Goods-market Desirability of Minimum Wages

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Abstract

This paper develops a general-equilibrium model with heterogeneous firms and a perfectly competitive labor market to explore the effects of minimum wages on welfare, inequality, and unemployment. We find that a low minimum wage may increase the social welfare, in which all labor groups are weighted equally. This is because minimum wages may reduce the market distortions caused by the weak firm-selection effect in the goods market without minimum wages. Furthermore, we show that a minimum wage is also desirable for reducing income inequality and decreasing the unemployment rate.

Keywords: minimum wage, firm selection, welfare, inequality
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1 Introduction

This paper aims to examine the effects of minimum wages on welfare, inequality (or antipoverty), and unemployment. Using a general-equilibrium model incorporating heterogeneous firms and a competitive labor market, we find that reducing the market distortions is a new rationale for the desirability of minimum wages. Moreover, a low minimum wage can reduce inequality and decrease unemployment simultaneously.

Minimum wages are widely used by governments to reduce income inequality. Such an instrument is also considered to be related to unemployment in the labor market. However, the policy debate over the desirability of minimum wages is long and remains intense. In 2014, two open letters both signed by hundreds of economists, including several

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Nobel Prize winners, were sent to US federal policymakers, expressing opposite opinions on the effect of raising minimum wages.[1] Proponents emphasize the antipoverty benefits as a minimum wage increases the wages of low-skilled workers. Opponents argue that minimum wages reduce employment, thereby worsening the welfare of workers who lose their jobs. However, such a negative effect is not supported by recent empirical literature (e.g., Card and Kruger, 1994; Cengiz, Dube, Lindner, and Zipperer, 2019).

For a long time, the desirability of minimum wages was proposed based on two rationales: their efficiency in reducing the losses from monopsony power in the labor market (e.g., Ahlfeldt, Roth, and Seidel, 2022) and redistributing the economic output from high-income workers to low-income workers (e.g., Berger, Herkenhoff, and Mongey, 2022). Two assumptions are crucial for these discussions. First, the labor market is limited to a monopsonistic one. Second, the desirability of minimum wages in redistribution depends on the relative weights of labor groups in the social welfare. It is not known whether minimum wages are still desirable without those assumptions.

To fill the theoretic gap, we take another approach by focusing on the goods market. We use a closed-economy version of the heterogeneous-firms model of Melitz and Ottaviano (2008). There are two types of workers (high- and low-skilled) and two sectors in the model. The labor market is perfectly competitive. Two types of workers can work in either sector. In the numeraire good sector (sector A henceforth), two types of workers employ themselves to produce homogeneous good A. We assume that high-skilled workers have a higher productivity level in sector A. In the manufacturing sector (sector M henceforth), firms produce differentiated goods under monopolistic competition. Firms employ high-skilled workers as entry costs and low-skilled workers as the only marginal input. The minimum wage is binding in sector M for firms to employ low-skilled workers. Specifically, after observing the policy of minimum wages, firms decide whether to employ high-skilled workers to enter the goods market. After paying this entry cost, firms observe their marginal cost and decide whether to employ low-skilled workers.

To explain our findings, we highlight two driving forces in the model. The first one is a selection effect: an increase in the minimum wage results in a lower cutoff cost level for producing. Evidence for the selection effect is found in China (Bai, Chatterjee, Krishna, and Ma, 2021). The second effect is a reallocation effect: an increase in minimum wages enlarges the employment of low-cost firms and shrinks that of high-cost firms. Evidence for the reallocation effect is found in recent literature. (Clemens, Kahn, and Meer, 2021; Dustmann, Lindner, Schonberg, Umkehrer, and Vom Berge, 2022).

Our first important result is that a country can gain from the introduction of minimum

[1] See Gerritsen and Jacobs (2020). The letters are available at `www.epi.org/minimum-wage-statement` and `http://nebula.wsimg.com/faf44fea2172ad008b46a64835ae2492?AccessKeyId=D2418B43C2D698C15401&disposition=0&alloworigin=1`.

wages. According to Nocco, Ottaviano, and Salto (2014), in the market without minimum wages, the firm selection in equilibrium is weaker than that in optimum. Furthermore, low-cost firms are smaller and high-cost firms are larger than optimal in the market equilibrium without minimum wages. Introducing a minimum wage results in a lower cutoff cost level for firms to be active. Compared with the situation without minimum wages, more firms are expelled from the market, which reduces the intensity of firm competition in sector M. Moreover, the reallocation effect may occur by which (relatively) low-cost firms produce more and (relatively) high-cost firms shrink their production. Thus, minimum wages may reduce distortions in the goods market and increase the efficiency of production, which ultimately increases welfare. Nevertheless, the minimum wage system is not beneficial to all workers. We show that only low-skilled workers working in section M are net winners, while other workers are net losers.

To investigate the efficiency of minimum wages in decreasing inequality, we look at two aspects of income inequality, namely, the inter-group inequality between low-skilled and high-skilled workers and the inequality among all individuals. The inter-group inequality is measured by the ratio of high-skilled workers' average income to low-skilled workers' average income, while the Gini coefficient is used to measure the inequality among all individuals. We show that inter-group inequality decreases under minimum wages. The Gini coefficient decreases with a low minimum wage if the preference intensity for differentiated products relative to the numeraire is large.

Furthermore, we find the positive effects of minimum wages on the employment rate using an alternative model that considers the costs of work. Workers with positive net income choose to work. Minimum wages affect employment rate via two channels: the first is a negative one, increasing the competition tightness of the labor market; the second is a positive one, encouraging an additional labor supply. We show that a low minimum wage reduces the unemployment rate.

This paper contributes to three streams of literature. The first one clarifies a new effect of minimum wages on welfare from the goods market. Most previous papers examined the efficiency of minimum wages on welfare by combining the competitive labor markets with an optimized redistributive tax (Allen, 1987; Guesnerie and Roberts, 1987; Dreze and Gollier, 1993; Marceau and Boadway, 1994; Boadway and Cuff, 2001; Gerritsen and Jacobs, 2020; Lavecchia, 2020). All of these papers show that minimum wages increase welfare only if they can reduce the labor market distortions caused by redistributive income taxation. In a setting with competitive labor markets, Lee and Saez (2012) show that minimum wages increase welfare as long as the welfare weight on low-skilled workers is greater than the weight on high-skilled workers, the demand elasticity of low-skilled labor is finite, the supply elasticity of low-skilled labor is positive, and the rationing of unemployment is efficient (the lowest-surplus workers are hit by unemployment first).

Simon and Wilson (2021) focus on centralized and decentralized minimum wage policy setting by extending the Lee and Saez (2012) model to a two-jurisdiction framework with mobile agents, regional governments, and a federal government. They derive sufficient conditions for the desirability of a binding minimum wage from the perspective of both a local and central government. Unlike the previous studies focusing on labor market distortions, we present a mechanism showing that minimum wages are desirable if selecting firms brings the goods market closer to its efficient level.

The second stream is the theoretical study of the effects of minimum wages on inequality or antipoverty. Previous empirical studies suggest that minimum wages helped reduce wage inequality in the United States (DiNardo, Fortin, and Lemieux, 1996; Lee, 1999; David, Manning, and Smith, 2016; Dube, 2019), Germany (Dustmann, Lindner, Schonberg, Umkehrer, and Vom Berge, 2022), and Brazil (Neumark, Cunningham, and Siga, 2006). In contrast, MaCurdy (2015), after examining various mechanisms to understand who pays for the minimum wages, argues that the introduction of minimum wages is an ineffectual antipoverty policy. Our paper enriches the theoretical study by establishing a general equilibrium including both the labor market and the goods market. We offer novel analysis of the antipoverty effect by two dimensions: inequalities across groups and among individuals.

The third stream is the theoretical study of the effect of minimum wages on unemployment, which remains one of the most widely studied and most controversial topics in labor economics. Card and Kruger (1994) show a positive effect of minimum wages on employment by examining fast-food employment in New Jersey. More empirical studies support their work and report a non-negative employment effect of minimum wages (Card and Krueger, 2000; Zavodny, 2000; Dube, Naidu, and Reich, 2007; Dube, Lester, and Reich, 2010; Allegretto, Dube, and Reich, 2011; Cengiz, Dube, Lindner, and Zipperer, 2019). Some authors argue that labor market frictions play an important role in the near-zero employment effect of minimum wages (Rebitzer and Taylor, 1995; Bhaskar, Manning, and To, 2002; Van Den Berg, 2003; Flinn, 2011; Dube, Lester, and Reich, 2016; Clemens, and Strain, 2021). In contrast, our paper reveals that the positive employment effect of minimum wages might result from the fact that more low-skilled workers are encouraged to search for jobs, which increases the labor supply.

This paper is also related to recent studies on the market distortions of monopolistic competition models when consumer preferences are not CES (constant elasticity of substitution). Assuming additively separable preferences with a general subutility function form, Dhingra and Morrow (2019) consider a one-sector monopolistic competition model with heterogeneous firms. They show that the market equilibrium coincides with the social optimum under the CES demand, but market allocations do not match the optimum under any non-CES demand. Their work is further extended to a setting with multiple sectors by Behrens, Mion, Murata, and Suedekum (2020). Our analysis of welfare is consistent with theirs, although we choose a quasi-linear quadratic utility rather than an additively separable one.

The rest of the paper is organized as follows. Section 2 presents the basic framework without minimum wages. In Section 3, we analyze the welfare and Gini coefficient changes in minimum wages. In Section 4, we take into account costs of work and analyze the impacts of minimum wages on unemployment rates. Finally, Section 5 concludes.

2 The model

We revise the model of Melitz and Ottaviano (2008) by incorporating minimum wages in a closed economy. There are two sectors: a differentiated good sector (sector M) and a numeraire good sector (sector A).

2.1 Preference and demand

All individuals share the same preferences given by

$$U = q_0^c + \alpha \int_{i\in\Omega} q_i^c di - \frac{\gamma}{2}\int_{i\in\Omega}(q_i^c)^2 di - \frac{\eta}{2}\left(\int_{i\in\Omega} q_i^c\right)^2, \quad \alpha > 0,\ \beta > 0,\ \gamma > 0, \tag{1}$$

where q_0^c and q_i^c represent the individual consumption levels of the numeraire good and variety $i \in \Omega$, respectively. Parameter α represents the preference intensity for the differentiated products relative to the numeraire, γ is the degree of love for variety, and η describes substitutability between the differentiated varieties. In particular, varieties are perfect substitutes and consumers only care about their consumption level over all varieties when γ is close to 0.

The budget constraint is written as:

$$I + \bar{q}_0 = \int_{i\in\Omega} q_i^c p_i di + q_0^c,$$

where I is the individual's net income, $\bar{q}_0$ is the initial endowment. We assume that the initial endowment is large enough, (e.g., $\bar{q}_0 > 1$), that the demand of each consumer for the numeraire good is always positive ($q_0^c > 0$).

From the FOC of the utility maximization problem, the inverse demand for each variety i is given by

$$p_i = \alpha - \gamma q_i^c - \eta Q^c, \quad Q^c = \int_{i\in\Omega} q_i^c di, \tag{2}$$

whenever $q_i^c > 0$. Let $\Omega^* \subset \Omega$ be the subset of varieties that are consumed (i.e., $q_i^c > 0$) and $N = |\Omega^*|$. Equation (2) can be inverted to yield the linear market demand system for these varieties:

$$q_i \equiv Lq_i^c = \frac{\alpha L}{\gamma + \eta N} - \frac{L}{\gamma} p_i + \frac{\eta N}{\gamma + \eta N} \frac{L\bar{p}}{\gamma}, \qquad \bar{p} = \frac{1}{N} \int_{i \in \Omega^*} p_i di. \tag{3}$$

Let

$$p_{\max} \equiv \frac{\alpha\gamma + \eta N \bar{p}}{\gamma + \eta N}.$$

This represents the price by which the demand for a variety (3) is driven to 0. Note that (2) implies $p_{\max} \leq \alpha$.

2.2 Labor supply and production

The labor market has two types of workers: $(1 - \lambda)L$ high-skilled workers and λL low-skilled workers. The costs to individuals of work are 0, thus all individuals will choose to work.[2] In sector A, workers are employed by themselves. One unit of low-skilled labor can produce one unit of the numeraire good and the wages are $w_{Al} = 1$. Meanwhile, one unit of high-skilled labor can produce $a > 1$ units of the numeraire good and the wages are $w_{Ah} = a$.

In sector M, there is a continuum of monopolistically competitive firms. Following Melitz and Ottaviano (2008), firms are ex-ante identical. After making the irreversible investment of one unit of high-skilled labor as the sunk cost of entry into the market, a firm observes its marginal unskilled-labor requirement c_l, which is randomly drawn from a cumulative distribution $G(c_l)$.[3] Each firm then decides whether to produce or exit.

We also follow the previous literature by assuming a Pareto distribution of c_l such that

$$G(c_l) = \left(\frac{c_l}{c_M}\right)^{\kappa}, \qquad c_l \in (0, c_M], \quad \kappa \geq 1, \tag{4}$$

where κ is the shape parameter describing the degree of firm heterogeneity and $c_M > 0$ represents the upper bound of c_l. Furthermore, we impose some technical assumptions:

$$L > 2(\kappa + 1)(\kappa + 2)\frac{\gamma a}{c_M^2}, \tag{5}$$

$$\alpha > 2c_M, \tag{6}$$

$$\eta > \frac{\alpha^2(\kappa + 1)}{4(\kappa + 2)}, \tag{7}$$

$$\frac{4c_M\kappa}{\alpha(\kappa + 1)}\left(1 - \frac{c_M}{\alpha}\right) < \lambda < 1 - \frac{4c_M}{a\alpha(\kappa + 1)}\left(1 - \frac{c_M}{\alpha}\right). \tag{8}$$

[2]Section 4 relaxes this assumption using an alternative model with working cost.

[3]Since workers are homogeneous in Melitz and Ottavianao (2008), that model does not distinguish the labor inputs of marginal costs and entry costs.

Inequality (5) requires a large total population. Inequality (6) requires that the preference intensity of differentiated goods is large enough. By (7), the substitutability between varieties is large enough. Melitz and Ottaviano (2008) explain this condition in terms of α in their footnote 7. The inequalities of (8) imply that the amounts of low-skilled labor and high-skilled labor are not extremely large or small. Given $a > 1$, it is easy to verify that its left-hand side is smaller than its right-hand side.

Let w_{Ml} be the wage rate of low-skilled labor working in sector M. Without a minimum wage system, $w_{Ml} = w_{Al} = 1$ holds because of the free mobility of low-skilled workers between sectors M and A. However, $w_{MI} = \underline{w}$ if a minimum wage policy of $\underline{w}$ is imposed. In contrast, $w_{Al} = 1$ remains true because sector A is an informal labor market sector in which workers employ themselves.

The marginal cost of a firm with c_l is given as $c(c_l) = c_l w_{Ml}$. The gross profit of a firm in sector M is $\pi(c_l) = [p(c_l) - c_l w_{Ml}]q(c_l)$. The FOC to maximize the profit gives the following relationship between the optimal price $p(c_l)$ and the output level $q(c_l)$:

$$q(c_l) = (L/\gamma)[p(c_l) - c_l w_{Ml}].$$

Using (2), the optimal price and the optimal gross profit are

$$p(c_l) = \frac{c_{lD} + c_l}{2} w_{Ml} \quad \text{and} \quad \pi(c_l) = \frac{L}{4\gamma}[c_{lD} - c_l]^2 w_{Ml}^2, \tag{9}$$

respectively, where c_{lD} is the cutoff marginal labor requirement at which the firm is indifferent between remaining in the industry and exiting. The expression of c_{lD} is given by

$$p(c_{lD}) = c(c_{lD}) = c_{lD} w_{Ml} = p_{\max} = \frac{\alpha\gamma + \eta N \bar{p}}{\eta N + \gamma}. \tag{10}$$

The above equations give the mass of active firms

$$N = \frac{\gamma}{\eta} \frac{\alpha - c_{lD} w_{Ml}}{c_{lD} w_{Ml} - \bar{p}}. \tag{11}$$

Before the entry, the expected profit of a firm is $\int_0^{c_{lD}} \pi(c_l) dG(c_l) - w_{Mh}$, where w_{Mh} is the wage rate of high-skilled labor in sector M. We have $w_{Mh} = w_{Ah} = a$ because of the free mobility of high-skilled workers between two sectors. The free entry condition gives

$$\int_0^{c_{lD}} \pi(c_l) dG(c_l) = \frac{L}{4\gamma} \int_0^{c_{lD}} [c(c_{lD}) - c(c_l)]^2 dG(c_l) = a. \tag{12}$$

2.3 Market equilibrium without minimum wage system

In an economy without a minimum wage requirement, we have $w_{Ml} = 1$. The marginal cost and price of each firm are given by

$$c(c_l) = c_l, \qquad p(c_l) = \frac{c_{lD} + c_l}{2}. \tag{13}$$

Combining equations (3), (10), (12), and (13), the equilibrium cutoff c_{lD} and the mass of firms N are determined. The cutoff marginal labor requirement is given by

$$c_{lD}^0 \equiv \Big[\frac{2(\kappa+1)(\kappa+2)\gamma a(c_M)^\kappa}{L}\Big]^{\frac{1}{\kappa+2}}. \tag{14}$$

Assumption (5) gives

$$c_{lD}^0 < c_M \tag{15}$$

immediately.

According to (11), the mass of surviving firms is

$$N^0 = \frac{2(\kappa+1)\gamma}{\eta}\Big(\frac{\alpha}{c_{lD}^0} - 1\Big),$$

where the positiveness of N^0 (i.e., $\alpha > c_{lD}^0$) is ensured by (6) and (15). The low-skilled labor demand of firm c_l is $q(c_l)c_l$. Therefore, the amount of low-skilled labor in sector M, L_{Ml}^0, is given by

$$\begin{aligned} L_{Ml}^0 =& \frac{N^0}{G(c_{lD})}\int_0^{c_{lD^0}} q(c_l)c_l dG(c_l) = \frac{\kappa(c_{lD}^0)^2 L N^0}{2(\kappa+1)(\kappa+2)\gamma} = \frac{\kappa(\alpha - c_{lD}^0)c_{lD}^0 L}{(\kappa+2)\eta} \\ <& \lambda L, \end{aligned} \tag{16}$$

where inequality (16) is ensured by (5)—(8).[4] Thus, the low-skilled labor supply is large enough to meet the demand in sector M. Meanwhile, the low-skilled workers who are not employed by firms in sector M choose to employ themselves in sector A. The technology of producing the numeraire good is constant returns to scale, which generates sufficient demand for low-skilled labor in sector A. Thus, the amount of low-skilled workers in sector A is $L_{Al}^0 = \lambda L - L_{Ml}^0$.

Each firm employs one high-skilled labor as the entry cost. Therefore, the demand for high-skilled workers in sector M equals the mass of entrants given by

$$\begin{aligned} L_{Mh}^0 =& \frac{N^0}{G(c_{lD})} = \frac{2(\kappa+1)\gamma(c_M)^\kappa}{\eta {c_{lD}^0}^{\kappa+2}}\left(\alpha - c_{lD}^0\right)c_{lD}^0 = \frac{\left(\alpha - c_{lD}^0\right)c_{lD}^0 L}{a(\kappa+2)\eta} \\ <& (1-\lambda)L, \end{aligned}$$

where the inequality is ensured by (6), (7), and (8) for reasons similar to those given in footnote 4. Thus, the high-skilled labor supply is large enough to meet the demand in sector M. Note that the remaining skilled labor works in sector A, whose amount is $L_{Ah}^0 = (1-\lambda)L - L_{Mh}^0$.

[4] Inequalities (6) and (15) give $(\alpha - c_{lD}^0)c_{lD}^0 < (\alpha - c_M)c_M$. From (7) and (8), we have $L_{Ml}^0 < L[4\kappa/(\kappa+1)](1-c_M/\alpha)c_M/\alpha < \lambda L$.

3 Minimum wage system

Now we assume that the government imposes a minimum wage rate $\underline{w} > 1$,[5] which is binding in sector M. In sector A, individuals are employed by themselves, thus minimum wages are not guaranteed.[6]

Following Lee and Saez (2012) and Lavecchia (2020), we mostly focus on minimum wages just above the market equilibrium wages earned by low-skilled workers. Specifically, we assume

$$\underline{w} \leq \min\left\{a, \left(\frac{c_M}{c_{lD}^0}\right)^{\frac{\kappa+2}{\kappa}}\right\}. \tag{17}$$

The first part implies that wages of high-skilled workers are not directly related to the minimum wage rate. Accordingly, the high-skilled workers' wages remain a. The second part is imposed to ensure the existence of active firms. In fact, $p_{\max} \leq \alpha$ holds from (2). No firm is active if the labor costs are too high to allow for positive profits in sector M.

Given $w_{Ml} = \underline{w}$, after substituting $G(c_l)$ using (4), $\pi(c_l)$ using (9), and $c(c_{lD})$ using (10) into equation (12), the cutoff marginal labor requirement level c_{lD} is derived as

$$c_{lD}^1 \equiv c_{lD}^0 \underline{w}^{-\frac{2}{\kappa+2}} \ \left(< c_{lD}^0 < c_M < \frac{\alpha}{2}\right), \tag{18}$$

where inequalities are obtained from $\underline{w} > 1$ and the results of Section 2.3.

The averages of firm-level performance measures are

$$\bar{p} = \frac{2\kappa+1}{2\kappa+2} c_{lD}^1 \underline{w}, \quad \bar{c} = \frac{\kappa}{\kappa+1} c_{lD}^1 \underline{w}, \quad \bar{q} = \frac{L}{2\gamma} \frac{1}{\kappa+1} c_{lD}^1 \underline{w}. \tag{19}$$

The mass of surviving firms is

$$N^1 = \frac{2(\kappa+1)\gamma}{\eta}\left(\frac{\alpha}{c_{lD}^1 \underline{w}} - 1\right), \tag{20}$$

where the positiveness of N^1 is ensured by (17) and (18).

3.1 Labor market

The demand for low-skilled workers employed in sector M is

$$L_{Ml}^1 = \frac{N^1}{G(c_{lD}^1)} \int_0^{c_{lD}^1} q(c_l) c_l dG(c_l) = \frac{\kappa L}{(\kappa+2)\eta} c_{lD}^1 (\alpha - c_{lD}^1 \underline{w}) \tag{21}$$

[5]For convenience, with slight abuse of notation, the system without a minimum wage is also treated as a special case of $\underline{w} = 1$ later.

[6]Sector M can be regarded as the formal labor market sector, and sector A is the informal labor market sector in the two-sector models of Welch (1974); Gramlich, Flanagan, and Wachter (1976); and Mincer (1976).

$$<L^0_{Ml} < \lambda L,$$

where the first inequality is from $\underline{w} > 1$ and (18) directly, and the second inequality is from (16). The amount of low-skilled labor in sector A is $L^1_{Al} = \lambda L - L^1_{Ml}$. The demand for high-skilled workers in sector M is given by

$$L^1_{Mh} = \frac{N^1}{G(c^1_{lD})} = \frac{\alpha^2}{a(\kappa+2)\eta}\left(1 - \frac{c^1_{lD}\underline{w}}{\alpha}\right)\frac{c^1_{lD}\underline{w}}{\alpha}L < (1-\lambda)L,$$

where the inequality is ensured by (6), (7), (8), and (17). The amount of high-skilled labor in sector A is $L^1_{Ah} = (1-\lambda)L - L^1_{Mh}$. The average income of all groups of individuals is given by

$$\bar{I} = \frac{1}{L}\left[L^1_{Al} + \underline{w}L^1_{Ml} + a(L^1_{Ah} + L^1_{Mh})\right] = I_0 + (\underline{w}-1)\frac{L^1_{Ml}}{L}, \tag{22}$$

where $I_0 \equiv 1 + (1-\lambda)(a-1)$ represents the average income without minimum wages, L^1_{Ml}/L is the share of workers covered by minimum wages, and $(\underline{w}-1)$ is the additional increase in income of a low-skilled worker working in sector M.

3.2 Welfare

In the heterogeneous-firms model considered here, we first check the effect of a minimum wage system on entering firms. A higher minimum wage increases the marginal costs, so firms must be more efficient to survive. The result below follows from (18) directly.

Lemma 1. *(Selection effect): An increase in minimum wages strengthens the selection effect monotonically:* $dc^1_{lD}/d\underline{w} < 0$.

Bai et al. (2021) also predict the selection effect of minimum wages by considering heterogeneous firms under perfect competition in a Heckscher-Ohlin setting. Firms are monopolistic competition in this paper. Our finding is consistent with and complementary to theirs.

This selection effect on high-cost firms decreases the mass of active firms and reduces the intensity of competition in sector M. This results in a larger amount of production by more productive firms and a smaller amount of production by less productive firms.

Lemma 2. *(Reallocation effect): An increase in minimum wages enlarges the employment of low-cost firms* $(c_l < c^1_{lD}\kappa/(\kappa+2))$ *and shrinks that of high-cost firms* $(c_l > c^1_{lD}\kappa/(\kappa+2))$.

Proof: Let $\iota(c_l) = q(c_l)c_l$ be the unskilled-labor input of firm with cost c_l. The result follows immediately from the derivative below.

$$\frac{d\iota(c_l)}{d\underline{w}} = \frac{Lc_l}{2\gamma}\left(\frac{c^1_{lD}\kappa}{\kappa+2} - c_l\right).$$

□

This reallocation effect is supported by recent empirical evidence in Germany (Dustmann et al., 2022). Berger et al. (2022) also predict the reallocation effect of minimum wages by considering a general-equilibrium model with an oligopsony labor market. Although the labor market is perfectly competitive in this paper, our finding is consistent with and complementary to theirs.

These two lemmas are informative if we compare them with the results in Nocco et al. (2014), which shows that the selection effect is weaker in equilibrium than in optimum. Furthermore, low-cost firms are smaller and high-cost firms larger in the market equilibrium without minimum wages than in the optimal allocation. In contrast, we show that the new selection effect in Lemma 1 and the reallocation effect in Lemma 2 generated by minimum wages may reduce the bias between the market equilibrium and the social optimum, which inspires us to investigate the welfare change due to the minimum wages.

Welfare can be evaluated using the indirect utility function associated with equation (1):

$$W = \bar{V} = \bar{q}_0 + \bar{I} + \frac{1}{2}\left(\eta + \frac{\gamma}{N^1}\right)^{-1}(\alpha - \bar{p})^2 + \frac{1}{2}\frac{N^1}{\gamma}\sigma_p^2, \tag{23}$$

where

$$\sigma_p^2 = \frac{1}{N^1}\int_{i\in\Omega}(p_i - \bar{p})^2 di = \frac{\kappa\left(\underline{w}c_{lD}^1\right)^2}{4(\kappa+1)^2(\kappa+2)}. \tag{24}$$

Using (18), (19), (23), and (24), we rewrite the expression for welfare as

$$W = \bar{q}_0 + \bar{I} + \text{CS}, \tag{25}$$

where CS is the consumer surplus

$$\text{CS} = \frac{\alpha - \underline{w}c_{lD}^1}{2\eta}\left(\alpha - \underline{w}c_{lD}^1\frac{\kappa+1}{\kappa+2}\right). \tag{26}$$

Equations (23) and (25) indicate that minimum wages affect the social welfare via two channels: the first one is changing the average income directly; the second one is changing consumer surplus through price and the mass of varieties.

Thus the derivative of W with respect to $\underline{w}$ is divided into two parts as follows.

$$\frac{dW}{d\underline{w}} = \frac{d\bar{I}}{d\underline{w}} + \frac{d\text{CS}}{d\underline{w}} \tag{27}$$

$$= \frac{c_{lD}^0\kappa\underline{w}^{-\frac{\kappa+4}{\kappa+2}}}{\eta(\kappa+2)^2}\left[(2+\kappa\underline{w})(\alpha - 2c_{lD}^0\underline{w}^{\frac{\kappa}{\kappa+2}}) + (2+\kappa)c_{lD}^0\underline{w}^{\frac{\kappa}{\kappa+2}}\right] \tag{27a}$$

$$+ \frac{c_{lD}^0\kappa\underline{w}^{-\frac{2}{\kappa+2}}}{2\eta(\kappa+2)^2}\left[2(1+\kappa)c_{lD}^0\underline{w}^{\frac{\kappa}{\kappa+2}} - (3+2\kappa)\alpha\right]. \tag{27b}$$

Assumptions (6) and (17) give the positiveness of (27a) and the negativeness of (27b). Specifically, the minimum wage raises the wages of low-skilled workers, which links the minimum wage to a gain in social welfare. At the same time, the higher wages reduce consumer surplus by the decrease in the mass of varieties and the increase in the price of differentiated goods. The total effect on welfare becomes ambiguous. However, we are able to show that the positive effect dominates the negative one when the minimum wage is small.

Proposition 1. *A low minimum wage increases the social welfare.*

Proof: The derivative of (27) at $\underline{w} = 1$ is

$$\left.\frac{dW}{d\underline{w}}\right|_{\underline{w}=1} = \left.\frac{d\bar{I}}{d\underline{w}}\right|_{\underline{w}=1} + \left.\frac{d\text{CS}}{d\underline{w}}\right|_{\underline{w}=1} = \frac{c_{lD}^0 \kappa(\alpha - 2c_{lD}^0)}{2\eta(\kappa+2)^2} \tag{28}$$
$$>0,$$

where the inequality follows from (6) and (15). □

The inequality of $\alpha > 2c_{lD}^0$ is crucial for Proposition 1. Noting that c_{lD}^0 of (14) decreases with L and increases with a, c_M, and γ, we know that the positive effect of a low minimum wage is more likely in a larger country with a smaller productivity gap between workers and across firms, a lower degree of love for variety, and a higher demand intensity for the differentiated goods.

The technical assumptions of (5) to (8) are imposed to derive Proposition 1. These conditions are not related to the sufficient conditions of Lee and Saez (2012) derived from a competitive labor market, where they conclude that for the minimum wage to be desirable, the government needs to redistribute the weights in the social welfare from high-skilled workers toward low-skilled workers. In our paper, the weights of all workers are equal in the social welfare function; in other word, there is no redistribution effect.

Ahlfeldt et al. (2022) also predict a welfare gain from a low minimum wage using a quantitative spatial model with heterogeneous firms and a monopsonistic labor market. Their welfare gain directly depends on the monopsony power of the labor market, namely, the efficiency in reducing the losses from the monopsony power. In contrast, the labor market is perfectly competitive, and therefore, there is no monopsony power in the current paper.

Without a minimum wage, the market is distorted and biased from the social optimum in the Melitz-Ottaviano Model (Nocco et al., 2014). The market provides too much entry if the demand parameter α is high. The introduction of a low binding minimum wage generates higher costs of employing low-skilled labor in sector M, which increases the selection effect that expels low-productivity firms from the market. This stronger selection effect reduces the distortion of the market and increases welfare. Bagwell and Lee (2020) also find that starting at the market equilibrium, a new entrant generates a negative externality, which decreases welfare if α is higher than a critical level based on the model of Melitz and Ottaviano (2008). Similar facts are found in models based on additively separable preferences. Dhingra and Morrow (2019) consider a one-sector monopolistic competition model with heterogeneous firms and show that the market equilibrium is the social optimum under the CES demand while market allocations do not maximize individual welfare under any non-CES demand. Furthermore, Behrens et al. (2020) extend Dhingra and Morrow'(2019) analysis to a setting with multiple sectors and show that intersectoral distortions exist with regard to labor allocation and firm entry even in the CES form. Our finding is related and complementary to theirs. We show that imposing minimum wages can be an efficient policy to reduce the aforementioned market distortions.

We have three groups receiving different impacts from the minimum wage policy: low-skilled workers working in sector M, low-skilled workers working in sector A, and high-skilled workers. Having clarified the overall impact in Proposition 1, we now explore another important question: which group gains and which loses as a result of the minimum wages?

Among all workers, it is clear that L_{Ml} low-skilled workers working in sector M benefit from the minimum wage directly; their nominal wages increase from 1 to $\underline{w}$. However, the amount of L_{Ml} decreases with $\underline{w}$.

In the goods market, the increase in wage costs has two effects. The first is an increase in prices. From (19), the average prices of differentiated goods increase with minimum wages, which is passed to the consumers directly. The second effect is the selection of firms. Low-productivity firms are expelled from the market under minimum wages. It is easy to verify that the mass of surviving firms (as well as the mass of varieties) decreases with minimum wages from equation (20). However, the expected profits of firms are still zero under the free-entry condition, which implies that the reduction in varieties is ultimately passed on to the consumers which decreases the consumer surplus. The quasi-linear utility function assumed in this paper indicates that all individuals share the same level of consumption of differentiated goods. Therefore, the reduction in consumer surplus is identical and given by (27b) for all individuals.

The derivative of utility of low-skilled workers in sector M, W_{Ml}, with respect to $\underline{w}$ at $\underline{w} = 1$ is given by

$$\left.\frac{dW_{Ml}}{d\underline{w}}\right|_{\underline{w}=1} = 1 - \frac{c_{lD}^0 \kappa}{2\eta(\kappa+2)^2}[(3+2\kappa)\alpha - 2(1+\kappa)c_{lD}^0] \tag{29}$$

$$>0, \tag{30}$$

and inequality (30) is given in Appendix A.

The derivative of utility of low-skilled workers in sector A and high-skilled workers, W_{Al} and W_h with respect to $\underline{w}$, is given by

$$\frac{dW_{Al}}{d\underline{w}} = \frac{dW_h}{d\underline{w}} = \frac{d\mathrm{CS}}{d\underline{w}} < 0.$$

The above equations indicate that L_{Ml} low-skilled workers are net winners under a low minimum wage. Meanwhile, all high-skilled workers and other low-skilled workers are net losers.

We use a numerical simulation to show the features of the effects of a general $\underline{w}$ on the utility of each group. The parameters are set as

$$\alpha = 4, \eta = 4, \gamma = 2, a = 2, L = 400, \kappa = 2, c_M = 1, \lambda = 0.7, q_0 = 1.1.$$

It is easily verified that these parameters satisfy assumptions (5), (6), (7), and (8).[7] Meanwhile, $(c_M/c^0_{lD})^{(\kappa+2)/\kappa} \approx 2.04124 > 2 = a$ holds. According to (17), the upper bound of $\underline{w}$ is 2.

Figure 1 plots the average income and consumer surplus of all individuals. The solid curve is the consumer surplus of (26), whose monotonicity shows that raising the minimum wage is harmful to consumers. The dashed curve is the average income of (22), whose monotonicity shows that raising the minimum wage improves the average income of all individuals.

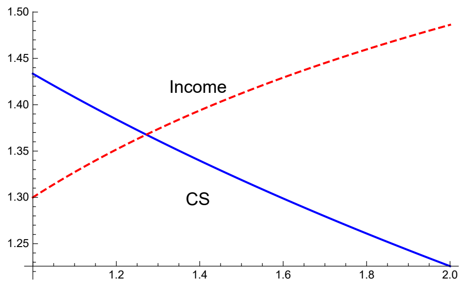


Figure 1: Consumer surplus and average income

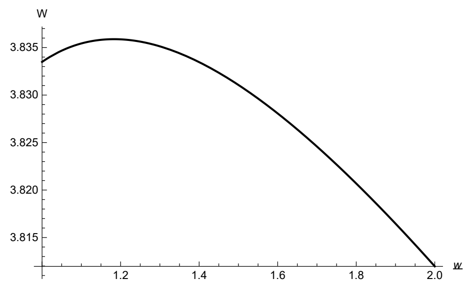


Figure 2: Welfare

Figure 2 shows how the minimum wage changes the social welfare. We observe a bell shape. Although a low minimum wage improves the society, a high minimum wage is not desirable. This nonmonotonicity provides an explanation for the long debate over the minimum wage policy introduced in footnote 1.

[7] Some empirical literature estimates that the average κ is close to 2. See footnote 6 of Melitz and Ottaviano (2008). Meanwhile, in the CES framework, κ is supposed to be larger: $\kappa > 4$.

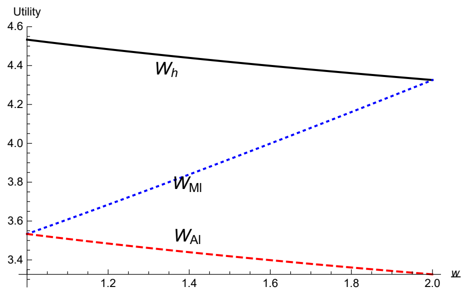


Figure 3: Utility of each group of workers

Figure 3 provides three utility curves: one for high-skilled workers, one for low-skilled workers in sector M, and one for low-skilled workers in sector A. They are the dotted (W_{Ml}), dashed (W_{Al}), and solid (W_h), respectively. All of them are monotonic. The rise in W_{Ml} dominates the fall in W_{Al} and W_h when $\underline{w}$ is small, but it is dominated when $\underline{w}$ is large. The interaction among groups results in the nonmonotonic property observed in Figure 2.

3.3 Income distribution and inequality

Evidently, minimum wages have an effect on wage inequality. Three groups of workers have incomes, a, $\underline{w}$, and 1. Following Kohl (2020), we first look at the inequality between high- and low-skilled workers by calculating the ratio of their average income. The income of high-skilled workers is independent of minimum wages and equal to a. The income of low-skilled workers employed by themselves in sector A is still equal to 1, while sector M's workers benefit from a minimum wage system and get $\underline{w}$. The inter-group inequality is given by

$$\text{InterIneq} = \frac{a}{\underline{w}L^1_{Ml}/L_l + L^1_{Al}/L_l} = \frac{a\lambda L}{\lambda L + (\underline{w}-1)L^1_{Ml}}. \tag{31}$$

The derivative of this inter-group inequality with respect to $\underline{w}$ is given by

$$\begin{aligned} \frac{d\text{InterIneq}}{d\underline{w}} = &- \frac{a\lambda L^2 c^0_{lD}\kappa \underline{w}^{-\frac{\kappa+4}{\kappa+2}}}{\eta[\lambda L + (\underline{w}-1)L^1_{Ml}]^2(\kappa+2)^2} \\ &\times \left[(2+\kappa\underline{w})(\alpha - 2c^0_{lD}\underline{w}^{\frac{\kappa}{\kappa+2}}) + (2+\kappa)c^0_{lD}\underline{w}^{\frac{\kappa}{\kappa+2}}\right] \end{aligned} \tag{32}$$

Assumptions (6) and (17) give the negativeness of (32). Therefore, an increase in the minimum wage decreases the inter-group inequality monotonically.

One more inequality source is the inequality within low-skilled workers, which evidently increases with $\underline{w}$. To see the whole effect, we plot the Lorenz curves in Figure 4. The dotted curve represents the inequality without minimum wages and the solid line shows the inequality with minimum wages.

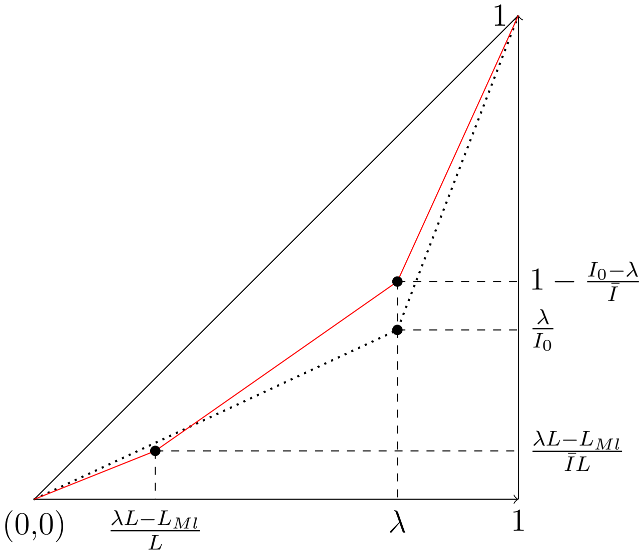


Figure 4: Lorenz curve

The Gini coefficient is the ratio of the area that lies between the 45-degree line and the Lorenz curves. With a minimum wage, the Gini coefficient is given by

$$\begin{aligned}
\text{Gini} =& 2\lambda - 1 - \frac{L^1_{Ml}(\bar{I}-I_0)}{\bar{I}L} + \frac{(1-\lambda)I_0-\lambda}{\bar{I}} \\
=& 2\lambda - 1 + \frac{\eta(2+\kappa)[I_0-(1+I_0)\lambda]\underline{w}^{\frac{4}{\kappa+2}}}{c^0_{lD}\kappa(\underline{w}-1)\left(\alpha\underline{w}^{\frac{2}{\kappa+2}} - c^0_{lD}\underline{w}\right) + \eta I_0(\kappa+2)\underline{w}^{\frac{4}{\kappa+2}}} \\
&- \frac{(c^0_{lD})^2\kappa^2(\underline{w}-1)\left(\alpha - c^0_{lD}\underline{w}^{\frac{\kappa}{\kappa+2}}\right)^2}{\eta(\kappa+2)\left[c^0_{lD}\kappa(\underline{w}-1)\left(\alpha\underline{w}^{\frac{2}{\kappa+2}} - c^0_{lD}\underline{w}\right) + \eta I_0(\kappa+2)\underline{w}^{\frac{4}{\kappa+2}}\right]},
\end{aligned}$$

where the last equality is from (18), (21), and (22).

Without the minimum wage requirement, the Gini coefficient is simply

$$\text{Gini}\Big|_{\underline{w}=1} = \frac{\lambda(I_0-1)}{I_0}.$$

Taking the derivative of Gini with respect to $\underline{w}$ at $\underline{w}=1$ gives

$$\frac{d\text{Gini}}{d\underline{w}}\Bigg|_{\underline{w}=1} = \underbrace{\frac{{c^0_{lD}}^2\kappa^2(\alpha - c^0_{lD})}{(2+\kappa)^2 I_0\eta^2}}_{+} \times (\alpha_1 - \alpha), \tag{33}$$

where

$$\alpha_1 \equiv c^0_{lD} + \frac{(2+\kappa)\eta}{c^0_{lD}\kappa}\left(\lambda - 1 + \frac{\lambda}{I_0}\right).$$

Equation (33) immediately leads to the following result.

Proposition 2. *A low minimum wage reduces the Gini coefficient if $\alpha > \alpha_1$.*

Note that the value of α_1 depends on the share of low-skilled workers λ and the substitutability between the differentiated varieties η. α_1 is large if λ and η are large. Propositions 1 and 2 tell us that the minimum wage policy may be desirable in terms of both welfare and inequality when α is large.

An intuitive explanation of Proposition 2 is obtained if we notice that there are some low-skilled workers self-employed in the numeraire sector and not covered by the minimum wage system. Recall that α represents the preference intensity for differentiated products relative to the numeraire. A large α indicates a large demand for differentiated products, which leads to a large share of low-skilled labor working in sector M who benefits from the minimum wages. Therefore, the average income of low-skilled workers becomes large, which narrows the inter-group inequality. Furthermore, the effects on inter-group inequality dominate and the overall inequality decreases.

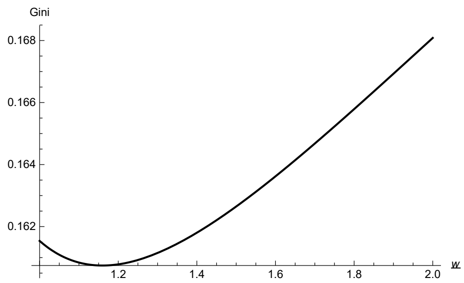


Figure 5: Gini coefficient

Figure 5 shows how the minimum wage changes the Gini coefficient, using the same parameters as in Section 3.2. Noting that $\alpha_1 \approx 3.42549 < 4 = \alpha$ holds, the Gini coefficient decreases in the beginning. We observe a nonmonotonic shape again. A low minimum wage decreases the inequality, while a high minimum wage is not desirable. This is a natural result because the inter-group inequality decreases and the inequality within the low-skilled workers increases with $\underline{w}$.

4 Working costs and unemployment

To investigate the employment effect of minimum wages, we borrow the idea of working cost in Lee and Saez (2012) and Simon and Wilson (2021). Note that the employment of skilled workers is not related to the minimum wages in our model, we simply assume that only low-skilled workers face cost θ of work, which is randomly drawn from a distribution $H(\theta)$ on interval $[0, \bar{\theta}]$, where $\bar{\theta} > 1$. The low-skilled workers pay these costs only if they work.[8] There are three low-skilled labor supply options: (a) not working and earning zero, (b) working in sector A with wages 1, and (c) working in sector M with wages $\underline{w}$. We assume that firms can not observe the individual cost θ of work. Workers with positive net income choose to work.

To ensure a sufficient supply of unskilled labor, we assume that $\bar{\theta}$ is not too large. Specifically, the inequality

$$\lambda > \frac{1}{H(1)} \times (\text{LHS of (8)}) \tag{34}$$

is assumed to replace the first inequality of assumption (8). Meanwhile, the demand side of labor does not change since the wages are the same as in the case of no working costs. Finally, in this alternative model, we add one more assumption

$$\underline{w} \leq \bar{\theta} \tag{35}$$

in addition to (17).

4.1 Unemployment rate without minimum wages

Without the minimum wage policy, wages are 1 for all low-skilled workers and a for all high-skilled workers. All low-skilled workers whose costs are smaller than 1 choose to work. Therefore, the labor supplies of two types of workers are

$$L_l = \lambda L H(1), \qquad L_h = (1 - \lambda)L.$$

Inequality $L_l > L^0_{Ml}$ is ensured by (5)—(7), and (34).[9] The technology of producing the numeraire good is constant returns to scale, whose market results in an infinity demand for two types of labor in sector A. Therefore, all workers who choose to work can get jobs. The unemployment rate in the economy can be easily calculated as follows.

$$\mu_0 = \lambda(1 - H(1)).$$

4.2 Unemployment rate with minimum wages

Under a minimum wage policy, the wages for low-skilled workers in sector M become $\underline{w}$. The wages remain 1 for low-skilled workers in sector A and a for all high-skilled workers.

All low-skilled workers whose costs θ are smaller than $\underline{w}$ will search for jobs in sector M. Meanwhile, the low-skilled labor demand in sector M is the L^1_{Ml} of (21). Firms do

[8] Therefore, these costs are different from the participation cost of Lavecchia (2020), which needs to be paid as long as workers start searching jobs.

[9] Inequalities (6) and (15) give $(\alpha - c^0_{lD})c^0_{lD} < (\alpha - c_M)c_M$. From (7) and (34), we have $L^0_{Ml} < L[4\kappa/(\kappa + 1)](1 - c_M/\alpha)c_M/\alpha < H(1)\lambda L$.

not know the working costs θ of each individual. Therefore, all low-skilled workers with $\theta < \underline{w}$ share an identical possibility to get a job in sector M. The probability of a successful match is given by

$$e = \frac{L_{Ml}^1}{\lambda L H(\underline{w})} < 1, \tag{36}$$

where the inequality comes from the fact of $L_{Ml}^1 (< L_{Ml}^0 < L_l) < \lambda L H(\underline{w})$. Meanwhile, $H(\underline{w})$ increases and L_{Ml}^1 decreases with $\underline{w}$, which indicates that e decreases with $\underline{w}$. Note that e also represents the tightness of the labor market in sector M.[10] Thus, raising $\underline{w}$ reduces the tightness and increases the difficulty for unskilled workers to get jobs.

Low-skilled workers who failed to match with a firm in sector M choose to work in sector A if their costs θ are smaller than 1. Thus, the amount of low-skilled labor in sector A is

$$L_{Al}^{wc} = (1 - e) H(1) \lambda L. \tag{37}$$

All high-skilled workers choose working. Therefore, the high-skilled labor supply is $L_h = (1 - \lambda) L$. The unemployment rate in the economy is easily calculated as

$$\mu_1 = \mu_0 - \underbrace{e}_{\text{tightness}} \times \underbrace{\lambda [H(\underline{w}) - H(1)]}_{\text{supply}}. \tag{38}$$

Equations (36) and (38) indicate that the unemployment rate is jointly affected by two effects of minimum wages: the first one is increasing the competition tightness of the labor market (lower e) through $H(\underline{w})$ and L_{Ml}^1; the second one is encouraging additional labor supply through $H(\underline{w})$.

Equation (38) implies that $\mu_1 < \mu_0$ as long as $\underline{w} > 1$. Moreover, the derivative of μ_1 with respect to $\underline{w}$ is

$$\frac{d\mu_1}{d\underline{w}} = \underbrace{\lambda [H(\underline{w}) - H(1)] \left(-\frac{de}{d\underline{w}} \right)}_{+} - \underbrace{e\lambda \frac{dH(\underline{w})}{d\underline{w}}}_{+}, \tag{39}$$

where

$$\left. \frac{d\mu_1}{d\underline{w}} \right|_{\underline{w}=1} = -e\lambda \left. \frac{dH(\underline{w})}{d\underline{w}} \right|_{\underline{w}=1} < 0. \tag{40}$$

Thus, a low minimum wage decreases the unemployment rate from (40), although the relationship between μ_1 and $\underline{w}$ is ambiguous for a general $\underline{w}$ according to (39).

In summary, we have the following proposition:

Proposition 3. *A minimum wage policy generates a lower unemployment rate.*

[10] The tightness is defined as the ratio of the number of vacancies to the number of job seekers in Lavecchia (2020). The number of job vacancies equals to the total low-skilled labor demand in this paper.

When a minimum wage is introduced, more low-skilled workers are encouraged to search for jobs in sector M, which increases the low-skilled labor supply. The low-skilled workers with $\theta \in (1, \underline{w})$ become new entrants in the job market. They face a possibility e of being employed in sector M. Meanwhile, all low-skilled workers with costs $\theta \leq 1$ can still be employed either by firms in sector M or by themselves in sector A, which is the same as the case without minimum wages. Therefore, the new job searchers ($\theta \in (1, \underline{w})$) who are successfully employed in sector M become additional employees, contributing to the reduction in the unemployment rate.

The theoretical result of Proposition 3 is supported by a series of empirical studies. Non-negative or positive employment effects of minimum wages are reported by Card and Krueger (1994,2000); Zavodny (2000); Dube, Naidu, and Reich (2007); Dube, Lester, and Reich (2010); Allegretto, Dube, and Reich (2011); and Cengiz, Dube, Lindner, and Zipperer (2019).

We conduct numerical examples by assuming $H(\underline{w}) = \theta/\bar{\theta}$ and using following parameters:

$$\alpha = 2, \eta = 5, \gamma = 0.001, a = 10, L = 100000, \kappa = 2, c_M = 0.01, \lambda = 0.1, \bar{\theta} = 5.$$

It is easily verified that these parameters satisfy assumptions (5), (6), (7), (8), and (34). The upper bound of $\underline{w}$ in this model is 5 because $\bar{\theta} = 5 < (c_M/c_{lD}^0)^{(\kappa+2)/\kappa} \approx 6.45497 < 10 = a$ holds. Figure 6 shows how the unemployment rate changes with minimum wages. We observe a nonmonotonic shape again. A low minimum wage is desirable to reduce the unemployment rate, while a large one is not. This nonmonotonicity gives an explanation for the empirical evidence of both negative (e.g., Clemens and Wither, 2019) and non-negative employment effects of minimum wages.

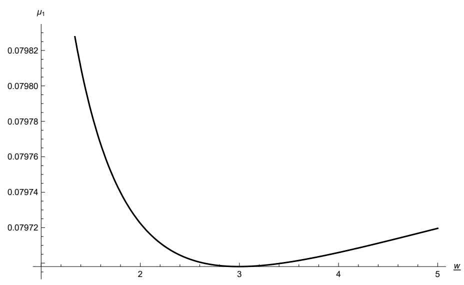


Figure 6: Unemployment rate

4.3 Robustness

We now examine the robustness of the welfare and inequality results in Section 3 by considering the working costs. For convenience, we simply assume that the working cost is randomly drawn from a uniform distribution $H(\theta) = \theta/\bar{\theta}$.

4.3.1 Welfare

Given minimum wages $\underline{w}$, the average net income of each group is

$$\bar{I}_{Al} = 1 - \frac{1}{H(1)} \int_0^1 \theta dH(\theta), \quad \bar{I}_{Ml} = \underline{w} - \frac{1}{H(\underline{w})} \int_0^{\underline{w}} \theta dH(\theta),$$
$$\bar{I}_h = a, \quad \bar{I}_u = 0.$$

Using the uniform distribution of θ, the average net income of all groups of individuals is given by

$$\begin{aligned}\bar{I} &= \frac{1}{L}\left(\bar{I}_{Al}L_{Al}^{wc} + \bar{I}_{Ml}L_{Ml}^{1} + L_h\bar{I}_h\right)\\ &= \frac{1}{2L}(L_{Al}^{wc} + \underline{w}L_{Ml}^{1} + 2aL_h),\end{aligned}$$

where L_{Al}^{wc} is the amount of low-skilled labor in sector A from (37). Similarly, welfare is given by (25). The derivative of W with respect to $\underline{w}$ is given by

$$\begin{aligned}\frac{dW}{d\underline{w}} =& \frac{d\bar{I}}{d\underline{w}} + \frac{d\text{CS}}{d\underline{w}}\\ =& \frac{c_{lD}^0 \kappa \underline{w}^{-\frac{2\kappa+6}{\kappa+2}}}{2\eta(\kappa+2)^2} \times \left\{(2+\kappa)\alpha + (2+\kappa\underline{w}^2)(\alpha - 2c_{lD}^0 \underline{w}^{\frac{\kappa}{\kappa+2}})\right\}\\ &+ \frac{c_{lD}^0 \kappa \underline{w}^{-\frac{2}{\kappa+2}}}{2\eta(\kappa+2)^2} \times \left\{2c_{lD}^0(1+\kappa)\underline{w}^{\frac{\kappa}{\kappa+2}} - (3+2\kappa)\alpha\right\}.\end{aligned}$$

We then obtain the effect of minimum wages at $\underline{w} = 1$:

$$\left.\frac{dW}{d\underline{w}}\right|_{\underline{w}=1} = \left.\frac{d\bar{I}}{d\underline{w}}\right|_{\underline{w}=1} + \left.\frac{d\text{CS}}{d\underline{w}}\right|_{\underline{w}=1} = \frac{c_{lD}^0 \kappa(\alpha - 2c_{lD}^0)}{2\eta(\kappa+2)^2},$$

which is the same as equation (28). Thus, the results of Proposition 1 are robust even when working costs are included. These results also extend the sufficient condition for the desirability of minimum wages given by Lee and Saez (2012). Their efficient rationing is not required here. In fact, all low-skilled workers with working cost $\theta < \underline{w}$ have the same possibility of being employed in sector M.

Furthermore, Figure 7 shows how the minimum wage changes the welfare in the case of working costs, using the same parameters as in Section 3.2 and $\bar{\theta} = 1.3$. It is easily verified that these parameters also satisfy assumption (34). Noting that $\bar{\theta} = 1.3 < a = 2 < (c_M/c_{lD}^0)^{(\kappa+2)/\kappa} \approx 2.04124$ holds, the upper bound of $\underline{w}$ is 1.3 according to (35). Figure 7 has the same bell shape as Figure 2. a low minimum wage increases welfare, and a high minimum wage decreases welfare.

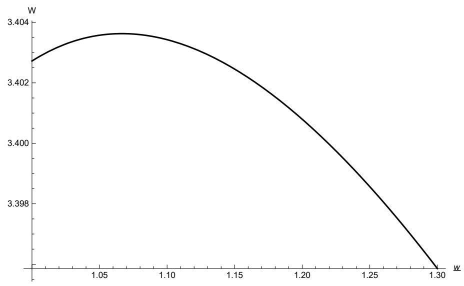


Figure 7: Welfare

4.3.2 Income inequality

The inter-group inequality is given by

$$\text{InterIneq} = \frac{a}{I_{Ml}L^1_{Ml}/L_l + I_{Al}L^{wc}_{Al}/L_l} = \frac{2a\bar{\theta}\lambda L}{\lambda L + \bar{\theta}(\underline{w} - \frac{1}{\underline{w}})L^1_{Ml}}.$$

The above equation leads to a counterpart of (32):

$$\begin{aligned}\frac{d\text{InterIneq}}{d\underline{w}} = &- \frac{2a\bar{\theta}^2\lambda L^2 c^0_{lD}\kappa\underline{w}^{-\frac{2}{\kappa+2}}}{\eta[\lambda w L + \bar{\theta}(\underline{w}^2 - 1)L^1_{Ml}]^2(\kappa+2)^2} \\ &\times \left[(2+\kappa\underline{w}^2)(\alpha - 2c^0_{lD}\underline{w}^{\frac{\kappa}{\kappa+2}}) + (2+\kappa)\alpha\right]\end{aligned} \tag{41}$$

Assumptions (6) and (17) give the negativeness of (41). Thus, an increase in minimum wages decreases the inter-group inequality monotonically again.

All high-skilled workers are homogeneous since they share the same wage rate a and work without costs. However, low-skilled workers are heterogeneous along two dimensions: working place (sector M or A) and working cost (θ). Let $\xi(I)$ denote the bottom percentiles of the population whose net income is I. Therefore, $\xi(I) = 0$ implies that I is the lowest income and $\xi(I) = 1$ means that I is the highest income.

We obtain[11]

$$\xi(I) = \begin{cases} [0, \mu_m], & \text{if } I = 0, \\ \mu_m + \frac{\lambda}{\overline{\theta}}I, & \text{if } 0 < I \leq 1, \\ \mu_m + \frac{\lambda}{\overline{\theta}} + \frac{L^1_{Ml}}{L}\frac{I-1}{\underline{w}}, & \text{if } 1 < I \leq \underline{w}, \\ (\lambda, 1], & \text{if } I = a. \end{cases} \tag{42}$$

[11]See Appendix B for the proof.

The reverse function of (42) is the income schedule of all individuals:

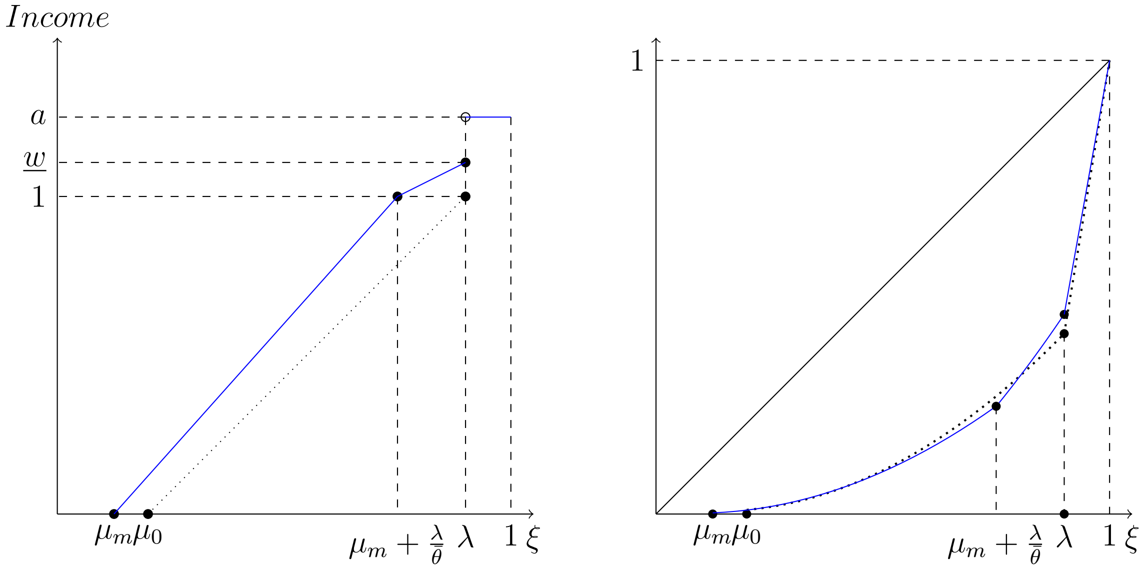
$$I(\xi) = \begin{cases} 0, & \text{if } 0 \le \xi \le \mu_m, \\ \dfrac{\bar{\theta}}{\lambda}(\xi - \mu_m), & \text{if } \mu_m < \xi < \mu_m + \dfrac{\lambda}{\bar{\theta}}, \\ 1 - \dfrac{L\underline{w}(\lambda + \mu_m\bar{\theta})}{L_{Ml}^1\bar{\theta}} + \dfrac{L\underline{w}}{L_{Ml}^1}\xi, & \text{if } \mu_m + \dfrac{\lambda}{\bar{\theta}} < \xi \le \lambda, \\ a, & \text{if } \lambda < \xi \le 1, \end{cases}$$

which is plotted in Figure 8(a).

Thus, the Lorenz curve is

$$Q(\xi) = \begin{cases} 0, & \text{if } 0 \le \xi \le \mu_m, \\ \dfrac{1}{\bar{I}}\displaystyle\int_{\mu_m}^{\xi} \dfrac{\bar{\theta}}{\lambda}(\xi - \mu_m)d\xi, & \text{if } \mu_m < \xi < \mu_m + \dfrac{\lambda}{\bar{\theta}}, \\ \dfrac{1}{\bar{I}}\Big\{\displaystyle\int_{\mu_m}^{\mu_m + \frac{\lambda}{\bar{\theta}}} \dfrac{\bar{\theta}}{\lambda}(x - \mu_m)dx & \\ \quad + \displaystyle\int_{\mu_m + \frac{\lambda}{\bar{\theta}}}^{\xi} \Big[1 - \dfrac{L\underline{w}(\lambda + \mu_m\bar{\theta})}{L_{Ml}^1\bar{\theta}} + \dfrac{L\underline{w}}{L_{Ml}^1}\xi\Big]d\xi\Big\}, & \text{if } \mu_m + \dfrac{\lambda}{\bar{\theta}} < \xi \le \lambda. \\ \dfrac{1}{2\bar{I}}\left[\dfrac{(\underline{w}^2 - 1)L_{Ml}^1}{\underline{w}L} + \dfrac{\lambda}{\bar{\theta}}\right] + \dfrac{a(\xi - \lambda)}{\bar{I}}, & \text{if } \lambda < \xi \le 1, \end{cases}$$

which is illustrated in Figure 8(b).



(a): Income schedules (b): Lorenz curve

Figure 8: Income schedule and Lorenz curve

The dotted line indicates the inequality without minimum wages, and the solid line shows the inequality with minimum wages. The corresponding Gini coefficient can then

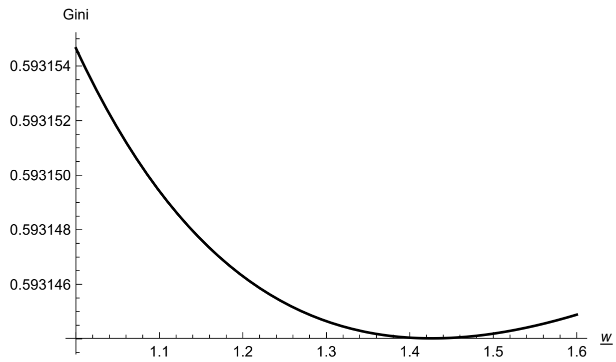


Figure 9: Gini coefficient

be calculated as

$$\begin{aligned}\text{Gini} =& 1 - 2\int_0^1 Q(\xi)d\xi \\ =& 2\lambda - 1 + \frac{2[a\bar{\theta}\underline{w}L(1-\lambda)^2 - \lambda(\underline{w}-1)L_{Ml}^1]}{\bar{\theta}L_{Ml}^1(\underline{w}^2-1) + \underline{w}L[2a\bar{\theta}(1-\lambda)+\lambda]} \\ &- \frac{2[\lambda^2\underline{w}^2L^2 + \bar{\theta}^2(\underline{w}^3 - 3\underline{w} + 2)(L_{Ml}^1)^2]}{3\bar{\theta}\underline{w}L\left\{\bar{\theta}L_{Ml}^1(\underline{w}^2-1) + \underline{w}L[2a\bar{\theta}(1-\lambda)+\lambda]\right\}}. \end{aligned} \tag{43}$$

Substituting L_{Ml}^1 of (21) into (43) and taking its derivative with respect to $\underline{w}$ at $\underline{w} = 1$, we obtain

$$\left.\frac{d\text{Gini}}{d\underline{w}}\right|_{\underline{w}=1} = -\frac{2L_{Ml}^1}{3L[2a\bar{\theta}(1-\lambda)+\lambda]^2}[6a\bar{\theta}^2(1-\lambda)^2 + 6a\bar{\theta}\lambda(1-\lambda) + \lambda^2] < 0.$$

Thus, a low minimum wage decreases inequality again. Compared with Proposition 2, in the model of working costs, the antipoverty effects of a low minimum wage always happen without any additional condition regarding α.

We conduct a simulation to see how the minimum wage changes the Gini coefficient in the case of working costs, using the following parameters:

$$\alpha = 2, \eta = 5, \gamma = 0.001, a = 2, L = 100000, \kappa = 2, c_M = 0.01, \lambda = 0.99, \bar{\theta} = 1.6.$$

It is easily verified that these parameters satisfy assumptions (5), (6), (7), (8), and (34). Noting that $\bar{\theta} = 1.6 < a = 2 < (c_M/c_{lD}^0)^{(\kappa+2)/\kappa} \approx 4.56435$ holds, the upper bound of $\underline{w}$ is 1.6. The result is plotted in Figure 9. We observe that a low minimum wage decreases the inequality, but a high minimum wage may not be desirable. The nonmonotonic property observed in Figure 5 is robust when working costs are incorporated.

5 Conclusion

We analyze the efficiency of a minimum wage policy in a closed economy with heterogeneous workers using a general-equilibrium framework. We show that a country can gain from a minimum wage policy from the viewpoint of the goods market. Under the market equilibrium, the firm selection is weaker than that in the social optimum. The new selection effect under minimum wages is more efficient. The market distortions are reduced; therefore, welfare is increased. Moreover, we investigate the desirability of minimum wages in terms of inequality. The Gini coefficient decreases if the demand intensity for the manufactured varieties is large. Finally, we discuss the effects of minimum wages on unemployment rates using an alternative model with working costs. It is shown that a minimum wage reduces the unemployment rate by encouraging more low-skilled workers to search for jobs. The desirability in terms of welfare and inequality remains true when working costs are included in the model.

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DISCLOSURE

The authors have no financial arrangements that might give rise to conflicts of interest with respect to the research reported in this paper. The views expressed here are solely those of the authors and do not reflect those of any institution with which they are affiliated.

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Appendices

A. Proof of (30)

The inequality

$$\begin{aligned}\frac{(\kappa+1)(\kappa+2)}{2\kappa}\alpha^2 &- (3+2\kappa)\alpha c_{lD}^0 + 2(1+\kappa)(c_{lD}^0)^2 \\ &= 2(1+\kappa)\left[c_{lD}^0 - \frac{3+2\kappa}{4(1+\kappa)}\alpha\right]^2 + \alpha^2\frac{8+11\kappa+4\kappa^2}{8\kappa(1+\kappa)} \\ &> 0\end{aligned}$$

leads to

$$\frac{\alpha^2(\kappa+1)(\kappa+2)}{2\kappa} > c_{lD}^0[(3+2\kappa)\alpha - 2(1+\kappa)c_{lD}^0]. \tag{A.1}$$

Then (29) gives

$$\begin{aligned}\left.\frac{dW_{Ml}}{d\underline{w}}\right|_{\underline{w}=1} &= 1 - \frac{c_{lD}^0\kappa}{2\eta(\kappa+2)^2}[(3+2\kappa)\alpha - 2(1+\kappa)c_{lD}^0] \\ &> 1 - \frac{2\kappa c_{lD}^0}{\alpha^2(\kappa+1)(\kappa+2)}[(3+2\kappa)\alpha - 2(1+\kappa)c_{lD}^0] \\ &> 0,\end{aligned}$$

where the first inequality is from (7) and the second inequality is from (A.1).

B. The derivation of (42)

First, noting that only unemployed workers have income 0, we have $\xi(0) = [0, \mu_m]$, where μ_m is the unemployment rate.

Second, we consider the situation of $I \in (0, 1]$. The working costs of low-skilled workers are randomly drawn from a uniform distribution, which implies that the net income of low-skilled workers within a sector is also drawn from a uniform distribution. In sector M, the net income range of low-skilled workers is $(0, \underline{w}]$. Accordingly, the number of workers whose net income is smaller than I is $L_{Ml}^1 I/\underline{w}$ in sector M. Similarly, the number of workers whose net income is smaller than I is $L_{Al}^{wc} I$ in sector A, where L_{Al}^{wc} is given by (37). Combining the working labor in two sectors and the unemployed labor, $\xi(I)$ is given by

$$\xi(I) = \mu_m + \frac{L_{Ml}^1 I}{\underline{w}L} + \Big(\frac{\lambda}{\bar{\bar{\theta}}} - \frac{L_{Ml}^1}{\underline{w}L}\Big)I = \mu_m + \frac{\lambda}{\bar{\bar{\theta}}}I, \qquad \text{if } I \in (0, 1].$$

Third, we consider the situation of $I \in (1, \underline{w}]$. In sector M, the number of workers whose net income is smaller than I is $L_{Ml}I/\underline{w}$ in sector M. Meanwhile, in sector A, the income of each low-skilled worker is smaller than I. Combining the working labor in two sectors and the unemployed labor, $\xi(I)$ is given by

$$\xi(I) = \mu_m + \frac{L_{Ml}^1 I}{\underline{w}L} + \left(\frac{\lambda}{\bar{\bar{\theta}}} - \frac{L_{Ml}^1}{\underline{w}L}\right) = \mu_m + \frac{\lambda}{\bar{\bar{\theta}}} + \frac{L_{Ml}^1}{L}\frac{I-1}{\underline{w}}, \qquad \text{if } 1 < I \leq \underline{w}.$$

Finally, all high-skilled workers have the same wages, a, which implies that $\xi(I) = (\lambda, 1]$ if $I = a$.

Combining all situations above, function $\xi(I)$ is given by

$$\xi(I) = \begin{cases} [0, \mu_m], & \text{if } I = 0, \\ \mu_m + \frac{\lambda}{\bar{\bar{\theta}}} I, & \text{if } 0 < I \leq 1, \\ \mu_m + \frac{\lambda}{\bar{\bar{\theta}}} + \frac{L_{Ml}^1}{L}\frac{I-1}{\underline{w}}, & \text{if } 1 < I \leq \underline{w}, \\ (\lambda, 1], & \text{if } I = a. \end{cases}$$