Skill Formation and the Production of Environmental Goods: 

The Role of Public Education

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Abstract
We construct a model to examine the impacts of an increase in the skilled labor input to public education on the output of final goods, environmental goods as well as the amount of pollution emissions, in which skill formation is incorporated into the traditional RV model. Due to the skill formation, one of the specific factors and mobile factor which are represented by skilled labor and unskilled labor are endogenously determined compared to the traditional RV model. We then reveal that an increase in public education service may bring negative effects to the output above due to the change in wages of skilled labor and unskilled labor as well as the rental rate of real capital which are caused by the skill formation process. However, we also find some conditions for the role of public education to ensure the policy to achieve the goals, in particular environmental protection.

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1. Introduction

The role of education in the issue of sustainable development has long been discussed actively on a global scale.¹ In the context of sustainable development, environment is one of the most important issues for the sustainability. On the other hand, according to the United States Environmental Protection Agency (EPA), the environmental education can play a great role to protect and improve the environment.

One of the main contents of the environmental education is to learn the skills for identifying and resolving environmental challenges.² Then, environmental goods (EG) such as soil/water separation systems, catalytic converters, and chemical remediation and bioremediation can be produced with not only unskilled labor but also skilled. For example, bioremediation means that using microorganisms, fungi, plants, or their enzymes restore contaminated land or groundwater to restore the natural environment contaminated with harmful substances. Professional expertise on the environment are essential for its supply. Hence, in this paper, we consider the role of

¹ For example, UNESCO World Conference on Education for Sustainable Development (ESD) held in Aichi-Nagoya, Japan, from 10 to 12 November 2014, discussed the role of education that fosters the leadership needed to build a sustainable society. For further information, see the official website of Japanese National Commission for UNESCO (http://www.mext.go.jp/en/unesco/index.htm).
² According to the information provided by United States Environmental Protection Agency (https://www.epa.gov/), environmental education includes various activities and programs, besides the professional development.
education for environmental preservation which affects the level of supply for skilled labor employed in EG sector, that is, the environmental industry. As related papers, Merrifield (1988), Copeland (1991), Chua (2003), Abe and Sugiyama (2010), and Sugiyama and Saito (2016) examine the vertical relationship between upstream EG sector and downstream final goods (FG) sector which discharges emissions or dumps wastes. However, these papers above do not include skill formation.³

Although we follow the basic concept of Findlay and Kierzkowski (1983) to describe the skill formation, our model incorporates the publicly provided education into the traditional Ricardo-Viner (RV) Model.⁴ In particular, we focus on the skill formation through the public education provided by the central and local governments, instead of privately provided one, because such education courses are sometimes offered by national and public universities.⁵

Wong and Chong (1999), Janeba (2003) and Kreickemeier (2009) also modeled skill formation in HO or RV model. However, environmental-related issues are not included in these

³ Wan, et.al (2018) considered the impact of trade liberalization in final goods as EGs with or without the pollution taxes on dirty upstream sectors.
⁴ Findlay and Kierzkowski (1983) assume that education is provided by the private sector instead of the public sector, and incorporates the skill formation into the traditional Hechsher-Ohlin (HO) Model, instead of RV Model.
⁵ In this meaning, the public education can be considered as public intermediate goods in our model. Refer to Tawada and Abe (1984), Abe (1990), and Yanase and Tawada (2012), in which public intermediate good is incorporated into HO Model and RV Model. See also Ishikawa (2000), where RV model is applied with a sector-specific (private) intermediate good.
studies. As explained below, our model assume that skilled labor can either be employed in EG sector or in the public education sector as an “educator”. Then, unlike the traditional RV model, the supply of skilled labor used in EG sector that is one of the specific factors is endogenously determined due to skill formation under the public education.

We then reveal the impacts of an increase in skilled labor toward public education, in other words, an increase in the educator service on the output of EG, FG, the wage differential between skilled and unskilled labor, and the national income in addition to the amount of pollution emissions. One of the important results we obtain in our analysis is, the policy that increases the educator service may not help to increase the output of EG, even if the supply of skilled labor employed in EG sector can be increased by the policy. On the other hand, this policy also decreases the output of FG necessarily, and the amount of emissions decreases under some conditions. The relative wage of unskilled labor to skilled labor can always be improved by this policy. However, the national income may decline with a decrease in the output of FG.

The remainder of the paper is organized as follows. Section 2 presents our model. Section 3 gives preliminary analysis for comparative statics. Section 4 confirms the impact of an increase

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6 In this paper, the supply of education service provided the government depends only on the number of educators employed in the education sector. Hence, an increase in the number of educators implies an increase in the education service as well. We refer this as “educator service”.

7 EG may increase or decrease in our analysis.
in the educator service on the supply side of the economy, that is, skilled and unskilled wages, the rental rate of capital, the supply of skilled labor, the output of EG and FG, and the amount of emissions. Moreover, we discuss the effect of this policy on national income. Section 5 provides some concluding remarks.

2. The Model

We consider a small open economy consisting of two private sectors and one public sector which provides public education. Private sectors correspond to a final goods (FG) sector and an environmental goods (EG) sector, that is, an environmental industry. Let the output of FG and EG be denoted by $X$ and $A$, respectively. The production process for FG generates pollution emissions as a by-product, whereas EG produced in the upstream environmental industry are used for abating emissions in FG sector. In this sense, two private sectors are vertically related. We assume that the government imposes an emission tax, $t$, on polluting downstream producers to reduce pollution emissions.

There are three factors of production in our model. Let unskilled labor be denoted by $L$, which can move between FG and EG sectors. while $K$ denotes the real capital, is a specific factor

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8 In general, environmental industries can supply either the environmental goods or the environmental services. Contrary to our model, abatement programs or services such as consulting, monitoring, and auditing in the polluting FG sector can also be provided by EG sector, with skilled workers who possess advanced environmental knowledge. However, for simplicity and tractability, we stick only our attention on the skilled labor who contributes the production of EG.
of production for FG. $H_A$ denotes the skilled labor employed in EG sector who serves as human capital with professional environmental knowledge, which is a specific factor of production for EG.\(^9\) The assumptions above are exactly same as those in the traditional RV model.

In addition, the government provides public education to produce the skilled labor. Since the government also employs the skilled labor as educators for the public education, the skilled labor is employed not only in EG sector as input for the production but also in the public education sector as educators.\(^10\) Let $H_G$ denote the educator service, that is, the skilled labor employed in the public education sector. These two parts of skilled labor, then, can be expressed as

$$H = H_G + H_A,$$

where $H$ denotes the whole supply of the skilled labor in the economy, which can also be referred as the whole size of “human capital” through the skill formation.

Hence, unlike the conventional specific factor model, the input of skilled labor as a specific factor used in the production of EG, $H_A$, is immobile between two private sectors, i.e. EG and FG sectors, but is mobile between EG and public education sectors. Moreover, as explained in the

\(^9\) Notice that skilled labor is not a number of workers in this paper.

\(^10\) Unlike the traditional RV model, the skilled labor supply is not inelastic, and in fact, it can move between EG and public education sectors. Moreover, the unskilled labor supply is also not inelastic in our model. Since the skilled labor acquires special environmental knowledges via the skill formation process, that is, the public education, we stick on the terminology of “skilled labor” consists of both the “quality” and the “amount” of individuals, instead of “the number of workers”. We will see this more later.
next subsection, the whole supply of skilled labor, $H$, is also a production function of skilled labor produced in the public education sector. As a result, $H_A$ is endogenously determined in our model.

We represent the production functions for FG and for EG as

$$X = F_X(K_X, L_X, A_X, Z),$$

$$A = F_A(H_A, L_A),$$

respectively, where $L_i$, $(i = X, A)$ is the input of unskilled labor for both goods, $K_X$ is the input of real capital, $A_X$ denotes the input demand for EG, and $Z$ is the amount of pollution emissions. We assume that $F_X(\cdot)$ exhibits constant returns to scale in four variables including emissions $Z$, and $F_A(\cdot)$ is also constant returns to scale in two variables, and both are assumed to be quasi-concave. In the following analysis, we assume the substitutability between unskilled labor and skilled labor, (i.e., $L_A$ and $H_A$) in EG sector, as well as unskilled labor and real capital, (i.e., $L_X$ and $K_X$) in FG sector. We will explain in detail whether $Z$ and $L_X$ or $K_X$ is substitutive or complementary are critical when we derive the results of comparative statics with respect to an increase in the educator service, $H_G$.

2.1 Skill formation

In this economy, $N$ individuals are born at each period and are uneducated in the beginning. Assume that each of them lives for two periods, then the population of the economy at each period
is $2N$. At each period, $N$ individuals are split into two parts right after being born, in which they can choose to either become skilled labor or unskilled labor at the first period. Those who choose to become skilled labor, need to be educated through the public education for free, which is provided by the government, but they need to forgo the opportunity to participate in the market for unskilled labor during the education period (i.e., the first period). In other words, educated individuals can only earn their income at the second period as skilled labor.

Let $U_G$ denote those who decide to earn their income under wage rate $w_H$ as skilled labor at the second period after having enjoyed the public education at the first period, which can also be referred as “student”. On the other hand, those who decide not to enjoy the education, can choose to participate in the unskilled labor market, to earn their income under wage rate $w_L$, at the first period and the second period (i.e., their lifetime). Let the number of individuals choose to become unskilled labor be denoted by $U_L$. Hence, the population in this economy can be expressed as $2N = 2(U_G + U_L)$. This equation is equivalent to

$$N = U_G + U_L.$$  \hspace{1cm} (1)

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11 The assumption for two periods is for tractability. If we assume any $T$ periods, there is no essential difference. Similarly, our main results do not change if we assume constant population growth rate.

12 At each period, the unskilled labor consists of 2 types of $U_L$, since $U_L$ born at the present period and $U_L$ born at the previous period will work together at the present period, which results in $2U_L$. In contrast, at each period, only $U_G$ born at the previous period participate in the skilled labor market at the present period, while $U_G$ born at the present period who are being educated do not participate.
As mentioned above, $H$ is the whole supply of skilled labor in the economy, the skill formation in the public education sector can be expressed with the production function as follows:

$$H = F^G(H_G, U_G), \quad (2)$$

$F^G(\cdot)$ is also assumed to exhibit constant returns to scale in two variables, where $H_G$ and $U_G$ are used as the input factors for the skill formation in the public education. Noting this assumption, $F^G(\cdot)$ is rewritten as

$$h = f(h_G),$$

where $h$ and $h_G$ are defined as $h \equiv H/U_G$ and $h_G \equiv H_G/U_G$, respectively. The former represents the skill acquired by each individual of $U_G$, and the latter denotes the educator-student ratio. In addition, we assume $f(0) > 0$, $f'(h_G) > 0$, and $f''(h_G) < 0$.

Next, we present the lifetime income of educated and uneducated individuals, and then the arbitrary condition. For simplicity, we assume that the domestic capital stock is constant at each period which is denoted by $\bar{K}$ and owned by all the individuals. Moreover, there is perfectly equality in distribution of the capital stock.\(^{13}\) Then, in each period, each individual receives $r\kappa$ equally, where $\kappa \equiv \bar{K}/2N$ and $r$ is the factor price (i.e., rental rate) of the real capital.

On the other hand, the government imposes an emission tax for FG sector where pollution emissions are generated. For simplicity, we also assume that the tax revenue is distributed to all

\(^{13}\) Many studies assume this, for example, see Gupta (1994).
individuals equally at each period, which can be expressed as $tz$, where $t$ denotes the emission tax rate, and $z \equiv {Z} / {2N}$.

Let $w_L$ denotes the wage for each uneducated individual (i.e., $U_L$) earn in period 1 and period 2 as unskilled labor (i.e., $L$), while $w_Hf(h_G)$ is earned by each individual in period 2 for EG production if he or she decides to acquire the skill in period 1 through the public education, where $w_H$ denotes the wage for a unit of skill.$^{14}$

Assume that the public education provided by the government is financed by equal income tax among all individuals, which can be shown as

$$w_H H_G = \tau \{ w_H (H_A + H_G) + w_L L + rK \},$$

where $\tau$ denotes the income tax rate.$^{15}$ Therefore, the lifetime income of $U_L$ and $U_G$ at present value are, respectively, can be expressed as follows:

$$V_L = (1 - \tau) \{ r\kappa + w_L + (r\kappa + w_L)/(1 + \rho) \} + tz + tz/(1 + \rho),$$

$$V_G = (1 - \tau) \{ r\kappa + (r\kappa + w_Hf(h_G))/(1 + \rho) \} + tz + tz/(1 + \rho),$$

where $\rho$ is the fixed interest rate.$^{16}$

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$^{14}$ Notice that $w_H$ is not the wage for an individual, $U_G$, but for a unit of $f(h_G)$ which is owned by each individual, $U_G$. As a result, each of them earns $w_Hf(h_G)$ in period 2.

$^{15}$ See Abe (1990). Notice also that $\tau$ is determined with this equation alone if other endogenous variables are all determined in the model. In addition, the emission tax revenue distributed to the individuals is assumed not subject to the income taxation. However, even without this assumption, we will obtain the same result under the arbitrary condition.

$^{16}$ Note that the interest rate, $\rho$, does not necessarily coincide the price of real capital since we assume a
When the arbitrary condition holds for an individual's choice regarding whether he or she enjoys the education or not, that is, $V_L = V_G$, the condition is expressed as follows:\(^{17}\)

$$f(h_G) = \left(\frac{w_L}{w_H}\right)R,$$ (3)

where $R(\equiv 2 + \rho)$ is constant. As explained above, given $R$ and the educator service, that is, the skilled labor employed in the public education sector, $H_G$, the number of individuals who decide to become skilled labor, $U_G$, is determined from equation (3) as the function of the relative wage of unskilled relative to skilled, $w_L/w_H$. Moreover, after $U_G$ is determined, the number of uneducated individuals, $U_L$, who work as unskilled labor, $L$, is determined from equation (1), given individuals $N$ born in each period.

Moreover, considering $h_G = H_G/U_G = H_G/(N - U_L)$ from equation (1), equation (3) is rewritten as

$$f\left(\frac{H_G}{N - U_L}\right) = \left(\frac{w_L}{w_H}\right)R.$$ (3)'

From equation (3)', we obtain the relation between the number of uneducated individuals, $U_L$, and the relative wage of unskilled relative to skilled, $w_L/w_H$, given the educator service, $H_G$, and $R(\equiv 2 + \rho)$.

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\(^{17}\) Since $dV_G/dU_G < 0$ holds, $U_G$ must be determined under the arbitrary condition, as well as $U_L$. 

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2.2 Equilibrium Conditions

In the following analysis, we designate the price of FG as the numeraire and the relative price of EG as $p$. Considering a small open economy and the assumption of constant returns-to-scale technology about production functions, $F^i(\cdot), (i = X, A)$, the zero-profit conditions for the production of FG and EG are represented as

$$1 = c^X(w_L, r, p^*, t), \quad (4)$$

$$p^* = c^A(w_L, w_H), \quad (5)$$

where $c^i(\cdot), (i = X, A)$ is the unit cost function for private sectors, and $p^*$ denotes the world relative price of EG.\(^{18}\) Note that each unit cost function is homogeneous of degree one and concave with respect to the vector of corresponding variables.

Next, we provide full employment conditions of production factors. Note that we assume two periods and unskilled labor is the input for the production of EG and FG. Then, the supply of unskilled workers, $L$, at each period is expressed as follows:\(^{19}\)

$$L = 2U_L. \quad (6)$$

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\(^{18}\) An asterisk denotes the corresponding foreign variables in this paper.

\(^{19}\) Since unskilled labor does not acquire skill at all, the supply of unskilled labor is just the number of the individuals. In contrast, the supply of the skilled labor is not the number of educated individuals but the total skill they acquire. Notice also that the skilled input in the public sector, namely the educator service is not represented as the number.
On the other hand, as mentioned above, noting the skilled labor are used for the production of EG and the public education (i.e., $H = H_A + H_G$), and considering $h \equiv H/U_G$ and $h = f(h_G)$, we obtain the following equation:

$$H_A = f(h_G)U_G - H_G. \quad (7)$$

Since the educator service, that is, the input of skilled labor in the public education, $H_G$, is given, and then the number of uneducated individuals, $U_G$, is determined from equation (3), the input of skilled labor, $H_A$, used in the production of EG is also determined from equation (7).

In the following analysis, $c_j^i, (i = X, A; j = H, L, r, p^*, t)$ denotes the partial derivatives of the unit cost functions with respect to the factor prices, which are the skilled and unskilled wages, the rental rate of real capital, the world price of EG, and the emission tax rate. Considering equation (7), the full employment condition for skilled labor is then expressed as follows:

$$H_A = c_H^A(w_L, w_H)A. \quad (8)$$

Moreover, considering $h_G = H_G/U_G = H_G/(N - U_L)$ in equation (7), then equation (8) is rearranged as

$$f \left( \frac{H_G}{N-U_L} \right)(N - U_L) - H_G = c_H^A(w_L, w_H)A. \quad (8)'$$

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20 From Shephard’s lemma and the envelope theorem, $c_j^i, (i = X, A; j = H, L, r, p^*, t)$ is equal to skilled, unskilled, real capital and EG requirements for producing one unit of the $i$-th good, and the emissions per FG output for the partial derivative with respect to the emission tax. For example, $c_L^X(\cdot) \equiv \partial c^X/\partial w_L$ is the unskilled labor requirements for producing one unit of FG.
Similarly, the full employment conditions for unskilled labor is expressed as

\[ L = c_L^X(w_L, r, p^*, t)X + c_L^A(w_L, w_H)A, \quad (9) \]

which yields the following equation from equation (6):

\[ 2U_L = c_L^X(w_L, r, p^*, t)X + c_L^A(w_L, w_H)A. \quad (9)' \]

The full employment conditions for real capital is represented as

\[ \bar{K} = c_r^X(w_L, r, p^*, t)X. \quad (10) \]

On the other hand, since we assume a small open economy which allows FG to be tradable in the world market, the equilibrium condition for EG is expressed as \( A + A' = A_X \), which is represented as

\[ A + A' = c_p^X(w_L, r, p^*, t)X, \quad (11) \]

where \( A' \) denotes the traded volume of EG in the world market.\(^{21}\)

Finally, noting that \( c_l^X(w_L, r, p^*, t) \) denotes the emissions per output of FG, the amount of pollution emissions is expressed as follows:

\[ Z = c_l^X(w_L, r, p^*, t)X. \quad (12) \]

There are 12 endogenous variables \( L, U_L, U_G, H, H_A, w_L, w_H, r, X, A, A' \), and \( Z \) from equations (1) to (12) except for (3)', (8)', and (9)'. We then confirm the economic and environmental impacts of the educator service, that is, an increase in the input of skilled labor

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\(^{21}\) Notice that if we assume EG is to be imported, \( A' > 0 \), and vice versa.
toward the public education, $H_G$. Moreover, these equilibrium conditions above are summarized into the equations (3)', (4), (5), (8)', (9)', and (10) to solve the variables of $U_L$, $w_L$, $w_H$, $r$, $X$ and $A$.

3. Preliminaries

Totally differentiating the equations (3)', (4), (5), (8)', (9)', and (10), we obtain the following simultaneous equation system:

$$
\begin{bmatrix}
\frac{h_G}{v_G} f' - \frac{R}{w_H} & \frac{w_L R}{w_H} & 0 & 0 & 0 \\
0 & c_L^X & 0 & c_L^X & 0 & 0 \\
0 & c_R^A & c_H^A & 0 & 0 & 0 \\
(1 - \varepsilon_H) f & c_{HL}^A & c_{HH}^A & 0 & 0 & c_H^R \\
-2 & -\delta_L & c_{LH}^A & c_{Lr}^X & c_L^X & c_L^A \\
0 & c_{rL}^X & 0 & c_{rr}^X & c_r^X & 0
\end{bmatrix}
\begin{bmatrix}
\frac{dU_L}{v_g} \\
\frac{dw_L}{v_g} \\
\frac{dw_H}{v_g} \\
\frac{dr}{v_g} \\
\frac{dX}{v_g} \\
\frac{dA}{v_g}
\end{bmatrix}
= \begin{bmatrix}
\frac{-f'}{v_g} \\
0 \\
0 \\
1 - \frac{\varepsilon_H}{\theta_G} \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
dH_G
\end{bmatrix},
$$

where $\delta_L \equiv -c_{Lk}^X - c_{Lk}^A A$, $\theta_G \equiv \frac{H_G}{H_G + H_A} = \frac{h_G}{f}$, and $\varepsilon_H \equiv f' \cdot \frac{h_G}{f}$, which also imply that $\delta_L > 0$, $0 < \theta_G < 1$, and $0 < \varepsilon_H < 1$ since $F^G(H_G, U_G)$ is a linear homogeneous function.

$c_{jk}^i, (i = X, A; j, k = H, L, r, p^*, t)$ is the second-order partial derivatives for $c_j^i$. As the unit cost function $c_j^i, (i = X, A)$, is concave with respect to the vector of corresponding variables, the signs of the $c_{jk}^i$ are negative when $j = k$. In particular, we have, $c_{LL}^X < 0$, $c_{rr}^X < 0$, and $c_{HH}^A < 0$, which also implies that $\delta_L > 0$. On the other hand, when $j \neq k$, the signs of $c_{jk}^i$ depend on the substitutability between factor $j$ and $k$. In our model, $c_{HL}^A > 0$ and $c_{rL}^X > 0$ are satisfied because unskilled labor is substitutive to real capital in FG sector and to skilled labor in EG sector. In addition, $c_{LL}^X > (<=) 0 \text{, } (j = L, r)$ means that the unskilled labor or real capital is substitutive.
(complementary) to the emissions in FG sector.\textsuperscript{22}

Notice also that $\varepsilon_H$ denotes the output (i.e., $H$) elasticity of the educator service (i.e., $H_g$) in the public education sector, whereas $\theta_G$ is the share of the educator service from the whole supply of human capital. Since $\varepsilon_H \equiv f' \cdot \frac{h_g}{f}$ and $\theta_G \equiv \frac{H_g}{H_g + H_A} = \frac{H_g}{H} \frac{U_g}{U_g}$, which results in $\theta_G = \frac{h_g}{f}$, we also have $\frac{\varepsilon_H}{\theta_G} = f'(h_g)$. Hence, $1 - \frac{\varepsilon_H}{\theta_G} = 1 - f'(h_g)$ tends to be positive if $h_g$ is sufficiently large which brings a significant small size of $f'$ or $\varepsilon_H$, and vice versa. We conclude as follows:

**Lemma 1**: If educator-student ratio ($h_g$) is sufficiently large (small), $\varepsilon_H < \theta_G$ ($\theta_G < \varepsilon_H$) holds, thus we have $1 - \frac{\varepsilon_H}{\theta_G} > 0$ ($1 - \frac{\varepsilon_H}{\theta_G} < 0$).

Let $J$ represent the coefficient matrix on the left-hand side of equation (13). Then, noting $p^* = c^A(w_L, w_H) = w_H c^A_H + w_L c^A_L$, the determinant of $J$, that is, $|J|$ results in the following equation:

$$|J| = \frac{h_g}{U_g} f' \psi + \frac{p^* c^X_R}{w_H} \{(1 - \varepsilon_H) f c^A_L + 2 c^A_H \} > 0,$$

where $\psi \equiv c^A_H X \Delta^X + c^A_L X \Delta^A > 0$, $\Delta^X \equiv c^X_L \Delta^X_1 + c^X_R \Delta^X_2 > 0$, and $\Delta^A \equiv c^A_L \Delta^A_1 + c^A_H \Delta^A_2 > 0$, since $\Delta^X_1 \equiv c^X_T c^X_R - c^X_L c^X_R > 0$, $\Delta^X_2 \equiv c^X_T c^X_L - c^X_R c^X_R > 0$, $\Delta^A_1 \equiv c^A_H c^A_H - c^A_L c^A_H > 0$ and $\Delta^A_2 \equiv c^A_L c^A_L - c^A_H c^A_H > 0$.

\textsuperscript{22} Without the analysis for the change in $p^*$, the discussion for $c^X_{Lp^*}$ is omitted.
4. Comparative Statics

In this section, we first reveal the impacts of the change in the employment of skilled labor as educators in the public education sector (i.e., the role of public education), on the output of FG and EG, as well as the amount of emissions. Before this, we first confirm the effects on the wages of skilled and unskilled labor as well as the rental rate of the real capital, which will change the skill formation as well as the supply of skilled labor employed in EG sector.

4.1 Wages, rental rate and supply of skilled labor

In order to see the change in the supply of skilled labor employed in EG sector due to the change in the educators employed in the public education, we need to see also the change in \( U_G \), where \( dU_G = -dU_L \), as well as the change in \( w_L, w_H \), and \( r \), simultaneously.

From equation (13), we obtain

\[
\frac{dw_L}{dH_G} = \frac{|J|^{-1} f_U}{u_G} c_H^x c_L^A c_H^A [2c_H^A + c_L^A f(1 - \theta_G)] > 0
\]

(14)

\[
\frac{dw_H}{dH_G} = -\frac{|J|^{-1} f_U}{u_G} c_H^x c_L^A c_H^A [2c_H^A + c_L^A f(1 - \theta_G)] < 0
\]

(15)

\[
\frac{dr}{dH_G} = -\frac{|J|^{-1} f_U}{u_G} c_H^x c_L^A c_H^A [2c_H^A + c_L^A f(1 - \theta_G)] < 0
\]

(16)

These results with respect to the impacts on factor prices are summarized in the following proposition.
PROPOSITION 1: An increase in the educator service, always brings unambiguous effects on the factor prices, in which the wage of unskilled labor rises, whereas the wage of skilled labor and the rental rate of real capital decline. As a result, the relative wage of unskilled labor to skilled labor always rises.

These results are so familiar compared to the results in the traditional RV model, in which an increase in one of the specific factors, always increases the factor price of mobile factor while decreases the factor prices of immobile factors. In this paper, unskilled labor is a mobile factor whereas skilled labor and real capital are immobile factors between EG and FG sector, respectively. However, in fact, skilled labor is actually mobile between EG and public education sector. Moreover, the supplies of skilled labor and unskilled labor which are equal to the amount employed in FG and EG sector are not inelastic compared to the traditional RV model.

Since $H_G$ is only an increase in educator service instead of the supply of skilled labor employed in EG sector, the change in the $H_A$ needs to be revealed as well.

From equation (13), we obtain

$$
\frac{dU_L}{dH_G} = -|J|^{-1} \left[ \frac{f'}{v_G} \psi + \left( 1 - \frac{\epsilon_H}{\theta_G} \right) c_L c_r x^2 \bar{p} \right] < 0. \tag{17}
$$

As we have discussed in Lemma 1, if the educator-student ratio is sufficiently large, $\epsilon_H < \theta_G$ holds, which implies $1 - \frac{\epsilon_H}{\theta_G} > 0$, then we have $\frac{dU_L}{dH_G} < 0$. 

Moreover, considering equation (17) and \( dU_G = -dU_L \) from equation (1), we can show the effect on the unskilled labor, which implies

\[
\frac{dU_G}{dH_G} = |J|^{-1} \left[ \frac{f'}{U_G} \psi + \left( 1 - \frac{\epsilon_H}{\theta_G} \right) c_L^2 c_f^2 \left( 1 - \frac{\epsilon_H}{\theta_G} \right) \right] > 0. 
\] (18)

Similarly, \( \frac{dU_G}{dH_G} > 0 \) must hold if the educator-student ratio is sufficiently large.

**LEMMA 2:** If educator-student ratio is sufficiently large, an increase in educator service in the public education sector decreases the unskilled labor supply, while increases students in the public education sector.

Totally differentiating equation (7) and rearrange it, we have

\[
\frac{dH_A}{dH_G} = \left( 1 - \epsilon_H \right) f \frac{dU_G}{dH_G} - \left( 1 - \frac{\epsilon_H}{\theta_G} \right). 
\] (19)

The first term and the second term in the RHS of equation (16) are the indirect effect and the direct effect of \( H_G \) on \( H_A \), which are positive and negative, respectively.\(^{24}\) Paying attention to \( 1 - \epsilon_H > 1 - \frac{\epsilon_H}{\theta_G} \) in the case of the educator-student ratio is sufficiently large, if \( f \frac{dU_G}{dH_G} > 1 \) holds, we can obtain

\[
\frac{dH_A}{dH_G} > 0, 
\]

\(^{23}\) Concretely, \( \frac{dH_A}{dH_G} = \left( 1 - \epsilon_H \right) f |J|^{-1} \left[ \frac{f' \psi}{U_G} + \left( 1 - \epsilon_H / \theta_G \right) c_L^2 c_f^2 \left( 1 - \epsilon_H / \theta_G \right) \right] - \left( 1 - \epsilon_H / \theta_G \right). \)

\(^{24}\) Recalling \( dU_G/dH_G > 0 \) may not be satisfied if \( \epsilon_H < \theta_G \) does not hold.
In particular, equation (19) can also be expressed as

\[
\frac{dH_A}{dH_G} = (1 - \varepsilon_H) \sigma_G^U \frac{H_G}{\sigma_G^{U}} - \left(1 - \frac{\varepsilon_H}{\theta_G}\right),
\]

(19)'

where \( \sigma_G^U \equiv \frac{H_G}{\sigma_G^{U}} \) denotes the output elasticity of educator service input, which is smaller than 1. This can be proved by totally differentiating equation (3)' and using the results obtained from equations (14) and (15), in which the impacts of the educator service on the wages of skilled and unskilled have been discussed. From the results we know that an increase in \(H_G\) raises the ratio of \( \frac{w_L}{w_H} \) then \( f'\left(\frac{H_G}{U_G}\right) \) increases as well, which also implies that the ratio of \( \frac{H_G}{U_G} \) rises, hence we know that \( U_G \) is diminishing returns to \( H_G \). As a result, \( \sigma_G^U \) must be smaller than 1. Notice also that if \( H_G \) is too large, \( U_G \) turns to decrease, but \( \sigma_G^U < 1 \) still holds.

Hence, we can conclude that if \( \sigma_G^U > \theta_G \), then \( \frac{dH_A}{dH_G} > 0 \) is satisfied, which can be concluded as the following proposition:

**PROPOSITION 2:** If the output elasticity of educator service input is larger than the educator-student ratio, an increase in educator service in the public education sector increases the skilled labor supply for EG sector.

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25 One may argue that if \( 1 - \frac{\varepsilon_H}{\theta_G} < 0 \), then \( \frac{dH_A}{dH_G} \) becomes positive unambiguously, which is not true, since \( \frac{dU_G}{dH_G} > 0 \) requires the condition in the opposite way (i.e., \( 1 - \frac{\varepsilon_H}{\theta_G} > 0 \)) to be determined unambiguously.

26 Considering \( 0 < \theta_G < 1 \) and \( \sigma_G^U < 1 \), we have also \( 0 < \theta_G < \sigma_G^U < 1 \).
Table 1. The impacts on wages, rental rate, students, and supply of skilled labor for EG

<table>
<thead>
<tr>
<th></th>
<th>$w_L$</th>
<th>$w_H$</th>
<th>$r$</th>
<th>$U_G$</th>
<th>$H_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_G$</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>+§</td>
<td>+**</td>
</tr>
</tbody>
</table>

The results from equations (14) to (19) are summarized in Table 1. Signs with (*) and (**) in Table 1 are determined under conditions, which require a significantly large $h_G$ and $\sigma_G^H > \theta_G$, respectively.

4.2 The Output of FG and EG

In this section, we reveal the impacts on the output of FG and EG, which are shown and summarized in Table 2. Here, (*) in Table 2 is determined under conditions. We will confirm the condition in detail.

Table 2. The impacts on environmental goods and final goods

<table>
<thead>
<tr>
<th></th>
<th>$X$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_G$</td>
<td>+</td>
<td>+§</td>
</tr>
</tbody>
</table>

From equation (13), we obtain the effects on FG and EG. The effects of the educator service on FG is as follows:

$$\frac{dx}{dt} = -|f|^{-1} c_H XA^X \cdot \frac{\ell'}{\ell G} \left(2c_H + \frac{c_L}{\ell G} \left[(1 - \varepsilon_H)H \left(1 - \frac{e_H}{\ell G} \right)H_G \right] \right) < 0, \quad (20)$$

since $H > H_G$ and $(1 - \varepsilon_H) > \left(1 - \frac{e_H}{\ell G} \right)$. That is, the educator service definitely decreases the
output of FG.

**PROPOSITION 3:** An increase in educator service in the public education sector always decreases the output of FG.

On the other hand, the effects of the educator service on EG is as follows:

\[
\frac{dA}{dH_G} = |J|^{-1} \left[ \frac{f_t}{u_G} \Delta \xi + 2c_r \Delta^2 \left\{ \frac{f_t}{u_G} A \Delta^4 - \left(1 - \frac{\varepsilon_{H}}{\theta_G}\right)p^* \frac{R}{w_H} \right\} \right],
\]

(21)

where \( \Delta \equiv c_r X \Delta^{X} + c_r X \Delta^{X} > 0 \) and \( \xi \equiv \left\{ (1 - \varepsilon_{H})H - (1 - \frac{\varepsilon_{U}}{\theta_G})H_G \right\} > 0 \) as we have argued previously.

Reminding the production function of EG which is represented as \( A = F^A(H_A, L_A) \), we know that the change in output of EG can be decomposed into two parts (i.e., \( dH_A \) and \( dL_A \)). As we have revealed, an increase in \( H_G \) always decreases the output of FG which brings a positive effect on \( L_A \) which is similar to the result in the traditional RV model.

However, on the other hand, as we have seen, an increase in \( H_G \) increases \( H_A \) but also decreases \( L \) which also brings a negative effect on \( L_A \). As a result, the input demand of \( L \) in EG sector which is represented by \( L_A \) may increase or decrease due to many reasons such as the change in wages, wage elasticity of unskilled labor demand, substitutability between factors, outputs, etc. If this disturbance brings a negative total effect which is significantly large, the
output of EG may decrease.\textsuperscript{27}

**PROPOSITION 4:** *Even though an increase in educator service in the public education sector can raise up the level of the supply of skilled labor employed in EG sector, it may decrease the output of EG due to the decrease in the whole supply of unskilled labor and hence the unskilled labor demand in EG sector.*

### 4.3 The output of emissions and national income

In this section, we reveal the impacts on the output of emission and the national income. This can be done by using the results what we have examined above. The effects are ambiguous as shown in Table 3. However, some arguments for the conditions required are discussed in this section.

<table>
<thead>
<tr>
<th>$H_G$</th>
<th>$Z$</th>
<th>$Y$</th>
</tr>
</thead>
</table>

Table 3. The output of emissions and national income

Totally differentiating equation (12), we have as follows:\textsuperscript{28}

\[
\frac{dZ}{dH_G} = \left(c^X_tr \frac{dr}{dH_G} + c^X_L \frac{dw_L}{dH_G}\right)X + c^X_L \frac{dx}{dH_G}.
\] (22)

\textsuperscript{27} One may argue that $dA/dH_G > 0$ can also be obtained if we assume $1 - \epsilon_H/\theta_G < 0$, however, this violates the condition, $1 - \epsilon_H/\theta_G > 0$, which is required for the result simultaneously.

\textsuperscript{28} Substituting equations (14), (16) and (18) into equation (20), we will see the indeterminacy in more concrete form.
where \( dp^* = dt = 0 \).

The second term in the RHS is negative whereas the first term is not determined. In general, if \( c^X_{tr} < 0 \) and \( c^X_{tl} > 0 \) due to the complementary between real capital and emissions, and the substitutability between unskilled labor and emissions, respectively, the impacts on the emissions is ambiguous. On the contrary, if \( c^X_{tr} > 0 \) and \( c^X_{tl} < 0 \), which is the opposite case, then \( \frac{dZ}{dH_G} < 0 \) is satisfied.

**PROPOSITION 5**: An increase in the educator service decreases the emissions if the real capital is substitutable while the unskilled labor is complementary for the emissions. Otherwise, the emissions may increase.

Next, the national income can be expressed as follows:\(^{29}\)

\[
Y = X - p^*(A_X - A). \tag{23}
\]

Totally differentiating equation (23), we have

\[
\frac{dY}{dH_G} = \left(1 - p^*c^X_p\right) \frac{dX}{dH_G} - p^* X \left(c^X_p r \frac{dr}{dH_G} + c^X_p L \frac{dL}{dH_G}\right) + p^* \frac{dA}{dH_G}, \tag{24}
\]

where \( dK = dt = 0 \).

As we have revealed previously, the first term in the RHS of equation (24) is negative since

\[^{29}\text{The national income can also be defined as } Y = rK + w_h H_A + w_L L, \text{ which provides the same result and argument.}\]
$1 - p^*c_p^X > 0$ where $p^*c_p^X$ is the income share of the input demand of EG. $c_p^X$ and $c_p^L$ are positive if the real capital and the unskilled labor are both substitutable for EG, hence the sign of the second term in the RHS is not determined since $\frac{dr}{dH_G} < 0$ and $\frac{dw_L}{dH_G}$ > 0. As we have argued previously, an increase in the educator service may decrease the output of EG, the third term in the RHS can also be negative. As a result, an increase in the educator service may decrease the national income if the second term in the RHS turns to be negative.

5. Concluding Remarks

In this paper, we construct a skill formation model to investigate the role of public education which aims to promote the skilled labor in EG sector. Under this skill formation model, we study that even though the educator service in the public education sector can increase the whole supply of skilled labor, the amount of skilled labor that can be employed in EG sector does not increase necessarily if the share of skilled labor employed in the public education sector as educator is too large.

In addition, unlike the traditional RV model, even if the amount of skilled labor employed in EG sector increases, the output of EG does not increase necessarily as well due to the decrease in the whole supply of unskilled labor in the economy. Whether the increase in skilled labor as specific factor can increase the output of EG or not depends on some conditions which are not required in the traditional RV model. For example, if the wage elasticity of unskilled labor demand
in the environmental goods sector is sufficiently large, the educator service may decrease the output of EG.

On the other hand, the output of polluting FG always decreases. However, this may not help to decrease the emissions necessarily as well due to the disturbance from the substitutability between emissions and real capital or unskilled labor when the factor prices change. Although we study that the role of public education can decrease the emissions if the real capital is substitutable while the unskilled labor is complementary for the emissions, this assumption is quite perverse considering in most cases the pollution emissions are generated by real capital and unskilled labor is substitutable for the polluting real capital. Moreover, from this argument, we also study that the national income may decrease due to the policy.

The impacts on the wages of skilled and unskilled and the rental rate of real capital are very similar to those in the traditional RV model. The results remain robust even though we incorporate the skill formation into the RV model in which the supply of skilled labor is endogenously determined in this paper. In the sense of improving the relative wage of unskilled labor to skilled labor, the role of public education always brings a positive impact on this. However, this does not take the skill acquired by the individuals into account. In other words, even though the wage of skilled labor decline, the skilled labor may still be better off under the policy.

Further research is required to address the following points. First, as long as the lifetime
income of skilled and unskilled labor at present value are always equal in our model, it seems that to investigate the impacts on the lifetime income is more important compared to the wage differential. This analysis can solve for not only the optimum level of public education service but also the tax rate for emissions which takes more examination.

Second, in the context of international trade, the analysis for the impacts of change in the world price of EG can also be done. However, this has been omitted in this paper since we are focusing on the role of public education. By showing the impacts, we will be able to supplement the actual discussion on trade liberalization in EG.

Finally, the environmental industry is depicted to produce EG instead of environmental service, which does not make difference in our model. However, in the context of environmental service, if we consider that abatement activities in polluting FG are carried out through the service by the environmental experts solely, the model we construct in this paper can be modified.

References


