Lifetime Employment and a Mixed Duopoly with a Foreign Labour-Managed Firm

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Abstract
This paper investigates an international mixed duopoly market in which a state-owned firm coexists with a foreign labour-managed firm. Both firms are allowed to offer lifetime employment as a strategic commitment. The following timing of actions is considered. First, firms decide simultaneously and non-cooperatively whether to offer lifetime employment. If a firm offers lifetime employment, then it chooses an output level and enters into a lifetime employment contract with the number of workers necessary to achieve the output level. Second, firms choose actual outputs simultaneously and non-cooperatively. This study traces the firms’ reaction functions in the mixed duopoly model with lifetime employment.

Keywords: State-owned firm; Foreign labour-managed firm; Cournot model; Mixed duopoly; Lifetime employment
JEL classification: C72; D21; F23; L13; L30

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1. Introduction


There are also many other related researches (see, e.g. George and La Manna, 1996; Mujumdar and Pal, 1998; Pal, 1998; Matsumura, 2003; Chang, 2005; Matsumura and Kanda, 2005; Beladi and Chao, 2006; Chao and Yu, 2006; Lu, 2007; Lu and Poddar, 2007, 2009; Han and Ogawa, 2008; Kato, 2008; Ohnishi, 2008; Saha and Sensarma, 2008; Artz, Heywood and McGinty, 2009; Roy chowdhury, 2009; Wang and Wang, 2009; Wang, Wang and Zhao, 2009; Heywood and Ye, 2010; Ogawa and Matsumura, 2010; Wang and Lee, 2010; Pal and Saha, 2014; Cracau, 2015). However, all these researches consider mixed oligopoly markets where profit-maximizing capitalist firms coexist with state-owned firms, and do not include labour-managed firms.

After the pioneering contribution by Ward (1958), numerous researchers have investigated the behaviours of labour-managed firms. For instance, Cremer and Cremer (1992) investigate a two-stage game model where firms simultaneously and non-cooperatively choose both the capital stock and the employment level, and demonstrate that the profit-maximizing capitalist firm produces more than the
labour-managed firm in a Cournot-Nash duopoly model. Lambertini and Rossini (1998) consider a two-stage quantity-competition duopoly model with capital commitment, and demonstrate that the profit-maximizing firm optimally under-invests whereas the opposite holds for the labour-managed firm. Stewart (1991) explores strategic interactions both in a labour-managed duopoly and in a mixed duopoly with labour-managed and profit-maximizing firms using excess capacity to deter entry, and shows how the organizational form of potential entrant influences the strategy of an established firm. Ireland (2003) conducts a mixed oligopoly regime with imperfect consumer information, and demonstrates that in the free-entry mixed-strategy Nash equilibrium, profit-maximizing firms set higher prices than labour-managed firms. In addition, Ohnishi (2011a) examines a model in which a profit-maximizing capitalist firm and a labour-managed firm are allowed to offer lifetime employment as a strategic device, and shows that if the labour-managed firm does not offer lifetime employment, then its reaction function is upward sloping, whereas if it does, then its reaction function changes downward sloping.

There are also numerous other published papers (see, e.g. Law and Stewart, 1983; Mai and Hwang, 1989; Horowitz, 1991; Okuguchi, 1991; Stewart, 1992; Askildsen and Ireland, 1993; Ireland and Stewart, 1995; Futagami and Okamura, 1996; Lambertini, 1997, 2001; Neary and Ulph, 1997; Okamura and Futagami, 1997; Cuccia and Cellini, 2009; Luo, 2013; Kalashnikov et al, 2015). All these papers focus on mixed oligopoly markets where labour-managed firms compete against profit-maximizing capitalist firms, and do not include state-owned public firms.

Only a few studies investigate mixed oligopoly markets that consist of state-owned and labour-managed firms. For instance, Delbono and Rossini (1992) consider a Cournot mixed duopoly model with one state-owned firm and one labour-managed firm, and show that there is a unique Cournot-Nash solution in which the state-owned firm produces more than the labour-managed firm. Ohnishi (2009) investigates the behaviours of a labour-managed firm and a state-owned firm in a two-stage mixed duopoly game, and shows that if both firms are allowed to install capacity in stage one, then there is a subgame perfect Nash equilibrium where the labour-managed firm installs capacity whereas the state-owned firm does not. Ohnishi (2011b) investigates two three-stage games where a state-owned firm and a labour-managed firm can sequentially provide lifetime employment before competing in quantities, and demonstrates that introducing
lifetime employment into the three-stage model of mixed duopoly is beneficial for both firms. In addition, Ohnishi (2015a) examines a three-stage model where a state-owned firm and a labour-managed firm can sequentially offer a wage-rise contract as a strategic commitment before competing in quantities, and shows that there is an equilibrium that coincides with the Cournot solution with no wage-rise contract commitment.

We examine international mixed duopoly competition in which a state-owned firm and a foreign labour-managed firm are allowed to offer lifetime employment as a strategic commitment. We consider the following situation. In stage 1, both firms decide independently and simultaneously whether to offer lifetime employment. If a firm offers lifetime employment, then it chooses an output level and enters into a lifetime employment contract with the number of workers necessary to achieve the output level. In stage 2, both firms independently and simultaneously choose their actual outputs.

The main purpose of this study is to trace the firms’ reaction functions in the international mixed duopoly model with lifetime employment.

The organization of this paper is as follows. In the next section, the basic model of the paper is described. The third section presents the results of this study. The last section concludes the paper.

2. The basic setting

Let us consider a mixed duopoly economy consisting of one domestic state-owned firm (firm S) and one foreign labour-managed firm (firm L). In the remainder of this paper, subscripts S and L denote firm S and firm L, respectively. In addition, when $i$ and $j$ are used to refer to firms in an expression, they should be understood to refer to S and L with $i \neq j$. The market price is determined by the inverse demand function $P(Q)$, where $Q = q_S + q_L$ denotes the aggregate quantity. We assume that $P' < 0$ and $P'' < 0$.

The market will be modelled by means of following two-stage game. In stage 1, firms decide simultaneously and non-cooperatively whether to offer lifetime employment. If firm $i$ offers lifetime employment, then it chooses an output level $q_i^* > 0$ and enters into a lifetime employment contract with the number of employees necessary to achieve

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1 For details see Ohnishi (2001, 2002).
In stage 2, firms simultaneously and non-cooperatively choose actual outputs $q_s^*$ and $q_L^*$. Therefore, domestic economic welfare, which is the sum of domestic consumers’ surplus and firm S’s profit, is given by

$$W = \begin{cases} \int_0^0 P(x)dx - r(q_s) - w(q_s) - Pq_L - 2f & \text{if } q_s > q_s^* \\ \int_0^0 P(x)dx - r(q_s) - w(q_s^*) - Pq_L - 2f & \text{if } q_s \leq q_s^* \end{cases}$$

where $r$ denotes the capacity (capital) cost function, $w$ is the labour cost function, and $f > 0$ is the fixed cost. Let “$Pq_L - f$” be the sum of firm L’s variable cost and profit.

Firm L’s income per worker is given by

$$V_L = \begin{cases} \frac{P(Q)q_L - rq_L - f}{l(q_L)} & \text{if } q_L > q_L^* \\ \frac{P(Q)q_L^* - rq_L - f}{l(q_L^*)} & \text{if } q_L \leq q_L^* \end{cases}$$

where $l$ represents the labour input function. We assume that $l' > 0$ and $l'' > 0$. This assumption means that the marginal labour input is increasing. We assume that both firms have the same technology. In addition, we assume that $r' > 0$, $r'' > 0$, $w' > 0$, and $w'' > 0$. Throughout this paper, we adopt subgame perfection as our solution concept.

### 3. Reaction functions

First, we derive firm S’s best response from (1). If firm S’s marginal cost of production
is $r + w'$, then its reaction function is defined by

$$R_S^*(q_L) = \arg \max_{q_S} \left[ \int_0^q P(x)dx - r(q_s) - w(q_s) - Pq_L \right]$$

(3)

On the other hand, if firm S offers lifetime employment and reduces its marginal cost to $r'$, then its reaction function is defined by

$$R_S'(q_L) = \arg \max_{q_S} \left[ \int_0^q P(x)dx - r(q_s) - Pq_L \right]$$

(4)

Hence, if firm S selects $q_S^*$ and offers lifetime employment, then its best response is represented as follows:

$$R_S(q_L) = \begin{cases} R_S^*(q_L) & \text{if } q_s > q_S^* \\ q_S^* & \text{if } q_s = q_S^* \\ R_S'(q_L) & \text{if } q_s < q_S^* \end{cases}$$

(5)

We now prove the following proposition:

**Proposition 1:** Under quantity competition, both $R_S^*(q_L)$ and $R_S'(q_L)$ slope upwards, and the slope of $R_S'(q_L)$ is gentler than that of $R_S^*(q_L)$.

Proof: Firm S aims to maximize economic welfare with respect to its own output, given firm L’s output. The solution outcome must satisfy the following conditions: If firm S’s marginal cost of production is $r + w'$, then the first-order condition for welfare maximization is

$$P - r' - w' - P'q_L = 0$$

(6)

and the second-order condition is

$$P' - r'' - w'' - P''q_L < 0$$

(7)

On the other hand, if firm S’s marginal cost of production is $r'$, then the first-order condition for welfare maximization is

$$P - r' - P'q_L = 0$$

(8)

and the second-order condition is

$$P' - r'' - P''q_L < 0$$

(9)

Furthermore, we obtain

$$R_S^n(q_L) = \frac{P''q_L}{P' - r'' - w'' - P''q_L}$$

(10)

and
\[ R^i_S(q_L) = \frac{P^" q_L}{P' - r" - P^" q_L} \]  

(11)

Since \( P^" 0 \), \(-P^" q_L\) becomes positive. Hence, both \( R^u_S(q_L) \) and \( R^l_S(q_L) \) are upward sloping. Since \( w^" 0 \), the proposition follows immediately. QED

Second, we present the following proposition, which provides a characterization of lifetime employment as a strategic commitment.

**Proposition 2:** The offer of lifetime employment by firm S increases its welfare-maximizing output.


Proposition 2 means that firm S’s welfare-maximizing output is higher when its marginal cost of production is \( r' \) than when its marginal cost of production is \( r' + w' \).

Third, we derive firm L’s best response from (2). If firm L does not offer lifetime employment, then its reaction function is defined by

\[ R^u_L(q_S) = \arg \max_{q_L} \left[ \frac{P(Q)q_L - r(q_L) - f}{l(q_L)} \right] \]  

(12)

On the other hand, if firm L offers lifetime employment and produces \( q_L \leq q^*_L \), then its reaction function is defined by

\[ R^l_L(q_S) = \arg \max_{q_L} \left[ \frac{P(Q)q_L - r(q_L) - f}{l(q^*_L)} \right] \]  

(13)

Hence, if firm L offers lifetime employment, then its best response is represented as follows:

\[ R_L(q_S) = \begin{cases} 
R^u_L(q_S) & \text{if } q_L > q^*_L \\
q^*_L & \text{if } q_L = q^*_L \\
R^l_L(q_S) & \text{if } q_L < q^*_L 
\end{cases} \]  

(14)

We now state the following proposition:

**Proposition 2:** Under quantity competition, \( R^u_L(q_S) \) slopes upwards, while \( R^l_L(q_S) \) is downwards.
Proof: See Ohnishi (2011a, pp. 154-155).

For the remainder of this section, we illustrate both firms’ reaction curves by using Figures 1-5. For explanations, the figures are drawn simply.

Firstly, we illustrate firm S’s best response curve, which are drawn in Figure 1. Here, $R_i^n$ denotes firm $i$’s reaction function without lifetime employment, and $R_S^l$ is firm S’s reaction function with zero marginal labour costs. Both $R_i^n$ and $R_S^l$ are upward sloping. If neither firm offers lifetime employment in stage 1, then the stable solution occurs at $N$. Suppose that firm S unilaterally offers lifetime employment in stage 1. By strategic choice of lifetime employment, firm S’s best response becomes (5). The offer of lifetime employment by firm S thus creates kinks in the reaction curve at the level of $q_S^*$. Therefore, if firm S chooses $q_S^{*L}$ and offers lifetime employment, then its best response curve becomes the heavy broken lines as drawn in this figure. The solution outcome is decided in a Cournot fashion. That is, the intersection of these reaction curves gives us the equilibrium solution of the game. The reaction curves cross at $B$ in this figure. We can see easily that $B$ is a stable Cournot solution. In this case, there is a stable solution. Therefore, we see that the offer of lifetime employment by firm S changes the equilibrium solution of the game.

Secondly, we consider the case depicted in Figure 2, where $R_L^l$ is firm L’s reaction function with lifetime employment. $R_L^n$ is upward sloping, whereas $R_L^l$ is downward sloping. Suppose that firm L unilaterally offers lifetime employment in stage 1. By strategic choice of lifetime employment, firm L’s best response becomes (14). The offer of lifetime employment by firm L thus creates kinks in the reaction curve at the level of $q_L^*$. That is, if firm L offers lifetime employment, then its best response becomes the kinked curve shown in heavy lines in Figure 2. Both firms’ reaction curves cross at multiple points as in Figure 2. We see that both $E$ and $G$ are stable Cournot solutions. In this case, there are two stable solutions.

Thirdly, we consider the case depicted in Figure 3. If firm L chooses $q_L^{*II}$ and offers lifetime employment, then its best response becomes the kinked curve shown in heavy lines in Figure 3. If firm L chooses $q_L^{*II}$ and unilaterally offers lifetime employment, then the obvious outcome is that there is no stable solution.

Fourthly, we consider the case depicted in Figure 4. In this case, both firms offer
lifetime employment. When firm S chooses $q_{S}^{*K}$ and offers lifetime employment, its best response becomes the heavy broken lines. In addition, when firm L chooses $q_{L}^{*L}$ and offers lifetime employment, its best response curve becomes the heavy lines. In this figure, both firms’ reaction curves do not cross each other. That is, if firm S chooses $q_{S}^{*K}$ and firm L chooses $q_{L}^{*L}$, then the obvious outcome is that there is no stable solution.

Fifthly, we consider the case in Figure 5. When firm S chooses $q_{S}^{*X}$ and offers lifetime employment, its best response curve becomes the heavy broken lines. In addition, when firm L chooses $q_{L}^{*T}$ and offers lifetime employment, its best response curve becomes the heavy lines. The firms’ best response curves cross at multiple points as in Figure 5. We see that both $V$ and $W$ are stable Cournot solutions. In this case, there are multiple stable solutions.

### 4. Conclusion

We have studied mixed duopoly competition in which a state-owned firm and a foreign labour-managed firm are allowed to offer lifetime employment as a strategic commitment. Our results can be summarized as follows. (i) Though the state-owned firm’s reaction function is upward sloping, the slope is gentler when it offers lifetime employment than when it does not. (ii) If the foreign labour-managed firm does not offer lifetime employment, then its reaction function slopes upwards, whereas if it does, then its reaction function is downwards. (iii) There may be multiple stable Cournot solutions in the international mixed duopoly model.

### References


Cooperative Economics, 64, 367-393.


Figure 1: Firm S’s best response is kinked at the level of $q_s^*$. 
Figure 2: Firm L’s best response is kinked at the level of $q_L^*$. 
Figure 3: Firm L’s best response is kinked at the level of $q_L^{*H}$. 
Figure 4: There is no stable solution.
Figure 5: The reaction curves cross at multiple points.