Timing of Market Openings and Income Distribution

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Abstract

This study analyzes the dynamic effect of trade liberalization on the income distribution and the short- and long-run equilibria in a simple one-period model with intertemporal transfers. Existing related studies have discussed this issue for closed or small open economies by assuming initial differences in the wealth distribution. This study departs from previous studies by assuming heterogeneity in skills acquisition and an imperfect credit market. We show that the impact of market openness on income inequality depends on the model economy's stage of development. That is, the extent of income inequality increases as the market opens only when the economy is less developed. Furthermore, differences in development stages (i.e., differences in market opening timing) may result in different steady states. This study illustrates the possibility that an early market opening in the case of a less-developed economy may lead to a poverty trap.

***Keywords*:** income distribution, market opening, dynamics, skill acquisition

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Introduction

How do market openings (trade liberalization and international capital movement) affect income distribution through dynamic equilibria? Many studies focus on trade openness and inequality to understand the impact of market integration on income inequality.[1] A large part of these have focused solely on static theoretical analyses, a closed economy, or empirical analyses.[2] Many important aspects of the effect of globalization on an economy's educational choice, wage rate, and income distribution during its transitional path and in its long-run equilibrium remain unexplored. This study aims to analyze the dynamic effect of market openness on the income distribution by constructing a three-factor, two-good model. Unlike existing studies, this study theoretically analyzes market openings on transition path of closed economy, which incorporate heterogeneity in skills acquisition, educational choice, and imperfect credit markets. Our result regarding the poverty trap may, to some extent, explain some developing countries' choices to decelerate their steps toward market openings.[3]

One objective of this study is to discuss the possibility of long-run differences in income distributions. Galor and Zeira (1993) and Aghion and Bolton (1997) demonstrate that households' initial distributions of assets generate income inequality in the long run even without other forms of heterogeneity. Whereas these studies only consider either a closed or a small open economy, our study examines the relationship between the timing of opening the market from autarky and the long-run income distribution. The initial distribution of assets, which is determined by the timings of market opening, plays an important role in an open economy. We show that an early market opening, in which the initial assets are small, induces greater

[1]Although "Brexit" and the "withdrawal of the US from the Trans-Pacific Partnership " are well-known examples of anti-globalization, Harris and Robertson (2013) emphasize that the impact of market openness on the income distribution is also important for developing countries.

[2]Many existing studies show that an economy's long-run equilibrium depends on its transitional dynamics. For example, Romer (1986) and Matsuyama (1991) consider models of increasing returns, whereas Matsuyama (1992) and Redding (1999) consider a learning-by-doing model. This study does not assume these kinds of externalities but rather examines the effects of credit constraints.

[3]This fact can be seen from World Bank (2017) https://data.worldbank.org/.

income inequality in the long run. The importance of timing of market opening is pointed out by Artuc et al. (2008) and Falvey et al. (2010) as well, although these previous studies focus solely on an economy's transition process and, hence, do not discuss the long-run outcome.

Another objective of this study is to examine the dynamic impact of market integration on income inequality. Feenstra and Hanson (1996) empirically illustrate that globalization, especially outsourcing, may expand wage inequality between skilled and unskilled labor in the US economy. Meschi and Vivarelli (2009) find that trade with high-income countries increases income inequality in developing countries. In a recent study, Jaumotte et al. (2013) show that trade globalization shrinks inequality, whereas financial globalization expands it. As Lim and McNelis (2016) suggest, however, the relationship between market openness and income inequality is non-monotonic[4] in the development stage of the economy. To theoretically analyze this relationship, our study considers a priori heterogeneity of ability[5] and a posteriori heterogeneity of assets[6] in a dynamic model. In a closed economy, we show that the relationship between the economy's level of development and income inequality is similar to the well-known Kuznets (1955) curve.[7] Whether income inequality in an open economy is larger than that in a closed economy depends on the degree of economic development. In the early stage of an economy's development, income inequality is greater in an *open* economy than in a closed economy. On the contrary, in the later stage of an economy's development, income inequality is greater in a *closed* economy than in an open economy.

The remainder of this paper is organized as follows. We set up a tractable closed economy model with three factors and two goods in Section 1. The steady-state equilibrium of this

[4]They derive the relationship between economic openness and income inequality and suggest that openness is positively correlated with inequality across low-income countries and is negatively correlated with inequality across high-income countries.

[5]Theoretical trade models that focus on income inequality and consider heterogeneity of ability include those of Borsook (1987), Grossman and Maggi (2000), Bougheas and Riezman (2007), Helpman et al. (2010), Bombardini et al. (2014), Blanchard and Willmann (2016), and Grossman and Helpman (2018).

[6]Our study treats differences in assets as differences in transfers from parents to their offspring, following Galor and Zeira (1993), Aghion and Bolton (1997), Galor and Moav (2004), and Cavalcanti and Giannitsarou (2017).

[7]Barro (2000) points out, however, that the Kuznets curve is consistent with data in the medium term.

closed economy is discussed in Section 2, in which we prove the existence of a steady-state equilibrium and describe the derivation of the steady state in detail. In Section 3, the dynamics of the equilibrium path are clarified by considering economic development and imperfect credit markets. In Section 4, we introduce international trade and capital movement and examine the effects of these market openings. Numerical examples are given in Section 5, in which we intuitively illustrate the equilibrium transitional path. Finally, the last section provides concluding remarks.

1 The base model

To examine the effects of opening the market of a closed economy, we first consider a closed economy. We assume that the economy lasts infinitely and that time is discrete, that is, $t = 1, 2, \ldots, \infty$. In each period, there exists a continuum of individuals who live for one period. Each individual with skill acquisition ability a leaves some transfer amount and the same ability to her offspring. Thus, the population does not grow, and the distribution of individuals' abilities in each period is stable over time. There are two final goods: a consumption good, Y^A, and a consumable investment good, Y^M. Three factors, capital K, skilled labor H, and unskilled labor L, are needed for the production of final goods. Skills are acquired through costly educational investments and individuals' own abilities. All good and factor markets are competitive, but individuals face credit constraints in borrowing for educational investment.

1.1 Production

Agricultural goods Y^A are produced only by unskilled labor with a linear technology, whereas manufacturing goods Y^M are produced by capital and skilled labor with a constant elasticity of substitution technology. These technologies are time-invariant,[8] implying that

$$\begin{aligned} Y^A(L) &= AL, \\ Y^M(K,H) &= M\left[\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)H^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \\ &A, M > 0,\ \ 0 < \alpha < 1,\ \ 0 < \sigma < \infty. \end{aligned} \tag{1}$$

We assume a one-to-one conversion between manufacturing goods (capital) and capital (manufacturing goods) without any cost.

The manufacturing good is the numéraire, and p denotes the price of the agricultural good. Then, the wages of both types of labor, w^L and w^H, and the rental rate, r, are

$$w^L(p) = Ap, \tag{2a}$$

$$w^H(k) = M(1-\alpha)\left[\alpha k^{(\sigma-1)/\sigma} + (1-\alpha)\right]^{1/(\sigma-1)}, \tag{2b}$$

$$r(k) = M\alpha\left[\alpha + (1-\alpha)k^{(1-\sigma)/\sigma}\right]^{1/(\sigma-1)} - \delta, \tag{2c}$$

where $k \equiv K/H$ is the ratio of capital to the skilled labor supply and δ is the depreciation rate of capital. The factor prices are represented by p and k.

1.2 Individuals

The model includes a continuum of individuals with heterogeneous ability a. The distribution of ability follows the cumulative distribution function $G(a)$, $a \in [0,1]$ ($g(a)$ denotes the density function). The size of the population is normalized to one. Each individual lives for one period and has one offspring who lives for the subsequent period. An individual and his offspring have the same ability. Following Galor and Zeira (1993), Aghion and Bolton

[8]When $\sigma = 1$, we define $Y^M = MK^{\alpha}H^{(1-\alpha)}$.

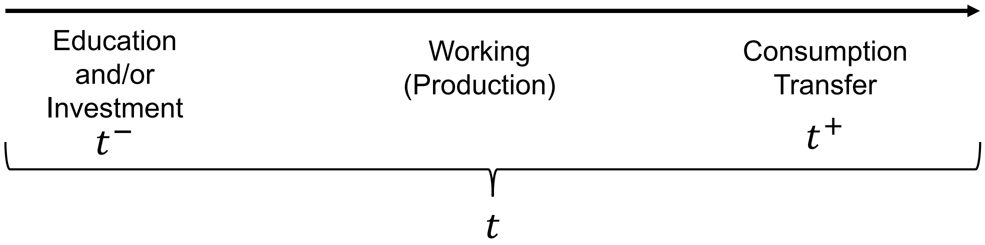


Figure 1: Time schedule of individuals

(1997), and Galor and Moav (2004)[9], we assume that individuals are motivated to leave transfers for their offspring. We also assume that the initial transfer, b_0, which is received by individuals born in the initial period $t = 0$, is identical across individuals of different abilities.

Figure 1 illustrates a time schedule of individuals. At the beginning of a period, each individual decides whether to obtain education depending on his or her ability and the transfer he or she received. Next, individuals supply capital and labor services according to their education choices and earn income. At the end of a period, individuals purchase two kinds of final goods for consumption and to transfer to their offspring.

The utility function is identical across individuals,

$$(1-\beta)\left[\gamma \ln c_t^A(a) + (1-\gamma)\ln c_t^M(a)\right] + \beta \ln b_t(a), \quad 0 < \beta, \gamma < 1, \tag{3}$$

where $c_t^A(a)$ and $c_t^M(a)$ are consumption of agricultural goods and manufacturing goods and $b_t(a)$ is transfers to offspring in units of manufacturing goods. Individuals are subject to the budget constraint

$$pc_t^A(a) + c_t^M(a) + b_t(a) \leq I_t(a), \tag{4}$$

where $I_t(a)$ is the income of an individual with ability a, which depends on his or her education decision.

[9]Galor and Zeira (1993) and Galor and Moav (2004) use a two-period overlapping generations model, whereas we use a one-period model in which individuals can obtain education at the beginning of a period.

Note that the indirect utility function is increasing with income and the education decision does not affect utility for a given income. Thus, the individual solution can be divided into the maximization of income and the maximization of utility for a given income. With a given income, an individual can obtain consumption and transfers.

$$
\begin{aligned}
c_t^A(a) &= (1-\beta)\gamma I_t(a)/p_t, \\
c_t^M(a) &= (1-\beta)(1-\gamma)I_t(a), \\
b_t(a) &= \beta I_t(a).
\end{aligned}
\tag{5}
$$

The sources of income are the wage from the labor supply and the return on assets. Individuals can provide one unit of unskilled labor without any effort. In addition, once individuals obtain education, they have the option to instead work as skilled labor. We assume that individuals with ability a can provide a units of efficient skilled labor if they obtain education. The cost of education is fixed and is defined in units of the manufacturing good. Recall that transfers are also defined in units of the manufacturing good, and each unit can be transformed into capital. Thus, received transfers can be used for two purposes: investment in capital through competitive banks and investment in education. If individuals do not obtain education and work as unskilled labor, they deposit the full amounts of their received transfers in banks and receive returns based on the rental rate. When individuals obtain education without borrowing, they deposit their remaining part of their received transfers in banks.

When individuals obtain education *with* borrowing, we assume that the credit market is imperfect, following Galor and Zeira (1993). Individual borrowers with debt d can avoid repayment unless banks pay a monitoring cost zd.[10] Incentive compatibility implies that banks pay zd as a monitoring cost. Because banks can alternatively lend capital to competitive

[10]We assume that banks execute the monitoring process after production activities occur. Thus, the opportunity cost of monitoring does not include the rental rate.

firms, the zero-profit condition on lending to individuals is

$$dr_t^d = dr_t + dz, \tag{6}$$

where r_t^d is the interest rate of individual borrowers. Thus, r_t^d is

$$r_t^d = r_t + z. \tag{7}$$

As $z \to 0$, the credit market becomes fully perfect, and the interest rate applied to borrowing from banks is r^d. Otherwise, this interest rate is equal to the rental rate r.

In this setting, the income of individuals with ability a at time t is

$$I_t(a) = \begin{cases} \max\{w_t^L + (1+r_t)b_{t-1}(a),\ aw_t^H + (1+r_t)(b_{t-1}(a) - e)\} & \text{if } b_{t-1}(a) \geq e, \\ \max\{w_t^L + (1+r_t)b_{t-1}(a),\ aw_t^H - (1+r_t^d)(e - b_{t-1}(a))\} & \text{if } b_{t-1}(a) < e, \end{cases} \tag{8}$$

where e is the cost of education. For a given factor price, the incomes of individuals are non-decreasing in their abilities a and received transfers b_{t-1}. When $b_{t-1}(a)$ is continuous, $I_t(a)$ is continuous owing to the max operator in (8).

Lemma 1. *Transfers are continuous and non-decreasing with respect to ability a for all t. That is,*

$$a' \geq a, \ \Rightarrow \ b_t(a') \geq b_t(a) \ \textit{for all} \ t.$$

For a given set of factor prices, we determine who obtains education in the following two steps. The first step is determining whether individuals need to borrow to obtain education. The second step involves comparing the benefit and cost of education for each individual. The cost depends on whether the individual must borrow to obtain education, which is determined in the first step.

In the first step, we determine that the cutoff ability for borrowing is

$$a_t^d \equiv \begin{cases} 0 & \text{if } \min_{a\in[0,1]} \{b_{t-1}(a)\} > e, \\ \infty & \text{if } \max_{a\in[0,1]} \{b_{t-1}(a)\} < e, \\ \min_a a \ \text{ s.t. } \ b_{t-1}(a) = e & \text{otherwise.} \end{cases} \tag{9}$$

Then, individuals born in period t can obtain education without borrowing if and only if their abilities are not less than a_t^d[11] because the received transfer $b_{t-1}(a)$ increases with ability, as in Lemma 1.

In the second step, we derive the minimum ability for obtaining education, defined as a_t^e. When an individual with ability a_t^d obtains education and $a_t^d w_t^H > w_t^L + (1+r_t)e$, there exists an ability that is lower than a_t^d and satisfies $aw_t^H - (1+r_t^d)(e - b_{t-1}(a)) > w_t^L + (1+r_t)b_{t-1}(a)$, at least in the neighborhood of a_t^d. Then, individuals with the minimum ability for obtaining education must borrow. In this situation, the cutoff ability for education a_t^e must satisfy the property that the income from providing skilled labor *with* borrowing is equal to the income from providing unskilled labor. However, when $a_t^d w_t^H \leq w_t^L + (1+r_t)e$, no individuals borrow to obtain education. In that situation, a_t^e must satisfy the property that the income from providing skilled labor *without* borrowing is equal to the income from providing unskilled labor.

$$a^e\left(p_t, k_t, \{b_{t-1}(a)\}_{a\in[0,1]}\right) = \begin{cases} \min\left\{\frac{w_t^L + (1+r_t)e + z[e - b_{t-1}(a^e(\cdot))]}{w_t^H}, 1\right\}, & \text{if } a_t^d w_t^H > w_t^L + (1+r_t)e. \\ \min\left\{\frac{w_t^L + (1+r_t)e}{w_t^H}, 1\right\}, & \text{if } a_t^d w_t^H \leq w_t^L + (1+r_t)e. \end{cases} \tag{10}$$

Note that a_t^e is implicitly determined when $a_t^d w_t^H > w_t^L + (1+r_t)e$ because the right-hand

[11] Not all of these individuals necessarily obtain education because education is less attractive for individuals with sufficiently low abilities.

side of the equation also includes a^e as part of the received transfer.

Before analyzing the factor supply based on the education decisions of individuals, we summarize the results of the above procedure. When $a_t^d w_t^H > w_t^L + (1 + r_t)e$, individuals whose abilities are close to but less than a_t^d want to obtain education even if they have to borrow. In this situation, (i) individuals with $a \leq a_t^e$ do not obtain education, (ii) individuals with $a_t^e < a < a_t^d$ obtain education with borrowing, and (iii) individuals with $a \geq a_t^d$ obtain education without borrowing. On the contrary, $a_t^d w_t^H \leq w_t^L + (1+r_t)e$ implies that individuals who cannot obtain education without borrowing never obtain education. In that situation, (i) individuals with $a \leq a_t^e$ do not obtain education, and (ii) individuals with $a \geq a_t^e$ obtain education without borrowing.

We express the factor supplies at t for given factor prices r_t, w_t^H, and w_t^L and a given transfer $b_{t-1}(a)$, $a \in [0,1]$. Because individuals use their received transfers for education, investment in capital, or both, the supply of capital at t is the aggregate transfer at $t-1$ that is not used for education.

$$K(a_t^e, B_{t-1}) = B_{t-1} - [1 - G(a_t^e)]e, \tag{11}$$

where $B_{t-1} \equiv \int_0^1 b_{t-1}(a)\, dG(a)$.

Because individuals with ability a who obtain education supply a units of skilled labor, the supply of skilled labor is

$$H(a_t^e) = \int_{a_t^e}^1 a\, dG(a). \tag{12}$$

The supply of unskilled labor is the mass of individuals who do not obtain education,

$$L(a_t^e) = G(a_t^e). \tag{13}$$

1.3 Dynamic equilibrium

Before defining the dynamic equilibrium, we represent the aggregate variable as functions of p_t, k_t, and transfers from individuals born in period $t-1$, $\{b_{t-1}(a)\}_{a\in[0,1]}$. The aggregate supplies of final goods are

$$Y^A\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right) = Y^A\left\{L\left[a^e\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right)\right]\right\},$$
$$Y^M\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right) = Y^M\left\{K\left[a^e\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right),B_{t-1}\right],H\left[a^e\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right)\right]\right\}, \tag{14}$$

where we assuming the full-employment condition holds. On the demand side, the aggregate demands for final goods are

$$C^A\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right) = \int_0^1 (1-\beta)\gamma I(w^L(p_t),w^H(k_t),r(k_t),b_{t-1}(a);a)\ dG(a),$$
$$C^M\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right) = \int_0^1 (1-\beta)(1-\gamma) I(w^L(p_t),w^H(k_t),r(k_t),b_{t-1}(a);a)\ dG(a). \tag{15}$$

We define the dynamic equilibrium as follows.[12]

Definition 1. *For a given b_0, if $\left(p_t,k_t,\{b_t(a)\}_{a\in[0,1]}\right)_{t=1}^{\infty}$ satisfies the following three conditions in addition to the agents' optimizations, the path is the dynamic equilibrium in a closed economy.*

- *Factors market equilibrium condition*

  $k_t = K\left[a^e\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right), \int_0^1 b_{t-1}(a)dG(a)\right] \Big/ H\left[a^e\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right)\right].$

- *Goods market equilibrium condition*

  $Y^A\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right) = C^A\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right).$

[12]We remove the market-clearing condition on the manufacturing good from Definition 1 because Walras' law implies that this condition must hold *in each period* whenever the condition on the agricultural good is satisfied. The condition on the manufacturing good is $Y^M\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right) + (1-\delta)K\left[a^e\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right),B_{t-1}\right] = C^M\left(p_t,k_t,\{b_{t-1}(a)\}_{a\in[0,1]}\right) + B_t + \int_0^1 (e-b_{t-1}(a))zdG(a)$.

- *Transition of transfers*

  $b_t(a) = \beta I(w^L(p_t), w^H(k_t), r(k_t), b_{t-1}(a); a)$ *for all* a.

To solve the equilibrium, we obtain the equilibrium price p_t and the factor ratio k_t for a given received transfer $\{b_{t-1}(a)\}_{a\in[0,1]}$. Once we obtain p_t and k_t, the factor prices derived from p_t and k_t in (2a), (2b), and (2c) determine the transfer at t $\{b_t(a)\}_{a\in[0,1]}$ using the third equilibrium condition in Definition 1. Combining this dynamic system and the initial condition on transfers at $t = 0$, the dynamic path of all endogenous variables is determined recursively.

First, we show that k_t can be rewritten as a function of p_t for a given transfer at $t-1$ using the first equilibrium condition.

Lemma 2. *There exists an unique ratio of capital to skilled labor* k_t *that satisfies the first equilibrium condition for all* $p_t \in (0, \bar{p}_t)$*, where* $\bar{p}_t = \sup\{p \in \mathbb{R}_{++} \mid \exists k \in \mathbb{R}_{++},\ a^e(p_t, k_t, \{b_{t-1}(a)\}_{a\in[0,1]}) < 1\}$. *This function,* $k_t(p_t)$*, is continuous and increasing with* p_t.

Proof. See Appendix A.1. □

Using Lemma 2, we can rewrite the cutoff ability on education as function of p_t, and $a_t^e(p_t) \equiv a^e(p_t, k_t(p_t), \{b_{t-1}(a)\}_{a\in[0,1]})$. This result implies that the aggregate supplies can be rewritten as functions of p_t only. From Walras' law, it is sufficient to analyze the market-clearing condition for one good. We focus only on the agricultural good.

Lemma 3. *The supply of agricultural goods in (14) is continuous and increasing in their price,* p_t. *As* $p_t \to 0$*, the supply approaches some finite level, and it approaches* A *as* $p_t \to \bar{p}_t$.

Proof. See Appendix A.2. □

Next, we discuss the demand for agricultural goods.

Lemma 4. *The demand for agricultural goods in (15) is continuous and decreasing in their price, p_t. As $p_t \to 0$, the demand is infinitely increasing, and it approaches $(1-\beta)\gamma A$ as $p_t \to \bar{p}_t$.*

Proof. See Appendix A.3. □

Using Lemma 2, Lemma 3, and Lemma 4, we can show the existence of the equilibrium price at t for a given bundle of transfers left by individuals born in period $t-1$.

Lemma 5. *For a given $\{b_{t-1}(a)\}_{a\in[0,1]}$, there exists a unique equilibrium price $p_t \in (0, \bar{p}_t)$ at t.*

Proof. From Lemma 2, Lemma 3, and Lemma 4, the aggregate excess demand function of the agricultural good in period t $(C_t^A - Y_t^A)$ is continuous and strictly decreasing with respect to p_t. In addition, the function is positive as $p_t \to 0$ and negative as $p_t \to \bar{p}_t$. Thus, applying the intermediate theorem to the market-clearing condition on agricultural goods yields the unique p_t in $(0, \bar{p}_t)$ as the equilibrium price at t. □

Thus, for a given $\{b_{t-1}(a)\}_{a\in[0,1]}$, all factor prices can be derived using the equilibrium price of goods p_t through Lemma 2 and (2a), (2b), and (2c). The factor prices in turn yield the amount of transfers left by individuals born in period t.

$$\text{If } \; a_t^d w_t^H > w^L + (1+r_t)e,$$

$$b_t(a) = \begin{cases} \beta[w_t^L + (1+r_t)b_{t-1}(a)], & \text{if } \; a \leq a_t^e. \\ \beta[a w_t^H - (1+r_t^d)(e - b_{t-1}(a))], & \text{if } \; a_t^e < a < a_t^d. \\ \beta[a w_t^H + (1+r_t)(b_{t-1}(a) - e)], & \text{if } \; a \geq a_t^d. \end{cases} \tag{16a}$$

If $a_t^d w_t^H \leq w^L + (1+r_t)e$,

$$b_t(a) = \begin{cases} \beta[w_t^L + (1+r_t)b_{t-1}(a)], & \text{if } a \leq a_t^e. \\ \beta[aw_t^H + (1+r_t)(b_{t-1}(a) - e)], & \text{if } a \geq a_t^e. \end{cases} \tag{16b}$$

Once we determine the equilibrium price at t for a given amount of transfers at $t-1$, we can easily show the existence and uniqueness of the dynamic equilibrium path in the recursive structure.

Proposition 1. *For a given initial transfer amount $b_0 > 0$, there exists a unique sequence of the equilibrium price and the factor ratio, $(p_t, k_t)_{t=1}^{\infty}$, that satisfies Definition 1. $b_t(a)$ follows (16a) and (16b).*

Proof. From Lemma 5, the equilibrium price at $t = 1$ uniquely exists corresponding to the initial bundle of transfers. This price determines the transfers left by individuals at $t = 1$ ($b_1(a), \forall\, a \in [0,1]$). Applying Lemma 5 once again, the same transformation holds between $t = s-1$ and s (s > 2). Thus, the vector of equilibrium prices $\{p_t\}_{t=1}^{\infty}$ is derived by induction. The factor ratio $\{k_t\}_{t=1}^{\infty}$ is derived from $\{p_t\}_{t=1}^{\infty}$. □

From Proposition 1, the dynamic path is unique once the initial conditions are determined. In other words, the long-run consequence depends only on the initial conditions in a closed economy. In an open economy, however, the long-run consequences may be determined by the timings of market openings because the distribution of transfers at the initial market opening depends on this timings. Thus, we define the equilibrium with trade liberalization and economic integration for small countries in Section 4.

2 Steady states of the base model

The steady-state equilibrium of the base model economy is defined as follows.

Definition 2. *On the equilibrium path, if every individual receives a transfer amount equal to the transfer amount he leaves to his offspring, that is,*

$$b_{t-1}(a) = b_t(a) \;\; \textit{for all} \;\; a \in [0,1],$$

then we call this equilibrium a steady-state equilibrium.

Once the economy reaches the steady state, the prices of goods and factors are fixed because the equilibrium price depends only on the distribution of transfers received in the previous period. This result is consistent with the property that transfers received and left by individuals with the same abilities are constant over time. Thus, Definition 2 is well-defined.

The objective of this section is to discuss the possibility of multiple steady states. Unlike in the economy with a perfect credit market ($z = 0$) discussed in Appendix A.7, there are three candidate steady states in the case of an imperfect credit market ($z > 0$). The first is the case in which the transfers of individuals who provide unskilled labor are larger than the cost of education and, thus, there are only two types of individuals: those who obtain no education and those who obtain education without borrowing. We call this type of steady state "*the richest*," and it reflects an economy without credit constraints. The second steady state includes all types of individuals (i.e., those who receive no education and those who receive education with or without borrowing). We call this steady state "*moderate*." The third steady state is the case in which transfer of individuals who provide unskilled labor are less than the cost of education, and no agents obtain education with borrowing. In this case, there may be multiple steady states. We call these steady states "*poverty traps*," and the state that the equilibrium path reaches depends on the initial conditions in closed economies and on the timing of market opening in open economies, as discussed in Section 4. Before discussing the existence of multiple steady states further, we show the existence of steady states.

2.1 Existence of steady states

We prove the existence of a fixed point of the mapping $\{b(a)_{t-1}\}_{a\in[0,1]} \to \{b(a)_t\}_{a\in[0,1]}$ as the equilibrium path in Definition 1. The following assumption prevents the equilibrium path from diverging.

Assumption 1. *The lower bound of the rental rate is sufficiently small that*

$$\lim_{k\to\infty} \frac{\beta}{1-\gamma+\beta\gamma}\left[1+r(k)\right] \leq 1.$$

However, the following assumption prevents the equilibrium path from shrinking too much.

Assumption 2. *There exists a b_0 such that $b_1(0) \geq b_0$.*

Proposition 2. *The equilibrium system in Definition 1 has fixed point(s) when Assumption 1 and Assumption 2 hold. These points are the steady states according to Definition 2.*

Proof. See Appendix A.4. □

2.2 Possibility of multiple steady states

Next, we use the (p,k) space to characterize the steady states. The difference from the other dynamic equilibrium path is that the transfers of individuals are no longer exogenous. Moreover, the distribution of transfers directly lead to the income distribution owing to the log-utilities of individuals. Thus, our exposition on steady states starts with transfers.

First, to express transfers in the steady states with respect to (p,k) and ability a, we define the transfers of each type of worker in the steady states. Because the transfer amount left by parents is equal to that received by offspring in the steady states, transfers in the steady states must satisfy $b_t(a) = b_{t-1}(a)$ in (16a) and (16b).

$$\begin{aligned} b^L(p,k) &= \frac{\beta w^L(p)}{1-\beta(1+r(k))}, \\ b^{Hd}(p,k;a) &= \frac{\beta\left[aw^H(k)-(1+r^d(k))e\right]}{1-\beta(1+r^d(k))}, \\ b^H(p,k;a) &= \frac{\beta\left[aw^H(k)-(1+r(k))e\right]}{1-\beta(1+r(k))}, \end{aligned} \tag{17}$$

where b^L represents the transfers of individuals who do not obtain education, b^{Hd} represents the transfer of individuals who obtain education with borrowing, and b^H represents the transfers of individuals who obtain education without borrowing.

Second, we define the cutoff abilities for obtaining education and borrowing in the steady states, similar to a_t^d in (9) and a_t^e in (10) on the dynamic equilibrium path in Section 1.

$$\begin{aligned} a^{d^*}(p,k) &= \min\left\{\frac{e}{\beta w^H(k)},1\right\}, \ \left(b^H(p,k;a) \geq e \text{ for all } a \geq a^{d^*}(p,k)\right) \\ a_d^{e*}(p,k) &= \frac{w^L(p)\left[1-\beta(1+r^d(k))\right]}{w^H(k)\left[1-\beta(1+r(k))\right]} + \frac{(1+r^d(k))e}{w^H(k)}, \ \left(b^L(p,k) \equiv b^{Hd}(p,k;a_d^{e*}(p,k))\right) \\ a^{e*}(p,k) &= \frac{w^L(p)+(1+r(k))e}{w^H(k)}. \ \left(b^L(p,k) \equiv b^H(p,k;a^{e*}(p,k))\right) \end{aligned} \tag{18}$$

$a^{d^*}(p,k)$ is similar to a_t^d but has one different point. $a > a^{d^*}(p,k)$ guarantees that the received transfer in the case of *skilled labor without borrowing* is larger than the cost of education, whereas $a > a_t^d$ focuses on given received transfers of all individuals $\{b_{t-1}(a)\}_{a\in[0,1]}$. a_d^{e*} and a^{e*} are the cutoff abilities for obtaining education in the steady state. a_d^{e*} is the ability for which the income from providing unskilled labor equals income from providing skilled labor with borrowing, which corresponds to the first case of a_t^e in (10). a^{e*} is the ability for which the income from providing unskilled labor is equal to the income from providing skilled labor without borrowing, which corresponds to the second case of a_t^e in (10). The transfer form (17) and these cutoff abilities (18) are components of the transfer function in terms of (p, k).

Third, we decompose the (p, k) space into three cases according to the candidate steady states to construct the transfer function. At the steady states, $\beta(1 + r) < 1$; otherwise, the transfers diverge. Assumption 1 guarantees this property. Moreover, if the steady states are stable, the slope of the transfers of all households must be less than one. After the discussion of steady states, we focus only on stable steady states so that $\beta(1 + \tilde{r}) < 1$ when some individuals borrow to obtain education.

The remainder of this section is devoted to the *conditional* phase diagram of transfers. "*Conditional*" means that the price p and the factor ratio k are treated as given. Thus, the phase diagram precisely shows only the transfer in the next period given the current (p, k). Along with the equilibrium path, p and k vary, as discussed in Section 3 and in the phase diagram. However, the *conditional* phase diagram of the steady state, in which (p, k) is fixed, is time-invariant. Thus, once we derive (p, k) in the steady state, the characteristics of the steady state can be analyzed using the *conditional* phase diagram of the steady state.

In the steady state we refer to as "*the richest*," the transfers of individuals who work as unskilled labor are greater than the cost of education ($b^L(\cdot) \geq e$, as shown in Figure 2).

$$\text{If } \; b^L(p, k) \geq e, \qquad b^*(p, k; a) = \begin{cases} b^L(p, k), & \text{if } \; a \leq a^{e*}(p, k). \\ b^H(p, k; a), & \text{if } \; a \geq a^{e*}(p, k). \end{cases} \tag{19}$$

In this case, no individual faces credit constraints. This type of steady state is the same as that in the economy without credit constraints discussed in Appendix A.7. Thus, this case does not involve multiple steady states.

At the steady state called "*moderate*," $b^L(\cdot) < e$ and $\beta(1 + \tilde{r}(\cdot)) < 1$, which guarantees that there exist individuals obtaining education with borrowing (see Figure 3).

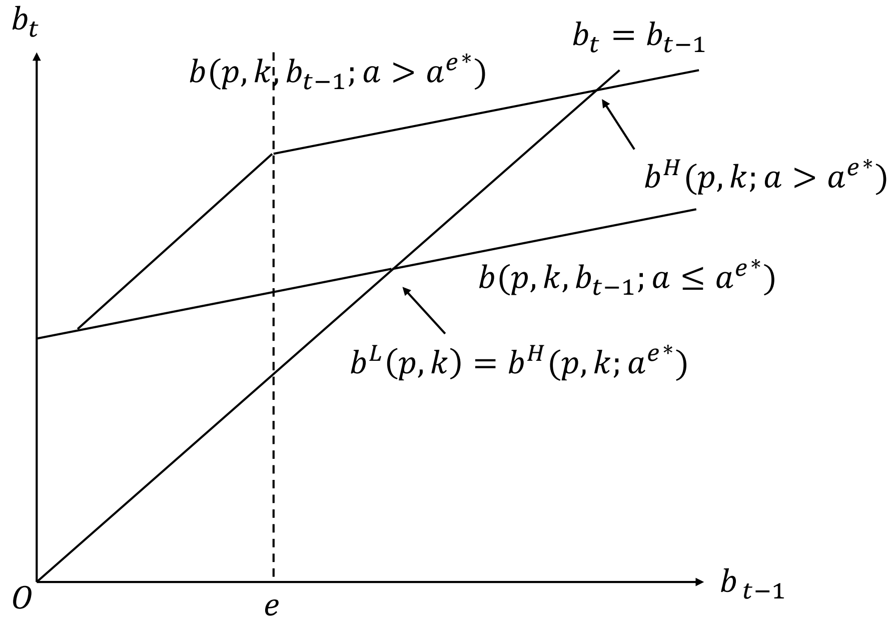


Figure 2: Phase diagram of transfers (*richest* steady state)

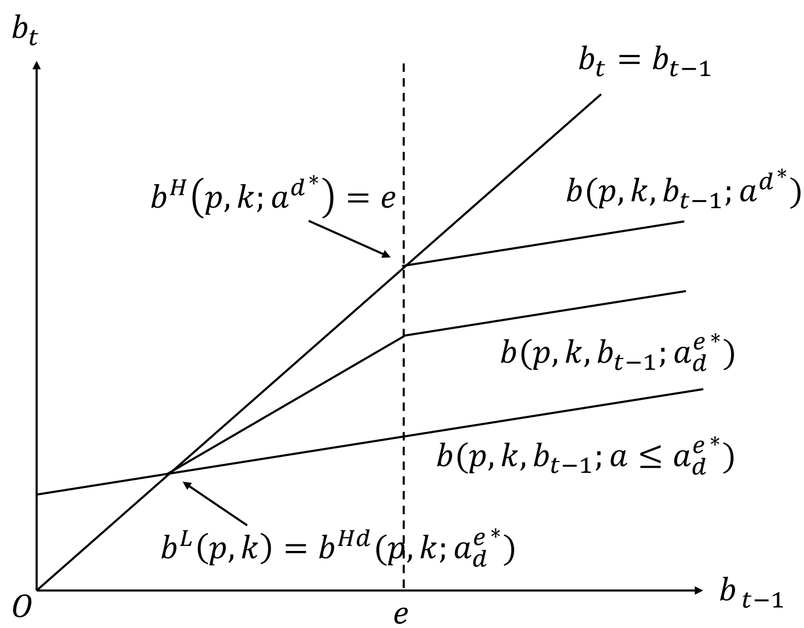


Figure 3: Phase diagram of transfers (*moderate* steady state)

If $b^L(p,k) < e$ and $\beta(1+r^d(k)) < 1$,

$$b^*(p,k;a) = \begin{cases} b^L(p,k), & \text{if } a \le a_d^{e*}(p,k) \\ b^{Hd}(p,k;a), & \text{if } a_d^{e*}(p,k) < a < a^{d*}(p,k) \\ b^H(p,k;a), & \text{if } a \ge a^{d*}(p,k). \end{cases} \tag{20}$$

The existence of individuals who borrow for education implies that the cutoff ability for borrowing to obtain education must be less than that for borrowing ($a_d^{e*}(\cdot) < a^{d*}(\cdot)$). This condition holds according to the two inequalities in (20).

Lemma 6. *A steady state* (p^*, k^*) *is "moderate" if the transfer amounts of individuals without education are smaller than the cost of education* ($b^L(p^*, k^*) < e$) *and if the slope of the transfers of individuals who borrow is less than* 1 ($\beta(1+\tilde{r}(k^*)) < 1$)*. Then, there exist individuals who borrowing in the steady state* ($a_d^{e*}(p^*, k^*) < a^{d*}(p^*, k^*)$)*.*

Proof. When $b^L(\cdot) < e$ and $\beta(1+\tilde{r}(\cdot)) < 1$,

$$\begin{aligned} a^{d*}(p,k) - a_d^{e*}(p,k) &\ge \frac{e}{\beta w^H(k)} - \left[\frac{1-\beta(1+r^d(k))}{\beta w^H(k)} b^L(p,k) + \frac{\beta(1+r^d(k))e}{\beta w^H(k)}\right] \\ &> \frac{e}{\beta w^H(k)} - \left[\frac{1-\beta(1+r^d(k))}{\beta w^H(k)} e + \frac{\beta(1+r^d(k))e}{\beta w^H(k)}\right] = 0. \end{aligned}$$

□

Comparing this type of steady state to the richest steady state, the mass of individuals working as skilled labor is smaller owing to the credit constraint, and the price of the agricultural good and the wage of unskilled labor are lower.

The conditions for a "poverty trap" steady state are $b^L(\cdot) < e$ and $\beta(1+\tilde{r}(\cdot)) \ge 1$. The

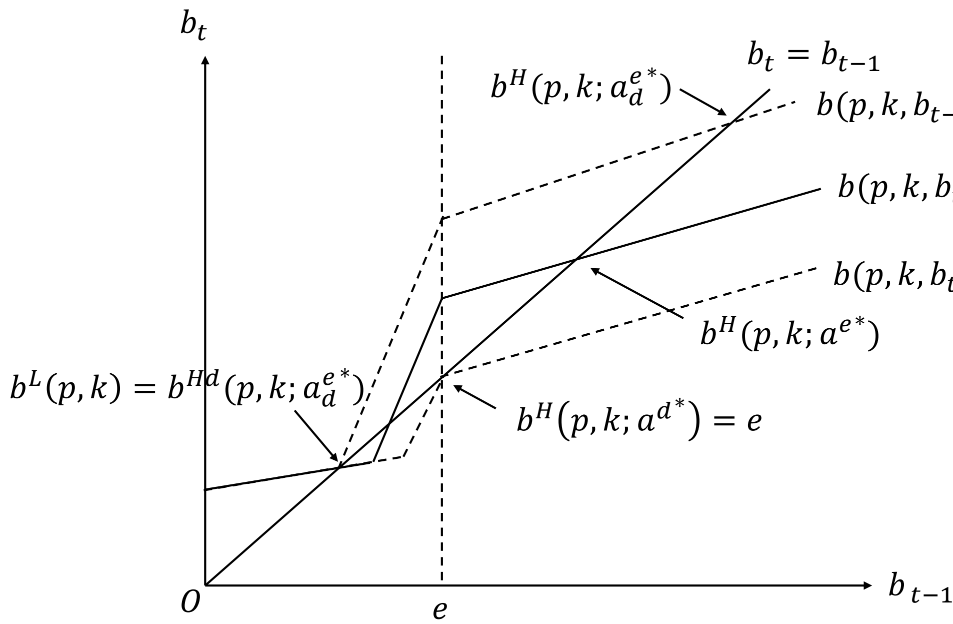


Figure 4: Phase diagram of transfers (*poorest* steady state)

proportion of individuals obtaining education is not determined for a given (p,k).

$$\text{If } b^L(p,k) < e \text{ and } \beta(1+r^d(k)) \geq 1,$$
$$b^*(p,k;a) = \begin{cases} b^L(p,k), & \text{if } a < a^{e*}, \\ b^H(p,k;a), & \text{if } a \geq a^{e*}, \end{cases} \text{ where } a^{d^*}(p,k) \leq a^{e*} \leq a_d^{e*}(p,k). \tag{21}$$

In this case, the transition of the transfers of individuals with abilities in $(a^{d^*}(\cdot), a_d^{e*}(\cdot))$ crosses the 45 degree line three times, and two of these crossings are stable points (see Figure 4). Thus, we cannot determine who obtains education. As we see later, a^{e*}, which determines the long-run economy, depends on the initial conditions, and thus, we call this steady state a “poverty trap." The following lemma shows that no individuals borrow for education in the stable “poverty trap."

Lemma 7. *When $b^L(\cdot) < e$ and $\beta(1+\tilde{r}(\cdot)) \geq 1$, no one obtains education in the steady state ($a_d^{e*}(\cdot) > a^{d^*}(\cdot)$).*

Proof. When $b^L(\cdot) < e$ and $\beta(1 + \tilde{r}(\cdot)) \geq 1$, we can easily show that $a_d^{e*}(\cdot) > a^{d^*}$ following the same logic as in the proof of Lemma 6. Then, no individuals obtain education with borrowing. □

Finally, using these functions of transfers with respect to (p, k), the steady states according Definition 2 can be rewritten in terms of (p, k).

Proposition 3. *All stable steady states satisfying $\{b^*(a)\}_{a\in[0,1]} \in \mathbb{B}$ in Definition 2 are derived as the solutions of the following system of (p, k).*

- *Transfers follow (19), (20), and (21).*

- *Factors market equilibrium condition*

  $k = K\left[a^e\left(p, k, \{b^*(p, k; a)\}_{a\in[0,1]}\right), \int_0^1 b^*(p, k; a)dG(a)\right] \Big/ H\left[a^e\left(p, k, \{b^*(p, k; a)\}_{a\in[0,1]}\right)\right].$

- *Goods market equilibrium condition*

  $Y^A\left(p, k, \{b^*(p, k; a)\}_{a\in[0,1]}\right) = C^A\left(p, k, \{b^*(p, k; a)\}_{a\in[0,1]}\right).$

Thus, if there exists a (p, k) that satisfies both the poverty trap condition, $b^L(p, k) < e$ and $\beta(1 + r^d(k)) > 1$, and the solution to Proposition 3, there are multiple steady states. We examine this finding in detail in Section 4.

In the next section, we consider the development of the economy, focusing on the transition of transfers (16a) and (16b). The main objective is determining the transitions of macro-level variables (i.e., factor prices, the fraction of individuals who obtain education, and the skill premium). Furthermore, these results yield the transition of inequality. The methods used are based on Section 1.

3 Transitional dynamics of the base model

In this section, we analyze the development process of the economy introduced in Section 1. We split the development process into two stages. In the first stage, transfers are growing for all ability levels ($b_t(a) > b_{t-1}(a)$ for all a). Assumption 2 guarantees that this stage lasts for at least one period. In the second stage, some transfers are decreasing, but the aggregate transfer is still growing.

3.1 Early stage of economic development

The early stage denotes an economy in which the transfers of all individuals are larger than those of their parents, that is, $b_t(a) > b_{t-1}(a)$ for all a. Because p_t and k_t affect factor prices and supplies through the cutoff ability for education, we highlight the relationship between the dynamics of p_t and k_t and the transition of transfers. The supply of agricultural goods depends only on the cutoff ability for education a_t^e in (10). An increase in transfers mitigates the credit constraint, and individuals find it easier to obtain education. Therefore, for a *fixed* price, the supply of skilled labor increases. In contrast, the supply of unskilled labor decreases over time.

Lemma 8. *If the aggregate transfer and the transfers of individuals who obtain education with borrowing in period $t-1$ are larger than those in period $t-1$ ($B_t > B_{t-1}$ and $b_t(a) > b_{t-1}(a)$ for all $a \in (a_t^e, a_t^d)$ when $a_t^d w_t^H > w_t^L + (1+r_t)e$), the supply of the agricultural good shifts down for a given goods price p_t: $Y^A(p_t, k_{t+1}(p_t), \{b_t(a)\}_{a\in[0,1]}) < Y^A(p_t, k_t(p_t), \{b_{t-1}(a)\}_{a\in[0,1]})$.*

Proof. See Appendix A.5. □

The demand for agricultural goods depends on domestic income. Intuitively, an increase in transfers increases aggregate income, although the direction of the transition in factor prices is ambiguous. The equivalence of the aggregate demand and the solution for a representative individual in the proof of Lemma 4 shows the correctness of this intuition; a larger transfer induces a larger income.

Lemma 9. *If the aggregate transfer and the transfers of individuals who obtain education with borrowing in period* $t-1$ *are larger than those in period* $t-1$ *(*$B_t > B_{t-1}$ *and* $b_t(a) > b_{t-1}(a)$ *for all* $a \in (a_t^e, a_t^d)$ *when* $a_t^d w_t^H > w_t^L + (1+r_t)e$*), the demand for the agricultural good shifts up for a given price:* $C^A(p_t, k_{t+1}(p_t), \{b_t(a)\}_{a\in[0,1]}) > C^A(p_t, k_t(p_t), \{b_{t-1}(a)\}_{a\in[0,1]})$.

Proof. See Appendix A.6. □

Combining Lemma 8 with Lemma 9, we can prove that the price of agricultural goods monotonically rises.

Proposition 4. *If the aggregate transfer and the transfers of individuals who obtain education with borrowing in period* $t-1$ *are larger than those in period* $t-1$ *(*$B_t > B_{t-1}$ *and* $b_t(a) > b_{t-1}(a)$ *for all* $a \in (a_t^e, a_t^d)$ *when* $a_t^d w_t^H > w_t^L + (1+r_t)e$*), the price of agricultural goods is increasing.*

Proof. This result is proved by the comparative statics in the market for agricultural goods, in which the equilibrium price is determined, using Lemma 8 and Lemma 9. □

According to Proposition 4, the wage of unskilled labor increases over time. The transitions of the rental and wage rates of skilled labor, however, are ambiguous. These factor prices are functions of k from (2c) and (2b). Because the equilibrium level of k can be decomposed into a function of $k(p)$ and the equilibrium level of p, which is unambiguous, we focus on the function $k(p)$. The sufficient condition for an increasing k is that the ratio of capital to skilled labor for a fixed p_t must be increasing: $k_{t+1}(p_t) \geq k_t(p_t)$. In detail, we can write this condition as

$$\frac{K(a^e(p_t, k_t, \{b_{t-1}(a)\}_{a\in[0,1]}), B_{t-1})}{H(a^e(p_t, k_t, \{b_{t-1}(a)\}_{a\in[0,1]}))} = k_t \leq \frac{K(a^e(p_t, k_t, \{b_t(a)\}_{a\in[0,1]}), B_t)}{H(a^e(p_t, k_t, \{b_t(a)\}_{a\in[0,1]}))}$$

When no individuals obtain education with borrowing ($a_t^e w_t^H \leq w_t^L + (1+r_t)e$), $a^e(p_t, k_t, \{b_{t-1}(a)\}_{a\in[0,1]}) = a^e(p_t, k_t, \{b_t(a)\}_{a\in[0,1]})$, and, thus, the inequality holds. When some individuals borrow for education, the inequality may not hold if the difference between $a^e(p_t, k_t, \{b_{t-1}(a)\}_{a\in[0,1]})$ and $a^e(p_t, k_t, \{b_t(a)\}_{a\in[0,1]})$ is large. Then, $k_{t+1}(p_t) < k_t(p_t)$. However, $k_{t+1}(p_{t+1}) > k_t(p_t)$ is a very rare case because $k'(\cdot) > 0$ and $p_{t+1} > p_t$ from Lemma 2 and Proposition 4. Furthermore, when the ratio of capital to skilled labor declines, $k_{t+1}(p_{t+1}) < k_t(p_t)$, the skill premium decreases and the fraction of individuals obtaining education increases: $a^e_{t+1}(p_{t+1}) < a^e_t(p_t)$.[13] Then, k increases.

At the end of the early stage of economic development, the price of agricultural goods and the wage of unskilled labor are increasing, and the ratio of capital to skilled labor increases when the economy is well developed. Then, the wage of skilled labor is increasing, and the rental rate is decreasing. Because both wages are increasing along with economic development, the transition of income inequality is ambiguous. We discuss this result in Section 5.

3.2 Late stage of economic development

In the early stage of economic development, the ratio of capital to skilled labor is eventually increasing, and the rental rate is decreasing. As a result, $\beta(1 + r)$ is less than 1, which means that the amount of received transfers without borrowing is negatively related to the growth in transfers across generations. In the late stage of economic development, individuals with abilities in some range may leave smaller transfers to their children than they received from their parents, although the aggregate transfer is still increasing. However, the transfers of individuals who borrow to obtain education must grow even in the late stage of economic development because their incomes including debt payments are negatively related to the rental rate. Thus, Lemma 8 and Lemma 9 and, hence, Proposition 4 hold even in the late stage of economic development. The wage of unskilled labor is therefore increasing

[13] $k_{t+1} < k_t$ and $B_t > B_{t-1}$ imply that $k_{t+1}(p_{t+1}) < K(a^e_{t+1}(p_{t+1}), B_{t-1})/H(a^e_{t+1}(p_{t+1})) < K(a^e_t(p_t), B_{t-1})/H(a^e_t(p_t)) = k_t(p_t)$. Then, $a^e_{t+1}(p_{t+1})$ must be less than $a^e_t(p_t)$ owing to the factor supplies (11) and (12).

throughout the economic development process, and the wage of skilled labor (the rental rate) is increasing (decreasing) in the long run.

The transition of inequality depends on the transitions of the rental rate and skill premium. A decrease in the rental rate from $\beta(1+r) \geq 1$ to $\beta(1+r) < 1$ implies the existence of a threshold timing of development at which the effect of the received transfer amount shifts from positive to negative. This effect implies that the transition of inequality is an inversed U-shaped curve. The other effect is that of the skill premium. When the skill premium is increasing over the economic development process, the effect increases inequality over the course of development. The joint effect is that inequality increases over some range but later decreases, and inequality in the long run is greater than it is in the initial economy. We demonstrate this scenario as a numerical example in Section 5. The impact of opening the market during the economic development process is examined in that section as well.

4 Open economies

4.1 Trade liberalization and economic integration

Suppose that world prices are not affected the focal country (i.e., the focal country is a small country.) We use the term *trade liberalization* to refer to an economy that faces world goods prices and *economic integration* to refer to an economy in which all prices, including factor prices, are at the world level.[14] In the following discussion, the superscript x^w means that the variable x describes the world. For both market opening types, individuals' problems, including factor supplies, do not change, but the equilibrium conditions are removed. In an economy with trade liberalization, all economic agents have access to the world market and, thus, the condition on goods markets in the home country does not hold.

[14]Economic integration allows free capital movement across countries and equalizes the rental rate. From (2c) and (2b), the factor ratio, which determines the wage of skilled labor, is also fixed. When the country's productivity is different from that of the rest of the world, the factor prices are also different.

Definition 3. *For a given* $b_{s=0}$ *or* $\{b_s(a)\}_{a\in[0,1]}$ *on the equilibrium path satisfying Definition 1 and* $\{p_t^w\}_{t=s+1}^{\infty}$*, if* $\left(k_t, \{b_t(a)\}_{a\in[0,1]}\right)_{t=s}^{\infty}$ *satisfies the following two conditions, the path is the dynamic equilibrium in a small economy with trade liberalization.*

- *Factors market equilibrium condition*
  $k_t = K\left[a^e\left(p_t^w, k_t, \{b_{t-1}(a)\}_{a\in[0,1]}\right), \int_0^1 b_{t-1}(a)dG(a)\right] \Big/ H\left[a^e\left(p_t^w, k_t, \{b_{t-1}(a)\}_{a\in[0,1]}\right)\right].$
- *Transition of transfers*
  $b_t(a) = \beta I(w^L(p_t^w), w^H(k_t), r(k_t), b_{t-1}(a); a)$ *for all* a*.*

Proposition 5. *For a given initial transfer level* $b_{s=0} > 0$ *or* $\{b_s(a)\}_{a\in[0,1]}$ *on the equilibrium path satisfying Definition 1 and for* $\{p_t^w\}_{t=s+1}^{\infty}$*, there exists an unique factor ratio sequence* $\{k_t\}_{t=s+1}^{\infty}$ *that satisfies Definition 3.* $b_t(a)$ *follows (16a) and (16b).*

Proof. From Lemma 2, $\{k_t\}_{t=s}^{\infty}$ is uniquely determined by $\{p_t\}_{t=s}^{\infty}$. Thus, the sequence of all factor prices is also uniquely determined. □

In an economy with economic integration, the factor market equilibrium condition is different from those under the other two regimes. Capital flows into or out of the home country unless the country's rental rate is equal to the world level. Thus, the ratio of capital to skilled labor is determined only by the world economy.

Definition 4. *For a given* $b_{s=0}$ *or* $\{b_s(a)\}_{a\in[0,1]}$ *on the equilibrium path satisfying Definition 1 and* $\{p_t^w\}_{t=s+1}^{\infty}$ *and for* $\{r_t^w\}_{t=s+1}^{\infty}$*, if* $\left(k_t, \{b_t(a)\}_{a\in[0,1]}\right)_{t=s}^{\infty}$ *satisfies the following two conditions, the path is the dynamic equilibrium in a small economy with economic integration.*

- *Equalization of the rental rate across countries*
  $k_t = r^{-1}(r_t^w)$
- *Transition of transfers*

$$b_t(a) = \beta I(w^L(p_t^w), w^H(k_t), r_t^w, b_{t-1}(a); a) \text{ for all } a,$$

Proposition 6. *For a given initial transfer* $b_{s=0} > 0$ *or* $\{b_s(a)\}_{a\in[0,1]}$ *on the equilibrium path satisfying Definition 1 and for* $\{p_t^w\}_{t=s+1}^{\infty}$ *and* $\{r_t^w\}_{t=s+1}^{\infty}$*, there exists an unique sequence of transfers that satisfies Definition 4 whenever* $r^{-1}(r_t^w)$ *exists.*

From Proposition 5 and Proposition 6, the equilibrium path is uniquely determined once the market opening timing is determined. Thus, the long-run equilibrium is also unique unless transfers grow infinitely. To understand whether the timing of market openings affects the steady state to which the economy converges, we apply numerical examples in the next section. First, however, we discuss the impact of openness.

4.2 The impact of market openness

To examine the impact of market openness, we divided this impact into static and dynamic effects. The static effect follows a simple Heckscher-Ohlin model. If the world economy is more developed (i.e., if the relative price of agricultural goods is higher than that of home economy from Proposition 5 and Proposition 6), the real wage of unskilled labor rises and that of skilled labor declines when the market opens. However, the dynamic effect is different from the static effect because market openness also affects transition of transfer. By this effect, early market opening makes certain individuals fall into poverty trap.

There are two reasons. The first reason is that transfer of individuals is small in the early stage of economic development. The second reason is that market opening to the developed world lowers the rental rate. Figure 5 shows that the two reasons load poverty trap. The right hand side of Figure 5 is the phase diagram of transfer in a small open economy with economic integration. Individuals who confront the possibility of poverty trap have intermediate ability; there are three intersections in the phase diagram, which means multiple steady states. Then,

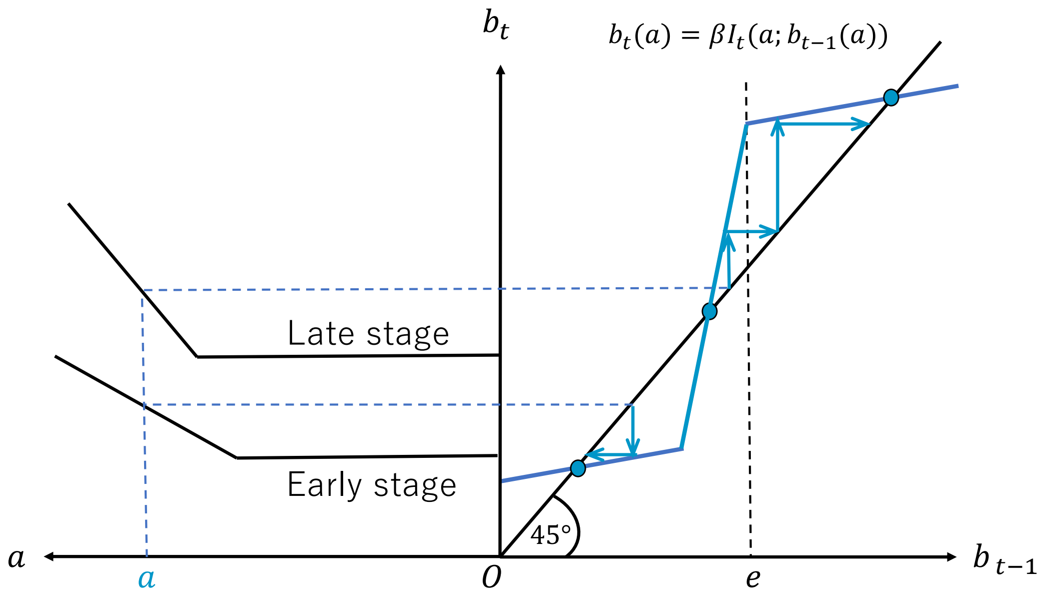


Figure 5: Early and late opening and long-run distribution

the individuals fall into poverty trap in the case of an early market opening, where their transfer is small. Early market opening lowers the rental rate and revenue from received transfer. This is expressed as flat slope of b_t without borrowing in Figure 5.

5 Numerical examples

Before describing the numerical example, we note the effect of opening a market on economic development. The range of the impact of openness depends on the abilities of individuals in the home country to access various items in the world market. The characteristics of the different open economy frameworks that we discuss are illustrated in Table 1.

Table 1: Economic regimes and prices

Regime	p	w^L	w^H	r
Trade liberalization	fixed	fixed	depends on k	depends on k
Economic integration	fixed	fixed	fixed	fixed

Table 2: Parameters in the numerical example

Parameters	A	M	α	β	γ	σ	δ	e	z	b_0
	3	3	0.5	0.5	0.5	1.2	0.04,	2,	0.5	0.2

5.1 Growth and inequality in different regimes

Parameters

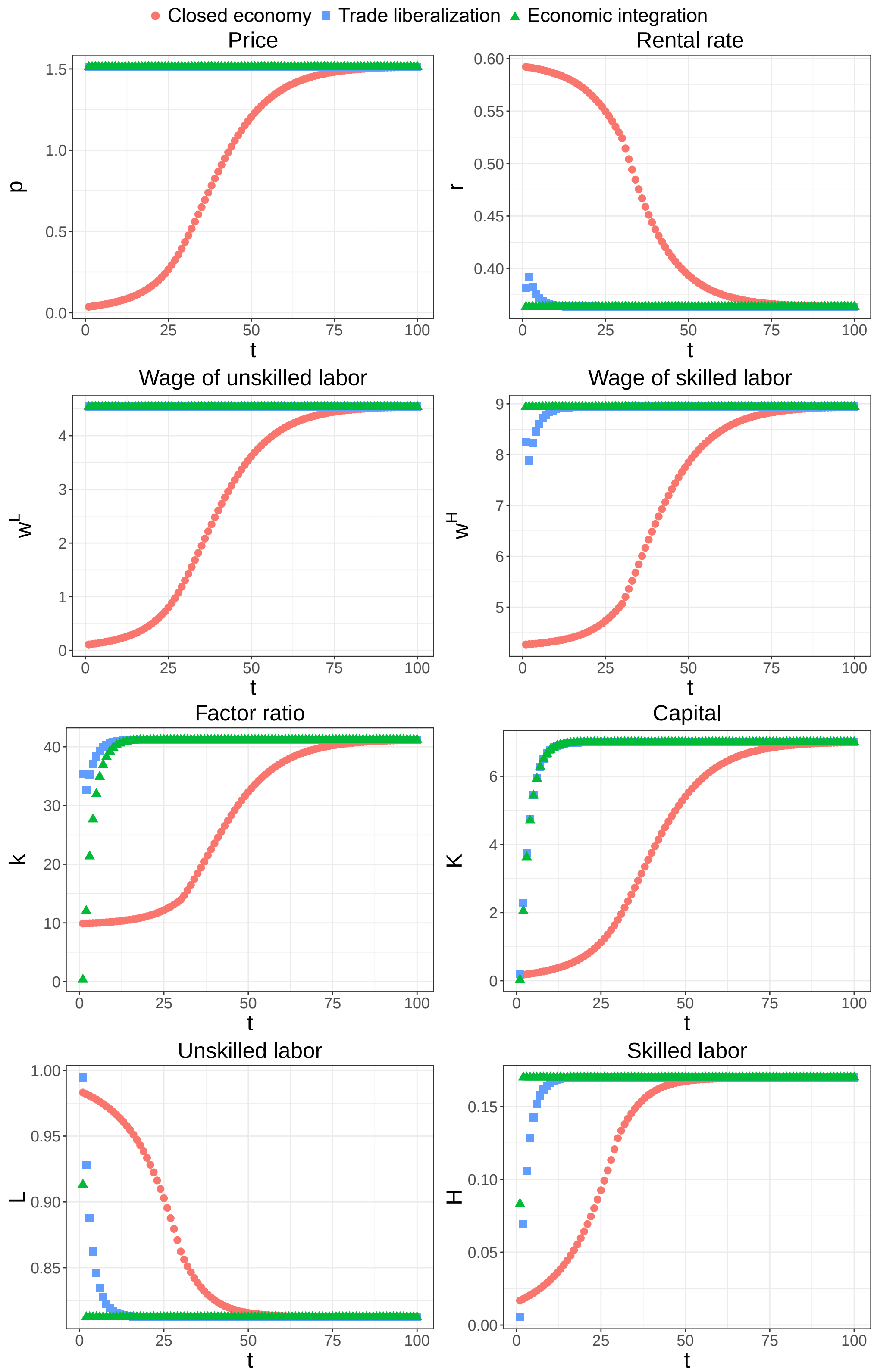
The parameters used in our simulation are based on those used by Harris and Robertson (2013) (HR, henceforth) and Lim and McNelis (2016) (LM, henceforth).[15] We assume that the depreciation rate, δ, is 0.04 (HR); the consumption share, γ, is 0.5 (HR and LM); and the technology parameter of manufacturing goods, α, is 0.5 (LM).[16] The Hicks-neutral technological parameter of both goods is assumed to be 3. We assume that the substitutability between capital and skilled labor in producing the manufacturing good, σ, takes a moderate level of 1.2, following HR. The cost of education is $e = 2$, the monitoring cost is $z = 0.5$, and the initial transfer is $b_0 = 0.2$ to examine economic development. Finally, the distribution of ability is assumed to be uniform, following Falvey et al. (2010). Table 2 summarizes these parameters.

Growth and inequality in different regimes

The transitional dynamics of the macro-level variables are shown in Figure 6. In a closed economy, development is slow during its early stage owing to the credit constraint. This constraint prevents poor individuals from obtaining education and suppresses any increase in the wage of unskilled labor owing to the excessive supply of unskilled labor. This effect also has a negative impact on the wage of skilled labor through the small accumulation of capital by poor individuals, although the credit constraint directly reduces the supply of skilled labor,

[15]HR focuses on a less developed economy in a representative agent model.
[16]LM assume that technology takes a Cobb-Douglas form.

Figure 6: Transitions of endogenous variables



which has a positive effect on the wage of skilled labor. Once the credit constraint is mitigated, the economy rapidly grows, finally converging to the steady state. In this case, the steady state is “the richest" state described in Section 2.

When trade liberalization occurs in the initial period, in which the transfers of all individuals equal b_0,[17] goods prices and the wage of skilled labor are fixed at a relatively higher level than they are in the closed economy. Thus, fewer individuals obtain education, and the supply of skilled labor is smaller in the initial period owing to smaller skill premium. During the period of economic growth represented by the accumulation of capital, however, the wage of skilled labor is much higher, as is the skill premium.[18] The supply of skilled labor is also larger than it is in the closed economy. The difference in the skill premium generates this different inequality trend, which we discuss later.

In the economy with economic integration in which all prices are fixed, the transition does not only degenerate in the factor supplies. The transition of the capital supply is similar to that in the case of trade liberalization, although the engine is different. Relative to the trade regime, the wage of skilled labor is higher in this regime, and the rental rate is lower. Combining both the positive and negative impacts of free capital movement, the speed of growth is similar across the two regimes (see the capital accumulation in Figure 6).

Next, we discuss transition of income inequality for the three economic regimes. Note that there are two sources of income inequality: the heterogeneity of efficiency among skilled labor (i.e., ability) and the transfers received from parents. Differences in ability affect the income distribution through the skill premium, which is strictly positive for all skilled labor. However, the difference in received transfers is generated by the skill premium. The effect through this channel is larger, as the rental rate is larger. Understanding the dominant source is important for understanding the impact of openness on the transition of inequality.

[17]The discussion still holds even when trade liberalization occurs in the early stage of economic development.

[18]The factor ratio k declines in the very early stage of economic development, when the mitigating effect of increasing transfers on the credit constraint is strong. Even so, the wage of skilled labor in the trade regime is higher than it is in the same stage in the closed economy.

Figure 7: Transition of inequality

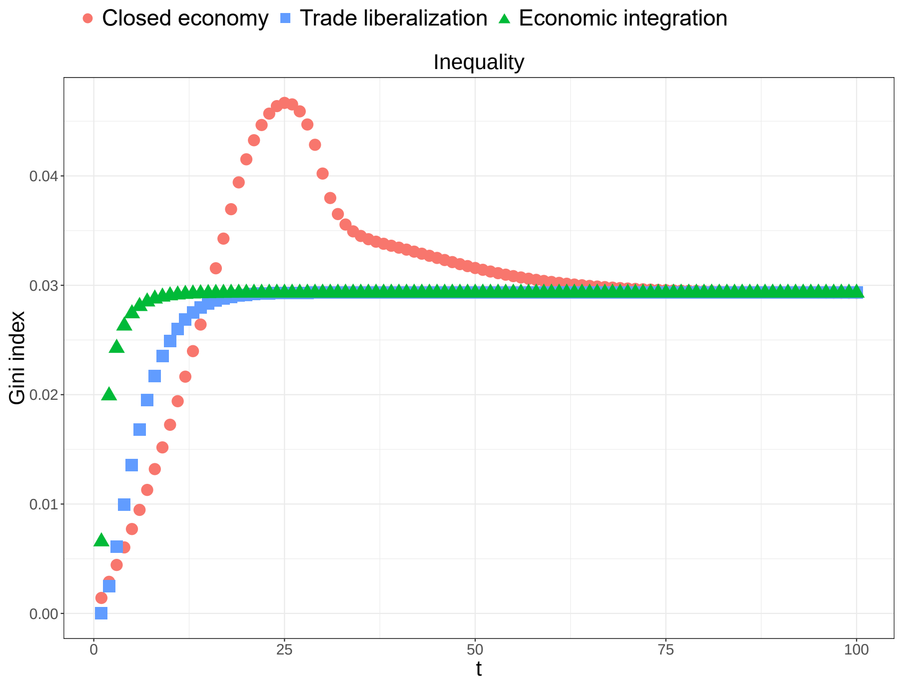


Figure 7 shows the transition of inequality in different regimes. Overall, scale of inequality is relatively small because we use a simple model in which ability provides the only innate heterogeneity across individuals. In a closed economy, the main source of inequality in the early stage of development is the difference in transfers because a small amount of transfers leads to a small skill premium and a large rental rate. After transfers have sufficiently accumulated, the skill premium becomes large and is the main source of inequality. As the economy develops further, the rental rate becomes so small that income inequality begins to shrink. The overall transition of inequality follows an inverse U shape. This non-monotonicity comes from the decrease in the rental rate and the small increase in the skill premium owing to high wage of unskilled labor in the late stage of development.

In an open framework in which all economic agents have access to the world market, the main sources of income inequality are different from those under autarky. In the case of trade liberalization, the rest of the world is assumed to be more developed than the home country is, which means that the price of the agricultural good is higher in the rest of the world, and the wage of unskilled labor is higher in the open regime. Intuitively, income inequality is smaller in the open regime owing to the decrease in the skill premium due to the increase in the wage of unskilled labor. This intuition only holds, however, when trade begins. After trade liberalization, transfers accumulate more to individuals providing unskilled labor. This accumulation leads to a large supply of capital and a high skill premium. Thus, the skill premium is the main source of the increase in income inequality in the early stage of development with trade. On the contrary, in the late stage, the large supply of capital induces a decline in the rental rate, and inequality is smaller.

Concluding remarks

This study discusses the impact of market opening on the development process and long-run outcomes in a developing economy. First, we examine the development process of a closed economy in which all prices, including factor prices, are endogenous. Compared with the autarkic equilibrium path, we show that the impact of market openings on income inequality depends on the stage of development. For less-developed countries, inequality is greater in open economies, in which capital is more accumulated, than it is in autarky. However, for well-developed countries, inequality is greater in closed economies, in which the rental rate is larger. To discuss long-run outcomes, we also consider the timing of openings and the possibility of multiple steady states. In the case of only commodity trades and no capital movement, early opening may prevent small countries from developing in the long run because the capital accumulation due to trade lowers the rental rate and reduces the revenue from capital. In the case of economic integration, which fixes all prices at the world level, the result is similar. The difference is the source of the reduction in the rental rate; economic integration leads to capital inflows from the rest of the world in the case of developing countries.

Several factors are missing from this model. First, individuals can be heterogeneous in various ways, such as preferences, initial wealth, residences, and so on, but this study focuses only on heterogeneity of ability. Thus, this study examines the role of innate ability in the transition of inequality. Nevertheless, the transition in a closed economy and the impact of openness is consistent with empirical results (e.g., Lim and McNelis (2016)). Thus, the accumulation of initially poor individuals, which is the main impact of openness in less developed countries, is important to explain inequality. Second, this study does not consider the knowledge spillover caused by foreign direct investment (FDI). If capital inflows due to economic integration bring spillovers, the economy will be more developed if it opens earlier. As a result, early economic integration requires spillovers to lead to more economic development.

Finally, we consider extensions to this study in several directions. First, income inequality can be better explained by adopting a continuum of tasks, each of which has a different complimentary with capital. Second, if FDI brings technological spillovers, the impact of opening on the long-run outcome depends on the degree of influence of spillovers and the outflow of rental revenue. Third, other credit constraints beyond the simple one used in this study may create a poverty trap in which borrowers exist in the stable steady state. With these extensions, the discussion of the relationship between the timing of opening and the development process and long-run outcomes will be richer.

Appendix

A.1 Proof of Lemma 2

$K\left[a^e(p_t, k_t, \{b_{t-1}(a)\}_{a\in[0,1]}), B_{t-1}\right]$ and $H\left[a^e(p_t, k_t, \{b_{t-1}(a)\}_{a\in[0,1]})\right]$ are continuous in k_t since the cumulative distribution function of individuals' ability $G(\cdot)$ is differential with respect to ability in addition to Assumption 1 and Lemma 1. Furthermore, $K(\cdot)$ ($H(\cdot)$) is strictly decreasing (increasing) with k_t for given p_t such that $K(\cdot) > 0$ because increase in k_t increases w_t^H and decreases r_t, and therefore decrease a_t^e, which means increase in k_t and decrease in H from (11) and (12). Define $\bar{k}_t(p_t) = \sup\{k \in \mathbb{R}_{++} | K[a^e(p_t, k_t, \{b_{t-1}(a)\}_{a\in[0,1]}), B_{t-1}] > 0\}$.

As $k_t \to 0$, $r_t \to \alpha^{\sigma/(\sigma-1)} - \delta$ and $w_t^H \to 0$ in the case of $\sigma < 1$, and $r_t \to \infty$ and $w_t^H \to (1-\alpha)^{\sigma/(\sigma-1)}$ in the case of $\sigma > 1$ from (2c) and (2b). Then, $a_t^e \to 1$ in both cases.[19] Therefore, $H \to 0$ and $K \to \int_0^1 b_{t-1}(a)dG(a)$ and hence, K/H positively diverges. This means that $k_t < K(\cdot)/H(\cdot)$.

As k_t increases from zero, the supply of capital decreases and the supply of skilled labor increases monotonically. Eventually, $k_t > K(\cdot)/H(\cdot)$ until k_t exceeds $\bar{k}_t(p_t)$ for all $p_t \in (0, \bar{p}_t)$. Thereby, the existence and uniqueness of K are prove by the intermediate value theorem. The existence guarantees $H(\cdot)$ since $k_t(p_t)$ is finite.

Next, we show the latter part. Note that $a^e(\cdot)$ is increasing with p_t and decreasing with k_t when the supplies of capital and skilled labor are positive. Thus, we can show that $k_t(p_t)$ is a continuous increasing function, using the implicit function theorem. □

A.2 Proof of Lemma 3

From the supply of unskilled labor (13), it is sufficient that a_t^e is continuous and increasing with p_t. As increasing in p_t, k_t must increase to hold the condition from Lemma 2. In the process, a_t^e also must increase since K/H is increasing with a_t^e. Since a^e is continuous in p and k, and $k_t(p_t)$ is continuous in p_t, $a^e(p_t, k_t(p_t), \{b_{t-1}(a)\}_{a\in[0,1]})$ is continuous in p_t.

Next, we show the latter part, focusing also on a_t^e. As $p_t \to 0$, a_t^e approaches zero in the case of $\lim_{p\to 0} \bar{k}(p) > 0$, and strictly positive but smaller than 1 in the case of $\lim_{p\to 0} \bar{k}(p) = 0$. As $p_t \to \bar{p}_t$, a_t^e approaches 1. □

[19]In the case of $\sigma = 1$, a_t^e also approaches 1 due to $r_t \to \infty$ and $w_t^H \to 0$.

A.3 Proof of Lemma 4

To analyze the property of the demand function, we find a function, which is equivalent to the aggregate income and an easier form. This function is domestic production function solved by social planner,

$$\mathrm{GDP}_t(p_t) = \max_{a^e} \; p_t Y^A \left(L(a^e)\right) + Y^M \left(K(a^e, B_{t-1}), H(a^e)\right) + (1-\delta) K\left(a^e, B_{t-1}\right) \\ - \int_{a^e}^{1} (e - b_{t-1}(a)) \mathbf{1}_{b_{t-1}(a) < e} dG(a),$$

subject to factor supplies (11), (12) and (13). We notice that the planner directly controls the cutoff ability on education a^e without information of factor prices. However, the planner is assumed not to be able to redistribute transfer. Thus, GDP_t is deducted of the monitoring cost in the last term.

First, we prove the equivalence. Since the relationship between a^e and GDP_t depends on whether a^e is smaller than a^d from the last term of GDP_t, the proof is also divided into two cases. We consider the case where optimal a^e is smaller than a_t^d. Then, the first order condition of $\mathrm{GDP}(\cdot)$ is[20]

$$p_t \frac{\partial Y^A}{\partial L} \frac{\partial L}{\partial a^e} + \left(\frac{\partial Y^M}{\partial K} \frac{\partial K}{\partial a^e} + \frac{\partial Y^M}{\partial H} \frac{\partial H}{\partial a^e} \right) + (e - b_{t-1}(a^e)) z g(a^e) = 0$$

where

$$\begin{aligned} \frac{\partial K}{\partial a^e} &= e g(a^e), \\ \frac{\partial L}{\partial a^e} &= g(a^e), \\ \frac{\partial H}{\partial a^e} &= -a^e g(a^e). \end{aligned}$$

[20]When the first order condition does not satisfy in $a^e \in (0, a^d)$, the other case is appropriate to maximize $\mathrm{GDP}_t(\cdot)$.

Then, the solution is

$$a^e = \frac{\frac{\partial Y^A}{\partial L} p_t + \left(1 + \frac{\partial Y^M}{\partial K} - \delta\right) e + (e - b_{t-1}(a^e))z}{\frac{\partial Y^M}{\partial H}},$$

which is identical to the cutoff ability in decentralized economy, where the net marginal productivities of factors are equal to factor prices in (10). Therefore, $\mathrm{GDP}_t(\cdot)$ is also equivalent to the aggregate income in this decentralized economy.

$$\begin{aligned}
\mathrm{GDP}_t(p_t) &= \max_{a^e} \Big\{ p_t Y^A \left(L(a^e)\right) + Y^M \left(K(a^e, B_{t-1}), H(a^e)\right) \\
&\qquad + (1-\delta)K\left(a^e, B_{t-1}\right) - \int_{a^e}^1 (e - b_{t-1}(a)) \mathbf{1}_{b_{t-1}(a)<e} dG(a) \Big\} \\
&= p_t \frac{\partial Y^A}{\partial L} L(a^e) + \left(\frac{\partial Y^M}{\partial K} K(a^e, B_{t-1}) + \frac{\partial Y^M}{\partial H} H(a^e) \right) \\
&\qquad + (1-\delta)K\left(a^e, B_{t-1}\right) - \int_{a^e}^1 (e - b_{t-1}(a)) \mathbf{1}_{b_{t-1}(a)<e} dG(a) \\
&= w_t^L L(a^e) + (1 + r_t) K(a^e, B_{t-1}) + w_t^H H(a^e) - \int_{a^e}^1 (e - b_{t-1}(a)) \mathbf{1}_{b_{t-1}(a)<e} dG(a) \\
&= \int_0^1 I(w_t^L, w_t^H, r_t, b_{t-1}(a); a) \, dG(a).
\end{aligned}$$

The second equality uses the Euler theorem, and the third and final equalities are based on the profit maximizing condition and the full-employment condition in the decentralized economy.

In the cases of $a^e \geq a_t^d$, the logic is analogous to the above case. Using the envelop theorem on $\mathrm{GDP}_t(\cdot)$, we can show that $d(\mathrm{GDP}_t(p_t)/p_t)/dp_t < 0$, and hence $dC_t^A(p_t, k_t(p_t))/dp_t < 0$. Furthermore, also at the border of two cases, the demand function is continuous due to continuity of a^e in (p, k).

Next, we show the latter part, considering the composition of the aggregate income. Since the aggregate income includes reward from the production of manufacturing goods, $C_t^A \to \infty$ as $p_t \to 0$. On the other hand, when p_t approaches $\bar{p}_t$, the aggregate income approaches to be equal reward from the production of agricultural goods. Thus, $C_t^A \to (1-\beta)\gamma$ as $p_t \to \bar{p}_t$. □

A.4 Proof of Proposition 2

On the equilibrium path, p_t and k_t are function of $\{b_{t-1}(a)\}_{a\in[0,1]}$ from Proposition 1. Thus, the equilibrium path can be represented by the self mapping from $\mathbb{R}^{\infty}_{++}$ into $\mathbb{R}^{\infty}_{++}$. For any initial condition b_0, the equilibrium path has an upper bound of transfer. To show this, suppose that transfer of some individual is increasing infinitely. Then, there are two possible cases: $\lim_{t\to\infty} \beta[1 + r(k_t)] > 1$ or $\lim_{t\to\infty} w^H(k_t) = \infty$. The first case is excluded by Assumption 1.[21] In the second case, $\lim_{t\to\infty} w^L(p_t) = \infty$ when $\lim_{t\to\infty} a^e_t = 1$ and otherwise the aggregate income of skilled labor, K, and Y^M diverge and $w^L(p)$ also diverges. Then, the credit constraint is not bounded in the long-run since both wages are growing. Finally, the transfer on the equilibrium path follows as the case without the credit constraint in Appendix A.7, where an existence and its uniqueness of the steady state are shown . We define the upper bound $\bar{b}$ as the supreme of upper bounds of the initial transfer.

Next, we consider the lower bound of transfer in the equilibrium system. In closed economy, Assumption 2 guarantees that the transfer of unskilled labor is not getting smaller than w^L_1 since the relative price of the agricultural good is increasing with transfer, which is a factor of the manufacturing good. Therefore, there exists the lower bound of transfer, and define $\underline{b}$ as the lower bound.

We consider the following subset $\mathbb{B} \subset \mathbb{R}^{\infty}_{++}$.

$$\mathbb{B} = \left\{ \{b(a)\}_{a\in[0,1]} \in \mathbb{R}^{\infty}_{++} \mid \forall a, a' \in [0,1], \underline{b} \le b(a) \le \bar{b} \text{ and } b(a') \ge b(a) \text{ if } a' \ge a \right\}$$

We define the self mapping $f : \mathbb{B} \to \mathbb{B}$ satisfying the condition on the equilibrium path in Definition 1. Since $\mathbb{B}$ is a nonempty, compact and convex set, and f is continuous mapping,[22] there exists at least one fixed point $f(\{b(a)\}_{a\in[0,1]}) = \{b(a)\}_{a\in[0,1]}$ from Schauder fixed point

[21] In the case, transfer of all individuals and the supply of capital grow infinitely though the supply of skilled labor has upper bound $\int_0^1 adG(a)$. That is why $\lim_{t\to\infty} k_t = \infty$.

[22] Since the a^e is continuous even when $a^e = a^d$, all macroeconomic variables are also continuous.

theorem. □

A.5 Proof of Lemma 8

Define $k(a^e, B) \equiv K(a^e, B)/H(a^e)$. When $B_t > B_{t-1}$, $K(a_t^e, B_t) > K(a_t^e, B_{t-1})$ and $k_t = k(a_t^e, B_{t-1}) < k(a_t^e, B_t)$. In addition, $b_t(a) > b_{t-1}(a)$ for all $a \in (a_t^e, a_t^d)$ with borrowers establishes $a_t^e = a^e(p_t, k_t, \{b_{t-1}(a)\}_{a\in[0,1]}) \geq a^e(p_t, k_t, \{b_t(a)\}_{a\in[0,1]})$. Then, $a_{t+1}(p_t) < a_t(p_t)$ from the first condition of Definition 1. This implies $L(a_{t+1}^e(p_t)) < L(a_t^e(p_t))$ from (13). □

A.6 Proof of Lemma 9

For fixed price p_t, the demand of the agricultural good depends only on the aggregate income GDP_t. GDP_{t+1} is shift up by the expansion of the factor supply frontier caused by $B_t > B_{t-1}$ and $b_t(a) > b_{t-1}(a)$ for all $a \in (a_t^e, a_t^d)$ with borrowers. □

A.7 Economy with perfect credit market

A.7.1 Steady state

When there is no credit constraint ($z = 0$), borrowers and lenders face on the common interest rate (r), and then the aggregate behavior of individuals can be represented from the solution of the representative individual's problem,

$$\begin{aligned} &\max_{a_t^e, C_t^A, C_t^M, B_t} (1-\beta)[\gamma \ln C_t^A + (1-\gamma)\ln C_t^M] + \beta \ln B_t, \\ &\text{s.t. } p_t C_t^A + C_t^M + B_t = w^L L(a_t^e) + w^H H(a_t^e) + (1+r_t)K(a_t^e, B_{t-1}). \end{aligned} \tag{22}$$

where prices including factor prices and the aggregate transfer at $t-1$ (B_{t-1}) are given for the representative individuals. This implies that it is sufficient to focus on the *aggregate transfer* B_t in order to examine the economic development without credit imperfection. The first order condition is

$$
\begin{aligned}
\frac{(1-\beta)\gamma}{C_t^A} &= \lambda_t p_t, \\
\frac{(1-\beta)(1-\gamma)}{C_t^M} &= \lambda_t, \\
\frac{\beta}{B_t} &= \lambda_t, \\
w_t^L g(a_t^e) - a_t^e w_t^H g(a_t^e) + (1+r_t) e g(a_t^e) &= 0.
\end{aligned} \tag{23}
$$

where λ_t is the Lagrange multiplier.

We show the equivalence between solutions of the representative individuals and independent individuals. From the maximization with respect to a_t^e, a_t^e is equal to the one in (10). From the maximization of consumption and transfer, C_t^A, C_t^M and B_t are equal to the ones in Section 1 due to homogeneity of the utility function. That is why these two problem are equivalent to each other.

Here, the equilibrium path according to Definition 1 can be represented by the aggregate transfer B_t and the ratio of getting education a_t^e, substituting factor prices (2a), (2b), (2c) and the aggregate supplies (14) into the first order condition (23).

$$
\begin{aligned}
a_t^e &= F(B_{t-1}), \\
B_t &= \frac{\beta}{1-\gamma+\beta\gamma} \left\{ Y^M \left[K(a_t^e, B_{t-1}), H(a_t^e) \right] + (1-\delta) K(a_t^e, B_{t-1}) \right\},
\end{aligned} \tag{24}
$$

where $F(\cdot)$ is derived from

$$
\frac{(1-\beta)\gamma}{G(a_t^e)} + \frac{(1-\gamma+\beta\gamma)}{Y^M(\cdot) + (1-\delta)K(\cdot)} \left[\left(1 - \delta + \frac{\partial Y^M}{\partial K} \right) e - \frac{\partial Y^M}{\partial H} a_t^e \right] = 0. \tag{25}
$$

Using this dynamic system (24) and (25), the development process and the long-run outcome are clarified.

A.7.2 Transitional dynamics

To yield the transition path through the economic development, we examine the relationship between B_t and B_{t-1}.

Proposition 7. *In the closed economy without credit constraint, the aggregate transfer at t (B_t) is increasing with the aggregate transfer at $t-1$ (B_{t-1}). In the case of satisfying Assumption 1 and Assumption 2, B_t monotonically converges to an unique level of the aggregate transfer where $B^* = B_t = B_{t-1}$. Then, $\{b(a)\}_{a\in[0,1]}$ in Definition 2 is also uniquely determined in the long-run.*

Proof. The equilibrium path of the economy without credit constraint is equivalent to the following problem solved by the successive social planner each of who care about the utility in (22) for given aggregate transfer at $t-1$.

$$\max_{a_t^e, C_t^A, C_t^M, B_t} (1-\beta)\left[\gamma \ln C_t^A + (1-\gamma)\ln C_t^M\right] + \beta \ln B_t,$$
$$\text{s.t. } Y^A(L(a_t^e)) = C_t^A,\ \ Y^M(K(a_t^e, B_{t-1}), H(a_t^e)) + (1-\delta)K(a_t^e, B_{t-1}) = C_t^M + B_t.$$

One can show the equivalence since the solution is equal to the recursive system (24). Increase in B_{t-1} relaxes the constraint on manufacturing good and hence, enhances the achievable utility. This implies that C_t^A rises or C_t^M, B_t rise. When C_t^A rises and C_t^M, B_t decline, it is contradicting to the first order condition (23) and Lemma 6. Therefore, larger B_{t-1} results in larger B_t, which means the monotonic transformation of B_t. Assumption 2 guarantees that B_t must be increasing if B_0 is sufficiently small.

To show the convergence, derive the upper bound of B_t to the following system,

$$\begin{aligned}\Gamma(B_{t-1}) &= \frac{\beta}{(1-\gamma+\beta\gamma)}\left\{Y^M\left[K(1,B_{t-1}),H(0)\right] + (1-\delta)K(1,B_{t-1})\right\},\\ &> \frac{\beta}{(1-\gamma+\beta\gamma)}\left\{Y^M\left[K(a_t^e,B_{t-1}),H(a_t^e)\right] + (1-\delta)K(a_t^e,B_{t-1})\right\},\end{aligned}$$

$\Gamma(\cdot)$ has the unique fixed point when $\lim_{k\to\infty}[\beta/(1-\gamma+\beta\gamma)](1+r(k)) < 1$. Thus, the original relationship also has the fixed point (B^*) as the upper limit. Since factor prices are constant and the slope of transfer is less than 1 under $B_t = B^*$, transfer of each individual converges to the level, where $b_t = b_{t-1}$ under the constant prices. This is the steady state. □

Note that there exists an unique $\{b(a)\}_{a\in[0,1]}$ satisfying Definition 2 though there are many candidates satisfying $B_t = B^*$. From the last part of the above proof, the other candidates are excluded in the long-run. That is why we focus on the steady states in Definition 2.

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