Leftrightarrow 12(\alpha_I - \alpha_X)X_1 a + 6\alpha_I X_1 \tau < 2\alpha_X\alpha_I w_{N1} a - 2(1-2z^*)\alpha_X^2\alpha_I\tau \\ &\Leftrightarrow 6(\alpha_I - \alpha_X)X_1 a + 3\alpha_I X_1 \tau < \alpha_X\alpha_I w_{N1} a - (1-2z^*)\alpha_X^2\alpha_I\tau \\ &\Leftrightarrow 2[2(1-2z^*)\alpha_X^2\alpha_I + 3z^*\alpha_I^3]\tau < \underbrace{[6(1-2z^*)\alpha_X^3 - 4(1-2z^*)\alpha_X^2\alpha_I + 15z^*\alpha_X\alpha_I^2 - 12z^*\alpha_I^3]}_{\equiv v(\alpha_X,\alpha_I)}a\end{aligned} \tag{A.14}$$

The left hand side of (A.14) is decreasing in τ. Differentiating $v(\alpha_X, \alpha_I)$ with respect to α_X, we have:

$$\frac{\partial v(\alpha_X,\alpha_I)}{\partial \alpha_I} = 18(1-2z^*)\alpha_X^2 - 8(1-2z^*)\alpha_X\alpha_I + 15z^*\alpha_I^2$$

If the labor coefficient of exporting industries is large enough ($\alpha_I \approx \alpha_X$), we have:

$$\frac{\partial v(\alpha_X,\alpha_I)}{\partial \alpha_I} = 10(1-2z^*)\alpha_X^2 + 15z^*\alpha_X^2 > 0$$

Thus, the right hand side of (A.14) is increasing in α_X when tthe labor coefficient of exporting industries is large enough ($\alpha_I \approx \alpha_X$). If we substitute $\tau = \frac{\alpha_I - \alpha_X}{2\alpha_I}a$ and $\alpha_I \approx \alpha_X$ into (A.14), we have:

$$0 < 2(1-2z^*)\alpha_X^3 + 3z^*\alpha_X^3$$

From this result, the inequality, $\omega_1 < w_{N4}$, holds in both cases, (A.12) and (A.13) with small τ and small enough α_I ($\alpha_I \approx \alpha_X$).

Combining the conditions that the fist inequality, $\omega_1 < 6\alpha_X\alpha_I < w_{N4}$, and the second inequality, $\omega_1 < w_{N4}$, hold, we require intermediate fixed costs of FDI (f), small trade costs (τ), and the large enough labor coefficient of exporting firms (α_X). Under these conditions, we have: $\Omega_{ct} < 0$.

A.6 Welfare analysis with $\alpha_X - \alpha_I > 0$

We show $\Omega_{ct} < 0$ holds with $\alpha_X - \alpha_I > 0$. Suppose $\Omega_{ct} \geq 0$ holds. We can rewrite $\Omega_{ct} \geq 0$ as:

$$\Omega_{ct} \geq 0 \Leftrightarrow \underbrace{\alpha_I[(1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2]\tau - \alpha_X\alpha_I(L - z^*f)}_{\equiv \omega_2} > 2(\alpha_X - \alpha_I)[(1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2]a$$

Around $s_{ct} = 0$, we have: $\bar{q}_X^{HF}$ in (18) is equal to q_X^{HF} in (7). The condition of positive q_X^{HF} in (7) is:

$$\begin{aligned} q_X^{HF} > 0 &\Leftrightarrow -z^*\alpha_I(\alpha_X - \alpha_I)a > [(1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2]\tau - \alpha_X(L - z^*f) \\ &\Leftrightarrow -z^*\alpha_I^2(\alpha_X - \alpha_I)a > \underbrace{\alpha_I[(1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2]\tau - \alpha_X\alpha_I(L - z^*f)}_{\equiv \omega_2} \end{aligned}$$

Combining these two conditions, we have:

$$\underbrace{-z^*\alpha_I^2(\alpha_X - \alpha_I)a}_{<0} > \omega_2 > \underbrace{2(\alpha_X - \alpha_I)[(1-2z^*)\alpha_X^2 + 2z^*\alpha_I^2]a}_{>0}$$

This inequality cannot hold. Therefore, with the condition that q_X^{HF} in (7) is positive, we must have: $\Omega_{ct} < 0$. This implies welfare decreases by the small subsidies financed by consumption taxes affect welfare negatively with $\alpha_X - \alpha_I > 0$.