Free Entry and Social Inefficiency under Vertical Oligopoly: Revisited*

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Abstract

In this paper, we examine social inefficiency in free entry under vertical oligopoly. We derive conditions for free entry in both upstream and downstream sectors to be excessive or insufficient. We demonstrate that irrespective of market structures in both sectors, upstream entry is insufficient (excessive) if the inverse demand is (not) strongly convex. Whether downstream entry is insufficient or excessive depends on both the market structure in both sectors and the curvature of the inverse demand. If upstream entry is not less than downstream entry, the downstream entry is insufficient (excessive) if the inverse demand is (not) strongly convex. If the upstream entry is less than downstream entry, and that the former is (not) one. Then, downstream entry is insufficient (excessive). Our analysis provides the fundamentals for industrial policy for vertical trade. The results imply that we must care the market condition and the numbers of entries in considering the policies under vertical oligopoly.

Keywords: Vertical oligopoly, Excess entry theorem, Cournot competition

JEL classifications: D43, L13

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1 Introduction

In this paper, we focus on vertical oligopoly, where there are multiple oligopolistic industries with vertical relationship. In recent times, vertical oligopoly has been quite common; for example, observed in automobile, construction, and computer industries.[1]

We consider social efficiency/inefficiency in free entry under vertical oligopoly. This question is originated from Mankiw and Whinston (1986) and Suzumura and Kiyono (1987), which establish that firms excessively enter in an oligopolistic market. This result is well-known as excess entry theorem. This context considers a single market, without vertical relationship.

Some subsequent studies examine whether the excess entry theorem holds in markets with vertical relationship. Ghosh and Morita (2007a) and Muhkerjee (2009) analyze the inefficiency in entry in a downstream sector, given the number of upstream firms. Ghosh and Morita (2007a) construct the oligoplistic situation with vertical relationship where an upstream firm produces the specific intermediate good for a downstream firm and its price is determined by bargaining between these firms. They established that insufficient entry may occur in the downstream sector if the upstream firm has a considerable bargaining power. Muhkerjee (2009) shows that Ghosh and Morita (2007a)'s result is not valid in the linear economy. Ghosh and Morita (2007b) consider the inefficiency in entry in an upstream sector, given the number of downstream firms. They focus on the effect that an entry in upstream sector enhances the profitability of the downstream firms through the decrease in the input price. They establish that insufficient entry can happen if such effect dominates the business stealing effect.

With respect to free entry permitted in both upstream and downstream sectors, Ghosh

[1]There are several other studies focus on vertical oligopoly. For example, Elberfeld (2003) consider the effect of vertical integration under vertical oligopoly. Reisinger and Schnitzer (2012) examine the effect of the two-part tariff and resale price maintenance on the endogenous market structure with differentiated goods.

and Morita (2007a) state the possibility of inefficiency. However, they do not derive the explicit condition for free entry to be excessive or insufficient. It is a quite important to know the condition for the free entry to be excessive or insufficient from the viewpoint from the policy. If we do not recognize it correctly, the policy may mitigate the economic welfare.

This paper aims at deriving the condition for free entry in both upstream and downstream sectors to be excessive or insufficient. We extend Ghosh and Morita (2007b)'s model by permitting free entry in an upstream sector. In particular, we focus on the relationship between inefficiencies in entry and market conditions. As Ohkawa, Shinkai, and Okamura (2012) point out, inefficiency in entry can depend on market condition.[2] We consider demand function with constant elasticity of the slope of demand and show the relationship between curvature condition of demand function and the inefficiency in entry in the both sectors.

We obtain the following results. First, the curvature of demand is crucial for the efficiency of the upstream entry (Proposition 1). Second, the numbers of upstream and downstream entries are important for the efficiency of the downstream entry, as well as the curvature of demand (Proposition 2). And, third, depending on the curvature of demand and the numbers of entries, entries can be excessive or insufficient (Proposition 3). We derive the conditions for the inefficiency in whole entries. In particular, we obtain conditions for insufficient entries in both upstream and downstream industries.

Our analysis provides the fundamentals for industrial policy in vertical trade. The results imply that we must care the market condition in considering the policy for downstream industries in the situation with vertical trade.

The rest of the paper is organized as follows. In section 2, we present our model. In section 3, we derive the equilibrium and clarify its properties. In section 4, we examine

[2]Ohkawa, Shinkai, and Okamura (2012) consider a common-resource model with vertical structure, to examine the tragedy of anti-commons in the long-run, that is, the insufficiency in the total output from the welfare viewpoint. They introduce perfect complementary inputs into a linear economy and show that excess entry prevails in the downstream sector when the demand parameter is large.

social inefficiency in free entry in upstream and downstream sectors, respectively, and then derive the condition for free entry to be excessive or insufficient under vertical oligopoly. Finally, in section 5, we have brief concluding remarks.

2 Model

We focus on an economy with vertically-related industries: upstream and downstream. Upstream firms produce intermediate goods and downstream firms produce final goods by using the intermediate goods. Firms are symmetric and supply homogeneous goods in the intermediate-good and final-good markets, respectively. Throughout this paper, we assume that there are no vertical integrations between upstream and downstream firms.[3]

In order to enter the market, upstream and downstream firms incur a fixed entry cost K_U and K_D, respectively. After entry, upstream firms face marginal cost c, and downstream firms face marginal cost r, which is equal to the price of intermediate goods. We assume that a unit of the final good can be produced by a unit of the intermediate good.

An inverse demand function in the final good market is given as follows:

$$p = p(Q) = \begin{cases} a - bQ^{\delta+1} & \text{if} \quad \delta \neq -1, \\ a - b \log Q & \text{if} \quad \delta = -1, \end{cases} \tag{1}$$

where a is a non-negative constant, b is a constant such that $b > 0$ ($b < 0$) if $\delta \geqslant -1$ ($\delta < -1$), p and Q are price and total output of the final good, and $\delta \equiv p''(Q)Q/p'(Q)$ is the price elasticity of the slope of the demand (Suzumura, 1995). Under the demand function (1), δ is constant. The demand function is convex if $\delta < 0$, linear if $\delta = 0$, and concave if $0 < \delta$, respectively. We further assume that δ is greater than $\underline{\delta} \equiv \max\{-n_U, -n_D\}$.[4]

Firms engage in the following three-stage game. In the first stage, both upstream and downstream firms determine whether to enter the market. In the second stage, each up-

[3] Each firm behaves as an independent oligopolist in a respective industry. A bunch of studies on vertically related industries, e.g., Greenhut and Ohta (1979) and Ishikawa and Spencer (1999), adopt this assumption.

[4] Under the assumption, firms' production strategies are strategic substitutes which leads to downward-sloping reaction functions, and the second order conditions for profit maximization are satisfied in both upstream and downstream industries. See Appendix.

stream firm simultaneously determines its output level of the intermediate good in order to maximize its profit. Finally, in the third stage, each downstream firm simultaneously determines its output level of the final good so as to maximize its profit. At this moment, we confine our attention to the situation where the number of firms in each industry is more than one.

3 Equilibrium

We now solve this game backward. First, we focus on equilibrium in the second and third stages as a short-run equilibrium where entries are exogenously given, and then consider equilibrium in the first stage as a long-run equilibrium in the sense that entries are endogenously determined.

3.1 Short-run equilibrium

In the second and third stages, firms engage in competition given numbers of firms. Suppose that n_U upstream firms and n_D downstream firms exist in respective markets.

In the third stage, downstream firm i ($i = 1, \ldots, n_D$)'s profit is given by

$$\pi_D^i = p(Q)q_i - rq_i - K_D, \tag{2}$$

where q_i is firm i's output. From (2), we obtain the first-order condition for profit maximization:

$$p'(Q)q_i + p(Q) - r = 0. \tag{3}$$

Focusing on symmetric equilibrium, we express $q_i = q$ for all i and thus $Q = n_D q$. We rewrite (3) as

$$p'(Q)\frac{Q}{n_D} + p(Q) \equiv r(Q; n_D) = r. \tag{3'}$$

From (3'), we obtain the equilibrium outputs in the final-good market.

In the second stage, making use of (3'), upstream firm j ($j = 1, \dots, n_U$)'s profit is given by

$$\pi_U^j = r(X; n_D)x_j - cx_j - K_U, \tag{4}$$

where X is the total output of the intermediate good, which equals to Q, and x_j is firm j's output. From (4), we obtain the first-order condition under symmetry:

$$r'(X; n_D)\frac{X}{n_U} + r(X; n_D) = \frac{(\delta + 1 + n_D)p'X}{n_D n_U} + r(X; n_D) = c, \tag{5}$$

where $r'(X; n_D) = dr/dX$. From (5), we obtain the equilibrium outputs in the intermediate-good market.

For analytical simplicity, we approximate the number of firms as a continuous variable.[5] Then, we can derive an effects of an increase in n_U or n_D on the total output of intermediate-good X. Equations (3) and (5) yields

$$\frac{dX}{dn_U} = \frac{X}{n_U(\delta + 1 + n_U)} > 0 \quad \text{and} \quad \frac{dX}{dn_D} = \frac{X}{n_D(\delta + 1 + n_D)} > 0. \tag{6}$$

(6) shows that quasi-competitiveness holds not only in final good market but in the intermediate good market. The equilibrium total output in the intermediate-good market is thus an increasing function in both n_U and n_D, i.e., $X = X(n_U, n_D)$.

Before proceeding, let us clarify the relationship between price-cost margins (PCMs) in upstream and downstream industries. From (3'), we express the PCM in the downstream industry as

$$p(Q) - r = \frac{p(Q)}{n_D \eta_D}, \tag{7}$$

where $\eta_D \equiv -p(Q)/p'(Q)Q$ is the price elasticity of demand in the downstream market. Similarly, from (5) we express the PCM in the upstream industry as

$$r(X) - c = \frac{r(X)}{n_U \eta_U}, \tag{8}$$

[5]This implies that we ignore the "integer problem." This approach is often adopted in the existing research on entry into oligopolistic markets (e.g., Suzumura and Kiyono, 1987).

where $\eta_U \equiv -r(X)/r'(X)X$ is the price elasticity of demand in the upstream market. Using (3') and (5), η_U is rewritten as $\eta_U = (n_D\eta_D - 1)/(\delta + n_D + 1)$. Thus, we can transform (8) into

$$r(X) - c = \frac{\delta + 1 + n_D}{n_U}(p(Q) - r). \tag{8'}$$

(8') explains the relationship between PCMs in the upstream and downstream markets.

Lemma 1

The difference between PCMs in upstream and downstream markets depends on market structures in both markets and curvature condition of demand function, that is, if $n_U \gtrless n_D + \delta + 1$, *then* $r - c \lessgtr p - r$, *and vice versa.*

Lemma 1 shows the following relationships: (1) Suppose that the demand curve of the downstream market is neither too concave nor too convex. If the market structure under the upstream market is highly (less) concentrated, and if the market structure under the downstream market is less (highly) concentrated, then the PCM under the upstream market is larger (smaller) than the PCM under the downstream market. (2) Suppose that the differences of the number of firms between under the upstream market and under the downstream one is not large. If the demand curve of the downstream market is too concave (too convex), then the PCM under the upstream market is larger (smaller) than the PCM under the downstream market.

3.2 Long-run Equilibrium

In the first stage, firms freely enter the market as far as they earn positive profits. Zero-profit conditions are thus satisfied in upstream and downstream industries, respectively:

$$\pi_U = [r(X(n_U, n_D), n_D) - c]\,\frac{X(n_U, n_D)}{n_U} - K_U = 0, \tag{9}$$

$$\pi_D = [p(X(n_U, n_D)) - r(X(n_U, n_D), n_D)]\,\frac{X(n_U, n_D)}{n_D} - K_D = 0. \tag{10}$$

The zero-profit conditions (9) and (10) determine equilibrium entries.[6] Further, using a superscript "e" to denote the equilibrium value, the equilibrium entries satisfy the following property:

Lemma 2

An increase in the fixed entry cost in an industry reduces not only entry in the industry but that in the other industry, i.e., $dn_h^e/dK_l < 0$, *for* $h, l = U, D$.

Proof. *See Appendix B.*

Lemma 2 illustrates the linkage between upstream and downstream industries. An increase (resp. decrease) in fixed entry cost is thus not preferable (resp. preferable) for both upstream and downstream firms.

4 Social Inefficiency in Free Entry

We now examine social inefficiency in free entry under vertical oligopoly. In the economy, social welfare W is defined as

$$W \equiv CS + n_D \pi_D + n_U \pi_U, \tag{11}$$

where CS is consumer surplus of the economy. Considering the effect of a marginal entry on the equilibrium social welfare, we regard entry in market k as *insufficient* if $\partial W^e/\partial n_h > 0$, *efficient* if $\partial W^e/\partial n_h = 0$, and *excessive* if $\partial W^e/\partial n_h < 0$ $(h = U, D)$, respectively.[7]

4.1 Inefficiency in upstream entry

First, we focus on upstream entry. Differentiating (11) with respect to n_U, we obtain

$$\frac{\partial W}{\partial n_U} = (p - r)\frac{\partial X}{\partial n_U} + n_U(r - c)\frac{\partial x}{\partial n_U} + \pi_U. \tag{12}$$

[6] We assume the existence and uniqueness of the equilibrium entries.
[7] Under the demand function (1), the second order condition is satisfied.

The first term of (12) express "business-creation effect" (Ghosh and Morita, 2007), which expresses the effect by an increase of total output created by the marginal entry. From (6), this effect is positive for welfare. The second term is "business-stealing effect" (Mankiw and Whinston, 1986), which shows the effect by a decrease of each firm's output by the marginal entry. This effect is negative for welfare. The third term is "direct effect", which corresponds to the profit of the marginal firm. Since the direct effects vanish under free entry, the welfare evaluation depends on "business-creation effect" versus "business-stealing effect."

Here, we introduce the following elasticities:

$$\Phi_U \equiv \frac{n_U}{X} \cdot \frac{\partial X}{\partial n_U} = \frac{1}{\delta + n_U + 1}, \tag{13}$$

$$\phi_U \equiv -\frac{n_U}{x} \cdot \frac{\partial x}{\partial n_U} = \frac{\delta + n_U}{\delta + n_U + 1} = 1 - \Phi_U. \tag{14}$$

Φ_U represents an upstream-entry elasticity of the total output, which explains how total output expands the entry into upstream market. ϕ_U is an upstream-entry elasticity of individual output, which illustrates how the upstream entry shrinks the incumbents' individual outputs.

Substituting these elasticities (13) and (14) into (12), we obtain the welfare effect of the upstream entry evaluating at the equilibrium as

$$\frac{\partial W^e}{\partial n_U} = x\left[(p-r)\Phi_U - (r-c)\phi_U\right]. \tag{15}$$

Whether upstream entry is excessive or insufficient depends on the sign of (15). That is, downstream entry is excessive (resp. insufficient) if the parenthesis in (15) is negative (resp. positive). Using (8'), the sign of the parenthesis in (15) is corresponding to

$$-\delta(\delta + 1 + n_D) - n_U(\delta + n_D) \equiv \Theta(\delta). \tag{16}$$

From (16) and Lemma 1, we establish the following result:

Proposition 1

(i) Suppose that the demand function in downstream market is concave, i.e., $\delta \geqslant 0$*. Then the upstream entry is excessive.*

(ii) Suppose that the demand function in downstream market is strictly convex, i.e. $\delta < 0$*. Then there exists a critical value* $\tilde{\delta}_U$ *in the interval* $(\underline{\delta}, 0)$*. Then, the upstream entry is insufficient if* $\underline{\delta} < \delta < \tilde{\delta}_U$*, and excessive if* $\tilde{\delta}_U < \delta < 0$*.*

Proof. *See Appendix C.*

Proposition 1 implies that the curvature condition of downstream demand function is critical for social inefficiency in upstream entry. This result is explained as follows: As shown in (12), welfare evaluation of the equilibrium upstream entry depends on the business-creation effect, which is positive for welfare, and the business-stealing effect, which is negative for welfare. The former is corresponding to the first term of (15), and the latter is corresponding to the second term of (15). Let us define the ratio of these two effects as

$$\theta_U(\delta) \equiv \frac{(r-c)\phi_U}{(p-r)\Phi_U} = \frac{(\delta + 1 + n_D)(\delta + n_D)}{n_U}. \tag{17}$$

$\theta_U(\delta)$ expresses the relative magnitude of the business-stealing effect caused by upstream entry. Note that $\theta'_U(\delta) > 0$ and $\theta''_U(\delta) > 0$. If $\theta_U(\delta) > 1$, the business-stealing effect dominates the business-creation effect, and thus the upstream entry is excessive. In contrast, if $\theta_U(\delta) < 1$, the business-creation effect overwhelms the business-stealing effect, and then the upstream entry is insufficient. We find that $\theta_U(0) = 1 + n_D > 1$, $\theta_U(\underline{\delta} = -n_D) = (-n_D + n_U)/n_U < 1$ when $n_D < n_U$, and $\theta_U(\underline{\delta} = -n_U) = 0$ when $n_D > n_U$. Thus, we find that the business-stealing effect always dominates the business-creation effect under concave demand, and that the business-creation effect can overwhelm the business-stealing effect under strongly convex demand.

4.2 Inefficiency in downstream entry

Second, we consider downstream entry. Differentiating (11) with respect to n_D, we obtain

$$\frac{\partial W}{\partial n_D} = (r-c)\frac{\partial Q}{\partial n_D} + n_D(p-r)\frac{\partial q}{\partial n_D} + \pi_D. \tag{18}$$

As in (12), the first term of (18) express "business-creation effect," the second term is "business-stealing effect," and the third term is "direct effect." Again, the "direct effect" vanishes at the equilibrium.

Similarly to the upstream entry, we introduce the following elasticities:

$$\Phi_D \equiv \frac{n_D}{Q} \cdot \frac{\partial Q}{\partial n_D} = \frac{1}{\delta + n_D + 1}, \tag{19}$$

$$\phi_D \equiv -\frac{n_D}{q} \cdot \frac{\partial q}{\partial n_D} = 1 - \Phi_D = \frac{\delta + n_D}{\delta + n_D + 1} = 1 - \Phi_D. \tag{20}$$

Φ_D is the downstream-entry elasticity of the total output, which express how the total output is expanded by the downstream entry. ϕ_D is the downstream-entry elasticity of the individual output, which shows how the incumbents' individual output is shrunk by the downstream entry.

Substituting these elasticities (19) and (20) into (18), we obtain the welfare effect of the upstream entry evaluating at the equilibrium as

$$\frac{\partial W^e}{\partial n_D} = q\left[(r-c)\Phi_D - (p-r)\phi_D\right]. \tag{21}$$

Whether downstream entry is excessive or insufficient depends on the sign of (21). Downstream entry is excessive (resp. insufficient) if the parenthesis in (21) is negative (resp. positive). Using (8'), the sign of the parenthesis in (21) is corresponding to

$$\delta + 1 + n_D - n_U(\delta + n_D) \equiv \Gamma(\delta). \tag{22}$$

From (22) and Lemma 1, we establish the following result:

Proposition 2

(i) Suppose that $n_U = 1$, then the downstream entry is insufficient.

(ii) Suppose that demand curve in the downstream market is strictly concave, i.e., $\delta > 0$, and that $n_U \geqslant 2$. Then the downstream entry is excessive.

(iii) Suppose that demand curve in the downstream market is convex, i.e., $\delta \leqslant 0$, and that $n_D > n_U \geqslant 2$. Then the downstream entry is excessive.

(iv) Suppose that the downstream market demand curve is convex and that $n_U \geqslant n_D \geqslant 2$. Then there exists a critical value $\tilde{\delta}_D$ in the interval $(\underline{\delta}, 0)$. Then the downstream entry is insufficient if $\underline{\delta} < \delta < \tilde{\delta}_D$, and excessive if $\tilde{\delta}_D < \delta < 0$.

Proof. *See Appendix D.*

Proposition 2 implies that the numbers of firms are crucial for social inefficiecy in downstream entry as well as the curvature condition of downstream demand function. As in the upstream case, this result is explained using the relative magnitude of the business-stealing effect. Let us define

$$\theta_D(\delta) \equiv \frac{(p-r)\phi_D}{(r-c)\Phi_D} = \frac{n_U(\delta + n_D)}{\delta + 1 + n_D}. \tag{23}$$

$\theta_D(\delta)$ expresses the relative magnitude of the business-stealing effect caused by downstream entry. Note that $\theta'_D(\delta) > 0$ and $\theta''_D(\delta) < 0$. When $n_U = 1$, $\theta_D < 1$ for any δ and n_D. Thus, in this case, the downstream entry is always insufficient. However, in the case with $n_U \geqslant 2$, the downstream entry can be insufficient and excessive depending on the values of γ, n_U, and n_D.

4.3 Inefficiency in whole entries

Finally, let us consider efficiency and downstream entry simultaneously. It is convenient to know that $\Theta(-1) = \Gamma(-1) = n_U + n_D - n_U n_D$.

First, we focus on the case with $n_U^e = 1$. In this case, we have threshold δ_U, but no threshold δ_D. Then, both entries are insufficient for $-1 < \delta < \delta_U$, and upstream entry is excessive while downstream entry is insufficient for $\delta > \delta_U$ (Figure 1 (i)).

Second, we state the case with $n_D^e = 1$. In this case, we have thresholds δ_U and δ_D such that $\delta_U < \delta_D$. Then, both entries are insufficient for $-1 < \delta < \delta_U$, upstream entry is excessive while downstream entry is insufficient for $\delta_U < \delta < \delta_D$, and both entries are excessive for $\delta > \delta_D$ (Figure 1 (ii)).

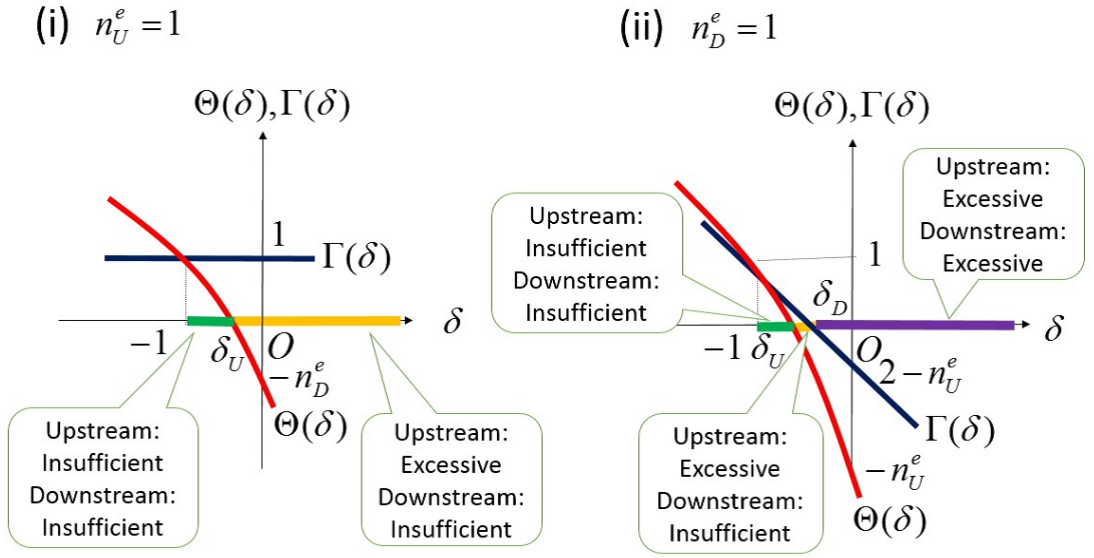

Figure 1: Efficiency in upstream and downstream entries (n_U^e or $n_D^e = 1$)

Third, we consider the case with $n_U, n_D \geqslant 2$. If $n_U > n_D$, $\Theta(-1) = \Gamma(-1) < 0$. In this case, we have thresholds δ_U and δ_D such that $\delta_D < \delta_U$. Then, both entries are insufficient for $\underline{\delta} = -n_D < \delta < \delta_D$, upstream entry is insufficient while downstream entry is excessive for $\delta_D < \delta < \delta_U$, and both entries are excessive for $\delta > \delta_U$ (Figure 2 (i)). If $n_U < n_D$, we have threshold δ_U, but no threshold δ_D. Then, upstream entry is insufficient while downstream entry is excessive for $\underline{\delta} = -n_U < \delta < \delta_U$, and both entries are excessive for $\delta > \delta_U$ (Figure 2 (ii)).

We summarize these results as the following proposition.

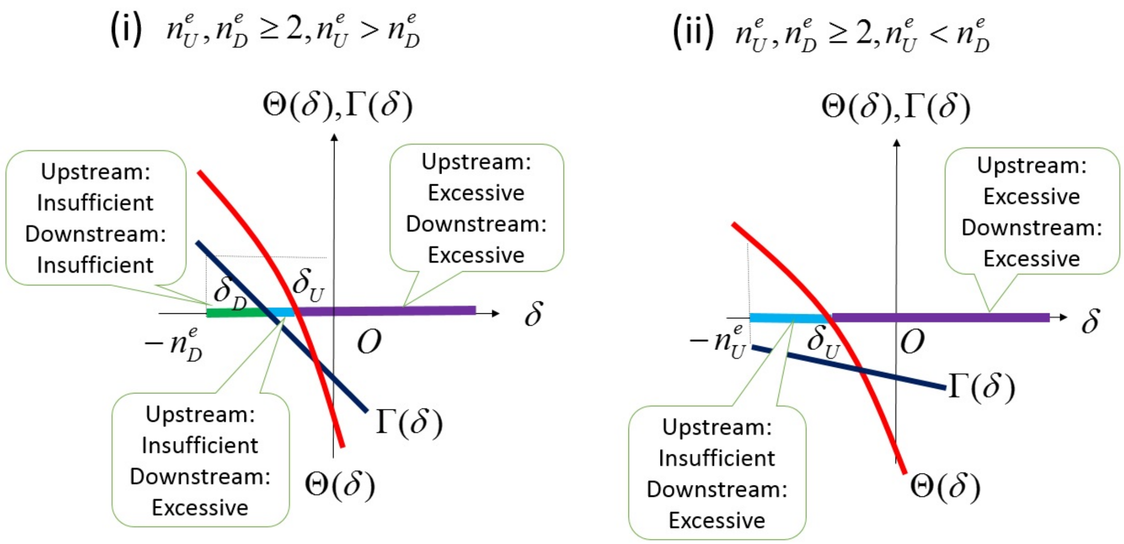

Figure 2: Efficiency in upstream and downstream entries ($n_U^e, n_D^e \geqslant 2$)

Proposition 3

(i) Suppose that $n_U = 1$ or $n_D = 1$. Then, both entries can be insufficient if the demand is strongly convex. Otherwise, upstream entry is more likely to be excessive than downstream entry.

(ii) Suppose that $n_U, n_D \geqslant 2$. Then, both entries can be insufficient if $n_U > n_D$ and the demand is strongly convex. Otherwise, downstream entry is more likely to be excessive than upstream entry.

5 Concluding Remarks

We have considered the social inefficiency of free entry in upstream and downstream industries. Considering both upstream and downstream entries, we have derived conditions for entries in both upstream and downstream sectors to be excessive or insufficient.

We obtain the following results: (i) Irrespective of market structures in both sectors, upstream entry is insufficient (excessive) if the inverse demand is (not) strongly convex. (ii) Whether downstream entry is insufficient or excessive depends on both the market structure in both sectors and the curvature of the inverse demand: Suppose that the upstream entry is not less than downstream entry. Then, downstream entry is insufficient (excessive) if the inverse demand is (not) strongly convex; Suppose that the upstream entry is less than downstream entry, and that the former is (not) one. Then, downstream entry is insufficient (excessive).

Our analysis provides the fundamentals for industrial policy for vertical trade. The results imply that we must care the market condition and the numbers of entries in considering the policies under vertical oligopoly.

Appendix A: Strategic substitutability

We consider a sufficient condition for strategic substitutabilities in both markets. Since strategic substitutability holds in the downstream (resp. the upstream) market if $p''q_i + p' < 0$ (resp. $r''x_i + r' < 0$), each of which is the sufficient condition to guarantee the second-order condition for profit maximization. At the symmetric equilibrium, the condition in downstream market is rewritten as

$$p'' \frac{Q}{n_D} + p' = \frac{p'}{n_D}(\delta + n_D) < 0. \tag{A1}$$

Since $p' < 0$, in order to satisfy the sign in (A1), the following inequality must be valid:

$$\delta + n_D > 0. \tag{A2}$$

Similarly, the condition under downstream market can be rewritten as

$$r''x_i + r' = \frac{\delta + 1 + n_D}{n_U n_D} p'(\delta + n_U) < 0. \tag{A3}$$

Since $p' < 0$ and (A2), in order for (A3) to hold, the following inequality holds:

$$\delta + n_U > 0. \tag{A4}$$

From (A2) and (A4), we derive a condition

$$\delta > \underline{\delta} = \max\{-n_U, -n_D\}$$

in order to ensure the strategic substitutabilities in both markets.

Appendix B: Proof of Lemma 2

Totally differentiating (9) and (10) with respect to n_U, n_D, K_U and K_D yields

$$\begin{pmatrix} \frac{\partial \pi_U}{\partial n_U} & \frac{\partial \pi_U}{\partial n_D} \\ \frac{\partial \pi_D}{\partial n_U} & \frac{\partial \pi_D}{\partial n_D} \end{pmatrix} \begin{pmatrix} dn_U \\ dn_D \end{pmatrix} = \begin{pmatrix} dK_U \\ dK_D \end{pmatrix}. \tag{B1}$$

Using Cramer's rule, we derive the followings from (B1).

$$\frac{dn_k}{dK_l} = \begin{cases} \frac{1}{\Delta}\frac{\partial \pi_k}{\partial n_l} & \text{if } k = l \\ -\frac{1}{\Delta}\frac{\partial \pi_k}{\partial n_l} & \text{if } k \neq l, \end{cases} \tag{B2}$$

where

$$\Delta = \left(\frac{\partial \pi_U}{\partial n_U}\right)\left(\frac{\partial \pi_D}{\partial n_D}\right) - \left(\frac{\partial \pi_U}{\partial n_D}\right)\left(\frac{\partial \pi_D}{\partial n_U}\right). \tag{B3}$$

From (B2) and (B3), in order to investigate the sign of dn_k^e/dn_l, we focus on effects of a change in the number of upstream firms on each upstream or downstream firm's profit. Differentiating (9) and (10) with respect to n_U yields

$$\frac{\partial \pi_U}{\partial n_U} = \left[r'\frac{X}{n_U} + (r-c)\frac{1}{n_U}\right]\frac{dX}{dn_U} - (r-c)\frac{X}{n_U^2} = \frac{p'X^2(\delta+2n_U)(\delta+1+n_D)}{n_U^3 n_D(\delta+1+n_U)} < 0, \tag{B4}$$

$$\frac{\partial \pi_D}{\partial n_U} = \left[p'\frac{X}{n_D} - r'\frac{X}{n_D} + (p-r)\frac{1}{n_D}\right]\frac{dX}{dn_U} = -\frac{p'X^2(\delta+2)}{n_U n_D^2(\delta+1+n_U)} > 0. \tag{B5}$$

Noting that $\partial r/\partial n_D = -p'X/n_D^2$ derived from (3) and differentiating (9) and (10) with respect to n_D yield

$$\frac{\partial \pi_U}{\partial n_D} = \left[r'\frac{X}{n_U} + (r-c)\frac{1}{n_U}\right]\frac{dX}{dn_D} + \frac{\partial r}{\partial n_D}\frac{X}{n_U} = -\frac{p'X^2}{n_U^2 n_D^2} > 0, \tag{B6}$$

$$\frac{\partial \pi_D}{\partial n_D} = \left[(p'-r')\frac{X}{n_D} + (p-r)\frac{1}{n_D}\right]\frac{dX}{dn_D} - \frac{\partial r}{\partial n_D}\frac{X}{n_D} + (p-r)\left(-\frac{X}{n_D^2}\right) = \frac{p'X^2(\delta+2n_D)}{n_D^3(\delta+1+n_D)} < 0. \tag{B7}$$

From (B3) through (B7), the sign of Δ is positive, i.e.,

$$\Delta = \frac{(p')^2 X^4}{n_U^3 n_D^4}\left[n_U(\delta+2n_U)(\delta+2n_D) + \frac{n_D(\delta+2)}{\delta+1+n_U}\right] > 0. \tag{B8}$$

Therefore, we have

$$\frac{dn_U}{dK_U} = \frac{1}{\Delta} \cdot \frac{p'X^2(\delta + 2n_D)}{n_D^3(\delta + 1 + n_D)} < 0, \quad \frac{dn_D}{dK_U} = \frac{1}{\Delta} \cdot \frac{p'X^2(\delta + 2)}{n_U n_D^2(\delta + 1 + n_U)} < 0,$$
$$\frac{dn_U}{dK_D} = \frac{1}{\Delta} \cdot \frac{p'X^2}{n_U^2 n_D^2} < 0, \quad \frac{dn_D}{dK_D} = \frac{1}{\Delta} \cdot \frac{p'X^2(\delta + 1 + n_D)(\delta + 2n_U)}{n_U^3 n_D(\delta + 1 + n_U)} < 0.$$

from (B2), (B4) through (B8). ■

Appendix C: Proof of Proposition 1

Proof of (i): If downstream demand function is concave, i.e., $\delta > 0$, the sign of (16), and thus, that of (15), is negative.

Proof of (ii): When $n_U \leqslant n_D$, we can express n_D as $n_U + \beta$ where $\beta \geqslant 0$. Substituting $n_D = n_U + \beta$ into (16) and arranging terms yields

$$-\delta^2 - (2n_U + \beta + 1)\delta - n_U(n_U + \beta) \equiv f(\delta) \tag{C1}$$

Since $f(0) = -n_U(n_U + \beta) < 0$, $f(-n_U) = n_U > 0$, and $f'(\delta) = -2(\delta + n_U) - (\beta + 1) < 0$ from (C1), there exists the critical value of $\tilde{\delta}_U$ such that $f(\tilde{\delta}_U) = 0$ in the interval $(-n_U, 0)$.

When $n_D \leqslant n_U$, we rewrite n_U as $n_D + \gamma$ where $\gamma \geqslant 0$. Substituting $n_U = n_D + \gamma$ into (16) and rearranging terms yield

$$-\delta^2 - (2n_D + \gamma)\delta + n_D(n_D + \gamma) \equiv g(\delta) \tag{C2}$$

Since $g(0) = -(n_D + \gamma)n_D < 0$, $g(-n_D) = n_D > 0$, and $g'(\delta) = -2(\delta + n_D) - (\gamma + 1) < 0$ from (C2), there exists the critical value of $\tilde{\delta}_U$ such that $g(\tilde{\delta}_U) = 0$ in the interval $(-n_D, 0)$. Note that $\underline{\delta} = -n_U$ (resp. $-n_D$) when $n_U \leqslant n_D$ (resp. $n_U \geqslant n_D$). ■

Appendix D: Proof of Proposition 2

Proof of (i): If $n_U = 1$, the sign of (22), and thus, that of (21), is positive.

Proof of (ii): If downstream demand function is concave, i.e., $\delta > 0$, and $n_U \geqslant 2$, then the sign of (22) is negative.

Proof of (iii): Denote n_D as $n_U + \beta$ where $\beta \geqslant 1$. Substituting $n_D = n_U + \beta$ into (22) and rearranging terms, we have

$$(1 - n_U)\delta + (1 - n_U)(n_U + \beta) + 1 \equiv h(\delta) \tag{D1}$$

Since $h(0) = (1-n_U)(n_U+\beta)+1 < 0$, $h(-n_U) = (1-n_U)\beta+1 \leq 0$, and $h'(\delta) = 1-n_U < 0$, we obtain $h(\delta) < 0$ for any $\delta \in (-n_U, 0]$.

Proof of (iv): When $n_D \leqslant n_U$, we rewrite n_U as $n_D + \gamma$ where $\gamma \geqslant 0$. Substituting $n_U = n_D + \gamma$ into (22) and rearranging terms yield

$$(1 - n_D - \gamma)\delta + (1 - n_D - \gamma)n_D + 1 \equiv i(\delta) \tag{D2}$$

Since $i(0) = (1 - n_D - \gamma)n_D + 1 < 0$, $i(-n_D) = 1 > 0$, and $i'(\delta) = 1 - n_D - \gamma < 0$ from (D2), there exists the critical value of $\tilde{\delta}_D$ such that $g(\tilde{\delta}_D) = 0$ in the interval $(-n_D, 0)$. ■

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