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Firm turnover in the export market and the case for fixed exchange rate regime*

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Abstract

This paper revisits the case for flexible vs. fixed exchange rate regime in a two-country model with firm heterogeneity and nominal wage rigidity under incomplete financial markets. Dampening nominal exchange rate fluctuations simultaneously stabilizes the firm turnover in the export market. When firms are homogeneous and low productive, the fixed exchange rate regime dominates the flexible one because it reduces the fluctuations in labor demand arising from entry and exit of exporters following a demand shock. We also show that an alternative regulation policy in the export market does not rule out the possible adoption of a managed floating regime.

JEL Classification Codes: F32, F41, E40.

Keywords: monetary policy, exchange rate regime, firm heterogeneity.

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1 Introduction

Over the past few years, policymakers have adopted measures aimed at protecting industries exposed to trade, in particular in countries where the economic activity is mainly driven by external demand. The main objective of these policies is to stabilize exports that are mainly realized by large and high productive firms. These policy decisions primarily take two forms: i) trade policies, like changes in trade tariffs, or ii) exchange rate policies, for instance moving from a fully floating regime to a more managed floating. This practice, sometimes dubbed "currency manipulation" in the public debate, has the advantage of reducing the fluctuations in the nominal exchange rate. In turn, the reduced fluctuations in the nominal exchange rate tend to dampen the fluctuations in exports. As shown in panel a) of figure 1, a lower volatility of the exchange rate is indeed associated with a lower volatility of exports. Moreover, the positive correlation between the volatility of exchange rate and exports is larger for countries adopting a fully floating regime (panel d) than a crawling peg (panel b) or a managed floating (panel c). As a result, policymakers may wish to move from a fully floating regime to a less flexible one, in order to decrease both the volatility of exchange rate and the volatility of exports. In addition, a partial management of the exchange rate has the advantage of limiting the fluctuations of profits from trade, hence regulating the turnover of exporters.

In this paper, we revisit the case for flexible vs. fixed exchange rate regime in a framework where firm dynamics respond to demand shocks. In the model, a stochastic shift in world demand generates fluctuations in trade and firm dynamics. The conduct of monetary policy determines the exchange rate regime, and therefore acts as a stabilization tool. In particular, we explore the impact of the choice of the exchange rate regime on the firm turnover in the export market. In order to do so, we provide a two-country dynamic stochastic general equilibrium model with firm heterogeneity and nominal wage rigidity, in a framework simple enough to have a closed form analytical solution. We first compare the welfare between the two polar cases of fixed vs. flexible exchange rate regimes. We then characterize the optimal monetary policy that maximizes the welfare of households.

The presence of wage rigidities generates a trade-off for monetary authorities in designing the exchange rate regime. In the flexible exchange rate regime, a positive (negative) demand shift on goods produced at Home induces a trade surplus (deficit) and subsequent nominal exchange rate appreciation (depreciation). This nominal appreciation (depreciation) results in a lower (higher) sales and profits in Home currency unit, leading to the exit (entry) of Home exporters. On the domestic market instead, there is a relatively small adjustment both in the entry of new firms and overall production since the nominal appreciation largely absorbs the demand shock. Flexible exchange rate regime thus dampens the fluctuations in the domestic market at the expense of the export market. At the same time, the above mentioned adjustment under flexible regime creates undesirable allocation of domestic and tradable goods. Indeed, the nominal appreciation favors the consumption of less expensive foreign goods at the expense of domestic goods, which is opposite to the preference shift. In contrast, under a fixed exchange rate regime, following a positive (negative) demand shift for Home produced goods, the monetary intervention increases (decreases) aggregate demand and investment in Home while it sterilizes the turnover of exporters leaving the exchange rate unaffected. Choosing a fixed exchange rate regime thus inevitably has a characteristic of "currency manipulation" in our setting. Moreover, the fixed regime realizes a desirable allocation of goods, as the consumption of domestic goods co-moves with the preference shift.

The main novelty of this paper is to introduce a new dimension in the well-known trade-off on the choice of the exchange rate regime. Specifically, we emphasize the role played by firm heterogeneity and the selection into exporting market. In an economy where firm productivity is highly dispersed, only a few highly productive firms are able to export their products. In this case, external demand shocks for the Home produced good do not translate in large fluctuations in labor demand at the extensive margin of trade, because only few exporter firms are subject to those fluctuations. As a consequence, the flexible regime absorbs the fluctuations in labor demand for the Home produced goods. Since households set wages one period in advance, based on their expectations on labor demand, the equilibrium wage can be lower because of the limited fluctuations in labor demand. On the other hand, when firm productivity is less dispersed, the fluctuations on external demand may induce a larger fraction of firms to enter or exit the exporting market. As households expect the labor demand to move substantially in response to demand shocks, they set higher wages. However, in this economy a fixed exchange rate regime sterilizes the fluctuations on labor demand due to the entry and exit of firms in the export market, and therefore relatively dampens the increase in wages. Our results suggest that the fixed regime deals better with wage rigidities when the dispersion of firm productivity is low, that is when many firms at the extensive margin are subject to fluctuations in external demand. The reverse is true for an economy where firm productivity is highly dispersed.

We then depart from the two polar cases of exchange rate regimes, and derive the optimal monetary policy under demand uncertainty. In line with the previous results, we find that the optimal variability in nominal exchange rate is smaller in an economy where firm productivity is less dispersed. We also explore alternative regulation policies aimed at reducing the fluctuations in the exporting market. When comparing the welfare under such trade policy and the fixed exchange rate regime, we show that the temptation to fix the exchange rate cannot be removed, as this policy still provides the higher welfare when firm productivity is highly homogeneous.

This paper belongs to the literature on open economy with firm heterogeneity, which seminal paper is Ghironi and Melitz (2005). In line with our setting, several papers emphasize the trade adjustments occurring at both the intensive and extensive margins of trade with or without firm heterogeneity (see among others Corsetti et al. (2007), Corsetti et al. (2013), Pappadà (2011), and Hamano (2014)). In this paper, we depart from them as we introduce a nominal rigidity that allows us to discuss the choice of exchange rate regime and the firm dynamics. While our unique source of nominal rigidity is the wage rigidity, an alternative source of nominal rigidity that has been extensively studied in the literature is the price rigidity. Moreover, the practice of pricing to market and dollar pricing by exporters has also been emphasized in the literature (Betts and Devereux (1996), Devereux and Engel (2003), Corsetti et al. (2010) and Gopinath et al. (2010) among others). Importantly this price rigidity in exporting market breaks down the "expenditure switching effect" of the nominal exchange rate, and allows deviations from the “divine coincidence” where flexible exchange rate regime dominates serving as “shock absorber” (Friedman (1953) and Mundell (1961)). We do not introduce this type of distortion related to the pricing behavior of exporters. Instead, they freely adjust prices in exporting market accordingly with the exchange rate fluctuations (producer currency pricing). However, the fixed exchange rate regime may dominate because of financial market incompleteness as shown by Devereux (2004) and Hamano and Picard (2017). While Devereux (2004) highlights the role of the elasticity of labor supply and Hamano and Picard (2017) emphasizes the role of preference for product variety in ranking the exchange rate regime, we study how the heterogeneity in firm productivity shapes the response of the economy to demand shocks. Finally our paper is related to the recent debate on protectionism. Barattieri et al. (2018) study a temporary tariff shock in a model with heterogeneous firms and cast a doubt for its effectiveness as a macroeconomic stabilization tool. Auray et al. (2019) investigate the optimal tariff policy under different exchange rate regimes and analyze the cyclical properties of protectionism. While this literature studies trade policies, we rather focus on the ability of the monetary policy to act as a powerful macroeconomic stabilization tool for the export market. In this respect, our paper is reminiscent of Bergin and Corsetti (2019) which study the allocation effect of monetary policy that might change the comparative advantage of countries.

The paper is structured as follows. In the next section, we introduce a two country model with external demand shocks and provide an analytical solution of our model. In section 3, we show how the monetary policy responds to external demand shocks when the exchange rate regime is fixed or flexible. Section 4 reports the welfare analysis and shows the optimal exchange rate regime as a function of the fundamentals of the economy. Section 5 provides one extension with an alternative trade policy for stabilizing the shocks on expected labor demand. Section 6 concludes.

2 The Model

In this section, we introduce a two country dynamic stochastic general equilibrium model with firm heterogeneity along the lines of Ghironi and Melitz (2005). Both Home and Foreign countries are inhabited by a unit mass of households which provide imperfectly-substituted labor. We denote Foreign variables with an asterisk (*). We introduce a nominal rigidity, as wages are set one period in advance based on the expectations of future labor demand. All goods are tradable but only a fraction of them are exported by firms operating in monopolistic competition, and the number of exporters is determined endogenously. We introduce demand shock to each countries' goods, and study how these shocks interacts with firm dynamics according to the conduct of monetary policy.[1]

2.1 Households

The representative household maximizes her life time utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t(j)$, where β $(0 < \beta < 1)$ is the exogenous discount factor. Utility of individual household j at time t depends on consumption $C_t(j)$ and labor supply $L_t(j)$ as follows

$$U_t(j) = \ln C_t(j) + \chi \ln \frac{M_t(j)}{P_t} - \eta \frac{[L_t(j)]^{1+\varphi}}{1+\varphi},$$

where χ and η represent the degree of satisfaction (unsatisfaction) from real money holdings and labor supply respectively, while the parameter φ measures the inverse of the Frisch elasticity of labor supply.

The basket of goods $C_t(j)$ is defined as

$$C_t(j) = \left(\frac{C_{H,t}(j)}{\alpha_t}\right)^{\alpha_t} \left(\frac{C_{F,t}(j)}{\alpha_t^*}\right)^{\alpha_t^*},$$

where α_t and α_t^* are the preference attached to the bundle of goods produced in Home

[1]Note that in our economy, the distribution of firm productivity represents a fundamental of the economy, which is taken by given by monetary authorities when choosing the exchange rate regime. We therefore disregard the potential feedback effects of the exchange rate regime on the distribution of firm productivity.

$C_{H,t}(j)$ and imported goods ($C_{F,t}(j)$), respectively. These preferences are assumed to be stochastic. Furthermore, these baskets are defined over a continuum of goods Ω as

$$C_{H,t}(j) = \left(\int_{\varsigma\in\Omega} c_{D,t}\left(j,\varsigma\right)^{1-\frac{1}{\sigma}} d\varsigma\right)^{\frac{1}{1-\frac{1}{\sigma}}},\ C_{F,t}(j) = \left(\int_{\varsigma^*\in\Omega} c_{X,t}\left(j,\varsigma^*\right)^{1-\frac{1}{\sigma}} d\varsigma^*\right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

In each time period, only a subset of variety of goods is available from the total universe of variety of goods Ω. We denote $N_{D,t}$ and $N^*_{X,t}$ as the number of domestic and imported product varieties, respectively. $c_{D,t}\left(j,\varsigma\right)$ and $c_{X,t}\left(j,\varsigma^*\right)$ represent the demand addressed for individual product variety indexed by ς and ς^*. σ denotes the elasticity of substitution among differentiated goods and is greater than 1.

The optimal consumption for each domestic basket, imported basket and individual product variety are found to be

$$C_{H,t}(j) = \left(\frac{P_{H,t}}{P_t}\right)^{-1} \alpha_t C_t(j), \quad C_{F,t}(j) = \left(\frac{P_{F,t}}{P_t}\right)^{-1} \alpha^*_t C_t(j),$$

$$c_{D,t}\left(j,\varsigma\right) = \left(\frac{p_{D,t}\left(\varsigma\right)}{P_{H,t}}\right)^{-\sigma} C_{H,t}(j), \quad c_{X,t}\left(j,\varsigma^*\right) = \left(\frac{p^*_{X,t}\left(\varsigma^*\right)}{P_{F,t}}\right)^{-\sigma} C_{F,t}(j).$$

$p_{D,t}\left(\varsigma\right)$ stands for the price of product variety ς which is domestically produced. In particular, $p^*_{X,t}\left(\varsigma^*\right)$ denotes the price of imported product variety ς^*, denominated in currency unit in Home. $P_{H,t}$ and $P_{F,t}$ are the price of basket of goods produced in Home and that of imported, respectively. P_t is the price of aggregated basket. Price indexes that minimize expenditures on each consumption basket are

$$P_t = P_{H,t}^{\alpha_t} P_{F,t}^{\alpha^*_t},$$

$$P_{H,t} = \left(\int_{\varsigma\in\Omega} p_{D,t}\left(\varsigma\right)^{1-\sigma} d\varsigma\right)^{\frac{1}{1-\sigma}}, \quad P_{F,t} = \left(\int_{\varsigma^*\in\Omega} p^*_{X,t}\left(\varsigma^*\right)^{1-\sigma} d\varsigma^*\right)^{\frac{1}{1-\sigma}}.$$

Similar expressions hold for Foreign. Crucially, the subset of goods available to Foreign during period t, $\Omega^*_t \in \Omega$, can be different from the subset of goods available to Home $\Omega_t \in \Omega$.

2.2 Production, Pricing and the Export Decision

There is a mass of $N_{D,t}$ number of firms in Home. Upon entry, firms draw their productivity level z from a distribution $G(z)$ on $[z_{\min}, \infty)$. Since there are no fixed production costs and hence no selection into domestic market, $G(z)$ also represents the productivity distribution of all producing firms. Prior to entry, however, these firms are identical and face sunk entry cost of $f_{E,t} = l_{E,t}$ amounts of labor. The sunk cost is composed of imperfectly differentiated labor services provided by households (indexed by i) such that

$$l_{E,t} = \left(\int_0^1 l_{E,t}(j)^{1-\frac{1}{\theta}} dj \right)^{\frac{1}{1-\frac{1}{\theta}}}, \tag{1}$$

where θ represents the elasticity of substitution among different labor services. We consider $f_{E,t}$ to be exogenous. By defining the nominal wage for type j labor as $W_t(j)$,total cost for a firm to setup is thus $\int_0^1 l_{E,t}(j) W_t(j) dj$. The cost minimization yields the following labor demand for type j labor service:

$$l_{E,t}(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta} l_{E,t}, \tag{2}$$

where W_t denotes the corresponding wage index, which is

$$W_t = \left(\int_0^1 W_t(j)^{1-\theta} dj \right)^{\frac{1}{}}.$$

Exporting requires an operational fixed cost of $f_{X,t} = l_{f_X,t}$ amount of labor defined in a similar way as (1). The cost minimization provides a similar demand for each specific labor service as (2).[2] Only a subset of firms whose productivity level z is above the cutoff level $z_{X,t}$ exports by charging sufficiently lower prices and earning positive profits despite the existence of fixed export cost $f_{X,t}$. Thus, non-tradeness in the economy arises endogenously with changes in the productivity cutoff $z_{X,t}$.

[2] These are specifically,

$$l_{f_X,t} = \left(\int_0^1 l_{f_X,t}(j)^{1-\frac{1}{\theta}} dj \right)^{\frac{1}{1-\frac{1}{\theta}}}, \qquad l_{f_X,t}(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta} l_{f_X,t}.$$

For production of each product variety, only composite labor basket is required as input. Thus the production function of firm with productivity z is given by $y_t(z) = z l_t(z)$ where

$$l_t(z) = \left(\int_0^1 l_t(z,j)^{1-\frac{1}{\theta}} \, dj \right)^{\frac{1}{1-\frac{1}{\theta}}}.$$

The cost minimization yields the demand for type j labor for production as

$$l_t(z,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta} l_t(z).$$

The firm faces a residual demand curve with constant elasticity σ. The production scale is thus determined by the demand addressed to the firm under monopolistic competition. Profit maximization yields the following optimal price $p_{D,t}(z)$ by firm with productivity z:

$$p_{D,t}(z) = \frac{\sigma}{\sigma - 1} \frac{W_t}{z}.$$

If the firm exports, its price of export is $p_{X,t}(z) = \tau p_{D,t}(z) \varepsilon_t^{-1}$ where ε_t is the nominal exchange rate defined as the price of one unit of foreign currency in terms of home currency units. $\tau > 1$ is iceberg trade cost. In our definition, $p_{X,t}(z)$ is thus denominated in terms of foreign currency units.

Total firm profits $D_t(z)$ can be decomposed into those from domestic sales $D_{D,t}(z)$ and those from exporting sales $D_{X,t}(z)$ (if the firm exports) as $D_t(z) = D_{D,t}(z) + D_{X,t}(z)$. Using the demand functions found previously and aggregate consumption defined as $C_t = \left(\int_0^1 C_t^{1-\frac{1}{\sigma}}(j) \, dj \right)^{\frac{1}{1-\frac{1}{\sigma}}}$, we can write the profits from each market as

$$D_{D,t}(z) = \frac{1}{\sigma} \left(\frac{p_{D,t}(z)}{P_{H,t}} \right)^{1-\sigma} \alpha_t P_t C_t, \tag{3}$$

$$D_{X,t}(z) = \frac{\varepsilon_t}{\sigma} \left(\frac{p_{X,t}(z)}{P_{H,t}^*} \right)^{1-\sigma} \alpha_t P_t^* C_t^* - W_t f_X, \qquad \text{if the firm } z \text{ exports} \tag{4}$$

2.3 Firm Averages

Given a distribution $G(z)$, the productivity level of a mass of $N_{D,t}$ domestically producing firms is distributed over $[z_{\min}, \infty)$. Among these firms, there are $N_{X,t} = [1 - G(z_{X,t})] N_{D,t}$ exporters in Home. Following Melitz (2003) and Ghironi and Melitz (2005), we define two average productivity levels, $\widetilde{z}_D$ for domestically producing firms and $\widetilde{z}_{X,t}$ for exporters as follows

$$\widetilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}, \quad \widetilde{z}_{X,t} \equiv \left[\frac{1}{1 - G(z_{X,t})} \int_{z_{X,t}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}.$$

These average productivity levels summarize all the information about the distribution of firm productivity. Given these averages, we define the average real domestic and exporting price as $\widetilde{p}_{D,t} \equiv p_{D,t}(\widetilde{z}_D)$ and $\widetilde{p}_{X,t} \equiv p_{X,t}(\widetilde{z}_{X,t})$, respectively. We also define average profits from domestic sales and exporting sales as $\widetilde{D}_{D,t} \equiv D_{D,t}(\widetilde{z}_D)$ and $\widetilde{D}_{X,t} \equiv D_{X,t}(\widetilde{z}_{X,t})$. Finally, average profits among all firms is given by $\widetilde{D}_t = \widetilde{D}_{D,t} + (N_{X,t}/N_{D,t}) \widetilde{D}_{X,t}$.

2.4 Firm Entry and Exit

New entrants need one time period to be built. Firm entry takes place until the expected value of entry is equalized with entry cost, leading to the following free entry condition:

$$\widetilde{V}_t = f_{E,t} W_t, \tag{5}$$

where $\widetilde{V}_t$ is the expected value of entry which is discussed below. For the tractability of the solution of the model, firms are assumed to depreciate by 100 % after production.

2.5 Parametrization of Productivity Draws

We assume the following Pareto distribution for $G(z)$:

$$G(z) = 1 - \left(\frac{z_{\min}}{z} \right)^{\kappa},$$

where $z_{\min}$ is the minimum productivity level, and $\kappa > \sigma - 1$ is the shape parameter. With this parametrization, we have

$$\widetilde{z}_D = z_{\min}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \qquad \widetilde{z}_{X,t} = z_{X,t}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}.$$

The share of exporters in the total number of domestic firms is then given by

$$\frac{N_{X,t}}{N_{D,t}} = z_{\min}^{\kappa}\left(\widetilde{z}_{X,t}\right)^{-\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}.$$

Finally, there exists a firm with a specific productivity cutoff $z_{X,t}$ that earns zero profits from exporting, as $D_{X,t}\left(z_{X,t}\right) = 0$. With the above Pareto distribution, this implies that

$$\widetilde{D}_{X,t} = W_t f_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}.$$

2.6 Household Budget Constraints and Intertemporal Choices

A household j in Home faces the following budget constraint at time t:

$$\begin{aligned} &P_t C_t(j) + B_t(j) + M_t(j) + x_t(j) N_{D,t+1}\widetilde{V}_t \\ &\quad = (1+\xi) W_t(j) L_t(j) + (1+i_{t-1}) B_{t-1}(j) + M_{t-1}(j) + x_{t-1}(j) N_{D,t}\widetilde{D}_t + T_t^f, \quad (6) \end{aligned}$$

where $B_t(j)$ and $x_t(j)$ denote bond holdings and share holdings of mutual funds, respectively. $1+\xi$ is the appropriately designed labor subsidy which aims to eliminate distortions due to monopolistic power in labor markets. i_t represents nominal interest rate between t and $t+1$ and T_t^f represents a transfer from domestic government, which can be positive or negative.

We assume that wages are sticky for one time period. Specifically, the household j sets wages at $t-1$ by maximizing her expected utility at t knowing the following demand for her labor:

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} L_t.$$

The first order condition with respect to $W_t(j)$ yields

$$W_t(j) = \frac{\eta\theta}{(\theta-1)(1+\xi)} \frac{\mathrm{E_{t-1}}\left[L_t(j)^{1+\varphi}\right]}{\mathrm{E_{t-1}}\left[\frac{L_t(j)}{P_t C_t(j)}\right]}. \tag{7}$$

Households set the wage so that the expected marginal cost of supplying additional labor services (left hand side in the below expression) equals the expected marginal revenue (right hand side):

$$\eta\theta W_t(j)^{-1}\,\mathrm{E_{t-1}}\left[L_t(j)^{1+\varphi}\right] = (\theta-1)(1+\xi)\,\mathrm{E_{t-1}}\left[\frac{L_t(j)}{P_t C_t(j)}\right].$$

Other choices occur within the same time period. The first order condition with respect to share holdings yields

$$\widetilde{V}_t = E_t\left[Q_{t,t+1}(j)\,\widetilde{D}_{t+1}\right], \tag{8}$$

where $Q_{t,t+1}$ is stochastic discount factor defied as $Q_{t,t+1}(j) = E_t\left[\frac{\beta P_t C_t(j)}{P_{t+1}C_{t+1}(j)}\right]$.

The first order condition with respect to bond holdings is given by

$$1 = (1+i_t)E_t\left[Q_{t,t+1}(j)\right].$$

Finally, the household maximizes its consumption and real money holdings. As a result, we have

$$P_t C_t(j) = \frac{M_t}{\chi}\left(\frac{i_t}{1+i_t}\right). \tag{9}$$

Nominal spending $P_t C_t(j)$ is tight down to the money supply M_t.

2.7 Balanced Trade and Labor Market Clearings

In equilibrium, there is a symmetry across households so that $C_t(j) = C_t$, $L_t(j) = L_t$, $M_t(j) = M_t$ and $W_t(j) = W_t$. Furthermore, we follow Corsetti et al. (2010) and Bergin and Corsetti (2019) and define monetary stance as

$$\mu_t \equiv P_t C_t.$$

Monetary stance is proportional to nominal expenditure.[3] Trade is assumed to be balanced, thus the value of Home exports is equal to the value of Home imports once they are converted to the same unit of currency: $\varepsilon_t P^*_{H,t} C^*_{H,t} = P_{F,t} C_{F,t}$. Combined with the demand system found previously, this implies

$$\varepsilon_t = \frac{\alpha^*_t}{\alpha_t} \frac{\mu_t}{\mu^*_t}. \tag{10}$$

It is assumed that the government has no power to directly control private lending and borrowing. The balanced budget rule is assumed as

$$M_t - M_{t-1} = T^f_t + \xi W_t L_t.$$

Under nominal wage rigidity, the aggregate labor supply L_t adjusts to its demand and the labor market clears as:

$$L_t = N_{D,t} \frac{\widetilde{y}_{D,t}}{\widetilde{z}_D} + N_{X,t} \left(\frac{\widetilde{y}_{X,t}}{\widetilde{z}_{X,t}} + f_{X,t} \right) + N_{D,t+1} f_{E,t} \tag{11}$$

In the above expression, $\widetilde{y}_{D,t}$ and $\widetilde{y}_{X,t}$ stand for production scale of each average domestic firms and average exporters.[4] The labor demand comes from producers selling their goods in the domestic and exporting markets (including fixed costs for exporting), and from resources used for the creation of new firms. A similar expression holds for the Foreign country.

Finally we assume the following process for the preference shift:

$$\alpha_t = \frac{1}{2} \alpha^{\rho}_{t-1} \upsilon_t, \quad \alpha^*_t = \frac{1}{2} \alpha^{*\rho}_{t-1} \upsilon^*_t,$$

with $\alpha_0 = \alpha^*_0 = 1$, $E_{t-1}[\upsilon_t] = E_{t-1}[\upsilon^*_t] = 1$, $\upsilon_t + \upsilon^*_t = 2$ and $0 \leq \rho \leq 1$. Indeed, υ_t and υ^*_t are defined as the i.i.d. shocks. Also we assume that $f_{E,t} = f^*_{E,t} = f_E$ and $f_{X,t} = f^*_{X,t} = f_X$

[3]Note that combining with the Euler equation with respect to the bond holdings, it is shown that

$$\frac{1}{\mu_t} = \mathrm{E}_t \lim_{s\to\infty} \beta^s \frac{1}{\mu_{t+s}} \prod_{\tau=0}^{s-1} (1 + i_{t+\tau}).$$

Monetary stance μ_t is expressed as a function of future expected path of interest rates. Or it can be expressed as a rule concerning money supply M_t as (9).

[4]$\widetilde{y}_{D,t} = (\sigma - 1) \frac{\widetilde{D}_{D,t} \widetilde{z}_D}{W_t}$ and $\widetilde{y}_{X,t} = (\sigma - 1) \frac{(\widetilde{D}_{X,t} + f_{X,t} W_t) \widetilde{z}_{X,t}}{W_t}$. See Online Appendix for more details.

for all time periods. The closed form solution of the model is reported in Table 1. We refer to the Online Appendix for the derivation of all the endogenous variables.

3 Exchange Rate Regimes

In this section, we study the behavior of the economy in presence of demand shocks under different exchange rate regimes. When the regime is fully floating, we assume that $\mu_t = \mu_t^* = \mu_0$ for all time periods. On the other hand, under fixed exchange rate regime, $\mu_t = 2\mu_0\alpha_t$ and $\mu_t^* = 2\mu_0\alpha_t^*$, as the monetary stance automatically responds to shocks in order to offset their impact on the exchange rate, as we detail below.

3.1 Flexible Exchange Rate Regime

In our setting, let first consider a preference shock such that households attach a higher utility to the consumption of Home produced goods. Under the flexible regime, following this relative demand shift for Home produced goods (a decrease in α_t^*/α_t), the nominal exchange rate ε_t appreciates for Home closing the trade surplus (trade deficit in Foreign) as $\varepsilon_t = \frac{\alpha_t^*}{\alpha_t}$. The adjustment takes place not only through the terms of trade fluctuations but also through the extensive margins of trade. Under our assumption of producer currency pricing, the appreciation improves the profitability of Foreign exporters in their currency units relative to those of Home (a decrease in $\widetilde{D}_{X,t}/\widetilde{D}_{X,t}^*$ on impact) and hence induces higher number of Foreign exporters relative to Home exporters (a decrease in $N_{X,t}/N_{X,t}^*$). Note that the higher number of imported varieties available for Home households does not immediately translates in a welfare gain because of the relative lower preference attached to goods produced in Foreign. At the same time, Foreign exporters become less efficient compared to Home exporters due to changes in cutoff productivity level for exporting (a rise in $\widetilde{z}_{X,t}/\widetilde{z}_{X,t}^*$). Accordingly, the price of goods produced by Foreign exporters increases (a decrease in $\widetilde{p}_{X,t}/\widetilde{p}_{X,t}^*$) and the quantity of average variety produced by them decreases (a rise in $\widetilde{y}_{X,t}/\widetilde{y}_{X,t}^*$). To sum up, using the equilibrium expressions in Table 1, and denoting implied variable X_t under flexible regime with X_t^{FL}, we have indeed,

$$\frac{N_{X,t}^{FL}}{N_{X,t}^{*FL}} = \frac{\varepsilon_t W_t^{*FL} f_X^*}{W_t^{FL} f_X}, \qquad \frac{\tilde{z}_{X,t}^{FL}}{\tilde{z}_{X,t}^{*FL}} = \left(\frac{N_{X,t}^{FL}}{N_{X,t}^{*FL}} \frac{N_{D,t}^{*FL}}{N_{D,t}^{FL}}\right)^{-\frac{1}{\kappa}}. \tag{12}$$

Since wages are rigid and the numbers of domestic firms is a state variable, we can easily compute the number of exporters and cutoff level of productivity under flexible regime given the equilibrium wage and nominal exchange rate. The equilibrium wage under flexible exchange rate regime is

$$W_t^{FL} = \Gamma \mu_0 \left\{ \frac{E_{t-1}\left[A_t^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]} \right\}^{\frac{1}{1+\varphi}} \tag{13}$$

where A_t embeds preferences and parameters as[5]

$$A_t \equiv \frac{\sigma-1}{\sigma}\alpha_t + \left(1 - \frac{\sigma-1}{\sigma\kappa}\right)\alpha_t^* + \frac{\beta}{\sigma}\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]$$

The equilibrium wage under flexible exchange rate regime inherits the uncertainty about the *future* demand shock because of the above mentioned inevitable adjustment in the export market. The expression (13) therefore shows that the nominal exchange rate is not a "shock absorber" in our setting with selection into export market.[6]

What happens in the domestic market after a positive demand shock for Home goods? Plugging the equilibrium expressions in Table 1, we may derive the equilibrium average production of output and new firms created at Home relative to Foreign:

---

[5]Note that with $\alpha_t = \frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t$, $\alpha_t^* = \frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^*$ and assuming a symmetric steady state across countries as $\alpha_{t-1} = \alpha_{t-1}^*$, we can express A_t as a function of fundamental shocks as

$$\begin{aligned}
A_t &= \frac{\sigma-1}{\sigma}\alpha_t + \left(1-\frac{\sigma-1}{\sigma\kappa}\right)\alpha_t^* + \frac{\beta}{\sigma}\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]. \\
&= \frac{1}{2}\frac{\sigma-1}{\sigma}\alpha_{t-1}^{\rho}\upsilon_t + \left(1-\frac{\sigma-1}{\sigma\kappa}\right)\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^* + \frac{\beta}{\sigma}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t\right)^{\rho}\upsilon_{t+1} + \frac{\sigma-1}{\kappa}\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t\right)^{\rho}\upsilon_{t+1}\right] \\
&= \frac{1}{2}\left\{\frac{\sigma-1}{\sigma}\upsilon_t + \left(1-\frac{\sigma-1}{\sigma\kappa}\right)\upsilon_t^* + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma}\left[\upsilon_t^{\rho}\upsilon_{t+1} + \frac{\sigma-1}{\kappa}\upsilon_t^{*\rho}\upsilon_{t+1}^*\right]\right\}
\end{aligned}$$

[6]Contrary to the models without firm heterogeneity in the literature - e.g. Devereux (2004) and Hamano and Picard (2017)) - the nominal exchange rate only partially absorbs the shock in our setting creating a substantial adjustment in the export market.

$$\frac{\widetilde{y}_{D,t}^{FL}}{\widetilde{y}_{D,t}^{*FL}} = \frac{\alpha_t \widetilde{z}_D}{\alpha_t^* \widetilde{z}_D^*} \frac{N_{D,t}^{*FL} W_t^{*FL}}{N_{D,t}^{FL} W_t^{FL}}, \qquad \frac{N_{D,t+1}^{FL}}{N_{D,t+1}^{*FL}} = \frac{W_t^{*FL} f_E^* E_t \left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]}{W_t^{FL} f_E E_t \left[\alpha_{t+1}^* + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]}. \tag{14}$$

In domestic market, following a relative demand shift for Home produced goods, the production scale of average domestic firms expands compared to that of Foreign (a rise in $\widetilde{y}_{D,t}/\widetilde{y}_{D,t}^*$). The persistence of preference shock $0 < \rho \leq 1$ induces *a modest* increase in domestic investment (a rise in $N_{D,t+1}/N_{D,t+1}^*$).

To sum up, on the one hand, the trade imbalance triggered by a relative positive demand shift for Home goods induces not only fluctuations in the nominal exchange rate ε_t but also abrupt fluctuations in extensive as well as intensive margins of trade. On the other hand, the flexible exchange rate regime triggers an adjustment in domestic market, the size of which is however relatively modest.

3.2 Fixed Exchange Rate Regime

When the nominal exchange rate is fixed ($\varepsilon_t = 1$), the allocation in the economy dramatically changes. A counter acting shift in monetary stance as $\mu_t = 2\mu_0\alpha_t$ and $\mu_t = 2\mu_0 (1 - \alpha_t)$ in both countries mitigates the profits fluctuations in the export market. As a result, the number of exporters as well as the production scales hence their prices remain constant in equilibrium following a demand shift. Using the equilibrium expressions in Table 1, and denoting implied variable X_t under flexible regime with X_t^{FX}, we have

$$\frac{N_{X,t}^{FX}}{N_{X,t}^{*FX}} = \frac{W_t^{*FX} f_X^*}{W_t^{FX} f_X}, \qquad \frac{\widetilde{z}_{X,t}^{FX}}{\widetilde{z}_{X,t}^{*FX}} = \left(\frac{N_{X,t}^{FX}}{N_{X,t}^{*FX}} \frac{N_{D,t}^{*FX}}{N_{D,t}^{FX}}\right)^{-\frac{1}{\kappa}}. \tag{15}$$

As is clear from the above expressions, the fixed regime results in a sterilization in extensive and intensive margins of trade. However, it induces a drastic change in the domestic market due to the higher aggregate demand induced by the monetary expansion. The production scale of domestic firms and investment for future product varieties indeed rise substantially in Home compared to those in Foreign (a strong rise both in $\widetilde{y}_{D,t}/\widetilde{y}_{D,t}^*$ and $N_{D,t+1}/N_{D,t+1}^*$):

$$\frac{\widetilde{y}_{D,t}^{FX}}{\widetilde{y}_{D,t}^{*FX}} = \frac{\alpha_t^2 \widetilde{z}_D}{\alpha_t^{*2} \widetilde{z}_D^*} \frac{N_{D,t}^{*FX} W_t^{*FX}}{N_{D,t}^{FX} W_t^{FX}}, \qquad \frac{N_{D,t+1}^{FX}}{N_{D,t+1}^{*FX}} = \frac{\alpha_t}{\alpha_t^*} \frac{W_t^{*FX} f_E^* E_t \left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]}{W_t^{FX} f_E E_t \left[\alpha_{t+1}^* + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]}. \tag{16}$$

Comparing the above expressions with (14), one may notice that the volatility in domestic average production and the number of domestic entry (investment) are higher than those under the flexible regime. To sum up, the fixed exchange rate regime shifts the burden of adjustment away from exporting markets to domestic markets.

Finally, as expected, the equilibrium allocation under fixed regime influences the wage setting. The wage under fixed exchange rate regime is found to be

$$W_t^{FX} = 2\Gamma\mu_0 \left\{ \frac{E_{t-1}\left[(A_t\alpha_t)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]} \right\}^{\frac{1}{1+\varphi}}.$$

Because of the monetary policy intervention in response to the demand shock, $\mu_t = 2\mu_0\alpha_t$, and the equilibrium wage under the fixed regime depends on the expected interaction between labor demand fluctuations and monetary shock which is captured by the term $(A_t\alpha_t)^{1+\varphi}$ in the expectation operator. W_t^{FX} therefore depends on the level of each component (A_t and α_t) and the covariance ($\text{Cov}(A_t, \alpha_t)$) augmented by the elasticity of labor supply, φ. On the one hand, monetary intervention increases wage in level because of a higher aggregate demand, captured by α_t in expectations. On the other hand, since labor demand and monetary policy shock can be *negatively* correlated ($\text{Cov}(A_t, \alpha_t) < 0$), a monetary policy that aims at fixing the exchange rate simultaneously dampens the fluctuations in labor demand and hence uncertainty specifically in the export market. Intuitively, under the fixed exchange rate regime, the profitability of exporters remains constant, whereas the domestic production and investment rise more abruptly than under a flexible regime. As we will describe later in detail, the negative correlation between labor demand and demand shift is a function of the fundamentals of the economy, among them the firm productivity distribution, and is crucial in deriving the welfare ranking between fixed and flexible exchange rate regime.

4 Welfare Analysis

In this section, we explore the welfare implication of policy intervention in the presence of demand shock. Monetary authority can have various policy objectives based on the political economic environment. Among the policy objectives, the policymakers may target the extent of desired variability of the exchange rate which can be achieved through monetary interventions. In what follows, we compare the welfare outcome under fixed and flexible exchange rate regime.

4.1 Expected Utility

First, we characterize the expected utility of the households as a function of exogenous disturbances and monetary stance. Although the expected discounted sum of utility is defined over an infinite horizon of time, policy intervention at time t has impact just for two consecutive time periods due to the assumption of one period to build and produce, and the wage stickiness. In deriving the welfare metrics, we thus express the expected utility only for two consecutive periods without loss of generality. The expected utility of the Home representative household at time t and $t+1$ being at $t-1$ is presented therefore as

$$\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] \equiv \mathrm{E}_{t-1}\left[U_t\right] + \beta \mathrm{E}_{t-1}\left[U_{t+1}\right] \\
= E_{t-1}\left[\alpha_t \ln C_{H,t} + \alpha_t^* \ln C_{F,t}\right] + \beta E_{t-1}\left[\alpha_t \ln C_{H,t+1} + \alpha_t^* \ln C_{F,t+1}\right] \\
= \mathrm{E}_{t-1}\left[\alpha_t \left(\frac{\sigma}{\sigma-1}\ln N_{D,t} + \ln \widetilde{y}_{D,t}\right) + \alpha_t^* \left(\frac{\sigma}{\sigma-1}\ln N_{X,t}^* + \ln \widetilde{y}_{X,t}^*\right)\right] \\
+ \beta \mathrm{E}_{t-1}\left[\alpha_{t+1}\left(\frac{\sigma}{\sigma-1}\ln N_{D,t+1} + \ln \widetilde{y}_{D,t+1}\right) + \alpha_{t+1}^*\left(\frac{\sigma}{\sigma-1}\ln N_{X,t+1}^* + \ln \widetilde{y}_{X,t+1}^*\right)\right] \quad (17)
\end{aligned}$$

The expected utility is a function of the current number of domestic and imported varieties ($N_{D,t}$ and $N_{X,t}^*$) and their production scales ($\widetilde{y}_{D,t}$ and $\widetilde{y}_{X,t}^*$) at time t and the expected number of them at time $t+1$.[7]

[7] Note that the sum of utility in any two consecutive time periods can be expressed as the above expression without loss of generality.

Furthermore, plugging the equilibrium expression in Table 1 and shock process discussed previously, the equation (17) becomes (see Online Appendix for derivation.)

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] = & \frac{1}{2}\left\{\mathrm{E}_{t-1}\left[\upsilon_t \ln \mu_t\right] - \frac{1}{1+\varphi} \ln E_{t-1}\left[\left(A_t \mu_t\right)^{1+\varphi}\right]\right\} \\
& + \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^* \ln \mu_t^*\right] - \frac{1}{1+\varphi} \ln E_{t-1}\left[\left(A_t^* \mu_t^*\right)^{1+\varphi}\right]\right\} \\
& + \left(\frac{1}{2}\right)^{1+\rho} \frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho} \ln \mu_t\right] - \frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{1+\varphi} \ln E_{t-1}\left[\left(A_t \mu_t\right)^{1+\varphi}\right]\right\} \\
& + \left(\frac{1}{2}\right)^{1+\rho} \frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \ln \mu_t^*\right] - \frac{E_{t-1}\left[\upsilon_t^{*\rho}\right]}{1+\varphi} \ln E_{t-1}\left[\left(A_t^* \mu_t^*\right)^{1+\varphi}\right]\right\} + \mathrm{cst} \qquad (18)
\end{aligned}
$$

The expected utility can therefore be expressed as a function of shocks and monetary stances in Home and Foreign.

4.2 Fixed vs. Flexible Exchange Rate Regimes

Given the above expected utility, the welfare difference between the fixed and flexible exchange rate regime is found as[8]

$$
\begin{aligned}
& \mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right] \\
& \quad = \frac{1}{2}\left(\frac{1}{\sigma-1} + 2 - \frac{1}{\kappa}\right)\left\{E_{t-1}\left[\upsilon_t \ln \upsilon_t\right] - \Delta \ln W_t\right\} \\
& \quad + \left(\frac{1}{2}\right)^{1+\rho} \beta\left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right)\left\{E_{t-1}\left[\upsilon_t^{\rho} \ln \upsilon_t\right] - E_{t-1}\left[\upsilon_t^{\rho}\right] \Delta \ln W_t\right\} \qquad (19)
\end{aligned}
$$

[8]See Online Appendix for more details.

where $\Delta \ln W_t \equiv \ln W_t^{FX} - \ln W_t^{FL}$ represents the wage difference between the fixed and flexible regime:[9]

$$\begin{aligned}\Delta \ln W_t &\equiv \ln W_t^{FX} - \ln W_t^{FL} \\ &= \frac{1}{1+\varphi}\left[\ln E_{t-1}\left[(A_t \upsilon_t)^{1+\varphi}\right] - \ln E_{t-1}\left[A_t^{1+\varphi}\right]\right] \quad (20)\end{aligned}$$

In the expression of welfare ranking (19), both $E_{t-1}[\upsilon_t \ln \upsilon_t]$ and $E_{t-1}[\upsilon_t^{\rho} \ln \upsilon_t]$ are greater than 0. This means that there is a better congruence between the preference shock and the amount of goods produced at the intensive and extensive margin in both domestic and imported markets under fixed exchange rate regime. However, the fluctuations of monetary stance in response to the stochastic preference shocks under fixed regime are costly and detrimental to welfare as $\Delta \ln W_t > 0$ as we explain in detail in the next subsection.

We now discuss the role played by the fundamentals of our economy in the welfare comparison (19). First, we discuss the welfare implications arising from current and future number of varieties. Second, we explore the role of firm heterogeneity on the sign and magnitude of the wage difference $\Delta \ln W_t$.

4.2.1 Variety effect with selection into exporting market

In the expression (19), the term $\frac{1}{\sigma-1} - \frac{1}{\kappa} > 0$ captures the balance between preference for the *current* number of imported varieties and the price of those varieties. Other things

[9] To get the expression (20),

$$\begin{aligned}\Delta \ln W_t \equiv \ln W_t^{FX} - \ln W_t^{FL} &= \ln \Gamma \left\{\frac{E_{t-1}\left[(A_t 2\mu_0 \alpha_t)^{1+\varphi}\right]}{E_{t-1}[A_t]}\right\}^{\frac{1}{1+\varphi}} - \ln \Gamma \left\{\frac{E_{t-1}\left[(A_t \mu_0)^{1+\varphi}\right]}{E_{t-1}[A_t]}\right\}^{\frac{1}{1+\varphi}} \\ &= \frac{1}{1+\varphi} \ln \left\{\frac{E_{t-1}\left[(A_t 2\mu_0 \alpha_t)^{1+\varphi}\right]}{E_{t-1}[A_t]}\right\} - \frac{1}{1+\varphi} \ln \left\{\frac{E_{t-1}\left[(A_t \mu_0)^{1+\varphi}\right]}{E_{t-1}[A_t]}\right\} \\ &= \frac{1}{1+\varphi} \ln E_{t-1}\left[(A_t 2\mu_0 \alpha_t)^{1+\varphi}\right] - \frac{1}{1+\varphi} \ln E_{t-1}\left[(A_t \mu_0)^{1+\varphi}\right] \\ &= \frac{1}{1+\varphi}\left[\ln E_{t-1}\left[(A_t \upsilon_t)^{1+\varphi}\right] - \ln E_{t-1}\left[A_t^{1+\varphi}\right]\right].\end{aligned}$$

equal, the gain under fixed regime that realizes a better congruence between preferences and imported number of varieties increases with a higher preference for variety (lower value of σ) and decreases with a higher firm dispersion (lower value of κ). This is because when the number of imported varieties goes up, these varieties are produced by less efficient firms that charge expensive prices on average. Given the love for variety, the welfare gain in consuming a higher number imported varieties is fully realized when exporters are homogeneous ($\kappa = \infty$) and hence there is no increase in price of import.

In a similar way, the term $\left(\frac{1}{2}\right)^{1+\rho} \beta \left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right)$ in the expression (19) scales the welfare impact on the number of *future* domestic varieties and the *future* cutoff level of imported goods. Other things equal, a rise in the future number of domestic products provides a higher utility gain when the love for variety is high (lower value of σ). Note that the impact is amplified by a lower discount factor (a higher value of β). A higher persistence of the shock (a higher value of ρ) also amplifies this effect, as a current positive shock will result in a higher number of future varieties. Furthermore, a rise in the number of varieties in the next period increases competition. As a result, the future cutoff level increases due to selection and the price of imported varieties become cheaper. From this channel, the higher the value of κ, the lower is the welfare gain because of the survival of less efficient producers that charge higher prices in the future.[10]

4.2.2 The first and second order effect of the fixed regime on labor demand fluctuations

In determining the welfare ranking in (19), in addition to the variety effect, the sign of $\Delta \ln W_t$ is crucial. In turn, the sign of $\Delta \ln W_t$ depends on the covariance terms in the equilibrium wage under fixed regime W_t^{FX}. Under the fixed regime, the monetary intervention increases the expected labor demand (*first order effect*), but at the same time it dampens labor demand fluctuations at the extensive margin (*second order effect*).

[10] The above welfare improving effect under fixed regime through the demand congruence is similar to Devereux (2004) without extensive margins and Hamano and Picard (2017) with extensive margins. Here the mechanism is more elaborated due to the selection into exporting market among heterogeneous firms.

The intuition can be best described by setting $\varphi = 0$ (the case of infinitely elastic labor supply). In such a case, the wage difference equation (20) is expressed as

$$\Delta \ln W_t \mid_{\varphi=0} = [\ln E_{t-1}[(A_t \upsilon_t)] - \ln E_{t-1}[A_t]]$$
$$= \ln \left[1 + \frac{E_{t-1}[\upsilon_t] + Cov(A_t, \upsilon_t)}{E_{t-1}[A_t]} \right]. \quad (21)$$

The term $E_{t-1}[\upsilon_t]$ is the level *first order effect* of the monetary intervention under the fixed regime whereas $Cov(A_t, \upsilon_t)$ captures the *second order effect* stemming from the covariance between the labor demand and monetary shock under fixed regime.

Assuming a symmetric steady state across countries as $\alpha_{t-1} = \alpha^*_{t-1}$, by deriving the expression A_t with respect to the monetary shock, we have

$$\frac{\partial A_t}{\partial \upsilon_t} = -\frac{1}{2\sigma}\left(1 - \frac{\sigma - 1}{\kappa}\right)\left[1 - \left(\frac{1}{2}\right)^{\rho} \beta\rho\upsilon_t^{\rho-1}\right] < 0. \quad (22)$$

The expression is strictly negative indicating a negative covariance between labor demand and monetary intervention. Importantly, the extent of the negative covariance depends on the degree of firm productivity dispersion. When κ is high, firms are less dispersed and less productive, and the relative number of exporter over domestic firms is higher. Labor demand is higher because there are more exporter firms potentially affected by the demand shock (larger extensive margin of trade). In such a situation, the active monetary policy under the fixed regime stabilizes firm turnover in the export market and hence the volatile labor demand, improving welfare. In this case, the *second order* covariance effect mitigates the *first order* level effect on higher labor demand. As a consequence, the equilibrium wage therefore decreases with a lower firm dispersion. Figure 2 reports a numerical simulation for different values of κ and σ. As discussed, when κ is high, the wage difference between the regimes is reduced and the fixed exchange rate regime provides a higher welfare.

The elasticity of substitution among goods σ also determines the size of negative covariance. With a higher value of σ, the covariance is increasing and the welfare improving mitigation effect of monetary intervention is thus weaker. In fact, with a higher value of σ, labor demand is low in exporting sector due to a tougher competition and the welfare improving effect of monetary intervention is lower. As shown in panel b) of Figure 2, the wage gap between the two regimes is increasing with σ, and the fixed exchange rate regime is less supported.[11]

Similarly, when the shock persistence ρ is lower or the elasticity of labor supply $1/\varphi$ is higher, the covariance decreases giving more support to the fixed exchange rate regime. The numerical simulations in Figure 5 show that under lower persistence or higher labor supply elasticity the welfare implications of the *current* monetary intervention are indeed muted.[12] The same mechanism applies for a higher discount factor β. In this case, workers put a lower weight on the *current* monetary intervention which does not have a persistent impact on their welfare in future periods.

4.3 Optimal Monetary Policy

We now depart from the suboptimal polar cases of fixed and flexible regimes and study the optimal monetary policy, and the implied fluctuations in the nominal exchange rate. Given the welfare metrics derived in equation (17), the first order condition with respect to μ_t is found as

$$\frac{1}{2}\left\{\frac{\upsilon_t}{\mu_t}-\frac{1}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}\frac{(A_t\mu_t)^{1+\varphi}}{\mu_t}\right\} + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\frac{\upsilon_t^{\rho}}{\mu_t}-\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}\frac{(A_t\mu_t)^{1+\varphi}}{\mu_t}\right\}=0.$$

Under the above optimal policy, the exchange rate is expressed as

$$\varepsilon_t=\frac{\alpha_t^*}{\alpha_t}\frac{\mu_t}{\mu_t^*}=\frac{\upsilon_t^*}{\upsilon_t}\frac{A_t^*}{A_t}\left[\frac{\upsilon_t+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\upsilon_t^*+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{*\rho}}\right]^{\frac{1}{1+\varphi}}.$$

[11] To be precise, by deriving (22) with respect to σ, we get indeed

$$\frac{\partial A_t/\partial\upsilon_t}{\partial\sigma}=\frac{1}{2\sigma}\left(\frac{1}{\kappa}+\frac{1}{\sigma}\left(1-\frac{\sigma-1}{\kappa}\right)\right)\left(1-\left(\frac{1}{2}\right)^{\rho}\beta\rho\upsilon_t^{\rho-1}\right)<0.$$

[12] The result of the numerical simulations with respect to the elasticity of labor supply $1/\varphi$ is consistent with the results in Devereux (2004) and Hamano and Picard (2017).

Note that monetary stance comoves with the preference shock hence limiting the fluctuations in the nominal exchange rate. Figure 3 documents the variability of the nominal exchange rate under the optimal policy with respect to different value of κ and σ. As discussed in the previous section, as the firm productivity dispersion decreases (κ increases), the wage difference is reduced and the cost related to fixed regime decreases. Accordingly, the optimal fluctuations in the nominal exchange rate also decrease with κ. With respect to the elasticity of substitution among goods, as σ increases, the wage difference increases, and therefore the cost related to the fixed exchange rate regime. Accordingly, the optimal volatility of the nominal exchange rate increases with σ.

5 Regulation Policy in the Export Market

In this section, we explore an alternative option for policymakers willing to limit the fluctuations of firm turnover in the export market. Instead of a fixed exchange rate regime, we assume that policymakers can directly intervene on the fixed cost of exporting $f_{X,t}$. We dub this intervention in response to demand shock as a regulation policy and specify it as follows

$$f_{X,t} = \frac{f_X}{\alpha_t}, \qquad f^*_{X,t} = \frac{f_X}{\alpha^*_t}.$$

The aim of the above policy reaction is to sterilize the volatility in the export market which is detrimental to welfare in our setting. In this respect, this regulation policy may be interpreted as an alternative policy to fixed exchange rate. The relative number of exporters under the implementation of the above policy is

$$\frac{N_{X,t}}{N^*_{X,t}} = \frac{\mu_t}{\mu^*_t}\frac{W^*_t}{W_t}.$$

In the above expression, since wages are fixed in the previous period, the relative number of exporters is invariant as $\mu_t = \mu^*_t = \mu_0$.Thanks to the regulation policy, monetary policy is now free from the pressure of stabilizing the export market and can let the exchange rate fluctuated freely.

We then compare the choice between fixed exchange rate regime as previously argued and the above regulation policy regime. Denoting the expected utility under regulation policy as $\mathrm{E}_{t-1}\left[\mathcal{U}^{RG}\right]$, the welfare difference is expressed as (see Online Appendix):

$$\begin{aligned}
&\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{RG}\right] \\
&\qquad = E_{t-1}\left[\upsilon_t \ln \upsilon_t\right]-\frac{1}{2}\left(\frac{1}{\sigma-1}+2-\frac{1}{\kappa}\right)\Delta \ln W_t \\
&\qquad\quad +\left(\frac{1}{2}\right)^{1+\rho}\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\left\{E_{t-1}\left[\upsilon_t^{\rho}\ln \upsilon_t\right]-E_{t-1}\left[\upsilon_t^{\rho}\right]\Delta \ln W_t\right\}
\end{aligned}$$

Note that under this regulation policy in the export market, the exchange rate floats freely and we therefore have the same expression for $\Delta \ln W_t$. As a result, the fixed exchange rate regime may still dominate as $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] > \mathrm{E}_{t-1}\left[\mathcal{U}^{RG}\right]$ for high levels of κ.[13] The regulation policy is slightly preferred over the flexible exchange rate regime since it realizes a better congruence between preference and intensive as well as extensive margins at least for traded goods. However it is not able to nail down the economy wide uncertainty stemming from stochastic demand. Figure 4 shows the above welfare ranking with different value of κ and σ. The main take away of this exercise is that a "one fits all" policy like adopting a fixed regime dominates a more specific regulation policy when the firm productivity distribution is less dispersed.[14]

6 Conclusion

This paper explores the choice of exchange rate regime in a model with firm heterogeneity and endogenous selection into export markets. A fixed regime does not only realizes a better congruence between preference and the variety consumed but also substantially reduces uncertainty in labor demand that arises from entry and exit of exporters. In our setting, fixed exchange rate regime can be superior to flexible exchange rate regime depending on the fundamentals of the economy, among them the firm productivity distribution. When firms are more homogeneous, the effects of the demand shocks on labor

[13]Note that $\mathrm{E}_{t-1}\left[\mathcal{U}^{RG}\right] > \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ because of sterilized flutuations in the export market in case of regulation policy.

[14]In the Online Appendix A.1, we provide additional empirical evidence on exchange rate regimes and country-level firm heterogeneity.

demand fluctuations are amplified, because the response of the extensive margin of trade is larger. We also show that a regulation policy aimed at stabilizing firm turnover in the export market cannot remove the temptation of currency manipulation for low levels of firm dispersion.

References

Ahmad, Saad, Sarah Oliver, and Caroline Peters, "Using firm-level data to compare productivities across countries and sectors: possibilities and challenges," *Economics Working Paper Series, U.S. International Trade Commission*, July 2018, *2018-07-A.*

Auray, Stéphane, Michael B. Devereux, and Aurélien Eyquem, "Endogenous Trade Protection and Exchange Rate Adjustment," NBER Working Papers 25517, National Bureau of Economic Research, Inc, January 2019.

Barattieri, Alessandro, Matteo Cacciatore, and Fabio Ghironi, "Protectionism and the Business Cycle," NBER Working Papers 24353, National Bureau of Economic Research, Inc, February 2018.

Bergin, Paul R. and Giancarlo Corsetti, "Beyond Competitive Devaluations: The Monetary Dimensions of Comparative Advantage," NBER Working Papers 25765, National Bureau of Economic Research, Inc, April 2019.

Betts, Caroline and Michael Devereux, "The exchange rate in a model of pricing-to-market," *European Economic Review*, 1996, *40* (3-5), 1007–1021.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc, "Optimal Monetary Policy in Open Economies," in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, 1 ed., Vol. 3, Elsevier, 2010, chapter 16, pp. 861–933.

_ , Philippe Martin, and Paolo Pesenti, "Productivity, terms of trade and the 'home market effect'," *Journal of International Economics*, September 2007, *73* (1), 99–127.

_ , _ , and _ , "Varieties and the transfer problem," *Journal of International Economics*, 2013, *89* (1), 1–12.

Devereux, Michael and Charles Engel, "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility," *Review of Economic Studies*, 2003, *70* (4), 765–783.

Devereux, Michael B., "Should the exchange rate be a shock absorber?," *Journal of International Economics*, March 2004, *62* (2), 359–377.

Friedman, Milton, *Essays in Positive Economics*, University of Chicago Press, 1953.

Ghironi, Fabio and Marc J. Melitz, "International Trade and Macroeconomic Dynamics with Heterogeneous Firms," *The Quarterly Journal of Economics*, August 2005, *120* (3), 865–915.

Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon, "Currency Choice and Exchange Rate Pass-Through," *American Economic Review*, March 2010, *100* (1), 304–336.

Hamano, Masashige, "The Harrod-Balassa-Samuelson effect and endogenous extensive margins," *Journal of the Japanese and International Economies*, 2014, *31* (C), 98–113.

_ **and Pierre M. Picard**, "Extensive and intensive margins and exchange rate regimes," *Canadian Journal of Economics*, August 2017, *50* (3), 804–837.

Ilzetzki, Ethan, Carmen M Reinhart, and Kenneth S Rogoff, "Exchange Arrangements Entering the Twenty-First Century: Which Anchor will Hold?*," *The Quarterly Journal of Economics*, 01 2019, *134* (2), 599–646.

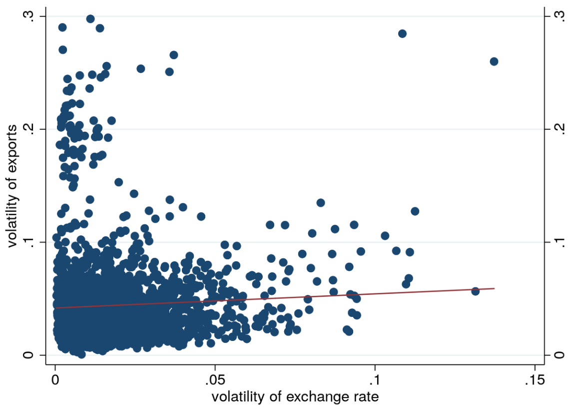
Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, November 2003, *71* (6), 1695–1725.

Mundell, Robert A., "A theory of optimum currency areas," *American Economic Review*, 1961, *51*, 657–665.

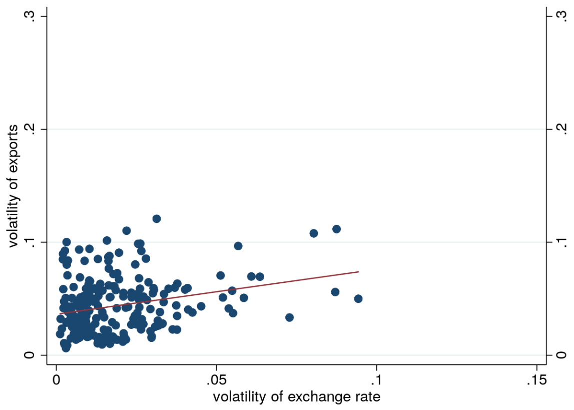
Pappadà, Francesco, "Real adjustment of current account imbalances with firm heterogeneity," *IMF Economic Review*, August 2011, *59* (3), 431–454.

Figures and tables

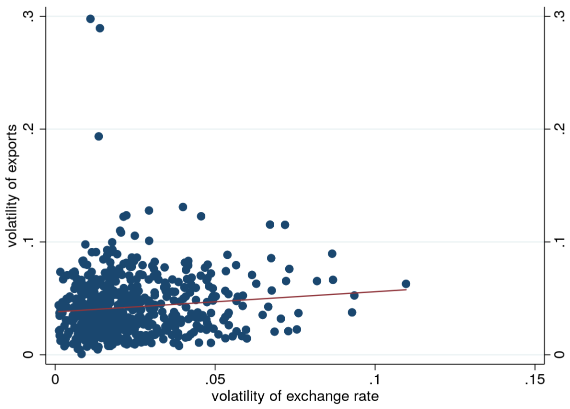
Figure 1: Volatility of exports and exchange rate.



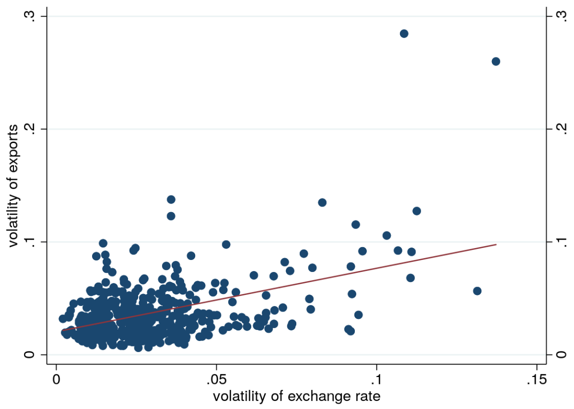
(a) Full sample.



(b) Crawling peg.



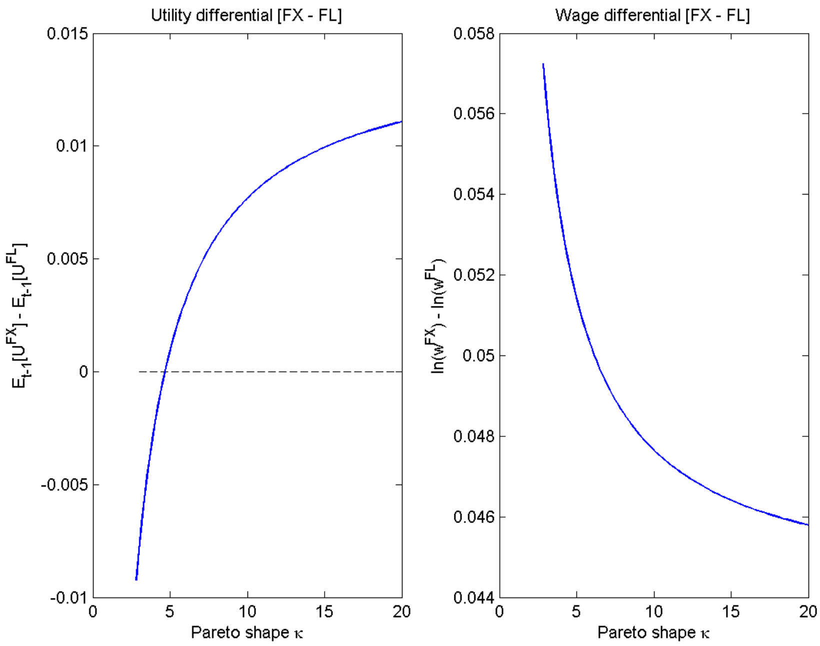
(c) Managed floating.



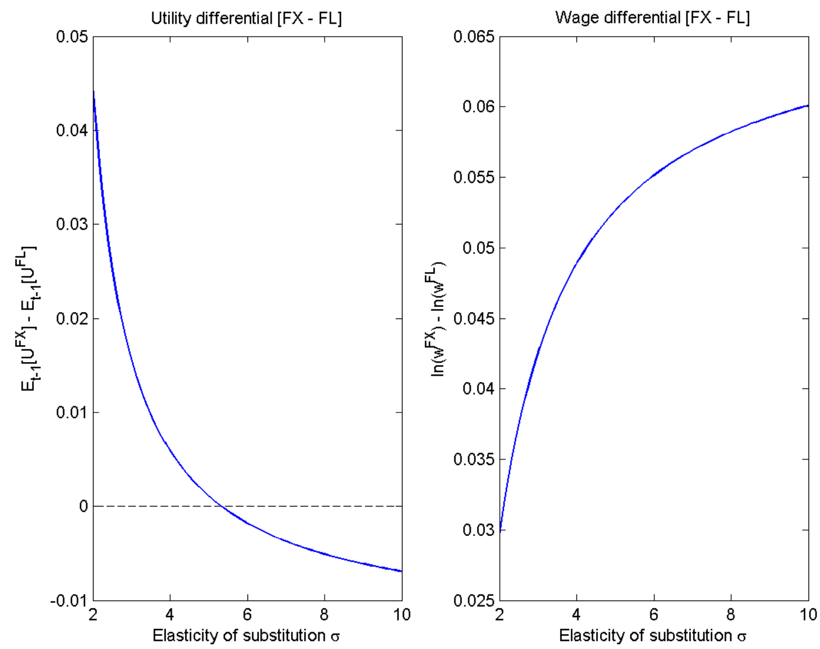
(d) Fully floating.

Notes: a) Full sample. OLS coefficient: .13*, average volatility of exchange rate (NEER): .018, observations: 2379. b) Crawling peg [index 2 in Ilzetzki et al. (2019)]. OLS coefficient: .40***, average volatility of exchange rate (NEER): .019, observations: 220. c) Managed floating [index 3 in Ilzetzki et al. (2019)]. OLS coefficient: .18*, average volatility of exchange rate (NEER): .023, observations: 631. d) Fully floating [index 4 in Ilzetzki et al. (2019)]. OLS coefficient: .56***, average volatility of exchange rate (NEER): .030, observations: 456.

Figure 2: Welfare ranking and wage differential: fixed vs. flexible e.r. regime.



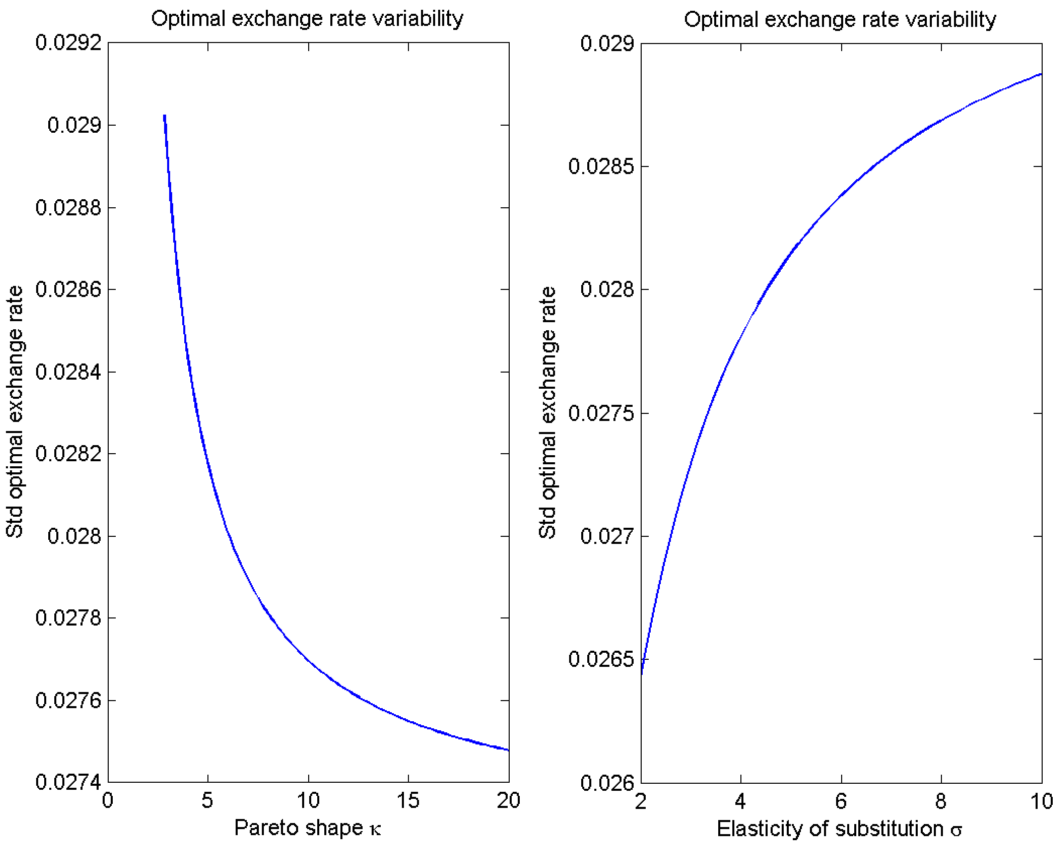
(a) Firm productivity distribution



(b) Elasticity of substitution

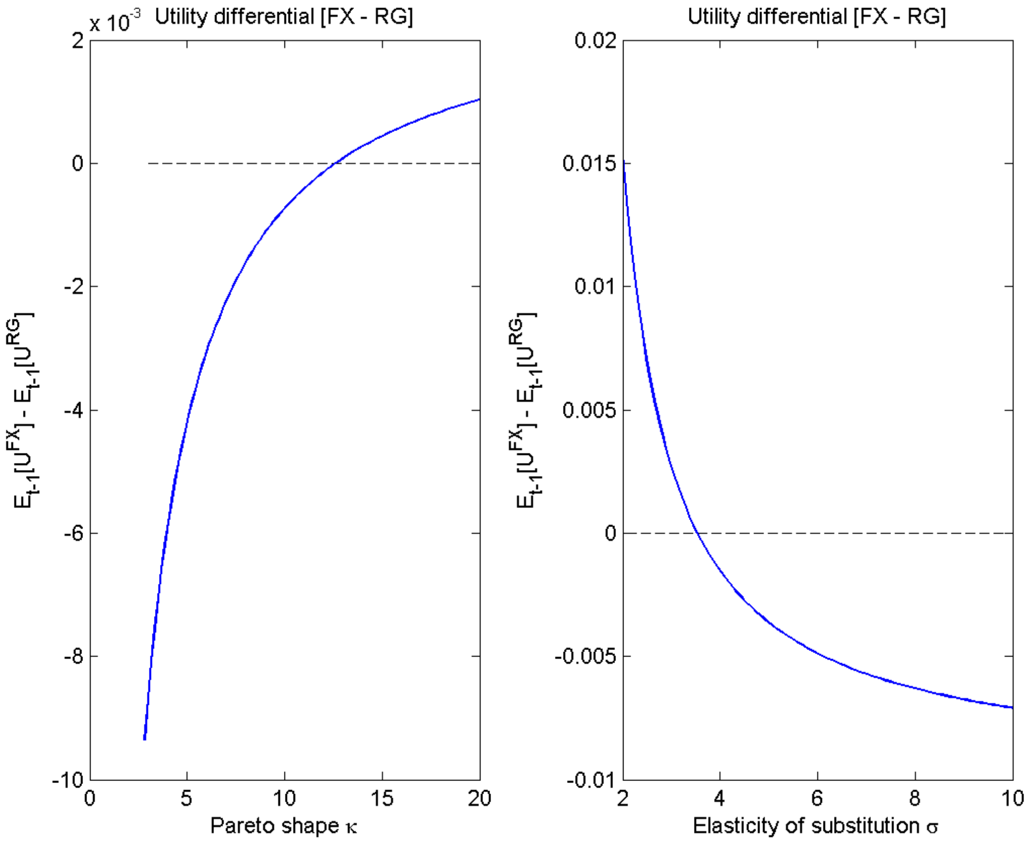
Notes. In the benchmark calibration, we set the value of $\sigma = 3.8$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$. In panel a) $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ and $\ln(W^{FX}) - \ln(W^{FL})$ are shown for different values of κ. In panel b) they are shown for different values of σ, while keeping $\kappa = (10-1) * 1.05$.

Figure 3: Optimal policy and the variance of the nominal exchange rate.



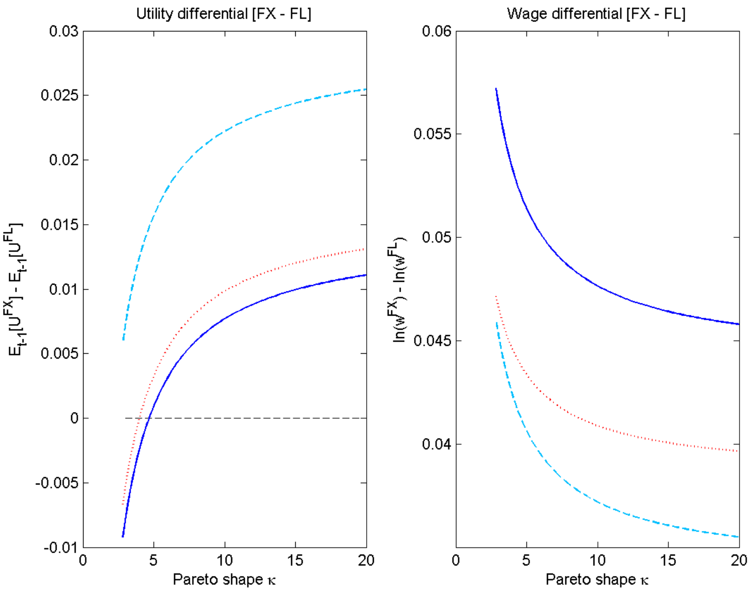
Notes. In the benchmark calibration, we set the value of $\sigma = 3.8$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$. In panel a) the variance of the nominal exchange rate is shown for different values of κ. In panel b) it is shown for different values of σ, while keeping $\kappa = (10 - 1) * 1.05$.

Figure 4: Welfare ranking: fixed exchange rate regime vs. regulation policy.

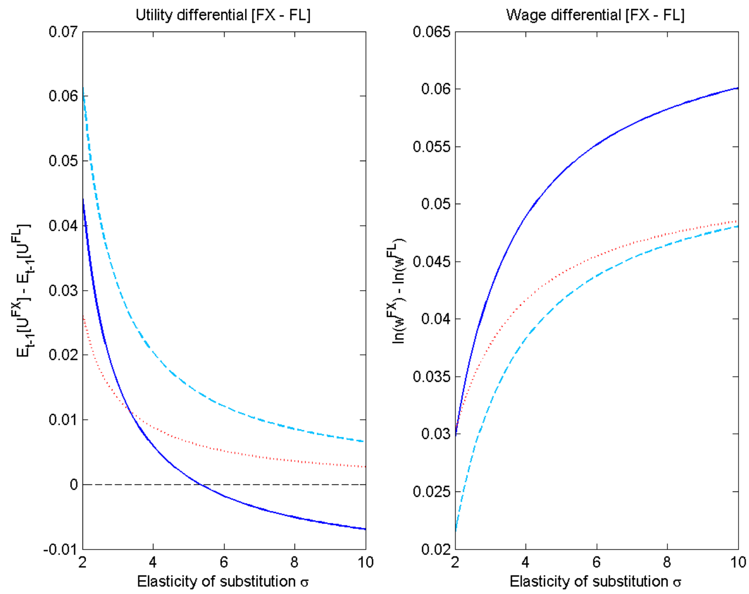


Notes. In the benchmark calibration, we set the value of $\sigma = 3.8$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$. In panel a) $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{RG}\right]$ and $\Delta \ln W_t$ are shown for different values of κ. In panel b) they are shown for different values of σ, while keeping $\kappa = (10-1) * 1.05$.

Figure 5: Welfare ranking: fixed vs. flexible e.r. regime - robustness.



(a) Firm productivity distribution



(b) Elasticity of substitution

Notes. With respect to the benchmark calibration (solid line), the dashed line refers to $\varphi^{-1} = 1$, and the dotted line to $\rho = 0.5$. In panel a) $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ and $\ln(W^{FX}) - \ln(W^{FL})$ are shown for different values of κ. In panel b) they are shown for different values of σ, while keeping $\kappa = (10-1) * 1.05$.

Table 1: The Model's Solution

Nb of Entrants	$N_{D,t+1} = \frac{\beta}{\sigma} \frac{\mu_t}{W_t f_{E,t}} E_t \left[\alpha_{t+1} + \frac{\sigma-1}{\kappa} \alpha^*_{t+1} \right]$	$N^*_{D,t+1} = \frac{\beta}{\sigma} \frac{\mu^*_t}{W^*_t f^*_{E,t}} E_t \left[\alpha^*_{t+1} + \frac{\sigma-1}{\kappa} \alpha_{t+1} \right]$
Nb of Exporters	$N_{X,t} = \frac{1}{\sigma}(1 - \frac{\sigma-1}{\kappa}) \frac{\alpha^*_t \mu_t}{W_t f_{X,t}}$	$N^*_{X,t} = \frac{1}{\sigma}(1 - \frac{\sigma-1}{\kappa}) \frac{\alpha_t \mu^*_t}{W^*_t f^*_{X,t}}$
Av. Exporters	$\widetilde{z}_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)} \right]^{\frac{1}{\sigma-1}} \left(\frac{N_{X,t}}{N_{D,t}} \right)^{-\frac{1}{\kappa}}$	$\widetilde{z}^*_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)} \right]^{\frac{1}{\sigma-1}} \left(\frac{N^*_{X,t}}{N^*_{D,t}} \right)^{-\frac{1}{\kappa}}$
Production	$\widetilde{y}_{D,t} = \frac{\sigma-1}{\sigma} \frac{\alpha_t \mu_t \widetilde{z}_D}{N_{D,t} W_t}, \quad \widetilde{y}_{X,t} = \frac{\sigma-1}{\sigma} \frac{\alpha^*_t \mu_t \widetilde{z}_{X,t}}{N_{X,t} W_t}$	$\widetilde{y}^*_{D,t} = \frac{\sigma-1}{\sigma} \frac{\alpha^*_t \mu^*_t \widetilde{z}^*_D}{N^*_{D,t} W^*_t}, \quad \widetilde{y}^*_{X,t} = \frac{\sigma-1}{\sigma} \frac{\alpha_t \mu^*_t \widetilde{z}^*_{X,t}}{N^*_{X,t} W^*_t}$
Average Price	$\widetilde{p}_{D,t} = \frac{\sigma}{\sigma-1} \frac{W_t}{\widetilde{z}_D}, \quad \widetilde{p}_{X,t} = \frac{\sigma}{\sigma-1} \frac{\tau_t \varepsilon_t^{-1} W_t}{\widetilde{z}_{X,t}}$	$\widetilde{p}^*_{D,t} = \frac{\sigma}{\sigma-1} \frac{W^*_t}{\widetilde{z}^*_D}, \quad \widetilde{p}^*_{X,t} = \frac{\sigma}{\sigma-1} \frac{\tau_t \varepsilon_t W^*_t}{\widetilde{z}^*_{X,t}}$
Price Indices	$P_{H,t} = N_{D,t}^{-\frac{1}{\sigma-1}} \widetilde{p}_{D,t}, \quad P_{F,t} = N_{X,t}^{*-\frac{1}{\sigma-1}} \widetilde{p}^*_{X,t}, \quad P_t = P_{H,t}^{\alpha_t} P_{F,t}^{\alpha^*_t}$	$P^*_{F,t} = N_{D,t}^{*-\frac{1}{\sigma-1}} \widetilde{p}^*_{D,t}, \quad P^*_{H,t} = N_{X,t}^{-\frac{1}{\sigma-1}} \widetilde{p}_{X,t}, \quad P^*_t = P_{F,t}^{*\alpha^*_t} P_{H,t}^{*\alpha_t}$
Consumption	$C_t = \left(\frac{C_{H,t}}{\alpha_t} \right)^{\alpha_t} \left(\frac{C_{F,t}}{\alpha^*_t} \right)^{\alpha^*_t}$	$C^*_t = \left(\frac{C^*_{F,t}}{\alpha^*_t} \right)^{\alpha^*_t} \left(\frac{C^*_{H,t}}{\alpha_t} \right)^{\alpha_t}$
Profits	$\widetilde{D}_{D,t} = \frac{\alpha_t}{\sigma} \frac{\mu_t}{N_{D,t}}, \quad \widetilde{D}_{X,t} = \frac{\sigma-1}{\kappa} \frac{\alpha_t}{\sigma} \frac{\varepsilon_t \mu^*_t}{N_{X,t}}, \quad \widetilde{D}_t = \widetilde{D}_{D,t} + \frac{N_{X,t}}{N_{D,t}} \widetilde{D}_{X,t}$	$\widetilde{D}^*_{D,t} = \frac{\alpha^*_t}{\sigma} \frac{\mu^*_t}{N^*_{D,t}}, \quad \widetilde{D}^*_{X,t} = \frac{\sigma-1}{\kappa} \frac{\alpha^*_t}{\sigma} \frac{\varepsilon_t^{-1} \mu_t}{N^*_{X,t}}, \quad \widetilde{D}^*_t = \widetilde{D}^*_{D,t} + \frac{N^*_{X,t}}{N^*_{D,t}} \widetilde{D}^*_{X,t}$
ZPC	$\widetilde{D}_{X,t} = W_t f_{X,t} \frac{\sigma-1}{\kappa-(\sigma-1)}$	$\widetilde{D}^*_{X,t} = W^*_t f^*_{X,t} \frac{\sigma-1}{\kappa-(\sigma-1)}$
Share Price	$\widetilde{V}_t = f_{E,t} W_t$	$\widetilde{V}^*_t = f^*_{E,t} W^*_t$
Labor Supply	$L_t = (\sigma-1) \frac{N_{D,t} \widetilde{D}_t}{W_t} + \sigma N_{X,t} f_{X,t} + N_{D,t+1} f_{E,t}$	$L^*_t = (\sigma-1) \frac{N^*_{D,t} \widetilde{D}^*_t}{W^*_t} + \sigma N^*_{X,t} f^*_{X,t} + N^*_{D,t+1} f^*_{E,t}$
Monetary Stance	$\mu_t = P_t C_t$	$\mu^*_t = P^*_t C^*_t$
Wages	$W_t = \Gamma \left\{ \frac{E_{t-1}\left[(A_t \mu_t)^{1+\varphi} \right]}{E_{t-1}[A_t]} \right\}^{\frac{1}{1+\varphi}}$	$W^*_t = \Gamma \left\{ \frac{E_{t-1}\left[(A^*_t \mu^*_t)^{1+\varphi} \right]}{E_{t-1}[A^*_t]} \right\}^{\frac{1}{1+\varphi}}$
Exchange Rate	$\varepsilon_t = \frac{\alpha^*_t}{\alpha_t} \frac{\mu_t}{\mu^*_t}$	
Definition of A_t	$A_t = \frac{\sigma-1}{\sigma} \alpha_t + \left(1 - \frac{\sigma-1}{\sigma\kappa} \right) \alpha^*_t + \frac{\beta}{\sigma} E_{t-1} \left[\alpha_{t+1} + \frac{\sigma-1}{\kappa} \alpha^*_{t+1} \right]$	$A^*_t = \frac{\sigma-1}{\sigma} \alpha^*_t + \left(1 - \frac{\sigma-1}{\sigma\kappa} \right) \alpha_t + \frac{\beta}{\sigma} E_{t-1} \left[\alpha^*_{t+1} + \frac{\sigma-1}{\kappa} \alpha_{t+1} \right]$
Shock Process	$\alpha_t = \frac{1}{2} \alpha_{t-1}^{\rho} \upsilon_t, \quad \alpha^*_t = \frac{1}{2} \alpha_{t-1}^{*\rho} \upsilon^*_t,\ \alpha_0 = \alpha^*_0 = 1, \quad E_{t-1}[\upsilon_t] = E_{t-1}[\upsilon^*_t] = 1, \quad E_{t-1}[\upsilon_t \upsilon_{t+1}] = 1, \quad \upsilon_t + \upsilon^*_t = 2, \quad 0 < \rho < 1$	

Online Appendix

A Data

In this section, we describe the data used for our empirical evidence. The source for aggregate exports is IMF-IFS, while the source for the nominal effective exchange rate is BIS over the period 1979-2017. Both the volatility of exports and the nominal effective exchange rate are computed as the within year standard deviation divided by the within year average.

The classification of high versus low heterogeneous countries is based upon the cross-country productivity distribution computed on the ORBIS database by Ahmad et al. (2018). The full sample includes the following high heterogeneous countries: Belgium, France, Greece, Ireland, Italy, Japan, Norway, Portugal, Spain, Sweden; and low heterogeneous countries: Australia, Austria, Canada, Finland, Germany, Netherlands, New Zealand, South Korea, Switzerland, United Kingdom, United States.

A.1 Additional evidence

Our theoretical model shows that the fixed exchange rate regime may be preferred over the flexible regime when the dispersion of firm productivity is low. Indeed, when the optimal policy is implemented, the implied volatility of the nominal exchange rate is lower for high levels of Pareto shape κ. These findings represent two "testable" predictions in the data.[15] First, we explore whether the volatility of exports is more sensitive to the volatility of the nominal exchange rate when firms are homogeneous. Second, we study whether the sensitivity of exports to exchange rate is in turn correlated with the degree of flexibility and the firm productivity distribution.

Figure A1 shows that the volatility of exports is positively correlated with the volatility of the exchange rate in countries with a more homogeneous firm productivity distribution. On the other hand, the correlation is much lower and not significant in heterogeneous firms

[15] See Online Appendix A. for additional details on the empirical evidence.

countries. This suggests that these countries have potentially less desire to intervene on the exchange rate fluctuations, as a highly managed exchange rate regime does not necessarily imply a lower volatility of exports.

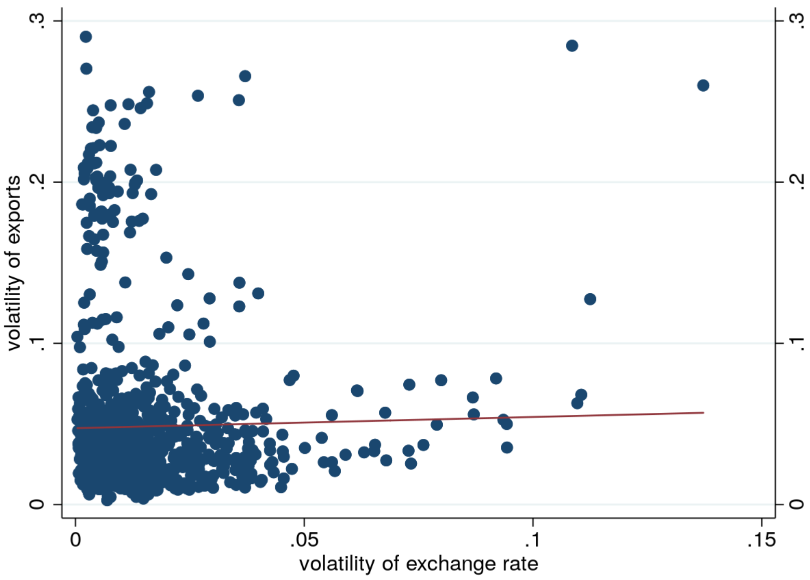
We then explore whether a higher ratio of the volatility of exports to the exchange rate is associated with a less flexible exchange rate regime. In order to do so, we run the following regression:

$$\text{regime}_{tc} = \alpha \frac{\text{vol}(\text{EXP})_{tc}}{\text{vol}(\text{TCEN})_{tc}} + \beta \frac{\text{vol}(\text{EXP})_{tc}}{\text{vol}(\text{TCEN})_{tc}} \times HOM_c + \delta_t + \gamma_c + \varepsilon_{tc},$$

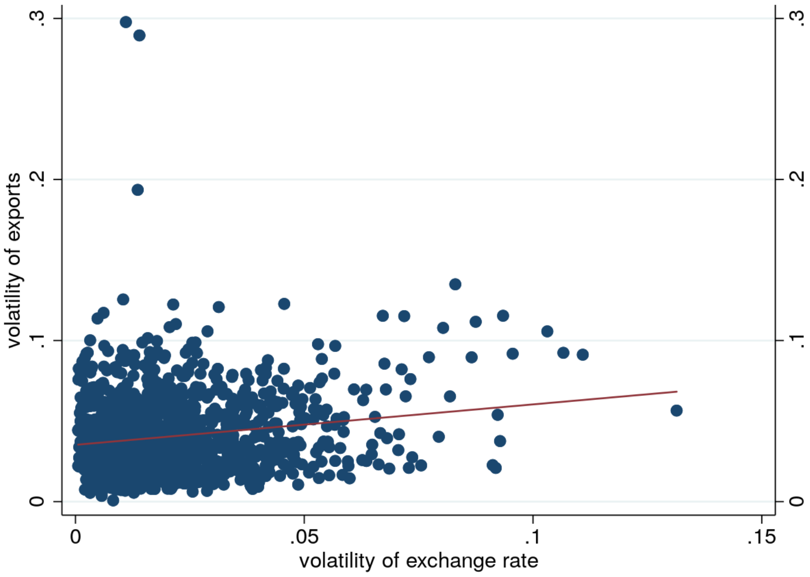
where regime_{tc} is de facto exchange rate regime defined in Ilzetzki et al. (2019), where a higher level refers to a more flexible regime. HOM_c is a dummy variable that takes value 1 fpr countries characterized by a more homogeneous firm productivity distribution. t indexes the period (quarter), c stands for the country, whereas δ_t captures time fixed-effects, γ_c the country fixed-effects, and ε_{tc} is the error term with standard errors clustered at the country level. The results are reported in table A1.

The results confirm that the degree of flexibility of the exchange rate regime is negatively associated with the ratio of volatility of consumption over nominal exchange rate. Then focus on the last column of table A1. We find that, in high homogeneous countries, the exchange rate regime tends to be more fixed when the ratio of volatility of exports over nominal exchange rate increases. This results is therefore in line with the model predictions presented above.

Figure A1: Volatility of exports and exchange rate - firm heterogeneity.



(a) Heterogeneous firms countries.



(b) Homogeneous firms countries.

Notes: a) Heterogeneous firms countries: Belgium, France, Greece, Ireland, Italy, Japan, Norway, Portugal, Spain, Sweden. OLS coefficient: .07, average volatility of exchange rate (NEER): .014, observations: 1088. b) Homogeneous firms countries: Australia, Austria, Canada, Finland, Germany, Netherlands, New Zealand, South Korea, Switzerland, United Kingdom, United States. OLS coefficient: .25***, average volatility of exchange rate (NEER): .020, observations: 1291.

Table A1: Exchange rate regime and volatility of exports to nominal exchange rate.

degree of flexibility exchange rate regime	(1)	(2)	(3)
$vol(EXP)_{tc}/vol(TCEN)_{tc}$	-.117	-.115	-.018
	(.008)	(.009)	(.004)
Observations	2379	2379	2379
Year FE	No	Yes	Yes
Country FE	No	No	Yes

degree of flexibility exchange rate regime	(1)	(2)	(3)
$vol(EXP)_{tc}/vol(TCEN)_{tc}$	-.119	-.115	-.012
	(.007)	(.009)	(.004)
$vol(EXP)_{tc}/vol(TCEN)_{tc}$ × homogeneous	.011	-.003	-.055
	(.016)	(.019)	(.009)
Observations	2379	2379	2379
Year FE	No	Yes	Yes
Country FE	No	No	Yes

B Solution of the Model

We derive here the closed form solution of the theoretical model presented in Table 1. The similar expressions hold for Foreign. First, note using average prices and the expressions of price indices, we have $P_{H,t} = N_{D,t}^{-\frac{1}{\sigma-1}} \widetilde{p}_{D,t}$ and $P_{F,t} = N_{X,t}^{*-\frac{1}{\sigma-1}} \widetilde{p}_{X,t}^{*}$. Plugging these expressions in the expression of domestic profits, profits from exporting and total profits on average, we have $\widetilde{D}_{D,t} = \frac{\alpha_t}{\sigma} \frac{\mu_t}{N_{D,t}}$, $\widetilde{D}_{X,t} = \frac{\alpha_t}{\sigma} \frac{\varepsilon_t \mu_t^*}{N_{X,t}} - f_{X,t} W_t$ and $\widetilde{D}_t = \widetilde{D}_{D,t} + \frac{N_{X,t}}{N_{D,t}} \widetilde{D}_{X,t}$. With zero cutoff profits (ZCP) condition, we have $\widetilde{D}_{X,t} = W_t f_{X,t} \frac{\sigma-1}{\kappa-(\sigma-1)}$. Note that by combining these two expressions of $\widetilde{D}_{X,t}$ we have $\widetilde{D}_{X,t} = \frac{\sigma-1}{\kappa} \frac{\alpha_t}{\sigma} \frac{\varepsilon_t \mu_t^*}{N_{X,t}}$. Also with ZCP and the expression of $\widetilde{D}_{X,t}$ previously found, we have $N_{X,t} = \frac{1}{\sigma}(1 - \frac{\sigma-1}{\kappa}) \frac{\alpha_t^* \mu_t}{W_t f_{X,t}}$. With the Pareto distribution as in the paper, it implies that $\widetilde{z}_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$.

We are now ready to derive the number of new entrant, $N_{D,t+1}$. Free entry implies that $\widetilde{V}_t = f_{E,t} W_t$. Combined with the expression of $\widetilde{D}_{t+1}$, the Euler equation about the share holdings, $\widetilde{V}_t = E_t \left[Q_{t,t+1} \widetilde{D}_{t+1}\right]$, is expressed as

$$E_t \left[\frac{\beta P_t C_t}{P_{t+1} C_{t+1}} \left(\widetilde{D}_{D,t+1} + \frac{N_{X,t+1}}{N_{D,t+1}} \widetilde{D}_{X,t+1} \right) \right] = f_{E,t} W_t.$$

Plugging the expression of $\widetilde{D}_{D,t+1}$, $\widetilde{D}_{X,t+1}$ and using the definition of monetary stance, it is rewritten as

$$E_t \left[\frac{\beta \mu_t}{\mu_{t+1}} \left(\frac{\alpha_{t+1}}{\sigma} \frac{\mu_{t+1}}{N_{D,t+1}} + \frac{N_{X,t+1}}{N_{D,t+1}} \frac{\sigma-1}{\kappa} \frac{\alpha_{t+1}}{\sigma} \frac{\varepsilon_{t+1} \mu_{t+1}^*}{N_{X,t+1}} \right) \right] = f_{E,t} W_t$$

Further, by plugging the expression of the equilibrium exchange rate $\varepsilon_t = \frac{\alpha_t^*}{\alpha_t} \frac{\mu_t}{\mu_t^*}$ and rearranging the terms, we have

$$\frac{\beta}{\sigma} \frac{\mu_t}{N_{D,t+1}} E_t \left[\left(\alpha_{t+1} + \frac{\sigma-1}{\kappa} \alpha_{t+1}^* \right) \right] = f_{E,t} W_t$$

which gives $N_{D,t+1} = \frac{\beta}{\sigma} \frac{\mu_t}{W_t f_{E,t}} E_t \left[\alpha_{t+1} + \frac{\sigma-1}{\kappa} \alpha_{t+1}^*\right]$.

Next we derive the labor demand in general equilibrium. Note that $\widetilde{D}_{X,t} = \frac{1}{\sigma} \frac{\varepsilon_t \widetilde{p}_{X,t}}{\tau} \widetilde{y}_{X,t} - f_{X,t} W_t$ and $\widetilde{D}_{D,t} = \frac{1}{\sigma} \widetilde{p}_{D,t} \widetilde{y}_{D,t}$. Also plugging the expression of prices into these profits, we have $\widetilde{y}_{D,t} = (\sigma-1) \frac{\widetilde{D}_{D,t} \widetilde{z}_D}{W_t}$ and $\widetilde{y}_{X,t} = (\sigma-1) \frac{\left(\widetilde{D}_{X,t} + f_{X,t} W_t\right) \widetilde{z}_{X,t}}{W_t}$. Putting these expression of intensive margins of average domestic and exporting firms in the labor market clearings (11), we have

$$L_t = N_{D,t}(\sigma-1)\frac{\widetilde{D}_{D,t}}{W_t} + N_{X,t}\left((\sigma-1)\frac{\widetilde{D}_{X,t}+f_{X,t}W_t}{W_t}+f_{X,t}\right) + N_{D,t+1}f_{E,t}$$

Plugging the expression of $\widetilde{D}_{D,t}$ and $\widetilde{D}_{X,t}$ found previously, the above expression becomes

$$L_t = \frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t}{W_t} + \frac{(\sigma-1)^2}{\sigma\kappa}\frac{\alpha_t\varepsilon_t\mu_t^*}{W_t} + \sigma N_{X,t}f_{X,t} + N_{D,t+1}f_{E,t}$$

Further, plugging $N_{D,t+1}$, $N_{X,t}$ and the exchange rate found previously, we have

$$L_t = \frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t}{W_t} + \frac{(\sigma-1)^2}{\sigma\kappa}\frac{\alpha_t^*\mu_t}{W_t} + (1-\frac{\sigma-1}{\kappa})\frac{\alpha_t^*\mu_t}{W_t} + \frac{\beta}{\sigma}\frac{\mu_t}{W_t}E_t\left[\alpha_{t+1}+\frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]$$

which can be further rewritten as

$$L_t = \frac{\mu_t}{W_t}\left[\frac{\sigma-1}{\sigma}\alpha_t + (1-\frac{\sigma-1}{\sigma\kappa})\alpha_t^* + \frac{\beta}{\sigma}E_t\left[\alpha_{t+1}+\frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]\right]$$

Finally, plugging the expression found in wage setting equation (7), we have

$$W_t = \Gamma\left\{\frac{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\}^{\frac{1}{1+\varphi}}.$$

B.1 Comparison of the Solution with Hamano and Picard (2017)

Stochastic labor demand followed by demand shock and its mitigation by monetary intervention is the key in deriving the main result of the paper. To see this, it would be useful to make a comparison with a model without selection into exporting market as described in Hamano and Picard (2017).

Note that by setting $f_{X,t}=0$, all firms export despite firm heterogeneity, hence $N_{X,t} = N_{D,t}$ and $\widetilde{z}_{X,t} = \widetilde{z}_D$. In such a specific case, we have $\widetilde{D}_{D,t} = \frac{\alpha_t}{\sigma}\frac{\mu_t}{N_{D,t}}$, $\widetilde{D}_{X,t} = \frac{\alpha_t}{\sigma}\frac{\varepsilon_t\mu_t^*}{N_{D,t}}$. Putting these expressions in t

$$E_t\left[\frac{\beta P_t C_t}{P_{t+1}C_{t+1}}\left(\widetilde{D}_{D,t+1}+\widetilde{D}_{X,t+1}\right)\right] = f_{E,t}W_t$$

$N_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}} E_t\left[\alpha_{t+1} + \alpha^*_{t+1}\right] = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}}$ with symmetric process of the shocks across countries.

The labor market clearings becomes

$$L_t = N_{D,t}\left(\frac{\widetilde{y}_{D,t}}{\widetilde{z}_D} + \frac{\widetilde{y}_{X,t}}{\widetilde{z}_D}\right) + N_{D,t+1} f_{E,t}$$

where $\widetilde{y}_{D,t} = (\sigma - 1)\frac{\widetilde{D}_{D,t}\widetilde{z}_D}{W_t}$ and $\widetilde{y}_{X,t} = (\sigma - 1)\frac{\widetilde{D}_{X,t}\widetilde{z}_D}{W_t}$. Plugging these expressions and $N_{D,t+1}$, we have

$$L_t = \frac{\mu_t}{W_t}\left[\frac{\sigma - 1}{\sigma} + \frac{\beta}{\sigma}\right]$$

This is the labor demand found in Hamano and Picard (2017) for their model called "lagged entry". The equilibrium wage is found to be

$$W_t = \Gamma\left\{E_{t-1}\left[\mu_t^{1+\varphi}\right]\right\}^{\frac{1}{1+\varphi}}.$$

C Expected Utility

The expected utility of Home representative household for any consecutive time period is given by

$$\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] \equiv{}& \mathrm{E}_{t-1}\left[U_t\right] + \beta \mathrm{E}_{t-1}\left[U_{t+1}\right] \\
&= E_{t-1}\left[\alpha_t \ln C_{H,t} + \alpha^*_t \ln C_{F,t}\right] + \beta E_{t-1}\left[\alpha_t \ln C_{H,t+1} + \alpha^*_t \ln C_{F,t+1}\right] \\
&\qquad \mathrm{E}_{t-1}\left[\alpha_t\left(\ln N_{D,t}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t}\right) + \alpha^*_t\left(\ln N_{X,t}^{*\frac{\sigma}{\sigma-1}}\frac{\widetilde{y}^*_{X,t}}{\tau}\right)\right] \\
&\qquad + \beta \mathrm{E}_{t-1}\left[\alpha_{t+1}\left(\ln N_{D,t+1}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t+1}\right) + \alpha^*_{t+1}\left(\ln N_{X,t+1}^{*\frac{\sigma}{\sigma-1}}\frac{\widetilde{y}^*_{X,t+1}}{\tau_t}\right)\right]
\end{aligned}$$

Plugging the equilibrium expression of $\widetilde{y}_{D,t}$, $\widetilde{y}^*_{X,t}$, $\widetilde{y}_{D,t+1}$ and $\widetilde{y}^*_{X,t+1}$,

$$\mathrm{E}_{t-1}[\mathcal{U}] = \mathrm{E}_{t-1}\left[\alpha_t\left(\ln N_{D,t}^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t\widetilde{z}_D}{W_t}\right)+\alpha_t^*\left(\ln N_{X,t}^{*\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t^*\widetilde{z}_{X,t}^*}{W_t^*\tau}\right)\right]$$
$$+\beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\left(\ln N_{D,t+1}^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_{t+1}\mu_{t+1}\widetilde{z}_D}{W_{t+1}}\right)+\alpha_{t+1}^*\left(\ln N_{X,t+1}^{*\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_{t+1}\mu_{t+1}^*\widetilde{z}_{X,t+1}^*}{W_{t+1}^*\tau}\right)\right]$$

Developing the expression, we have

$$\begin{aligned}\mathrm{E}_{t-1}[\mathcal{U}] = &\frac{1}{\sigma-1}\mathrm{E}_{t-1}[\alpha_t\ln N_{D,t}] + \mathrm{E}_{t-1}[\alpha_t\ln\alpha_t] + \mathrm{E}_{t-1}[\alpha_t\ln\mu_t] - \mathrm{E}_{t-1}[\alpha_t\ln W_t]\\ &+\frac{1}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_t^*\ln N_{X,t}^*\right] + \mathrm{E}_{t-1}[\alpha_t^*\ln\alpha_t]\\ &+\mathrm{E}_{t-1}[\alpha_t^*\ln\mu_t^*] + \mathrm{E}_{t-1}\left[\alpha_t^*\ln\widetilde{z}_{X,t}^*\right] - \mathrm{E}_{t-1}[\alpha_t^*\ln W_t^*]\\ &+\frac{\beta}{\sigma-1}\mathrm{E}_{t-1}[\alpha_{t+1}\ln N_{D,t+1}] + \beta\mathrm{E}_{t-1}[\alpha_{t+1}\ln\alpha_{t+1}]\\ &+\beta\mathrm{E}_{t-1}[\alpha_{t+1}\ln\mu_{t+1}] - \beta\mathrm{E}_{t-1}[\alpha_{t+1}\ln W_{t+1}]\\ &+\frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln N_{X,t+1}^*\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\alpha_{t+1}\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\mu_{t+1}^*\right]\\ &+\beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\widetilde{z}_{X,t+1}^*\right] - \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln W_{t+1}^*\right] + \text{cst}\end{aligned}$$

Plugging the equilibrium solution of $\widetilde{z}_{X,t}^*$ and $\widetilde{z}_{X,t+1}^*$ and relegating some terms as constant,

$$\begin{aligned}\mathrm{E}_{t-1}[\mathcal{U}] = &\mathrm{E}_{t-1}[\alpha_t\ln\mu_t] - \mathrm{E}_{t-1}[\alpha_t\ln W_t]\\ &+\frac{1}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_t^*\ln N_{X,t}^*\right] + \mathrm{E}_{t-1}[\alpha_t^*\ln\mu_t^*]\\ &+\mathrm{E}_{t-1}\left[\alpha_t^*\ln\left(\frac{N_{X,t}^*}{N_{D,t}^*}\right)^{-\frac{1}{\kappa}}\right] - \mathrm{E}_{t-1}[\alpha_t^*\ln W_t^*]\\ &+\frac{\beta}{\sigma-1}\mathrm{E}_{t-1}[\alpha_{t+1}\ln N_{D,t+1}] + \beta\mathrm{E}_{t-1}[\alpha_{t+1}\ln\mu_{t+1}] - \beta\mathrm{E}_{t-1}[\alpha_{t+1}\ln W_{t+1}]\\ &+\frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln N_{X,t+1}^*\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\alpha_{t+1}\right]\\ &+\beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\mu_{t+1}^*\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\left(\frac{N_{X,t+1}^*}{N_{D,t+1}^*}\right)^{-\frac{1}{\kappa}}\right] - \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln W_{t+1}^*\right] + \text{cst}\end{aligned}$$

Neglecting the terms for future policies, that is keeping constant μ_{t+1} and μ^*_{t+1} and the variables that depend on these policies W_{t+1}, W^*_{t+1} and $N^*_{X,t+1}$, and further rearranging,

$$
\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = \mathrm{E}_{t-1}[\alpha_t \ln \mu_t] - \mathrm{E}_{t-1}[\alpha_t \ln W_t] \\
+ \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right) \mathrm{E}_{t-1}\left[\alpha^*_t \ln N^*_{X,t}\right] + \mathrm{E}_{t-1}[\alpha^*_t \ln \mu^*_t] \\
- \mathrm{E}_{t-1}[\alpha^*_t \ln W^*_t] + \frac{\beta}{\sigma-1}\mathrm{E}_{t-1}[\alpha_{t+1} \ln N_{D,t+1}] \\
+ \frac{\beta}{\kappa}\mathrm{E}_{t-1}\left[\alpha^*_{t+1} \ln N^*_{D,t+1}\right] + \mathrm{cst}.
\end{aligned}
$$

Plugging the equilibrium solution of $N^*_{X,t}$, $N_{D,t+1}$and $N^*_{D,t+1}$, we have

$$
\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = \mathrm{E}_{t-1}[\alpha_t \ln \mu_t] - \mathrm{E}_{t-1}[\alpha_t \ln W_t] \\
+ \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right) \mathrm{E}_{t-1}\left[\alpha^*_t \ln \frac{\alpha_t \mu^*_t}{W^*_t f^*_{X,t}}\right] + \mathrm{E}_{t-1}[\alpha^*_t \ln \mu^*_t] \\
- \mathrm{E}_{t-1}[\alpha^*_t \ln W^*_t] + \frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1} \ln \frac{\mu_t}{W_t f_E}\right] \\
+ \frac{\beta}{\kappa}\mathrm{E}_{t-1}\left[\alpha^*_{t+1} \ln \frac{\mu^*_t}{W^*_t f^*_E}\right] + \mathrm{cst}
\end{aligned}
$$

Further rearranging,

$$
\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = \mathrm{E}_{t-1}[\alpha_t \ln \mu_t] - \mathrm{E}_{t-1}[\alpha_t \ln W_t] \\
+ \left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right) \{\mathrm{E}_{t-1}[\alpha^*_t \ln \mu^*_t] - \mathrm{E}_{t-1}[\alpha^*_t \ln W^*_t]\} \\
- \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right) \mathrm{E}_{t-1}\left[\alpha^*_t \ln f^*_{X,t}\right] \\
+ \frac{\beta}{\sigma-1}\{\mathrm{E}_{t-1}[\alpha_{t+1} \ln \mu_t] - \mathrm{E}_{t-1}[\alpha_{t+1} \ln W_t]\} \\
+ \frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\alpha^*_{t+1} \ln \mu^*_t\right] - \mathrm{E}_{t-1}\left[\alpha^*_{t+1} \ln W^*_t\right]\right\} + \mathrm{cst}
\end{aligned}
$$

Rearranging and plugging shock process, the expression becomes

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] = & \mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_{t}\ln\mu_{t}\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_{t}\ln W_{t}\right] \\
& + \left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_{t}^{*}\ln\mu_{t}^{*}\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_{t}^{*}\ln W_{t}^{*}\right]\right\} \\
& - \left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_{t}^{*}\ln f_{X,t}^{*}\right] \\
& + \frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_{t}\right)^{\rho}\upsilon_{t+1}\ln\mu_{t}\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_{t}\right)^{\rho}\upsilon_{t+1}\ln W_{t}\right]\right\} \\
& + \frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_{t}^{*}\right)^{\rho}\upsilon_{t+1}\ln\mu_{t}^{*}\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_{t}^{*}\right)^{\rho}\ln W_{t}^{*}\right]\right\} + \mathrm{cst}
\end{aligned}
$$

Note that monetary authority attempt to maximize the expected utility by optimally setting μ_t which has impact on for any two consecutive periods. With a symmetric steady state across countries we assume that $\alpha_{t-1} = \alpha_{t-1}^{*} = 1$, with which the expression becomes finally

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] = & \frac{1}{2}\mathrm{E}_{t-1}\left[\upsilon_{t}\ln\mu_{t}\right] - \frac{1}{2}\mathrm{E}_{t-1}\left[\upsilon_{t}\ln W_{t}\right] \\
& + \frac{1}{2}\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_{t}^{*}\ln\mu_{t}^{*}\right] - \mathrm{E}_{t-1}\left[\upsilon_{t}^{*}\ln W_{t}^{*}\right]\right\} \\
& - \frac{1}{2}\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\upsilon_{t}^{*}\ln f_{X,t}^{*}\right] \\
& + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_{t}^{\rho}\upsilon_{t+1}\ln\mu_{t}\right] - \mathrm{E}_{t-1}\left[\upsilon_{t}^{\rho}\upsilon_{t+1}\ln W_{t}\right]\right\} \\
& + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_{t}^{*\rho}\upsilon_{t+1}^{*}\ln\mu_{t}^{*}\right] - \mathrm{E}_{t-1}\left[\upsilon_{t}^{*\rho}\upsilon_{t+1}^{*}\ln W_{t}^{*}\right]\right\} + \mathrm{cst.}
\end{aligned}
$$

With symmetry of shocks as $\mathrm{E}_{t-1}\left[\upsilon_{t}^{\rho}\upsilon_{t+1}\ln\upsilon_{t}\right] = \mathrm{E}_{t-1}\left[\upsilon_{t}^{*\rho}\upsilon_{t+1}^{*}\ln\upsilon_{t}^{*}\right]$ and with no serial correlation across them such that $\mathrm{E}_{t-1}\left[\upsilon_{t}^{\rho}\ln\upsilon_{t}\right]\mathrm{E}_{t-1}\left[\upsilon_{t+1}\right] = \mathrm{E}_{t-1}\left[\upsilon_{t}^{\rho}\ln\upsilon_{t}\right]$, we have

$$
\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = \frac{1}{2}\mathrm{E}_{t-1}[\upsilon_t \ln\mu_t] - \frac{1}{2}\mathrm{E}_{t-1}[\upsilon_t \ln W_t] \\
+ \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}[\upsilon_t^* \ln\mu_t^*] - \mathrm{E}_{t-1}[\upsilon_t^* \ln W_t^*]\right\} \\
- \frac{1}{2}\left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\upsilon_t^* \ln f_{X,t}^*\right] \\
+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}[\upsilon_t^\rho \ln\mu_t] - \mathrm{E}_{t-1}[\upsilon_t^\rho \ln W_t]\right\} \\
+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}[\upsilon_t^{*\rho} \ln\mu_t^*] - \mathrm{E}_{t-1}[\upsilon_t^{*\rho} \ln W_t^*]\right\} + \mathrm{cst}
\end{aligned}
$$

Shutting down the fluctuations of fixed cost for exporting and plugging the expression of wages in equilibrium, the expression becomes (18).

D Fixed vs. Flexible Regime

Again with symmetry at the steady state and with $\Delta \ln W_t \equiv \ln W_t^{FX} - \ln W_t^{FL}$, the difference of the expected utility across different regime is

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FX}}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FL}}\right] = \frac{1}{2}\mathrm{E}_{t-1}[\upsilon_t \ln\upsilon_t] - \frac{1}{2}\Delta \ln W_t \\
+ \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}[\upsilon_t^* \ln\upsilon_t^*] - \Delta \ln W_t\right\} \\
+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}[\upsilon_t^\rho \upsilon_{t+1} \ln\upsilon_t] - \mathrm{E}_{t-1}[\upsilon_t^\rho \upsilon_{t+1}]\,\Delta \ln W_t\right\} \\
+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \upsilon_{t+1}^* \ln\upsilon_t^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \upsilon_{t+1}^*\right]\Delta \ln W_t\right\}
\end{aligned}
$$

With symmetry of shock $\mathrm{E}_{t-1}[\upsilon_t^\rho \upsilon_{t+1} \ln\upsilon_t] = \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \upsilon_{t+1}^* \ln\upsilon_t^*\right]$ and with no serial correlation of the shock such that $\mathrm{E}_{t-1}[\upsilon_t^\rho \ln\upsilon_t]\,\mathrm{E}_{t-1}[\upsilon_{t+1}] = \mathrm{E}_{t-1}[\upsilon_t^\rho \ln\upsilon_t]$, we have

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{F}\mathcal{X}}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{F}\mathcal{L}}\right] &= \frac{1}{2}\mathrm{E}_{t-1}\left[\upsilon_t \ln \upsilon_t\right]-\frac{1}{2}\Delta\ln W_t \\
&+\frac{1}{2}\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t \ln \upsilon_t\right]-\Delta\ln W_t\right\} \\
&+\beta\left(\frac{1}{2}\right)^{1+\rho}\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln\upsilon_t\right]-\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\right]\Delta\ln W_t\right\}.
\end{aligned}
$$

E The Optimal Policy

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] &= \frac{1}{2}\left\{\mathrm{E}_{t-1}\left[\upsilon_t \ln \mu_t\right]-\frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]\right\} \\
&+\frac{1}{2}\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*}\ln\mu_t^{*}\right]-\frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t^{*}\mu_t^{*}\right)^{1+\varphi}\right]\right\} \\
&+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln\mu_t\right]-\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{1+\varphi}\ln E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]\right\} \\
&+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\ln\mu_t^{*}\right]-\frac{E_{t-1}\left[\upsilon_t^{*\rho}\right]}{1+\varphi}\ln E_{t-1}\left[\left(A_t^{*}\mu_t^{*}\right)^{1+\varphi}\right]\right\}+\mathrm{cst}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{2}\left\{\frac{\upsilon_t}{\mu_t}-\frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\frac{\left(A_t\mu_t\right)^{1+\varphi}}{\mu_t}\right\} \\
&\qquad+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\frac{\upsilon_t^{\rho}}{\mu_t}-\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\frac{\left(A_t\mu_t\right)^{1+\varphi}}{\mu_t}\right\}=0
\end{aligned}
$$

$$
\begin{aligned}
&\left\{\upsilon_t-\frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\left(A_t\mu_t\right)^{1+\varphi}\right\} \\
&\qquad+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\left\{\upsilon_t^{\rho}-\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\left(A_t\mu_t\right)^{1+\varphi}\right\}=0
\end{aligned}
$$

$$
\left\{\frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\right\}\left(A_t\mu_t\right)^{1+\varphi}=\upsilon_t+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}
$$

$$
\left(A_t\mu_t\right)^{1+\varphi}=\frac{\upsilon_t+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}}
$$

$$\mu_t = \frac{1}{A_t}\left\{\frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\text{cst}}\right\}^{\frac{1}{1+\varphi}}$$

$$\varepsilon_t = \frac{\upsilon_t^*}{\upsilon_t}\frac{\frac{1}{A_t}\left\{\frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\text{cst}}\right\}^{\frac{1}{1+\varphi}}}{\frac{1}{A_t^*}\left\{\frac{\upsilon_t^* + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{*\rho}}{\text{cst}}\right\}^{\frac{1}{1+\varphi}}}$$

$$\varepsilon_t = \frac{\upsilon_t^*}{\upsilon_t}\frac{A_t^*}{A_t}\left[\frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\upsilon_t^* + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{*\rho}}\right]^{\frac{1}{1+\varphi}}$$

Note that when $\beta = 0$ and $A_t = A_t^* = cst$, we have

$$\varepsilon_t = \frac{\upsilon_t^*}{\upsilon_t}\left[\frac{\upsilon_t}{\upsilon_t^*}\right]^{\frac{1}{1+\varphi}}$$

The above is the expression found in Devereux (2004). When $\varphi = 0$, $\varepsilon_t = 1$.

F Regulation Policy

$$\begin{aligned}
&\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FX}}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{RG}}\right] = \\
&\quad \frac{1}{2}\left\{\left[\mathrm{E}_{t-1}\left[\upsilon_t \ln\mu_0\upsilon_t\right] - \mathrm{E}_{t-1}\left[\upsilon_t \ln W_t^{FX}\right]\right] - \left[\mathrm{E}_{t-1}\left[\upsilon_t \ln\mu_0\right] - \mathrm{E}_{t-1}\left[\upsilon_t \ln W_t^{FL}\right]\right]\right\} \\
&+\frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^* \ln\mu_0^*\upsilon_t^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^* \ln W_t^{*FX}\right] - \left[\mathrm{E}_{t-1}\left[\upsilon_t^* \ln\mu_0^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^* \ln W_t^{*FL}\right]\right]\right\} \\
&\qquad - \frac{1}{2}\left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^* \ln f_X^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^* \ln f_X^* \upsilon_t^{*-1}\right]\right\} \\
&+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho} \ln\mu_0\upsilon_t\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{\rho} \ln W_t^{FX}\right] - \left[\left[\mathrm{E}_{t-1}\left[\upsilon_t^{\rho} \ln\mu_0\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{\rho} \ln W_t^{FL}\right]\right]\right]\right\} \\
&+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \ln\mu_0^*\upsilon_t^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \ln W_t^{*FX}\right] - \left[\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \ln\mu_0^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \ln W_t^{*FL}\right]\right]\right\}
\end{aligned}$$

Further rearranging,

$$\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FX}}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{RG}}\right] = \frac{1}{2}\left\{\left[\mathrm{E}_{t-1}\left[\upsilon_t \ln \upsilon_t\right] - \Delta \ln W_t\right]\right\} \\
+ \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^* \ln \upsilon_t^*\right] - \Delta \ln W_t^*\right\} \\
- \frac{1}{2}\left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^* \ln \upsilon_t^*\right]\right\} \\
+ \left(\frac{1}{2}\right)^{1+\rho} \frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho} \ln \upsilon_t\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\right] \Delta \ln W_t\right\} \\
+ \left(\frac{1}{2}\right)^{1+\rho} \frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \ln \upsilon_t^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\right] \Delta \ln W_t\right\}
\end{aligned}$$

With symmetry as argued, we have

$$\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FX}}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{RG}}\right] = \\
\frac{1}{2}\left(\frac{1}{\sigma-1} + 2 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t \ln \upsilon_t\right] - \Delta \ln W_t\right\} \\
- \frac{1}{2}\left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t \ln \upsilon_t\right]\right\} \\
+ \left(\frac{1}{2}\right)^{1+\rho} \beta\left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho} \ln \upsilon_t\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\right] \Delta \ln W_t\right\}.
\end{aligned}$$

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