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Cultural Diversity Within a Country and Cultural Similarity Between Countries: An Effect of International Trade

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Abstract

We present an evolutionary model of cultural change to investigate how international trade promotes a "cosmopolitan culture," which is a mixture of local cultures. We find there is a stable stationary state in which both a cosmopolitan culture and a local culture coexist within a trading country. We interpret this as a theoretical reproduction of what is often observed: cultural diversity within a country and cultural similarity between countries. However, this result is dependent on the way in which parents transmit their cultural type to their children. When parents have perfect empathy with their children, local culture may go extinct due to international trade.
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Key Words: Cultural diversity, Cultural transmission, Trade

1 Introduction

For decades, there has been concern surrounding the relationship between globalization and cultural diversity. For example, Cowen (2002: pp.2-3) explained that some believe a market economy to be detrimental to culture and diversity. Proponents of this belief argue that international free trade is causing the worldwide spread of standardized cultures of large countries, particularly that of the United States. As a result, country-specific traditional local cultures in small countries are being destroyed by the cultures of large countries, and the world's culture is becoming increasingly uniform and homogeneous. Does international trade really destroy cultural diversity? Is the world's culture becoming homogenous? The objective of this study is to answer these questions using a theoretical evolutionary model of cultural change.

Of course, this is not the first theoretical study to tackle these questions. Bala and Long (2005) were one of the first to focus on the effect of international trade on the evolution of preferences. In their model, each country has two types of consumers, each with different preferences. The authors showed that if one country is sufficiently large compared to another,

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the preference type of the large country will dominate the world as a result of international trade.[1)]

In Bala and Long (2005), changes in the distribution of preference types are modeled using replicator dynamics imported from the literature on evolutionary biology. There is another line of research that uses the cultural transmission model developed by Bisin and Verdier (2001) to investigate cultural dynamics. Here, examples of studies include Bisin and Verdier (2014), Olivier et al. (2008), and Maystre et al. (2014). A main feature of the Bisin-Verdier framework is that it explicitly models how a parent-child relationship transmits cultural types. More specifically, parents choose how much effort to exert in socializing their children.

By incorporating such a cultural transmission model into the standard Heckscher-Ohlin framework, Bisin and Verdier (2014) showed that free trade leads to preference profile equalization: with initially different distributions of preference types under autarky, two trading countries ultimately have the same distribution of preferences in the long-run steady state.

On the other hand, Olivier et al. (2008) argued that when cultural externalities are taken into account, whether globalization leads to cultural convergence or divergence depends on the form of globalization. The authors showed that if globalization is in the form of social integration, it results in identical distributions of cultural types across countries.

Maystre et al. (2014) used a monopolistic competition model to consider the effect of increasing returns on cultural dynamics. The authors assumed that there are country-specific cultural goods and globally common cultural goods. The authors showed that international trade leads to cultural similarity across countries because the variety of common cultural goods is larger than that of country-specific goods as a result of market-size effects.

With the exception of Maystre et al. (2014), the studies cited above share two features. First, the cultural homogeneity resulting from international trade is *complete* homogeneity; that is, the distribution of cultural types is identical in all countries. Second, the culture of a country is characterized not by a country-specific culture, but by a distribution of cultural types. In these models, each country has the same cultural types, and the cultural differences between countries are expressed by different population sizes of cultural types.

In contrast to these studies, we investigate a case in which international trade may cause global cultural similarity, but not necessarily complete uniformity. Second, we characterize the culture of a country not by a distribution of cultural types, but by a distinct country-specific culture.

As such, this study is similar to Maystre et al. (2014), but there are some important differences. First, the model proposed here is much simpler than that of Maystre et al. (2014). Whereas Maystre et al. (2014) use a model of differentiated products, we consider an exchange economy with two goods. Second, although they modeled country-specific cultural goods and globally common cultural goods, they labeled them as such, and there is no intrinsic difference between these cultural goods in their model. In contrast, in our model, a country-specific culture (a "local culture") is explicitly tied to country-specific endowment. In addition, a globally common culture in our model is not completely different from a local culture, but is rather a mixture of local cultures. Therefore, we refer to a global culture as a "cos-

1) In the literature, preference types are typically interpreted as cultural types.

mopolitan" culture. Third, we show that international trade causes cultural similarity between countries and cultural diversity within a country. In our model, we suggest that an originally culturally uniform country under autarky may become a culturally diversified country as a result of international trade, whereas in Maystre et al. (2014), cultural diversity within a country is already set under autarky.

We believe that these points agree with real-world observations on globalization and culture. That is, a local culture is presumed to be a product of intrinsically country-specific factors, such as climate, landscape, biota, or history. In addition, global culture has its roots in various local cultures. As Cowen (2002) wrote:

> "A typical American yuppie drinks French wine, listens to Beethoven on a Japanese audio system, uses the Internet to buy Persian textiles from a dealer in London, watches Hollywood movies funded by foreign capital and filmed by a European director, and vacations in Bali; an upper-middle-class Japanese may do much the same." (p. 4)

Our aim is to incorporate these observations into a simple theoretical model and to derive an equilibrium at which cultural similarity across countries and cultural diversity within a country holds.

The remainder of the paper is organized as follows. In Section 2, we describe the economy and the mechanism of cultural change in our model. Then, in Sections 3 and 4, we analyze the cultural dynamics and characterize the steady-state equilibria under autarky and under free trade with different specifications for cultural transmission mechanisms. Lastly, Section 5 concludes the paper.

2 The model

2.1 The economy

We consider a two-good, two-country model of an exchange economy. Let the countries be called country A and country B and the goods be called good x and good y. Every consumer in country A is endowed with X units of good x and none of good y. Every consumer in country B is endowed with Y units of good y and none of good x. We interpret this to mean that good x and good y are "local goods" of country A and country B, respectively. The population of country B is normalized to one, and that of country A is denoted by $N \geq 1$. We assume that the population of each country is constant over time.

In each country, there are two types of consumers. A fraction q_i of the population in country i (i=A,B) are local types (type L). The utility functions of local types in country A and B are given by

$$u_{LA}(x,y) = k_A x,$$
$$u_{LB}(x,y) = k_B y,$$

respectively, where k_A and k_B are positive constants. That is, a local-type consumer derives utility from consuming the local good of their country only.

On the other hand, a fraction $1-q_i$ of the population in country i are cosmopolitan types (type C), who derive utility from consuming both local goods and foreign goods. The utility functions of cosmopolitans in country A and B are denoted by $u_{CA}(x,y)$ and $u_{CB}(x,y)$, respectively. Specifically, we assume that these utility functions are homothetic with the following form:

$$u_{CA}(x,y) = \phi_A\left(\frac{y}{x}\right)x,$$

$$u_{CB}(x,y) = \phi_B\left(\frac{x}{y}\right)y,$$

where $\phi_A(\cdot)$ and $\phi_B(\cdot)$ are increasing, strictly concave functions; that is, $\phi_i' > 0$ and $\phi_i'' < 0$, $i=A,B$.

Our specification of the utility functions is borrowed from the two-good, two-country models of Bisin and Verdier (2014): the utility functions of local types in our model are the same as Section 17.4 of their paper, and the utility functions of cosmopolitan types in our model are the same as Section 17.5 of their paper. A main difference between our model and that of Bisin and Verdier (2014) is in the interpretation of "culture of a country." Bisin and Verdier (2014) consider two preference types and assume that both types live in both counties. Then, in their model, culture of a country means a proportion of two preference types. On the other hand, in our model, culture of a country means a county-specific preference tied to endowment indigenous to the country. We adopt this interpretation of culture of a country because we want to examine whether such a country-specific local culture may disappear due to international trade.

Now, let us consider the consumers' decisions. Because there is only one type of good in each country, under autarky, no exchange takes place: everyone consumes the good they are endowed. Locals and cosmopolitans in country A obtain the utility $u_{LA}(X,0) = k_A X$ and $u_{CA}(X,0) = \phi_A(0)X$, respectively; and locals and cosmopolitans in country B obtain utility $u_{LB}(0,Y) = k_B Y$ and $u_{CB}(0,Y) = \phi_B(0)Y$, respectively. Moreover, even when international trade is allowed, the consumption choices of the local types are the same as those under autarky because they derive utility from consuming only their endowment of the local good. That is, $u_{LA}(X,0) = k_A X$ and $u_{LB}(0,Y) = k_B Y$ under international free trade also.

However, opportunities for international trade affect the consumption choices of cosmopolitans. Because they derive utility from consuming both goods, cosmopolitans want to exchange their local good for foreign good. To see this, let p denote the relative price of good x in terms of good y. Then, the utility-maximization problem of a cosmopolitan-type consumer in country A is

$$\max_{x,y} \phi_A\left(\frac{y}{x}\right)x$$

$$\text{s.t. } px + y \leq pX.$$

The condition for the marginal rate of substitution (MRS) to be equal to the relative price is given by

$$\frac{\phi_A(\theta_A)}{\phi'_A(\theta_A)} - \theta_A = p, \tag{1}$$

where $\theta_A = y/x$ denotes the consumption ratio of a cosmopolitan consumer in country A. Solving (1) for θ_A, we derive $\theta_A(p)$; that is, the consumption ratio depends only on the price. Using $\theta_A(p)$, we express the demand for good x and good y of a cosmopolitan type in country A as[2)]

$$x_{CA}(p) = \frac{1}{p+\theta_A(p)} pX, \tag{2}$$

$$y_{CA}(p) = \frac{\theta_A(p)}{p+\theta_A(p)} pX.$$

Similarly, for the cosmopolitans in country B, the condition for the MRS to be equal to the relative price is given by

$$\frac{\phi_B(\theta_B)}{\phi'_B(\theta_B)} - \theta_B = \frac{1}{p}, \tag{3}$$

where $\theta_B = x/y$ denotes the consumption ratio of a cosmopolitan consumer in country B. Solving (3) for θ_B, we express the demand for good x and good y of a cosmopolitan type in country B as

$$x_{CB}\left(\frac{1}{p}\right) = \frac{\theta_B\left(\frac{1}{p}\right)}{\frac{1}{p}+\theta_B\left(\frac{1}{p}\right)} \frac{1}{p} Y \tag{4}$$

$$y_{CB}\left(\frac{1}{p}\right) = \frac{1}{\frac{1}{p}+\theta_B\left(\frac{1}{p}\right)} \frac{1}{p} Y.$$

Having derived expressions for the demand in each country, we determine the world equilibrium price $p^* = p^*(q_A, q_B, N, X, Y)$ as a solution to the following market-clearing condition:

$$(1-q_A)Nx_{CA}(p^*) + (1-q_B)x_{CB}\left(\frac{1}{p^*}\right) = (1-q_A)NX. \tag{5}$$

To ensure that the demand curve is downward sloping, and so that the equilibrium price is unique, we assume the elasticity of substitution greater than or equal to unity. Then, we derive the following comparative statics for the equilibrium price:

2) Here, to make the notation simple, we do not explicitly write X as an argument of the demand functions.

$$\frac{\partial p^*}{\partial q_A} > 0,\ \frac{\partial p^*}{\partial q_B} < 0,\ \frac{\partial p^*}{\partial N} < 0,\ \frac{\partial p^*}{\partial X} < 0, \text{ and } \frac{\partial p^*}{\partial Y} > 0. \tag{6}$$

The intuitions are simple. The equilibrium relative price of good x rises as the proportion of the local types in country A increases because the supply of good x decreases. The equilibrium price falls as the proportion of local types in country B increases because the demand for good x decreases. The increases in N and X both lead to an increase in the supply of good x, resulting in a drop in the equilibrium price.[3] The larger the Y, the larger the income of cosmopolitans in country B, and the greater the demand for good x. Thus, the price of good x rises as Y increases.

When $q_A = 1$ or $q_B = 1$, there is no trade between country A and B; and thus, the equilibrium price does not exist. However, we can define p^* in these "limit" cases in the following way. For $0 \le q_B < 1$, as q_A increases and approaches 1, the supply of good x to the international market approaches zero, and the price increases infinitely. Therefore, we have $\lim_{q_A \to 1} p^* = \infty$ as long as $0 \le q_B < 1$. On the other hand, when $q_B=1$, the demand for good x in the international market is zero. Thus, let us define $p^*=0$ when $q_B=1$.

2.2 Cultural dynamics

In this study, we interpret the preference types in each country as cultural types. That is, we view the local-type preference as the "local culture" and the cosmopolitan-type preference as the "cosmopolitan culture." Then, we investigate the changes in the distribution of cultural types; that is, changes in q_A and q_B. More specifically, we consider the following scenario when interpreting the results of our analysis. We suppose that, initially, the economy is under autarky, and almost all consumers in each country are local types; the number of cosmopolitan types is small. Then, we attempt to answer the following questions. If the two countries are integrated through international trade, does the number of cosmopolitan types increase? If it does, then to what extent does this occur, and under what conditions? If the cosmopolitan culture prevails as a result of international trade, does this result in the disappearance of the local culture completely in each country?

The model of dynamics we use is a simplified version of the Bisin-Verdier framework (Bisin and Verdier (2001)) where the transmission of cultural types occurs as a result of socialization within and outside the family. The former case is called direct socialization, and the latter is called oblique socialization. Direct socialization is a process in which parents try to persuade their children to adopt their cultural type. Oblique socialization is a process whereby children meet adults other than their parents, adopt them as role models, and then learn their cultural traits.

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3) In fact, a change in N and a change in X has an identical effect on p^*. The reason is as follows. Because $x_{CA}(p) = \frac{1}{p+\theta_A(p)} pX$, the market-clearing condition is written as

$$(1-q_A)\frac{1}{p^*+\theta_A(p^*)} p^* NX + (1-q_B) x_{CB}\left(\frac{1}{p^*}\right) = (1-q_A) NX.$$

Therefore, N and X enters into the market-clearing equation in the same way.

Formally, we assume that one parent has exactly one child. Parents choose how much of an effort they are willing to make to socialize their children. If a parent of type L in country A makes an effort d_{LA} for direct socialization, the child will adopt the cultural traits of the parent with probability d_{LA}. With probability $1-d_{LA}$, this child is not socialized by the parent and will randomly choose an adult from society as a role model. Because there are q_A type-L adults and $1-q_A$ type-C adults in society, a child who is not socialized by the type-L parent adopts the type-L preference with probability $(1-d_{LA})q_A$ and adopts the type-C preference with probability $(1-d_{LA})(1-q_A)$. A similar argument applies to type-C parents and their children. Therefore, letting P_{tA}^{s} denote the probability that a parent is of type t (t=L, C) and that their child becomes type s (s=L, C) in country A, we have:

$$\begin{cases} P_{LA}^{L} = d_{LA} + (1-d_{LA})q_A \\ P_{LA}^{C} = (1-d_{LA})(1-q_A) \\ P_{CA}^{L} = (1-d_{CA})q_A \\ P_{CA}^{C} = d_{CA} + (1-d_{CA})(1-q_A) \end{cases} . \tag{7}$$

From these probabilities, the dynamics of q_A are derived as follows. Let q'_A denote the fraction of local types in the next period. Then,

$$\begin{aligned} q'_A &= P_{LA}^{L} q_A + P_{CA}^{L}(1-q_A) \\ &= q_A + q_A(1-q_A)(d_{LA}-d_{CA}). \end{aligned}$$

Thus, in the continuous limit, the change in q_A is given by

$$\dot{q}_A = q_A(1-q_A)(d_{LA}-d_{CA}) \tag{8}$$

where $\dot{q}_A = dq_A / dt$.

In a similar way, we derive the equation of motion for the fraction of local types in country B as

$$\dot{q}_B = q_B(1-q_B)(d_{LB}-d_{CB}). \tag{9}$$

Now, we consider the choice of direct socialization by a parent. When choosing the amount of effort they are willing to make, parents care about the well-being of their children. Formally, a type-t parent in country i values their child's welfare as V_{ti}^{s} when the child is type s. Since these V_{ti}^{s}'s depend on what the children will consume, which in turn depend on q_A and q_B when they are adults, the current parents must form expectations on the future values of q_A and q_B. Here, in line with previous studies such as Bisin and Verdier (2014), we simply assume that these expectations are myopic. That is, the current q_A and q_B are the predictors of future q_A and q_B.

The net expected value of the direct socialization of a type-L parent in country A is given by

$$W_{LA} = P_{LA}^{L} V_{LA}^{L} + P_{LA}^{C} V_{LA}^{C} - C\left(d_{LA}\right) \tag{10}$$

where $C(\cdot)$ is an increasing and strictly convex cost function for direct socialization with $C(0)=0$. We assume $\lim_{d_{LA}\to 1} C'\left(d_{LA}\right)=\infty$, so that d_{LA} satisfying the first order condition never exceeds 1.

A type-L parent chooses d_{LA} to maximize (10). The first order condition for the maximization problem of (10) with respect to d_{LA} defines the optimal choice of direct socialization by a type-L parent in country A:

$$\left(1-q_A\right)\Delta V_{LA} - C'\left(d_{LA}\right) \le 0 \text{ with } d_{LA} \ge 0, \tag{11}$$

where $\Delta V_{LA} = V_{LA}^{L} - V_{LA}^{C}$ is the utility gain of a type-L parent in country A when their child has the same cultural type rather than the different type. In general, we let $\Delta V_{ti} = V_{ti}^{t} - V_{ti}^{s}$ denote the utility gain of a type-t parent.

The optimal choice of direct socialization by a type-C parent is derived in a similar way:

$$q_A \Delta V_{CA} - C'\left(d_{CA}\right) \le 0 \text{ with } d_{CA} \ge 0. \tag{12}$$

Here, we assume that the cost functions for direct socialization effort are the same for both types because there is no special reason to believe that local types or cosmopolitan types incur a higher cost for socializing their children. Moreover, for simplicity, we assume that the cost functions for both types in country B are the same as those in country A.

By inspecting equation (11) and (12), we can easily see that d_{LA} is increasing in ΔV_{LA} and decreasing in q_A and that d_{CA} is increasing in ΔV_{CA} and increasing in q_A. The intuition is simple. A parent exerts more direct socialization effort when the utility gain of the effort is large; a parent exerts less direct socialization effort when the population of their type is high because it means a high probability of oblique socialization.

In a similar manner, we derive the optimal choices of direct socialization by type-L and type-C parents in country B. That is,

$$\left(1-q_B\right)\Delta V_{LB} - C'\left(d_{LB}\right) \le 0 \text{ with } d_{LB} \ge 0, \tag{13}$$

$$q_B \Delta V_{CB} - C'\left(d_{CB}\right) \le 0 \text{ with } d_{CB} \ge 0. \tag{14}$$

In the following sections, we consider two different specifications for the value of socialization to parents: imperfect empathy and perfect empathy.

3 Cultural transmission through imperfect empathy

In this section, we use a cultural transmission model based on imperfect empathy, as proposed by Bisin-Verdier (2001). More specifically, imperfect empathy means that parents care about their children's consumption choices and the resulting utility but evaluate them using their own utility. Formally, this is modeled by specifying that $V_{ti}^{s} = u_{ti}\left(x_{si}, y_{si}\right)$ where $\left(x_{si}, y_{si}\right)$ is the optimal consumption choice of a type-s consumer in country i. In other words, a type-t parent measures the well-being of their type-s child by the utility the parent would gain by consuming the consumption bundle chosen by the type-s child.

3.1 Autarky

As stated above, under autarky, everyone consumes the good they are endowed. In such a case, a parent receives no utility gain from direct socialization because the child consumes the same consumption bundle regardless of the child's type. That is, because $(x_{LA}, y_{LA}) = (x_{CA}, y_{CA}) = (X, 0)$ and $(x_{LB}, y_{LB}) = (x_{CB}, y_{CB}) = (0, Y)$ under autarky, $\Delta V_{ti} = 0$ for any $t = L, C$ and $i = A, B$. Thus, $d_{LA} = d_{CA} = 0$ and $d_{LB} = d_{CB} = 0$. Therefore, the dynamics of q_A and q_B are trivial in this case: for any initial values of q_A and q_B, it holds that $\dot{q}_A = 0$ and $\dot{q}_B = 0$. If we apply our interpretative scenario that under autarky almost all consumers are local types for this result, we see that the population size of each type remains the same. The small fraction of cosmopolitan types neither vanishes nor grows.

3.2 International trade

When international trade is allowed, type-C consumers in each country exchange their local goods internationally. As a result, their consumption bundles are different from those of type-L consumers. Then, we have

$$\begin{aligned}
\Delta V_{LA}(p^*) &= u_{LA}(X, 0) - u_{LA}\left(x_{CA}(p^*), y_{CA}(p^*)\right) = k_A\left(X - x_{CA}(p^*)\right), \\
\Delta V_{CA}(p^*) &= u_{CA}\left(x_{CA}(p^*), y_{CA}(p^*)\right) - u_{CA}(X, 0) = \phi_A\left(\theta_A(p^*)\right)x_{CA}(p^*) - \phi_A(0)X, \\
\Delta V_{LB}\left(\frac{1}{p^*}\right) &= u_{LB}(0, Y) - u_{LB}\left(x_{CB}\left(\frac{1}{p^*}\right), y_{CB}\left(\frac{1}{p^*}\right)\right) = k_B\left(Y - y_{CB}\left(\frac{1}{p^*}\right)\right), \qquad (15) \\
\Delta V_{CB}\left(\frac{1}{p^*}\right) &= u_{CB}\left(x_{CB}\left(\frac{1}{p^*}\right), y_{CB}\left(\frac{1}{p^*}\right)\right) - u_{CB}(0, Y) = \phi_B\left(\theta_B\left(\frac{1}{p^*}\right)\right)y_{CB}\left(\frac{1}{p^*}\right) - \phi_B(0)Y.
\end{aligned}$$

These utility gains are all positive as long as $0 \le q_A < 1$ and $0 \le q_B < 1$; that is, as long as international trade takes place. Recall that utility gain ΔV_{ti} measures how much a parent will gain by having their child inherit the parent's type instead of the other type. The utility gains are all positive because a parent assesses their child's well-being by evaluating the child's consumption bundle with their (the parent's) own utility function. Given that ΔV_{ti}'s are positive, the first order conditions for d_{ti} (equations (11), (12), (13), and (14) may hold in equality. Thus, now d_{ti}'s may not equal zero and may be different from each other, meaning that the dynamics of q_A and q_B are no longer trivial.

3.2.1 Optimal direct socialization

To examine the dynamics of q_A and q_B, we characterize the optimal choices of the direct socialization effort in the following lemma.

Lemma 1 *1. For any given $0 \le q_B < 1$, (i) d_{LA} is decreasing in q_A, (ii) d_{CA} is increasing in q_A, and (iii) $\lim_{q_A \to 0} d_{LA} > 0, d_{LA} = 0$ at $q_A = 1, d_{CA} = 0$ at $q_A = 0$, and $\lim_{q_A \to 1} d_{CA} > 0$. Therefore, (iv) there is the unique interior q_A such that $d_{LA} = d_{CA}$.*

2. For any given $0 \le q_A < 1$, (i) d_{LB} is decreasing in q_B, (ii) d_{CB} is increasing in q_B, and (iii) $\lim_{q_B \to 0} d_{LB} > 0$, $d_{LB} = 0$ at $q_B = 1$, $d_{CB} = 0$ at $q_B = 0$, and $\lim_{q_B \to 1} d_{CB} > 0$. Therefore, (iv) there is the unique interior q_B such that $d_{LB} = d_{CB}$.

Proof. See Appendix.■

Since the statement 1 and 2 in Lemma 1 are parallel, let us consider the intuition of statement 1. When q_A is close to 0, almost everyone is type C. Thus, a child who was not socialized by a parent is likely to become a type-C agent by oblique socialization. Under such a circumstance, type-L parents want to exert positive effort toward direct socialization to have their child become type L while type-C parents do not care to exert direct socialization effort. Therefore, $d_{LA}>0$ and $d_{CA} \approx 0$ when q_A is close to zero. As q_A increases, it is more likely that a child who was not socialized by a parent will become type L by oblique socialization. Because oblique socialization is a substitute for direct socialization, an increase in q_A decreases d_{LA} and increases d_{CA}.[4] Eventually, as q_A becomes 1, everyone becomes a type-L agent. Thus, the decreasing d_{LA} converges to 0 while the increasing d_{CA} reaches some positive value as $q_A \to 1$. Therefore, strictly between 0 and 1, there must be a unique point of q_A at which $d_{LA} = d_{CA}$.

Because of this uniqueness, we define a function $q_A(q_B)$ that gives the value of q_A such that $d_{LA} = d_{CA}$ for any given $0 \le q_B < 1$. By definition, $q_A(q_B)$ is continuous and $0 < q_A(q_B) < 1$ for any $0 \le q_B < 1$. Graphically, in the $q_A - q_B$ plane of $[0,1]\times[0,1]$, $q_A(q_B)$ is drawn as a curve from the bottom to the top. Similarly, we define a continuous function $q_B(q_A)$ as the value of q_B such that $d_{LB} = d_{CB}$ for any given $0 \le q_A < 1$. Because $0 < q_B(q_A) < 1$ for any $0 \le q_A < 1$, $q_B(q_A)$ is drawn as a curve from the left to the right in the $q_A - q_B$ plane (see Figure 1).

3.2.2 Stationary states

Having analyzed the optimal choices of d_{ti}'s under international trade, we proceed to examine the stationary states of the system of dynamic equations (8) and (9). By inspecting these two equations, we immediately see that there are seven possible cases in which $\dot{q}_A = 0$ and $\dot{q}_B = 0$ (see Figure 1).

(a) $q_A = 0$ and $q_B = 0$,
(b) $q_A = 0$ and $q_B = q_B(0)$,
(c) $q_B = 0$ and $q_A = q_A(0)$,
(d) $q_A = 1$ and any value of $0 \le q_B < 1$,
(e) $q_B = 1$ and any value of $0 \le q_A < 1$,

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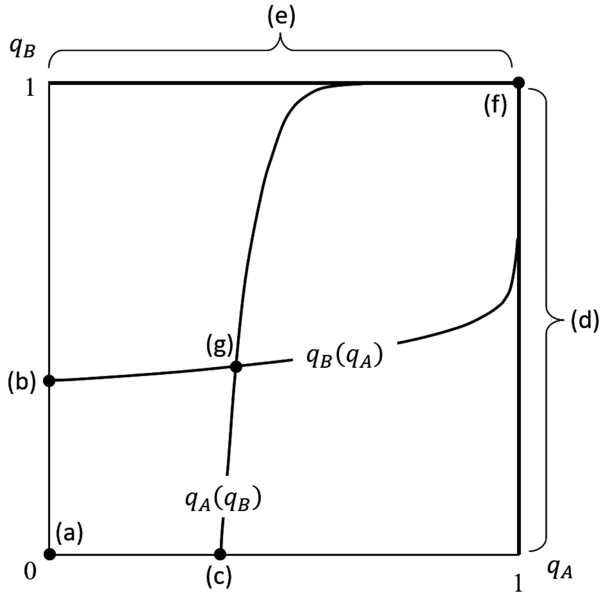
4) There is another second effect of q_A on d_{LA} and d_{CA} through an increase in the equilibrium price p^* affecting ΔV_{LA} and ΔV_{CA}. For d_{LA}, this second effect is counteracting the first effect explained above. When p^* increases, the consumption of good x by a type-C agent decreases. So, from the perspective of a type-L parent, if their child becomes type C, the child will look more miserable (namely, ΔV_{LA} is increasing in p^*). To avoid this event, a type-L parent wants to increase d_{LA}. Although this second effect counteracts the first effect of an increasing q_A to decrease d_{LA}, as shown in the proof of Lemma 1 in the Appendix, the first effect dominates the second. Thus, overall, as q_A increases, d_{LA} decreases. On the other hand, for d_{CA}, the second effect reinforces the first. An increase in p^* increases a type-C agent's income and raising the resulting utility $u_{CA}(x_{CA}, y_{CA})$. Thus, ΔV_{CA} increases as p^* increases. Therefore, a type-C parent wants to exert more direct socialization effort as q_A increases.

Figure 1: Seven possible cases for stationary states



(f) $q_A = 1$ and $q_B = 1$,
(g) the intersection of $q_A(q_B)$ and $q_B(q_A)$.

The cases from (a) to (f) are corner stationary states whereby q's are zero and/or one.[5] Only (g) is possibly the case for interior stationary states. Proposition 1 shows that with certain technical conditions on utility functions, there exist interior stationary states.

Proposition 1 If at least one of the following conditions is satisfied, interior stationary states exist.

1. $1 < \lim_{p \to 0} p \dfrac{\theta_A'}{\theta_A} < \infty$ and $\phi_A(0) > 0$.

2. $1 < \lim_{\frac{1}{p} \to 0} \dfrac{1}{p} \dfrac{\theta_B'}{\theta_B} < \infty$ and $\phi_B(0) > 0$.

Proof. See Appendix.■

The idea of Proposition 1 is as follows. Note that $q_A(q_B)$ lies strictly between zero and one for any $0 \le q_B < 1$ and so does $q_B(q_A)$ for any $0 \le q_A < 1$. Thus, if $\lim_{q_B \to 1} q_A(q_B) < 1$, or if $\lim_{q_B \to 1} q_A(q_B) < 1$ is guaranteed, the $q_A(q_B)$ curve and the $q_B(q_A)$ curve intersect at least once in the interior of $[0, 1] \times [0, 1]$. That is, there exists at least one interior stationary state.[6]

5) The reason why there are continuums of stationary states such as (d) and (e) is as follows. When $q_A = 1$, for example, there is no international trade and, thus, $d_{LB} = d_{CB} = 0$ holds for any $0 \le q_B < 1$.

6) Because the conditions in Proposition 1 are sufficient, interior stationary states can exist even if the conditions are not satisfied. For example, when the utility functions of cosmopolitan types in both countries

Next, we examine the stability of the stationary states. In Proposition 2, we show that the corner stationary states (a) through (f) are not stable in the sense that they are never reached as long as the initial points are interior.

Proposition 2 *If one of the conditions in Proposition 1 is satisfied, the corner stationary states of (a) through (f) are never reached when initial points* (q_A, q_B) *are such that* $0 < q_A < 1$ *and* $0 < q_B < 1$.

Proof. See Appendix.■

What about the stability of the interior stationary states? In Proposition 3, we provide a sufficient condition for uniqueness and stability of the interior stationary state.

Proposition 3 *Suppose that interior stationary states exist. If at least one of the following conditions is satisfied, the interior stationary state is unique and globally stable.*

1. $\dfrac{\Delta V'_{LA}(p^*)}{\Delta V_{LA}} \leq \dfrac{\Delta V'_{CA}(p^*)}{\Delta V_{CA}}$ *at the intersection of* $q_A(q_B)$ *and* $q_B(q_A)$.

2. $\dfrac{\Delta V'_{LB}(1/p^*)}{\Delta V_{LB}} \leq \dfrac{\Delta V'_{CB}(1/p^*)}{\Delta V_{CB}}$ *at the intersection of* $q_A(q_B)$ *and* $q_B(q_A)$.

Proof. See Appendix.■

Furthermore, in Corollary 1, we provide a more concrete sufficient condition for global stability of the interior stationary state.

Corollary 1 *If* $u_{CA}(x, y)$ *or* $u_{CB}(x, y)$ *is a CES function with an elasticity of substitution greater than one and not infinitely large, the interior stationary state is unique and globally stable.*

Proof. See Appendix.■

Using our interpretative scenario, the results above are illustrated as follows (see Figure 2). Let $\left(q_A^*, q_B^*\right)$ denote the unique interior stationary state. Suppose that in each country almost everyone is a local type under autarky, meaning that (q_A, q_B) is close to $(1, 1)$. Now, when international trade is allowed, the distribution of the cultural types in each country approaches $\left(q_A^*, q_B^*\right)$. That is, with almost all individuals being local types under autarky, opportunities for international trade causes the population of the cosmopolitan types to grow in each country.[7)]

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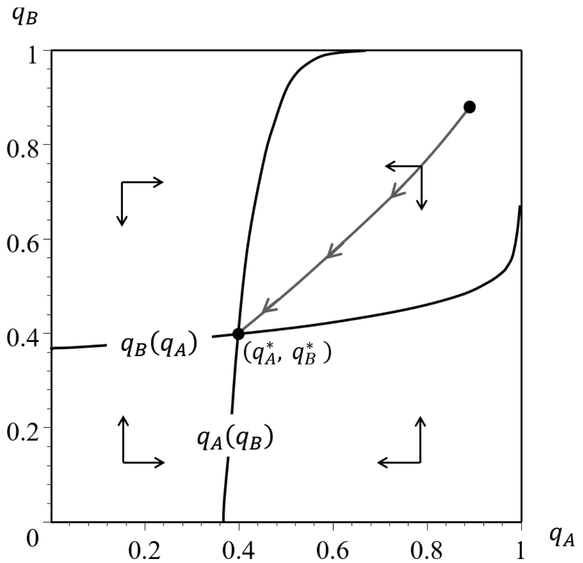
are Cobb–Douglas, the conditions in Proposition 1 are violated, but still the unique interior stationary state exists.

7) We may argue that the initial state under autarky should be exactly $(1, 1)$; that is, everyone is a local type because no one can develop a taste for a foreign good under autarky without an opportunity to try that good. In that case, the population stays at $(q_A, q_B) = (1, 1)$ even when international trade is allowed

Figure 2: The globally stable stationary state in the case of imperfect empathy: when $u_{LA}(x,y)=\frac{1}{2}x$, $u_{LB}(x,y)=\frac{1}{2}y$, $u_{CA}(x,y)=u_{CB}(x,y)=\left(\frac{1}{2}x^{\frac{1}{3}}+\frac{1}{2}y^{\frac{1}{3}}\right)^3$, $N=1$, and $X=Y=1$.



However, the population does not become completely cosmopolitan. Because the stationary state the world economy is heading toward is an interior state, neither q_A nor q_B goes to zero. That is, local types survive international trade. At the stationary state, there are both cosmopolitan types and local types in each country. Therefore, in our scenario, if the cultural distributions before trade and after trade are compared, we observe the following effects of international trade. On the one hand, within a country, international trade increases cultural diversity: a culturally uniform country where almost everyone is a local type will become a multi-cultural country with local types and cosmopolitan types. On the other hand, between countries, international trade increases cultural similarity: the countries that have totally different local cultures will begin to share each others' cultures through the increase in cosmopolitan types.

Our results are similar to those of existing literature, such as Bala and Long (2005), Olivier et al. (2008), and Bisin and Verdier (2014), in that international trade leads to cultural similarity between countries. However, what the previous studies have shown is *complete* cultural homogeneity, meaning that the distribution of cultural types is identical among trading countries. On the other hand, what we show here is that two countries become culturally similar but not completely homogeneous, implying that each country keeps its country-specific local culture even in the long-run equilibrium.

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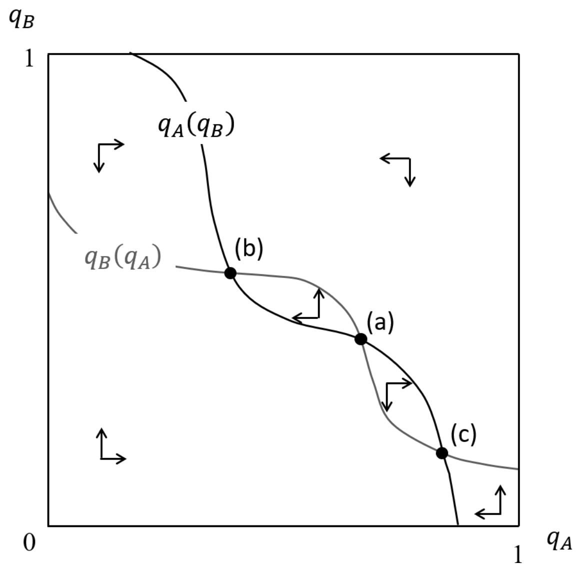
because it is a stationary state. However, if just a small fraction of the population in each country happens to have a cosmopolitan preference because of, for example, their experience working abroad, the cosmopolitan culture grows in each country until the population reaches the interior stationary state (q_A^*, q_B^*).

Although we have discussed our results in terms of our interpretative scenario that under autarky cultural distribution is close to $(1, 1)$, it is, of course, not a necessary condition for reaching the interior stationary state. In fact, whatever the cultural distribution is, as long as local and cosmopolitan types coexist in each country under autarky, international trade leads the economy to the unique interior stationary state $\left(q_A^*, q_B^*\right)$.

The main force of such global stability comes from the substitutability of direct socialization and oblique socialization. For example, when the fraction of local types is large in a county, local-type parents in the country can count on oblique socialization while cosmopolitan-type parents cannot. Thus, local parents exert less direct socialization effort than cosmopolitan parents. Therefore, the fraction of local types decreases when it is large. Conversely, if the fraction of cosmopolitan types is large, exactly the opposite will happen, and thus the fraction of cosmopolitans decreases. Because $0 < q_A(q_B) < 1$ for any q_B and $0 < q_B(q_A) < 1$ for any q_A, the mechanism explained above works in one country regardless of the cultural distribution of the other country. Essentially, this is the reason for global stability of the interior stationary state.

What would happen if both of the conditions in Proposition 3 are violated? First, notice that these conditions are sufficient. So, uniqueness and global stability of the interior stationary state may still hold without these conditions. That is, stability of a stationary state is lost only when neither of the conditions are satisfied. However, when stability of a stationary state is lost, it can be shown that there must be other interior stationary states and some of them must be locally stable (This is depicted in Figure 3). Therefore, whether the conditions in Proposition 3 are satisfied or not, the cultural distributions converge to some of the interior sta-

Figure 3: When there is an unstable interior stationary state such as (a), there must be other interior stationary states, and some of them must be locally stable. Here, (b) and (c) are stable. This occurs because the $q_A(q_B)$ curve must go through from the bottom to the top, and the $q_B(q_A)$ curve must go through from the left to the right, provided that $0 < q_A(q_B) < 1$ for any $0 \leq q_B < 1$ and that $0 < q_B(q_A) < 1$ for any $0 \leq q_A < 1$.



tionary states at which local types and cosmopolitan types coexist in both countries.

Then, what do the conditions in Proposition 3 do? To see this, consider, for example, that the world economy under autarky is at a point (q_A, q_B), just shy of a stationary state $\left(q_A^*, q_B^*\right)$, such that $q_A > q_A^*$ and $q_B < q_B^*$. In this case, when international trade is allowed, due to the substitutability of direct and oblique socialization, cosmopolitan types exert more direct socialization effort than local types in country A, and local types exert more direct socialization effort than cosmopolitan types in country B. This mechanism returns the economy to the stationary state $\left(q_A^*, q_B^*\right)$.

However, there is another mechanism, through the equilibrium price p^*, that affects the cultural dynamics. When $q_A > q_A^*$ and $q_B < q_B^*$, the equilibrium price is higher than it is at $\left(q_A^*, q_B^*\right)$. The higher the equilibrium price, the larger both ΔV_{CA} and ΔV_{LA}, and the smaller both ΔV_{CB} and ΔV_{LB} (see footnote 4 for an explanation). Thus, in response to the higher price, both types in country A exert more direct socialization effort than at $\left(q_A^*, q_B^*\right)$ while both types in country B exert less effort than at $\left(q_A^*, q_B^*\right)$. Now, suppose that both of the conditions 3 are violated. As ΔV_{LA} is more price elastic than ΔV_{CA}, d_{LA} increases more than d_{CA}. Similarly, if ΔV_{LB} is more price elastic than ΔV_{CB}, d_{LB} decreases more than d_{CB}. Thus, this second mechanism makes q_A larger and q_B smaller, moving the economy further away from the stationary state, $\left(q_A^*, q_B^*\right)$. If these forces are strong enough, the stationary state may become unstable. The conditions in Proposition 3 keep these forces in check.

3.2.3 Comparative statics

To see the characteristics of the stationary state, we examine how $\left(q_A^*, q_B^*\right)$ changes as certain parameters change. Recall that our motivation for this research is to see whether the culture of a large country may invade a small country through international trade and erode the local culture. Although we have found that the stable stationary state is an interior state, the equilibrium fraction of local types, say, q_B^* in country B, might be almost zero when country A is much larger than country B. Thus, we first examine the comparative statics of the stationary state $\left(q_A^*, q_B^*\right)$ with respect to N, the population size of country A.

Differentiating the system of equations $d_{LA}\left(q_A^*, q_B^*\right) = d_{CA}\left(q_A^*, q_B^*\right)$ and $d_{LB}\left(q_A^*, q_B^*\right) = d_{CB}\left(q_A^*, q_B^*\right)$ with respect to N, we have

$$\begin{bmatrix} \dfrac{\partial d_{LA}}{\partial q_A} - \dfrac{\partial d_{CA}}{\partial q_A} & \dfrac{\partial d_{LA}}{\partial q_B} - \dfrac{\partial d_{CA}}{\partial q_B} \\ \dfrac{\partial d_{LB}}{\partial q_A} - \dfrac{\partial d_{CB}}{\partial q_A} & \dfrac{\partial d_{LB}}{\partial q_B} - \dfrac{\partial d_{CB}}{\partial q_B} \end{bmatrix} \begin{bmatrix} \dfrac{\partial q_A^*}{\partial N} \\ \dfrac{\partial q_B^*}{\partial N} \end{bmatrix} = \begin{bmatrix} -\left(\dfrac{\partial d_{LA}}{\partial N} - \dfrac{\partial d_{CA}}{\partial N}\right) \\ -\left(\dfrac{\partial d_{LB}}{\partial N} - \dfrac{\partial d_{CB}}{\partial N}\right) \end{bmatrix}, \tag{16}$$

where

$$\frac{\partial d_{LA}}{\partial N}-\frac{\partial d_{CA}}{\partial N}=\frac{\left(\left(1-q_A^*\right)\Delta V_{LA}'\left(p^*\right)-q_A^*\Delta V_{CA}'\left(p^*\right)\right)\frac{\partial p^*}{\partial X}}{C''\left(d_{CA}\right)},$$

$$\frac{\partial d_{LB}}{\partial N}-\frac{\partial d_{CB}}{\partial N}=\frac{\left(\left(1-q_B^*\right)\Delta V_{LB}'\left(1/p^*\right)-q_B^*\Delta V_{CB}'\left(1/p^*\right)\right)\frac{\partial\left(1/p^*\right)}{\partial X}}{C''\left(d_{CB}\right)}.$$

The determinant of the matrix on the left-hand side of (16) is positive since we are considering the stable stationary state (see equation (21) in the Appendix). Thus, the sign of $\partial q_A^*/\partial N$ is the same as the sign of its numerator:

$$-\left(\frac{\partial d_{LA}}{\partial N}-\frac{\partial d_{CA}}{\partial N}\right)\left(\frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B}\right)+\left(\frac{\partial d_{LB}}{\partial N}-\frac{\partial d_{CB}}{\partial N}\right)\left(\frac{\partial d_{LA}}{\partial q_B}-\frac{\partial d_{CA}}{\partial q_B}\right)$$
$$=\frac{\left(\Delta V_{LB}+\Delta V_{CB}\right)\left(\left(1-q_A^*\right)\Delta V_{LA}'\left(p^*\right)-q_A^*\Delta V_{CA}'\left(p^*\right)\right)\frac{\partial p^*}{\partial N}}{C''\left(d_{CA}\right)C''\left(d_{CB}\right)}. \tag{17}$$

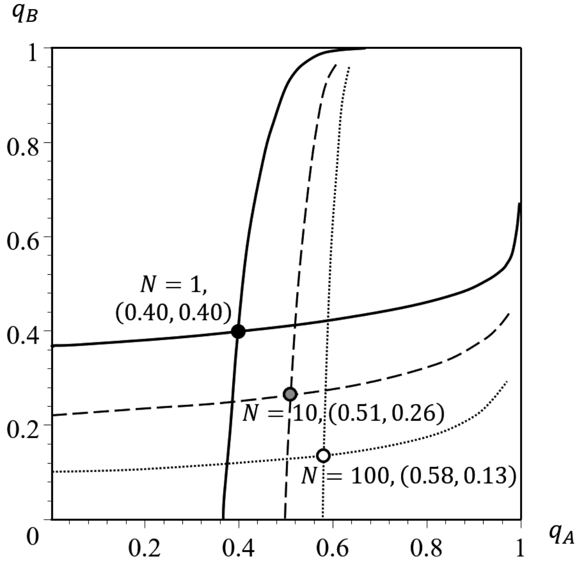
Similarly, the sign of $\partial q_B^*/\partial N$ is the same as the sign of its numerator:

$$-\left(\frac{\partial d_{LA}}{\partial q_A}-\frac{\partial d_{CA}}{\partial q_A}\right)\left(\frac{\partial d_{LB}}{\partial N}-\frac{\partial d_{CB}}{\partial N}\right)+\left(\frac{\partial d_{LB}}{\partial q_A}-\frac{\partial d_{CB}}{\partial q_A}\right)\left(\frac{\partial d_{LA}}{\partial N}-\frac{\partial d_{CA}}{\partial N}\right)$$
$$=\frac{\left(\Delta V_{LA}+\Delta V_{CA}\right)\left(\left(1-q_B^*\right)\Delta V_{LB}'\left(1/p^*\right)-q_B^*\Delta V_{CB}'\left(1/p^*\right)\right)\frac{\partial\left(1/p^*\right)}{\partial N}}{C''\left(d_{CA}\right)C''\left(d_{CB}\right)}. \tag{18}$$

As shown in the proof of Corollary 1, $\left(1-q_i^*\right)\Delta V_{Li}'-q_i^*\Delta V_{Ci}'<0$ holds if the utility function of cosmopolitan types in country i is CES. Therefore, as long as the utility functions of cosmopolitan types in both countries are CES, it holds that $\partial q_A^*/\partial N>0$ and $\partial q_B^*/\partial N<0$. Namely, as the population of country A gets larger, there are more local types in country A, and fewer local types in country B. This seems to support the statement that a local culture of a small country is eroded by a culture of a large country through international trade.

Let us consider the reason why this happens. When the population of country A increases, the equilibrium price p^* decreases because of the larger supply of good x. In response to the price decrease in good x, both local types and cosmopolitan types exert more direct socialization effort in country B. The reason is the following. Since good x is now cheaper, cosmopolitans in country B consume more of good x and less of good y. Therefore, local-type parents in country B now have a stronger incentive not to let their children become cosmopolitan types. At the same time, the price decrease in good x increases the real income of type-C consumers, and the resulting utility of type-C consumers is higher in country B. Thus, cosmopolitan-type parents, too, have a stronger incentive to ensure that their children become cosmopolitan types. However, given $\left(1-q_B\right)\Delta V_{LB}'\left(1/p^*\right)<q_B\Delta V_{CB}'\left(1/p^*\right)$, cosmopolitan types are more responsive to price changes than local types. Therefore, d_{CB} increases more than d_{LB}, resulting in a decrease in q_B^*. In essence, cultural erosion of a small country occurs

Figure 4: The stationary states with different N when $u_{LA}(x,y)=\frac{1}{2}x$, $u_{LB}(x,y)=\frac{1}{2}y$, $u_{CA}(x,y)=u_{CB}(x,y)=\left(\frac{1}{2}x^{\frac{1}{3}}+\frac{1}{2}y^{\frac{1}{3}}\right)^3$, $N=1$, and $X=Y=1$.



due to cheaper imports from a large foreign country.

How strong is the effect of asymmetric country size on cultural erosion of a smaller country? We investigate this question by conducting some numerical calculations. Other things being symmetric, let us change the population size of country A only. Figure 4 shows three stationary states: when two countries are the same in size; when country A is ten times as big as country B; and when country A is one hundred times as big as country B. At least in this numerical example, the effect of asymmetric country size is not so strong that local types in the smaller country virtually disappear. Even when country A is one hundred times as big as country B, more than 10% are local types in country B.

Note that the comparative statics result that $\partial q_B^* / \partial N < 0$ crucially depends on the assumption of $(1-q_B)\Delta V_{LB}'\left(1/p^*\right) < q_B \Delta V_{CB}'\left(1/p^*\right)$. In our model, we cannot exclude that the opposite, $(1-q_B)\Delta V_{LB}'\left(1/p^*\right) > q_B \Delta V_{CB}'\left(1/p^*\right)$, holds. Thus, $\partial q_B^* / \partial N > 0$ can happen. If local types are more responsive to the price change than cosmopolitan types, the fraction of local types increases when the population of country A grows.

Second, let us examine the effect of the endowment size, say, X, on the stationary state. Differentiating the system of equations $d_{LA}(q_A^*, q_B^*) = d_{CA}(q_A^*, q_B^*)$ and $d_{LB}(q_A^*, q_B^*) = d_{CB}(q_A^*, q_B^*)$ with respect to X, we have

$$\begin{bmatrix} \frac{\partial d_{LA}}{\partial q_A}-\frac{\partial d_{CA}}{\partial q_A} & \frac{\partial d_{LA}}{\partial q_B}-\frac{\partial d_{CA}}{\partial q_B} \\ \frac{\partial d_{LB}}{\partial q_A}-\frac{\partial d_{CB}}{\partial q_A} & \frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B} \end{bmatrix}\begin{bmatrix} \frac{\partial q_A^*}{\partial X} \\ \frac{\partial q_B^*}{\partial X} \end{bmatrix}=\begin{bmatrix} -\left(\frac{\partial d_{LA}}{\partial X}-\frac{\partial d_{CA}}{\partial X}\right) \\ -\left(\frac{\partial d_{LB}}{\partial X}-\frac{\partial d_{CB}}{\partial X}\right) \end{bmatrix},$$

where

$$\frac{\partial d_{LA}}{\partial X}-\frac{\partial d_{CA}}{\partial X}=\frac{\left(1-q_A^*\right)\frac{\partial \Delta V_{LA}}{\partial X}-q_A^*\frac{\partial \Delta V_{CA}}{\partial X}+\left(\left(1-q_A^*\right)\Delta V_{LA}'\left(p^*\right)-q_A^*\Delta V_{CA}'\left(p^*\right)\right)\frac{\partial p^*}{\partial X}}{C''\left(d_{CA}\right)},$$

$$\frac{\partial d_{LB}}{\partial X}-\frac{\partial d_{CB}}{\partial X}=\frac{\left(\left(1-q_B^*\right)\Delta V_{LB}'\left(1/p^*\right)-q_B^*\Delta V_{CB}'\left(1/p^*\right)\right)\frac{\partial\left(1/p^*\right)}{\partial X}}{C''\left(d_{CB}\right)}.$$

Here, note that $\left(1-q_A^*\right)\frac{\partial \Delta V_{LA}}{\partial X}-q_A^*\frac{\partial \Delta V_{CA}}{\partial X}=\left(\left(1-q_A^*\right)\Delta V_{LA}-q_A^*\Delta V_{CA}\right)/X=0.$ This is because X enters ΔV_{LA} and ΔV_{CA} linearly and because $\left(1-q_A^*\right)\Delta V_{LA}-q_A^*\Delta V_{CA}=0$ by the first order conditions (11) and (12), and $d_{LA}=d_{CA}$ at (q_A^*, q_B^*). Therefore, the comparative statics with respect to X are the same as with respect to N examined above. That is, $\partial q_A^*/\partial X>0$ and $\partial q_B^*/\partial X<0$ as long as we assume CES utility functions for cosmopolitan types. An increase in the endowment of the local good in a country increases the stationary-state fraction of local types in the country and decreases the stationary-state fraction of local types in the other country.

We are also interested in the effect of a change in the preferences of cosmopolitan types on the stationary state. In particular, we would like to discern what happens when the cosmopolitan types in a country become more "fond" of their local good. To derive a concrete result, we consider the Cobb-Douglas preferences $u_{CA}(x,y)=x^{\alpha}y^{1-\alpha}$ and $u_{CB}(x,y)=x^{1-\beta}y^{\beta}$. In this case, ΔV_{ti}'s are given by

$$\Delta V_{LA}=\left(1-\alpha\right)Xk_A,$$
$$\Delta V_{CA}=\left(1-\alpha\right)^{1-\alpha}\alpha^{\alpha}X\left(p^*\right)^{1-\alpha},$$
$$\Delta V_{LB}=\left(1-\beta\right)Yk_B,$$
$$\Delta V_{CB}=\left(1-\beta\right)^{1-\beta}\beta^{\beta}Y\left(\frac{1}{p^*}\right)^{1-\beta},$$

where $p^*=\frac{(1-q_B)(1-\beta)Y}{(1-q_A)(1-\alpha)NX}$. Suppose that the cosmopolitan types of country A become more fond of good x; that is, α increases. The effect on the stationary state is analyzed using

$$\begin{bmatrix} \frac{\partial d_{LA}}{\partial q_A}-\frac{\partial d_{CA}}{\partial q_A} & \frac{\partial d_{LA}}{\partial q_B}-\frac{\partial d_{CA}}{\partial q_B} \\ \frac{\partial d_{LB}}{\partial q_A}-\frac{\partial d_{CB}}{\partial q_A} & \frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B} \end{bmatrix}\begin{bmatrix} \frac{\partial q_A^*}{\partial \alpha} \\ \frac{\partial q_B^*}{\partial \alpha} \end{bmatrix}=\begin{bmatrix} -\left(\frac{\partial d_{LA}}{\partial \alpha}-\frac{\partial d_{CA}}{\partial \alpha}\right) \\ -\left(\frac{\partial d_{LB}}{\partial \alpha}-\frac{\partial d_{CB}}{\partial \alpha}\right) \end{bmatrix}.$$

Again, because the determinant of the matrix is positive, the sign of $\partial q_A^* / \partial \alpha$ is the same as the sign of its numerator:

$$-\left(\frac{\partial d_{LA}}{\partial \alpha}-\frac{\partial d_{CA}}{\partial \alpha}\right)\left(\frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B}\right)+\left(\frac{\partial d_{LB}}{\partial \alpha}-\frac{\partial d_{CB}}{\partial \alpha}\right)\left(\frac{\partial d_{LA}}{\partial q_B}-\frac{\partial d_{CA}}{\partial q_B}\right) \tag{19}$$

$$=-\frac{\left(\left(1-q_A^*\right)Xk_A+q_A^*\Delta V_{CA}\left(\ln\alpha-\ln\left(1-\alpha\right)-\ln p^*\right)\right)\left(\left(\Delta V_{LB}+\Delta V_{CB}\right)+q_B^*\Delta V_{CB}'\left(1/p^*\right)\frac{\partial\left(1/p^*\right)}{\partial q_B}\right)}{C''\left(d_{CA}\right)C''\left(d_{CB}\right)}$$

$$-\frac{\left(\Delta V_{LB}+\Delta V_{CB}\right)q_A\Delta V_{CA}'\left(p^*\right)\frac{\partial p^*}{\partial \alpha}}{C''\left(d_{CA}\right)C''\left(d_{CB}\right)}.$$

In general, we cannot determine the sign of (19) because of the term $\left(\ln\alpha-\ln\left(1-\alpha\right)-\ln p^*\right)$. However, if we consider the case in which two countries are symmetric ($\alpha=\beta$, $X = Y$, and $N = 1$), and cosmopolitan consumers are inclined to consume more of the local good than the foreign good $\left(\alpha>\frac{1}{2}\right)$, then $p^*=1$ and $\alpha>1-\alpha$ hold. Then, $\left(\ln\alpha-\ln\left(1-\alpha\right)-\ln p^*\right)>0$. Therefore, the sign of (19) is negative. Hence, in this case, we conclude that $\partial q_A^* / \partial \alpha<0$. Interestingly, as the cosmopolitan types become more fond of their local good, the proportion of local types decreases. This occurs partly because the cosmopolitan types' greater demand for the local good bids up its price, which raises the market value of their endowment and raises their resulting utility. This in turn encourages the direct socialization effort of the cosmopolitan types.

4 Cultural transmission through perfect empathy

In the previous section, we presented our main result showing that local types survive international trade when cultural transmission occurs through imperfect empathy. In this section, we examine a different specification for cultural transmission; that is, cultural transmission through perfect empathy. In the following analysis, we show that the equilibrium outcomes under perfect empathy are quite different from those under imperfect empathy. Particularly, under perfect empathy, the local culture of a small country may go extinct due to international trade with a large country. We explain the mechanism of such extinction of local culture in this section.

Here, by perfect empathy, we mean that parents care about their children's welfare just as it is. Formally, perfect empathy was modeled by specifying the utility gain as $\Delta V_{ti} = u_{ti}\left(x_{ti}, y_{ti}\right)-u_{si}\left(x_{si}, y_{si}\right)$. That is, the valuation by a type-t parent of a type-s child's utility is exactly the utility of the child. Thus, in the perfect empathy case, the utility gain of a type-L parent and that of a type-C parent are exactly the opposite: $\Delta V_{LA}=u_{LA}\left(X,0\right)-u_{CA}\left(x_{CA}, y_{CA}\right)$

$= -\Delta V_{CA}$ and $\Delta V_{LB} = u_{LB}(0, Y) - u_{CB}(x_{CB}, y_{CB}) = -\Delta V_{CB}$. Simply put, in the case of perfect empathy, a parent wants their child to become the type with higher utility.

Therefore, a parent whose utility gain of direct socialization is negative may want to choose a negative effort; that is, the parent may want to dissuade the child from inheriting their type. However, allowing for negative effort does not fit well in the current framework because the direct socialization effort is modeled as the probability that the child will inherit the parent's type. Since we would like to analyze the perfect empathy case and the imperfect empathy case within the same framework, we assume that the parental direct socialization effort should be nonnegative. Therefore, a parent exerts zero direct socialization effort when their utility gain from direct socialization is negative.

4.1 Autarky

Under autarky, everyone consumes the good they are endowed. That is, $(x_{LA}, y_{LA}) = (x_{CA}, y_{CA}) = (X, 0)$ and $(x_{LB}, y_{LB}) = (x_{CB}, y_{CB}) = (0, Y)$. Then, ΔV_{ti}'s under autarky are given by $\Delta V_{LA} = u_{LA}(X,0) - u_{CA}(X,0) = -\Delta V_{CA}$, and $\Delta V_{LB} = u_{LB}(0,Y) - u_{CB}(0,Y) = -\Delta V_{CB}$. Although $u_{LA}(X, 0) - u_{CA}(X, 0)$ and $u_{LB}(0, Y) - u_{CB}(0, Y)$ can be positive or negative, to set ideas, we assume $u_{LA}(X, 0) > u_{CA}(X, 0)$ and $u_{LB}(0, Y) > u_{CB}(0, Y)$. In other words, when consuming the local good only in the same amount, a local-type consumer has higher utility than a cosmopolitan-type consumer. We consider this assumption reasonable because local types, who derive utility only from consuming the local good of their country, should have a stronger taste for the local good than cosmopolitan types who enjoy the local good and the foreign good. In fact, this assumption is satisfied when the utility function of cosmopolitan types is Cobb Douglas: $u_{CA}(X,0) = X^{\alpha} 0^{1-\alpha} = 0 < k_A X = u_{LA}(X,0)$.

Based on this assumption, we now investigate the population dynamics under autarky. Because the two countries are separate under autarky, the dynamics of q_A and q_B are determined separately. Recalling that the equation of motion for $\dot{q}_A$ is given by $\dot{q}_A = q_A(1-q_A)(d_{LA} - d_{CA})$, we see there are three possible points for $\dot{q}_A = 0$: $q_A = 0$, q_A=1, and interior q_A such that $d_{LA} = d_{CA}$.

First, we show that interior q_A does not exist. By the assumption that $u_{LA}(X, 0) > u_{CA}(X, 0)$, both local types and cosmopolitan types want their children to become local types. Thus, local-type parents exert positive direct socialization effort, whereas cosmopolitan-type parents exert zero effort (because negative effort cannot be chosen). This means $d_{LA} > 0$ and $d_{CA} = 0$ for $0 < q_A < 1$. Therefore, there exists no interior q_A such that $d_{LA} = d_{CA}$.

Second, we examine the dynamics around $q_A = 0$. Note that $\lim_{q_A \to 0} \partial \dot{q}_A / \partial q_A = \lim_{q_A \to 0} (d_{LA} - d_{CA}) > 0$ because $d_{LA} > 0$ and $d_{CA} = 0$ for $0 < q_A < 1$. Therefore, $q_A = 0$ is not a stable point.

Finally, notice that $\dot{q}_A = q_A(1-q_A)(d_{LA} - d_{CA}) > 0$ for $0 < q_A < 1$ because of $d_{LA} > 0$ and $d_{CA} = 0$ for $0 < q_A < 1$. So, starting from any initial interior points, q_A converges to 1. Of course, this argument applies to country B also. In both countries, eventually everyone becomes a local type under autarky.[8)]

8) Rigorously speaking, starting from any interior points, q_A gets closer and closer to 1, but it never reaches

4.2 International trade

When international trade is allowed, type-C consumers in each country exchange local goods, which means their consumption bundles are different to those of type-L consumers. The utility gains under international trade are given by

$$\Delta\tilde{V}_{LA}\left(p^*\right) = u_{LA}\left(X, 0\right) - u_{CA}\left(x_{CA}, y_{CA}\right) = -\Delta\tilde{V}_{CA}\left(p^*\right),$$
$$\Delta\tilde{V}_{LB}\left(1/p^*\right) = u_{LB}\left(0, Y\right) - u_{CB}\left(x_{CB}, y_{CB}\right) = -\Delta\tilde{V}_{CB}\left(1/p^*\right),$$

where we put a tilde sign on V's to distinguish them from ΔV_{ti}'s in the case of imperfect empathy in the last section.

4.2.1 Cultural dynamics

In this section, we investigate the cultural dynamics of q_A and q_B geometrically by constructing a phase diagram in the $q_A - q_B$ plane of $[0,1] \times [0,1]$. First, consider the optimal direct socialization effort in country A. The first order conditions for d_{LA} and d_{CA} are respectively

$$(1-q_A)\Delta\tilde{V}_{LA}\left(p^*\right) - C'(d_{LA}) \leq 0, d_{LA} \geq 0,$$

and

$$-q_A\Delta\tilde{V}_{LA}\left(p^*\right) - C'(d_{CA}) \leq 0, d_{CA} \geq 0.$$

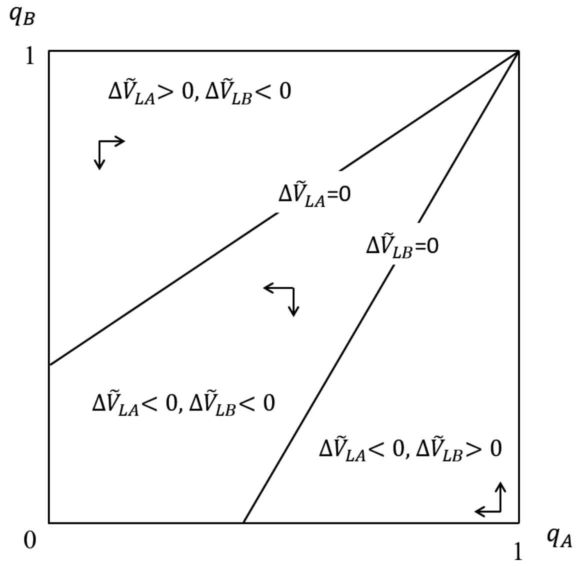
By inspecting these first order conditions, we immediately see that $d_{LA} > 0$ and $d_{CA} = 0$ if $\tilde{V}_{LA}\left(p^*\right) > 0$, that $d_{LA} = 0$ and $d_{CA} > 0$ if $\Delta\tilde{V}_{LA}\left(p^*\right) < 0$, and $d_{LA} = d_{CA} = 0$ if $\Delta\tilde{V}_{LA}\left(p^*\right) = 0$. These statements hold for any $0 < q_A < 1$ and $0 \leq q_B \leq 1$.[9] Given that $\dot{q}_A = q_A\left(1-q_A\right)\left(d_{LA} - d_{CA}\right)$, these results are straightforwardly translated into the dynamics of q_A: q_A is increasing when $\Delta\tilde{V}_{LA}\left(p^*\right) > 0$, decreasing when $\Delta\tilde{V}_{LA}\left(p^*\right) < 0$ and not changing when $\Delta\tilde{V}_{LA}\left(p^*\right) = 0$. Therefore, to characterize the dynamics of q_A in terms of q_A and q_B, we need to know the sign of $\Delta\tilde{V}_{LA}\left(p^*\right)$ on the $q_A - q_B$ plane.

Note that $\Delta\tilde{V}_{LA}\left(p^*\right)$ is affected by q_A and q_B only through p^*. Additionally, note that $\Delta\tilde{V}'_{LA}\left(p^*\right) < 0$ because u_{LA} $(X, 0)$ is independent of p^* whereas u_{CA} (x_{CA}, y_{CA}) is increasing in p^*. Then, combining this with $\partial p^* / \partial q_A > 0$ and $\partial p^* / \partial q_B < 0$, we have $\partial\Delta\tilde{V}_{LA}\left(p^*\right) / \partial q_A < 0$ and $\partial\Delta\tilde{V}_{LA}\left(p^*\right) / \partial q_B > 0$. Thus, when q_A is small and q_B is large, it is likely that $\Delta\tilde{V}_{LA}\left(p^*\right) > 0$; and when q_A is large and q_B is small, it is likely that $\Delta\tilde{V}_{LA}\left(p^*\right) < 0$. In fact, because u_{CA} (x_{CA}, y_{CA}) increases in p^* unboundedly and because $\lim_{q_A \to 1} p^* = \infty$, it must be that $\Delta\tilde{V}_{LA}\left(p^*\right) < 0$ as $q_A \to 1$. Additionally, because u_{CA} (x_{CA}, y_{CA}) approaches u_{CA} $(X, 0)$ as $p^* \to 0$ and because

1. This is because $\lim_{q_A \to 1} \partial\dot{q}_A / \partial q_A = -\lim_{q_A \to 1} d_{LA} + \lim_{q_A \to 1} d_{CA} = 0$.

9) Here, $q_A = 0$ and $q_A = 1$ are excluded because d_{LA} does not exist when $q_A = 0$, and d_{CA} does not exist when $q_A = 1$.

Figure 5: An example of a phase diagram in the case of perfect empathy



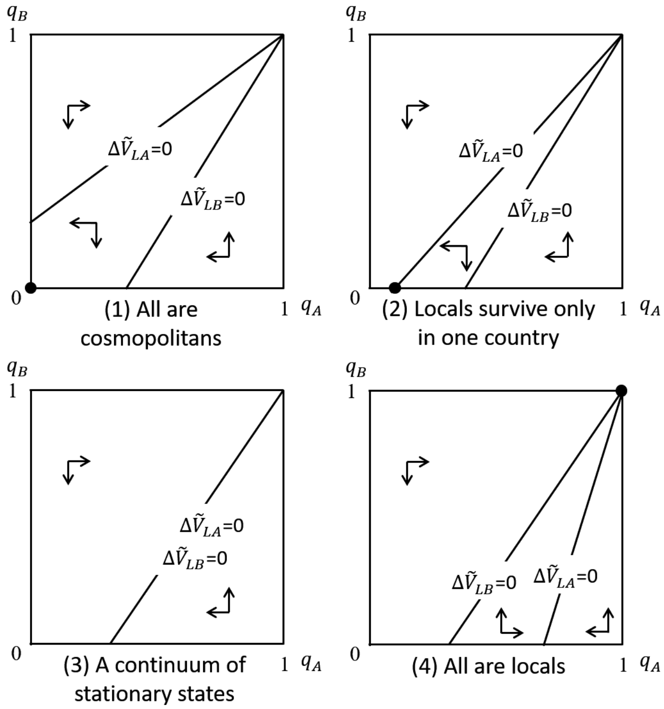
$\lim_{q_B \to 1} p^* = 0$, it must be that $\Delta\tilde{V}_{LA}\left(p^*\right) > 0$ as $q_B \to 1$. Therefore, in the $q_A - q_B$ plane of [0, 1] × [0, 1], there must be a positively sloped line representing $\Delta\tilde{V}_{LA}\left(p^*\right) = 0$ on which $\dot{q}_A = 0$, left of which $\Delta\tilde{V}_{LA}\left(p^*\right) > 0$ and thus $\dot{q}_A > 0$, and right of which $\Delta\tilde{V}_{LA}\left(p^*\right) < 0$ and thus $\dot{q}_A < 0$ (see Figure 5). As depicted in Figure 5, the $\Delta\tilde{V}_{LA}\left(p^*\right) = 0$ line divides the $q_A - q_B$ plane of [0, 1] × [0, 1] at the (1, 1) corner. This is because of $\Delta\tilde{V}_{LA}\left(p^*\right) < 0$ as $q_A \to 1$ and $\Delta\tilde{V}_{LA}\left(p^*\right) > 0$ as $q_B \to 1$. Moreover, the $\Delta\tilde{V}_{LA}\left(p^*\right) = 0$ line is, in fact, a straight line. We explain why it is a straight line in the Appendix.

The dynamics of q_B are characterized in a similar way. That is, the straight line representing $\Delta\tilde{V}_{LB}\left(1/p^*\right) = 0$ is positively sloped, and it divides the $q_A - q_B$ plane of [0, 1] × [0, 1] at the (1, 1) corner. On the $\Delta\tilde{V}_{LB}\left(1/p^*\right) = 0$ line, $\dot{q}_B = 0$; in the area below the $\Delta\tilde{V}_{LB}\left(1/p^*\right) = 0$ line, $\Delta\tilde{V}_{LB}\left(1/p^*\right) > 0$, and thus $\dot{q}_B > 0$; and in the area above the $\Delta\tilde{V}_{LB}\left(1/p^*\right) = 0$ line, $\Delta\tilde{V}_{LB}\left(1/p^*\right) < 0$, and thus $\dot{q}_B < 0$. This completes the phase diagram of q_A and q_B under international trade with perfect empathy. See Figure 5.

4.2.2 Stationary states

We can now discuss the stationary states using phase diagrams. As we found in Section 4.2.1, both the $\Delta\tilde{V}_{LA} = 0$ line and the $\Delta\tilde{V}_{LB} = 0$ line are straight lines and divide the $q_A - q_B$ plane of [0, 1] × [0, 1] at the (1, 1) corner. Depending on the slope of the $\Delta\tilde{V}_{LA} = 0$ line and that of the $\Delta\tilde{V}_{LB} = 0$ line, we have four possible configurations of the stationary states: (1) all are cosmopolitans; (2) locals survive only in one country; (3) there is a continuum of stable stationary states; and (4) all are locals. We show each case in Figure 6.

Figure 6: Four possible configurations of the stationary states in the case of perfect empathy: where $\Delta\tilde{V}_{LA} = 0$ and $\Delta\tilde{V}_{LB} = 0$ respectively represent $\dot{q}_A = 0$ and $\dot{q}_B = 0$.



In the first configuration, all agents are cosmopolitan types in both countries at the stable stationary state. That is, local cultures go extinct in both countries as a result of international trade. This occurs when both $\Delta\tilde{V}_{LA}$ and $\Delta\tilde{V}_{LB}$ are small, in the sense hat the area in which $\Delta\tilde{V}_{LA} > 0$ is small and so is the area in which $\Delta\tilde{V}_{LB} > 0$. In other words, this is the case where the utility of the local types is relatively small compared with that of the cosmopolitan types in both countries. In terms of parameters, this is likely to occur when both k_A and k_B are small because $\partial\Delta\tilde{V}_{LA} / \partial k_A = X > 0$ and $\partial\Delta\tilde{V}_{LB} / \partial k_B = Y > 0$.

The second configuration, "locals survive only in one country" is the case where one country, for example, country A, has both local types and cosmopolitan types while the other country, country B, has only cosmopolitans at the stationary state. This occurs when $\Delta\tilde{V}_{LA}$ is large and $\Delta\tilde{V}_{LB}$ is small. In other words, this is the case where the utility of the local types is relatively large compared with that of the cosmopolitan types in country A, but the opposite holds in country B. In terms of parameters, this is typically likely to occur when N, the population size of country A, is large because

$$\frac{\partial\Delta\tilde{V}_{LA}}{\partial N} = \Delta\tilde{V}'_{LA}\left(p^*\right)\frac{\partial p^*}{\partial N} > 0, \text{ and } \frac{\partial\Delta\tilde{V}_{LB}}{\partial N} = \Delta\tilde{V}'_{LB}\left(1/p^*\right)\frac{\partial\left(1/p^*\right)}{\partial N} < 0.$$

Intuitively, when the population of country A is large, the equilibrium price of its local good is low. Then, the cosmopolitan types in country A suffer from their low income while the local types in country A are not affected by the price. Thus, the local types tend to have higher utility than the cosmopolitan types in the large country and are more likely to survive. On the other hand, in the small country, country B, cosmopolitans can enjoy consuming cheap foreign good. Therefore, the utility of cosmopolitan types tends to be higher than that of local types, and eventually everyone becomes a cosmopolitan type in country B. This is the mechanism by which local culture goes extinct due to international trade when cultural transmission occurs through perfect empathy.

The third configuration is a continuum of stable stationary states where the $\Delta\tilde{V}_{LA} = 0$ line and the $\Delta\tilde{V}_{LB} = 0$ line coincide. In this case, any pairs (q_A, q_B) on the $\Delta\tilde{V}_{LA} = 0$ line are stable stationary states. This is the only case where local and cosmopolitan cultures coexist in both countries at the stationary state.

Finally, in the fourth configuration, all agents are local types in both countries at the stationary state, even when international trade is allowed. This occurs when both k_A and k_B are large so that the local-type consumers' utility is relatively high compared to the cosmopolitan types' utility.[10)]

Of these results, the third (i.e., a continuum of stable stationary states) is the only configuration in which there are interior stationary states: local types and cosmopolitan types coexist in both countries at the stationary state. However, because this configuration is a "knife-edge" case such that the $\Delta\tilde{V}_{LA} = 0$ line and the $\Delta\tilde{V}_{LB} = 0$ line happen to coincide, it is not likely to occur. Therefore, overall, when cultural transmission occurs through perfect empathy, at least one country loses its local culture as a result of international trade. This is in stark contrast to the case of imperfect empathy where both countries can preserve their local cultures at the stationary state of free-trade equilibrium.

Essentially, this difference in the long-run equilibria is understood as follows. When a parent has imperfect empathy, the parent has a biased preference to their own culture. So, whatever the economic conditions, the parent exerts some positive effort to ensure that their child inherit the parent's type. This is true for all parents. Therefore, both types of parents exert some positive effort toward direct socialization. As a result, what occurs at the long-run equilibrium is a sort of moderate, balanced outcome for which both cultural types exist. On the other hand, when a parent has perfect empathy, the parent wants their child to have a cultural type with higher utility, independent of the parent's own type. Therefore, the parents of a type with higher utility exert positive socialization effort toward ensuring their children inherit their own type while the parents of the other type with lower utility put zero socialization effort so that their children do not inherit the parents' type. Therefore, in the long-run equilibrium, the extreme outcome is reached where everyone is the same cultural type with higher utility.

5 Concluding remarks

In this study, we considered an evolutionary model of intergenerational cultural transmission to investigate the effect of international trade on the distribution of cultural types. We supposed that each country is endowed with its local good only, and that under autarky almost

10) The argument in footnote 8 applies here, too. Rigorously speaking, starting from any interior point, (q_A, q_B) gets closer and closer to $(1, 1)$ but never reaches it.

all agents are local types who consume only the local good of the country in which they live. Then, we examine the following questions. When a small fraction of the population in each country become cosmopolitan types who want to consume both local and foreign goods, how much does the population of cosmopolitan-type agents grow when international trade is allowed? Do local-type preferences go extinct as a result of international trade?

We find that the answers depend on the way in which intergenerational cultural transmission takes place. If parents evaluate their children's consumption choices in terms of their own utility (i.e., parents have imperfect empathy toward their children), there is a unique stable stationary state at which the local culture and the cosmopolitan culture coexist in each country. That is, we observe cultural diversity within a country and cultural similarity across countries as a result of international trade. However, if parents care about their children's utility exactly as it is (i.e., parents have perfect empathy toward their children), the coexistence of the local culture and the cosmopolitan culture at the trading equilibrium is unlikely to occur. The local culture can survive only in a country with a larger population.

Appendix

Proof of Lemma 1

First, we prove 1-(i). Given that $0 \le q_B < 1$, trade takes place, thus $\Delta V_{LA} > 0$. Then, unless $q_A = 1$, the first order condition for d_{LA} holds in equality: $(1-q_A)\Delta V_{LA} - C'(d_{LA}) = 0$. Then,

$$\frac{\partial d_{LA}}{\partial q_A} = \frac{-\Delta V_{LA} + (1-q_A)\Delta V'_{LA}\frac{\partial p^*}{\partial q_A}}{C''(d_{LA})},$$

where $\Delta V'_{LA}$ is the derivative of ΔV_{LA} with respect to p. The denominator is positive. With the expression for ΔV_{LA} given in equation (15), the numerator is calculated as follows.

$$\begin{aligned}
-\Delta V_{LA} + (1-q_A)\Delta V'_{LA}\frac{\partial p^*}{\partial q_A} &= -k_A\left(X - x_{CA}\left(p^*\right)\right) + \frac{(1-q_A)k_A\frac{\partial x_{CA}}{\partial p}N\left(X - x_{CA}\left(p^*\right)\right)}{(1-q_A)N\frac{\partial x_{CA}}{\partial p} + (1-q_B)\frac{\partial x_{CB}}{\partial p}} \\
&= \frac{-(1-q_B)\frac{\partial x_{CB}}{\partial p}}{(1-q_A)N\frac{\partial x_{CA}}{\partial p} + (1-q_B)\frac{\partial x_{CB}}{\partial p}} k_A\left(X - x_{CA}\left(p^*\right)\right) < 0.
\end{aligned}$$

This is because

$$\frac{\partial x_{CA}}{\partial p} = \frac{-\theta_A\left(\frac{p\theta'_A}{\theta_A} - 1\right)}{(p+\theta_A)^2}X \le 0 \text{ and } \frac{\partial x_{CB}}{\partial p} = -\frac{\frac{\theta'_B}{p^2} + \theta_B^2}{p^2\left(\frac{1}{p} + \theta_B\right)^2}Y < 0,$$

where $\theta'_A = \frac{d\theta_A}{dp} = -\frac{(\phi'_A)^2}{\phi_A \phi''_A} > 0$, $\frac{p\theta'_A}{\theta_A}$ is the elasticity of substitution assumed to be equal to or greater than one, and $\theta'_B = \frac{d\theta_B}{d(1/p)} = -\frac{(\phi'_B)^2}{\phi_B \phi''_B} > 0$. Therefore $\frac{\partial d_{LA}}{\partial q_A} < 0$.

Second, we show 1-(ii). Given that $0 \leq q_B < 1$, it holds that $\Delta V_{CA} > 0$. Then, unless q_A = 0, the first order condition for d_{CA} holds in equality: $q_A \Delta V_{CA} - C'(d_{CA}) = 0$. Then,

$$\frac{\partial d_{CA}}{\partial q_A} = \frac{\Delta V_{CA} + q_A \Delta V'_{CA} \frac{\partial p^*}{\partial q_A}}{C''(d_{CA})} > 0$$

because $\Delta V'_{CA} = \frac{\partial}{\partial p} \phi_A(\theta_A) x_{CA} = \theta_A \frac{\phi_A(\theta_A) X}{(p + \theta_A)^2} > 0$.

Third, we show 1-(iii). At q_A = 0, d_{CA} = 0 from equation (12). On the other hand, d_{LA} does not exist at q_A = 0 because there are no local types. However, we can calculate its limit as $q_A \to 0$. Note that $x_{CA}(p^*) < X$ when q_A = 0 and $0 \leq q_B < 1$. This is because cosmopolitans sell some of their endowment X under international trade. Therefore, $\Delta V_{LA} > 0$ when q_A = 0. This implies $\lim_{q_A \to 0} d_{LA} > 0$.

At q_A = 1, d_{LA} = 0 from equation (11). On the other hand, d_{CA} does not exist because there are no cosmopolitan types. However, we can calculate its limit as $q_A \to 1$. Recall that $\lim_{q_A \to 1} p^* = \infty$. Thus, as $q_A \to 1$, the income of a cosmopolitan type in country A increases infinitely. This leads to $\lim_{q_A \to 1} \Delta V_{CA} > 0$ because the utility of a cosmopolitan type $u_{CA}(x_{CA}, y_{CA})$ gets infinitely large. Therefore, $\lim_{q_A \to 1} d_{CA} > 0$.

Using (i), (ii), and (iii), we can show (iv). On the one hand, d_{LA} is positive when q_A is close to zero, monotonically decreasing in q_A, and reaches zero when q_A = 1. On the other hand, d_{CA} is zero when q_A = 0, monotonically increasing in q_A, and positive when q_A is close to 1. Therefore, there exists the unique q_A, $0 < q_A < 1$, at which $d_{LA} = d_{CA}$.

The statement 2 is proven in the similar way. ■

Proof of Proposition 1

We show that when condition 1 is satisfied, $\lim_{q_B \to 1} q_A(q_B) < 1$. Recall that $q_A(q_B)$ is defined as q_A such that $d_{LA} = d_{CA}$ for given $0 \leq q_B < 1$. From the first order conditions (11) and (12), it holds that $q_A(q_B) = \Delta V_{LA} / (\Delta V_{LA} + \Delta V_{CA})$. We know that as $q_B \to 1$, $p^* \to 0$. We also know that $\lim_{p^* \to 0} x_{CA} = X$, $\lim_{p^* \to 0} y_{CA} = 0$, and $\lim_{p^* \to 0} \theta_{CA} = 0$. Thus, we have that

$$\lim_{p^* \to 0} \Delta V_{LA} = \lim_{p^* \to 0} k_A(X - x_{CA}) = 0,$$
$$\lim_{p^* \to 0} \Delta V_{CA} = \lim_{p^* \to 0} \left(\phi_A(\theta_A) x_{CA} - \phi_A(0) X\right) = 0.$$

Hence, $\lim_{p^* \to 0} \Delta V_{LA} / (\Delta V_{LA} + \Delta V_{CA})$ is indeterminate. Then, to use L'Hopital's rule, we calculate

$$\Delta V_{LA}' = \frac{p^* \frac{\theta_A'}{\theta_A} - 1}{\left(p^* + \theta_A\right)^2} k_A \theta_A X.$$

Here, $p^* \frac{\theta_A'}{\theta_A}$ is the elasticity of substitution, which we assumed greater than or equal to one. Thus, $\Delta V_{LA}' \geq 0$. The derivative of ΔV_{CA} with respect to p is

$$\Delta V_{CA}' = \phi_A'\left(\theta_A\right)\theta_A' \frac{p^*}{p^* + \theta_A} X + \phi_A\left(\theta_A\right) \frac{\left(p^* + \theta_A\right) - p^*\left(1 + \theta_A'\right)}{\left(p^* + \theta_A\right)^2} X = \frac{\phi_A\left(\theta_A\right)}{\left(p^* + \theta_A\right)^2} \theta_A X,$$

where, in the last equality, we use $\phi_A\left(\theta_A\right) = \phi_A'\left(\theta_A\right)\left(p^* + \theta\right)$ from equation (1). Then,

$$\lim_{p^* \to 0} \frac{\Delta V_{LA}'}{\Delta V_{LA}' + \Delta V_{CA}'} = \lim_{p^* \to 0} \frac{\left(p^* \frac{\theta_A'}{\theta_A} - 1\right) k_A}{\left(p^* \frac{\theta_A'}{\theta_A} - 1\right) k_A + \phi_A\left(\theta_A\right)} = \frac{\left(\lim_{p^* \to 0} p^* \frac{\theta_A'}{\theta_A} - 1\right) k_A}{\left(\lim_{p^* \to 0} p^* \frac{\theta_A'}{\theta_A} - 1\right) k_A + \phi_A\left(0\right)}.$$

Therefore, $\lim_{q_B \to 1} q_A\left(q_B\right) < 1$ when condition 1 is satisfied.

In a similar way, it is proven that $\lim_{q_A \to 1} q_B\left(q_A\right) < 1$ when condition 2 is satisfied.

If at least one of condition 1 and condition 2 holds, $q_A(q_B)$ and $q_B(q_A)$ intersect at least once in the interior of $[0, 1] \times [0, 1]$. ■

Proof of Proposition 2

Let $\left(\bar{q}_A, \bar{q}_B\right)$ denote a stationary state. Linearizing the dynamic equations (8) and (9) at $\left(\bar{q}_A, \bar{q}_B\right)$, we have

$$\begin{aligned}
\dot{q}_A &= \left[\left(1 - \bar{q}_A\right)\left(d_{LA} - d_{CA}\right) - \bar{q}_A\left(d_{LA} - d_{CA}\right) + \bar{q}_A\left(1 - \bar{q}_A\right)\left(\frac{\partial d_{LA}}{\partial q_A} - \frac{\partial d_{CA}}{\partial q_A}\right)\right]\left(q_A - \bar{q}_A\right) \\
&\quad + \bar{q}_A\left(1 - \bar{q}_A\right)\left(\frac{\partial d_{LA}}{\partial q_B} - \frac{\partial d_{CA}}{\partial q_B}\right)\left(q_B - \bar{q}_B\right), \\
\dot{q}_B &= \bar{q}_B\left(1 - \bar{q}_B\right)\left(\frac{\partial d_{LB}}{\partial q_A} - \frac{\partial d_{CB}}{\partial q_A}\right)\left(q_A - \bar{q}_A\right) \\
&\quad + \left[\left(1 - \bar{q}_B\right)\left(d_{LB} - d_{CB}\right) - \bar{q}_B\left(d_{LB} - d_{CB}\right) + \bar{q}_B\left(1 - \bar{q}_B\right)\left(\frac{\partial d_{LB}}{\partial q_B} - \frac{\partial d_{CB}}{\partial q_B}\right)\right]\left(q_B - \bar{q}_B\right).
\end{aligned}$$

First, consider case (a) where $\left(\bar{q}_A, \bar{q}_B\right) = (0, 0)$. In this case, two characteristic roots of the linearized system are $(d_{LA} - d_{CA})$ and $(d_{LB} - d_{CB})$ evaluated at $(q_A, q_B) = (0, 0)$. From Lemma 1, we know that given $q_B = 0$, it holds that $\lim_{q_A \to 0} d_{LA} - \lim_{q_A \to 0} d_{CA} > 0$, and that given $q_A = 0$, it holds that $\lim_{q_B \to 0} d_{LB} - \lim_{q_B \to 0} d_{CB} > 0$. Therefore, two roots are both positive, meaning that the stationary state (0, 0) is unstable.

Second, consider case (b) where $(\bar{q}_A, \bar{q}_B) = (0, q_B(0))$. Recalling that $d_{LB} = d_{CB}$ at (0, $q_B(0)$), we have two characteristic roots ($d_{LA} - d_{CA}$) and $q_B(0)\ (1 - q_B(0)) \left(\frac{\partial d_{LB}}{\partial q_B} - \frac{\partial d_{CB}}{\partial q_B} \right)$ evaluated at (0, $q_B(0)$). Here, from Lemma 1, given $0 \leq q_B(0) < 1$, it holds that $\lim_{q_A \to 0} d_{LA} - \lim_{q_A \to 0} d_{CA} > 0$. Thus, the first root is positive. Additionally, from Lemma 1, we know that $\frac{\partial d_{LB}}{\partial q_B} < 0$ and $\frac{\partial d_{CB}}{\partial q_B} > 0$. Therefore, the second root is negative. Hence, the stationary state (0, $q_B(0)$) is a saddle point. Because q_A stays at zero for any points along $q_A = 0$, the saddle path toward (0, $q_B(0)$) is the vertical axis. By a similar argument, the stationary state in case (c), ($q_A(0)$, 0), is a saddle point whose saddle path is the horizontal axis.

Third, examine case (d) where $\bar{q}_A = 1$ and $0 \leq \bar{q}_B < 1$. Notice that $d_{LB} = d_{CB} = 0$ when $q_A = 1$ because no international trade occurs in this case. Two characteristic roots are then $-(d_{LA} - d_{CA})$ and $q_B(1 - q_B)\left(\frac{\partial d_{LB}}{\partial q_B} - \frac{\partial d_{CB}}{\partial q_B} \right)$ evaluated at $q_A = 1$ and $0 \leq q_B < 1$. Because $-\lim_{q_A \to 1} d_{LA} + \lim_{q_A \to 1} d_{CA} > 0$ for any $0 \leq q_B < 1$, the first root is positive. When $q_A = 1$, $d_{LB} = d_{CB} = 0$ for any q_B. Thus, it holds that $\frac{\partial d_{LB}}{\partial q_B} = \frac{\partial d_{CB}}{\partial q_B} = 0$ in this case. The second root is, therefore, zero. Hence, the stationary state in case (d) is unstable. By a similar argument, the stationary state in case (e) is unstable.

Finally, examine case (f), $(\bar{q}_A, \bar{q}_B) = (1, 1)$. Two characteristic roots in this case are $-(d_{LA} - d_{CA})$ and $-(d_{LB} - d_{CB})$ evaluated at (1, 1). Because there is no international trade in this case, $d_{LA} = d_{CA} = 0$ and $d_{LB} = d_{CB} = 0$. Therefore, two roots are zero. Now, consider interior points near (1, 1), such that $(1 - \varepsilon_A, 1 - \varepsilon_B)$ where ε_A and ε_B are positive and small. As shown in Proposition 1, if at least one of the conditions in Proposition 1 is satisfied, then $\lim_{q_B \to 1} q_A(q_B) < 1$ or $\lim_{q_A \to 1} q_B(q_A) < 1$ holds. Therefore, in the q_A–q_B plane of [0, 1] × [0, 1], there is a region to the right of $q_A(q_B)$ and above $q_B(q_A)$. Because $d_{LA} < d_{CA}$ and $d_{LB} < d_{CB}$ in this region, it holds that $\dot{q}_A < 0$ and $\dot{q}_B < 0$. Thus, any interior points $(1 - \varepsilon_A, 1 - \varepsilon_B)$ in this region are moving away from (1, 1).■

Proof of Proposition 3

Let (q_A^*, q_B^*) denote the intersection of $q_A(q_B)$ and $q_B(q_A)$. Linearizing the system of dynamic equations at (q_A^*, q_B^*), we have

$$\begin{bmatrix} \dot{q}_A \\ \dot{q}_B \end{bmatrix} = \begin{bmatrix} q_A^*\left(1-q_A^*\right)\left(\frac{\partial d_{LA}}{\partial q_A}-\frac{\partial d_{CA}}{\partial q_A}\right) & q_A^*\left(1-q_A^*\right)\left(\frac{\partial d_{LA}}{\partial q_B}-\frac{\partial d_{CA}}{\partial q_B}\right) \\ q_B^*\left(1-q_B^*\right)\left(\frac{\partial d_{LB}}{\partial q_A}-\frac{\partial d_{CB}}{\partial q_A}\right) & q_B^*\left(1-q_B^*\right)\left(\frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B}\right) \end{bmatrix} \begin{bmatrix} q_A - q_A^* \\ q_B - q_B^* \end{bmatrix}.$$

The sum of the two characteristic roots is negative:

$$q_A^*\left(1-q_A^*\right)\left(\frac{\partial d_{LA}}{\partial q_A}-\frac{\partial d_{CA}}{\partial q_A}\right)+q_B^*\left(1-q_B^*\right)\left(\frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B}\right)<0. \tag{20}$$

This is because $\frac{\partial d_{LA}}{\partial q_A}-\frac{\partial d_{CA}}{\partial q_A}<0$ and $\frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B}<0$ by Lemma 1. The product of the two characteristic roots is

$$q_A^*\left(1-q_A^*\right)q_B^*\left(1-q_B^*\right)\left[\left(\frac{\partial d_{LA}}{\partial q_A}-\frac{\partial d_{CA}}{\partial q_A}\right)\left(\frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B}\right)-\left(\frac{\partial d_{LA}}{\partial q_B}-\frac{\partial d_{CA}}{\partial q_B}\right)\left(\frac{\partial d_{LB}}{\partial q_A}-\frac{\partial d_{CB}}{\partial q_A}\right)\right], \tag{21}$$

whose sign is the same as the sign the of the square bracket term. It is calculated as follows.

$$\left(\frac{\partial d_{LA}}{\partial q_A}-\frac{\partial d_{CA}}{\partial q_A}\right)\left(\frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B}\right)-\left(\frac{\partial d_{LA}}{\partial q_B}-\frac{\partial d_{CA}}{\partial q_B}\right)\left(\frac{\partial d_{LB}}{\partial q_A}-\frac{\partial d_{CB}}{\partial q_A}\right)$$

$$=-\frac{\Delta V_{LB}+\Delta V_{CB}}{C''\left(d_{CB}\right)}\frac{\left(\left(1-q_A^*\right)\Delta V'_{LA}-q_A^*\Delta V'_{CA}\right)}{C''\left(d_{CA}\right)}\frac{\partial p^*}{\partial q_A}-\left(\frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B}\right)\frac{\Delta V_{LA}+\Delta V_{CA}}{C''\left(d_{LA}\right)} \tag{22}$$

$$=-\frac{\Delta V_{LA}+\Delta V_{CA}}{C''\left(d_{LA}\right)}\frac{\left(\left(1-q_B^*\right)\Delta V'_{LB}-q_B^*\Delta V'_{CB}\right)}{C''\left(d_{CB}\right)}\frac{\partial 1/p^*}{\partial q_B}-\left(\frac{\partial d_{LA}}{\partial q_A}-\frac{\partial d_{CA}}{\partial q_A}\right)\frac{\Delta V_{LB}+\Delta V_{CB}}{C''\left(d_{CB}\right)}, \tag{23}$$

where $\Delta V'_{LB}$ and $\Delta V'_{CB}$ are the derivatives of ΔV_{LB} and ΔV_{CB} with respect to $1/p$. Given that $\frac{\partial d_{LA}}{\partial q_A}-\frac{\partial d_{CA}}{\partial q_A}<0$ and $\frac{\partial d_{LB}}{\partial q_B}-\frac{\partial d_{CB}}{\partial q_B}<0$ (see proof of Lemma 1), the expression above is positive if

$$\left(1-q_A^*\right)\Delta V'_{LA}<q_A^*\Delta V'_{CA}, \tag{24}$$

or if

$$\left(1-q_B^*\right)\Delta V'_{LB}\le q_B^*\Delta V'_{CB} \tag{25}$$

Now, recall that at $\left(q_A^*, q_B^*\right)$, it holds that $d_{LA}=d_{CA}$. Combining this with the first order conditions (11) and (12), we have $1-q_A^*=q_A^*\Delta V_{CA}/\Delta V_{LA}$. Substituting this into (24) gives

$$\frac{\Delta V'_{LA}}{\Delta V_{LA}} \leq \frac{\Delta V'_{CA}}{\Delta V_{CA}}. \tag{26}$$

Similarly, (25) is equivalent to

$$\frac{\Delta V'_{LB}}{\Delta V_{LB}} \leq \frac{\Delta V'_{CB}}{\Delta V_{CB}}. \tag{27}$$

Therefore, (21) is positive if at least one of (26) and (27) is satisfied at $\left(q_A^*, q_B^*\right)$. This proves the stability of $\left(q_A^*, q_B^*\right)$.

Moreover, when at least one of (26) and (27) is satisfied, $\left(q_A^*, q_B^*\right)$ is unique. This is shown as follows. Suppose first that both (26) and (27) are satisfied at $\left(q_A^*, q_B^*\right)$. Then, both $q_A\,(q_B)$ and $q_B\,(q_A)$ are nonnegatively sloped at the intersection because

$$q'_A(q_B) = \frac{-\left(\dfrac{\partial d_{LA}}{\partial q_B} - \dfrac{\partial d_{CA}}{\partial q_B}\right)}{\dfrac{\partial d_{LA}}{\partial q_A} - \dfrac{\partial d_{CA}}{\partial q_A}} = \frac{-\dfrac{\left(1-q_A^*\right)\Delta V'_{LA} - q_A^*\Delta V'_{CA}}{C''\left(d_{CA}\right)}\dfrac{\partial p^*}{\partial q_B}}{\dfrac{\partial d_{LA}}{\partial q_A} - \dfrac{\partial d_{CA}}{\partial q_A}} \geq 0, \tag{28}$$

and

$$q'_B(q_A) = \frac{-\left(\dfrac{\partial d_{LB}}{\partial q_A} - \dfrac{\partial d_{CB}}{\partial q_A}\right)}{\dfrac{\partial d_{LB}}{\partial q_B} - \dfrac{\partial d_{CB}}{\partial q_B}} = \frac{-\dfrac{\left(1-q_B^*\right)\Delta V'_{LB} - q_B^*\Delta V'_{CB}}{C''\left(d_{CB}\right)}\dfrac{\partial (1/p^*)}{\partial q_A}}{\dfrac{\partial d_{LB}}{\partial q_B} - \dfrac{\partial d_{CB}}{\partial q_B}} \geq 0. \tag{29}$$

When both $q_A\,(q_B)$ and $q_B\,(q_A)$ are nonnegatively sloped at the intersection, the stability of $\left(q_A^*, q_B^*\right)$ implies that at the intersection the $q_A\,(q_B)$ curve is steeper than the $q_B\,(q_A)$ curve in the $q_A - q_B$ plane. Given that both $q_A\,(q_B)$ and $q_B\,(q_A)$ are continuous functions and they are non-negatively sloped at the intersection, if the intersections were multiple, there should be at least one intersection at which $q_A\,(q_B)$ is flatter than $q_B\,(q_A)$. This is a contradiction.

Second, suppose that (26) is violated and (27) is satisfied. Then, by (26) and (27), $q_A\,(q_B)$ is negatively sloped and $q_B\,(q_A)$ is nonnegatively sloped at the intersection. Given that both $q_A\,(q_B)$ and $q_B\,(q_A)$ are continuous functions, if there were another intersection, it should be the one at which $q_A\,(q_B)$ and $q_B\,(q_A)$ are both nonnegatively sloped or both nonpositively sloped. This is a contradiction. The same argument applies when (26) is satisfied and (27) is violated.

Therefore, the intersection of $q_A\,(q_B)$ and $q_B\,(q_A)$ is unique given that at least one of (26) and (27) is satisfied. Since there is no other stable stationary state, it is globally stable.■

Proof of Corollary 1

Let $u_{CA}\left(x, y\right) = \left(\alpha x^\rho + \left(1-\alpha\right) y^\rho\right)^{\frac{1}{\rho}}$. Then, $\phi_A\left(\theta_A\right) = \left(\alpha + \left(1-\alpha\right)\theta_A^\rho\right)^{\frac{1}{\rho}}$. We calculate

that

$$
\begin{aligned}
&\left(1-q_A^*\right)\Delta V_{LA}' - q_A^*\Delta V_{CA}' \\
&= \frac{\left(1-q_A^*\right)(\sigma-1)}{\left(p^*+\theta_A\right)^2}\theta_A k_A X - q_A^*\left(\frac{\phi_A'(\theta_A)\phi_A' p^*}{p^*+\theta_A} + \frac{\phi_A(\theta_A)\theta_A(1-\sigma)}{\left(p^*+\theta_A\right)^2}\right)X \\
&= \frac{q_A^* X}{\left(p^*+\theta_A\right)}\left[\left(\phi_A(\theta_A)-\phi_A(0)\right)(\sigma-1) - \phi_A'(\theta_A)\theta_A\sigma\right],
\end{aligned}
\tag{30}
$$

where $\sigma = 1/(1-\rho)$ is the constant elasticity of substitution parameter. Here, to derive the second equality, we use $\left(1-q_A^*\right)\Delta V_{LA} = q_A^*\Delta V_{CA}$ because $d_{LA} = d_{CA}$ at $\left(q_A^*, q_B^*\right)$.

Now, we examine the sign of the bracketed term of (30). Because q_A and q_B affect the bracketed term only through p^*, it suffices to show that the bracketed term is negative for any $p^* > 0$.

When $p^* = 0$, then $x_{CA} = X$, $y_{CA} = 0$, and $\theta_A = 0$. Thus, when $p^* = 0$, the bracketed term is zero because the first term in the bracket is zero when $\theta_A = 0$, and the second term

$\phi_A'(\theta_A)\theta_A = \dfrac{(1-\alpha)\theta_A^\rho}{\alpha+(1-\alpha)\theta_A^\rho}\phi_A(\theta_A) = 0$ when $\theta_A = 0$.

Taking the derivative of the bracketed term with respect to p, we have

$$
\frac{d}{dp}\left[\left(\phi_A(\theta_A)-\phi_A(0)\right)(\sigma-1) - \phi_A'(\theta_A)\theta_A\sigma\right] = -\frac{(1-\alpha)\theta_A^\rho}{\alpha+(1-\alpha)\theta_A^\rho}\phi_A'(\theta_A)\theta_A'.
$$

This is zero when $p^* = 0$ and negative for $p^* > 0$. Because q_A (q_B) lies strictly between zero and one for any $0 \le q_B < 1$, we have $p^* > 0$. Therefore, $\left(1-q_A^*\right)\Delta V_{LA}' - q_A^*\Delta V_{CA}' < 0$ at $\left(q_A^*, q_B^*\right)$.

Finally, confirm that the condition in Proposition 1 is satisfied. Notice that $\lim_{p^*\to 0} p^* \dfrac{\theta_A'}{\theta_A} = \sigma$, $1 < \sigma < \infty$ by assumption, and $\phi_A(0) = \alpha^{\frac{1}{\rho}} > 0$. So, condition 1 in Proposition 1 is satisfied.

Similarly, when $u_{CB}(x, y)$ is CES, we can show that $\left(1-q_B^*\right)\Delta V_{LB}' - q_B^*\Delta V_{CB}' < 0$ at $\left(q_A^*, q_B^*\right)$, and condition 2 in Proposition 1 is satisfied.■

The reason why $\Delta\tilde{V}_{LA}\left(p^*\right) = 0$ is represented by a straight line.

Consider the value of p^* that solves the equation $\Delta\tilde{V}_{LA}\left(p^*\right) = 0$. Then, substituting this value of p^* into the market-clearing condition, we have

$$\left(1-q_A\right)Nx_{CA}\left(p^*\right)+\left(1-q_B\right)x_{CB}\left(\frac{1}{p^*}\right)=\left(1-q_A\right)NX. \tag{31}$$

This equation determines the locus of (q_A, q_B) that makes the equilibrium price just equal to the value of $p*$ such that $\Delta\tilde{V}_{LA}\left(p^*\right)=0$. Since q_A and q_B enters linearly into (31), $\Delta\tilde{V}_{LA}\left(p^*\right)=0$ is represented by a straight line.

References

Bala, Venkatesh, and Ngo Van Long (2005), "International Trade and Cultural Diversity with Preference Selection," *European Journal of Political Economy* Vol. 21, No. 1, pp. 143–162.

Bisin, Alberto, and Thierry Verdier (2001), "The Economics of Cultural Transmission and the Dynamics of Preferences," *Journal of Economic Theory* Vol. 97, No. 2, pp. 298–319.

Bisin, Alberto, and Thierry Verdier (2014), "Trade and Cultural Diversity, in Victor A. Ginsburgh and David Throsby eds.," *Handbook of the Economics of Art and Culture* Vol. 2, Elsevier, pp. 439–484.

Cowen, Tyler (2002), *Creative Destruction: How Globalization Is Changing the World's Cultures*, Princeton University Press.

Maystre, Nicolas, Jacques Olivier, Mathias Thoenig, and Thierry Verdier (2014), "Product-Based Cultural Change: Is the Village Global?," *Journal of International Economics* Vol. 92, No. 2, pp. 212–230.

Olivier, Jacques, Mathias Thoenig, and Thierry Verdier (2008), Globalization and the Dynamics of Cultural Identity, *Journal of International Economics* Vol. 76, No. 2, pp. 356–370.