Trade, Capital Accumulation, and the Environment

Gang Li*
Hitotsubashi University
August 2015

Abstract

To highlight the role of endogenous capital accumulation in shaping the interaction between trade and the environment, I develop a two-sector Ramsey model featuring agriculture sector impaired by pollution from both agriculture itself and manufacturing sector. Trade raises capital rental rate and encourages investment. Under laissez faire, such scale effect necessarily leads to environmental degradation in the long run, even if the economy specializes in the relatively clean sector. The long-run specialization pattern depends upon parametrically determined pre-trade comparative advantage, which is revealed by comparing the world and the autarky steady-state relative prices. Welfare gains from trade is ambiguous. To achieve the social optimum, a dynamic version of the Pigouvian tax can be imposed, with lump-sum transfers of tax revenue to households. Under such optimal policy, trade does not necessarily harm the environment in the long run, and the long-run specialization pattern may also depend upon the initial condition.

Keywords: Capital accumulation; environmental degradation; pre-trade comparative advantage; dynamic Pigouvian tax
JEL classification: F18; Q20

---

*This paper is a revised version of a chapter of my PhD dissertation. I am indebted to the members of thesis committee, Kazumi Asako, Taiji Furusawa, and Jota Ishikawa for their constant support and invaluable advice at various stages of this project, and Motohiro Sato and Hidetoshi Yamashita for helpful comments. I am grateful to Emma Aisbett, Alan Woodland, and Akihiko Yanase for insightful discussions and suggestions. I also thank Ichiroh Diatoh, Russell Hillberry, Hayato Kato, James Markusen, Kaz Miyagiwa, Jun-ichi Nakamura, Raymond Riezman, E. Young Song, Rod Tyers, Anthony Venables, and seminar and conference participants at Hitotsubashi University, the 9th Asia Pacific Trade Seminars, the 72nd Annual Meeting of JSIE, 2013 Hitotsubashi-Sogang Conference on Trade, and the 9th Australasian Trade Workshop for useful comments on earlier drafts. Financial support from the Japanese Government (Monbukagakusho: MEXT) Scholarship is warmly acknowledged. All remaining errors are mine. E-mail: ligang.hitu@gmail.com
1 Introduction

The trade-off between economic development and environmental preservation has always been a challenging issue since the Industrial Revolution. Now, the issue becomes even more pressing thanks to the rapid process of trade liberalization over last few decades. In Latin American nations, exports have been driving huge expansion of cattle ranches and soybean plantations, which is, however, one of the main contributors behind vast deforestation around the Amazon basin (see, e.g., Austin, 2010). A research by Lathuillière et al. (2014) estimates that in the 2000s soybean production contributed to 65% of the deforestation in Brazil’s state of Mato Grosso, the largest soybean producer around the basin.

This vast deforestation around the Amazon basin, still ongoing, highlights two channels through which trade interacts with economic development and the environment. First, trade encourages economic development, which often comes at the expense of more pollution, wastes, and resource extractions. Second, trade promotes the spatial separation between production and consumption, aggravating environment related externality issues. The interaction can of course go the other way around. For example, environmental degradation harms those industries relying heavily on the environment, such as agriculture, fishery, and tourism. This may in turn discourage economic development and, by nullifying a country’s comparative advantages, alter trade patterns.

In this paper, I develop a two-sector general equilibrium model and attempt to address these close nexuses between trade, economic development, and the environment. The basic model setup is as follows. Households choose in a Ramsey fashion between consumption and investment, using the single final good. The final good is assembled using two intermediate goods, one from agriculture sector and the other from manufacturing sector. Manufacturing productivity is exogenously given whereas agriculture productivity relates positively to environmental quality, which evolves depending upon the difference between its natural growth and the flow of pollution arising from the use of capital in both agriculture and manufacturing.

With the setup above, the model represents economic development by capital accumulation and captures the following aspects of its interaction with trade and the environment. The model capture both the composition and scale effects of trade upon the environment. As for the composition effect, trade liberalization induces specialization, raising (reducing) pollution flow when the sector to specialize in is relatively dirty (clean). As for the scale effect, trade liberalization raises capital rental rate and encourages investment, stimulating capital accumulation but also pollution flow. On the other hand, the model captures the feedback of environmental change upon trade and economic development: environmental degradation (improvement) leads to a decline (an increase) in agriculture productivity, re-

\[1\]

Another well known channel is the technical effect, which emphasizes the benefit of trade in promoting cleaner technologies; see, among others, Grossman and Krueger (1994).
sulting in weaker (stronger) comparative advantage in agriculture and lower (higher) rental rate.

Both laissez faire and optimal policy are considered. The first set of results are about specialization patterns (and thus trade patterns). Under laissez faire, specialization pattern at every point in time depends upon environmental quality at the moment: if the environment is relatively good, the economy specializes in agriculture, otherwise specializing in manufacturing. This is because under laissez faire comparative advantage depends only upon agriculture productivity, and thus only upon the environment. However, since the environment evolves over time, a specialization pattern existing in the short run may not sustain in the long run. It turns out that, in the long run, specialization pattern depends upon pre-trade comparative advantage, which is parametrically determined and revealed by comparing the world and the autarky steady-state relative prices. If the economy has a pre-trade comparative advantage in agriculture, i.e., if agriculture good is relatively cheap in autarky steady state than that in the world, the economy specializes in agriculture in trade steady state; otherwise the economy specializes in manufacturing along a growth path. Under optimal policy, specialization pattern depends not only upon the environment, but also the level of pollution tax at every point in time. This is because the pollution tax increases more proportionately the cost in the relatively dirty sector, making it less competitive. It gets more complicated to characterize the long-run specialization pattern since pollution tax and environmental quality are endogenous. It is shown that pre-trade comparative advantage still serves, with some amendment, as the criterion for the long-run specialization pattern under optimal policy.

The second set of results reveal environmental impacts of trade and its welfare gains. Under laissez faire, the environment may improve or degrade in the short run, depending upon whether a relatively clean or dirty sector in which the economy specializes. In the long run, however, the environment necessarily degrades (compared to autarky steady state) even if the economy specializes in a relatively clean sector. The intuition comes as follows. First, capital reallocation is an instantaneous adjustment in the model whereas capital accumulation takes time, so in the short run the composition effect dominates, which can be either good or bad for the environment. Second, trade liberalization raises capital rental rate with others remaining the same, so capital continues accumulating if the environment is the same as in autarky, implying a worse environment (compared to autarky steady state) in the long run. Therefore, by introducing capital accumulation, the model is less optimistic about environmental impacts of trade. Under optimal policy, the long-run environmental consequence of trade is ambiguous thanks to the existence of pollution tax.

As for welfare gains, under laissez faire, trade is not necessarily welfare-improving due to production externalities. Although consumption increases right after trade liberalization (if the economy starts from autarky steady state), it may decline in steady state if agriculture is relatively dirty and the economy specializes in agriculture. In such case, trade may lead to
welfare loss given the time preference small enough. Under optimal policy, there are welfare gains from trade by definition.

The third set of results relate to the characterization of optimal policy. The social optimum can be achieved in a market-based economy through a pollution tax with lump-sum transfers of tax revenue to households. The pollution tax works in two directions. It corrects capital misallocation by raising more proportionately the cost in a dirtier sector. It also cuts excess investment by reducing rental rate. Moreover, the optimal pollution tax can be interpreted as a dynamic version of the Pigouvian tax.

This study is by no means the first effort to understand the relationship between trade, economic development, and the environment. In their excellent review, Copeland and Taylor (2004) provide a unified framework to discuss the environmental consequences of economic growth and international trade. Their framework, however, by treating economic growth as exogenous and examining its impacts through comparative statics, cannot capture the endogenous interactive linkage between the three.

While lacking models upon the interaction between the three, great efforts have been devoted to understand the linkages between pairs among the three. In the analysis of the linkage between trade and the environment, many authors focus, as in this study, on the damage of environmental degradation upon production. Brander and Taylor (1997, 1998) consider the intra-sector externalities in harvesting sector and show how trade can lead to welfare loss in the long run. Copeland and Taylor (1999) model the inter-sector externalities from manufacturing to agriculture and show how trade can provide a motive for trade, while Benarroch and Thille (2001) extend the model to the case of transboundary pollution. Kotsogiannis and Woodland (2013) formulate the idea of effective endowment in a very general framework and characterize Pareto-efficient and Pareto-improving policies. This study benefits directly from Copeland and Taylor (1999) by extending their framework to incorporate the Ramsey style of growth. I also allow agriculture to be polluting instead of purely clean, which is motivated by the fact that agriculture has become one of the most leading sources of pollution (EPA, 2009). The two extensions allow to formulate the endogenous responses of economic development to trade liberalization and environmental changes, through changes in capital rental rate. The insight derived from the resulting model is significant. That is, the long-run environmental consequences of trade could be largely underestimated by ignoring economic development through endogenous investment.

In the analysis of the linkage between trade and economic development, many authors develop the models by introducing capital accumulation into the Heckscher–Ohlin framework; see, among others, Stiglitz (1970), Baxter (1992), and Brecher et al. (2005). Then economic development affects comparative advantages since more capital lowers the price of

---

2 Another strand of literature focuses on the damage of environmental degradation upon utility rather than upon production; see, e.g., Markusen (1975a,b), Asako (1979), Copeland and Taylor (1994, 1995), and Ishikawa and Kiyono (2006).
capital-intensive goods. This study attempts to highlight the effect of capital accumulation upon comparative advantage through environmental changes, rather than through decreasing returns to scale. For this purpose, I assume a single factor with constant returns to scale. This study also adds to the huge body of literature upon the linkage between economic development and the environment; see Brock and Taylor (2005) for a careful review. One of the important topics in the field is the environmental Kuznets curve (EKC), which argues that the environment first degrades and then improves with per capita income (Stern et al., 1996). Although the empirical evidence remains inconclusive (e.g., Dinda, 2004; Stern, 2004), theoretically, as clearly shown in Copeland and Taylor (2004), the EKC arises in a model featuring disutility of environmental degradation, substitution between factor input and pollution, and policies maximizing the utility. This study, however, by focusing on production externalities and allowing no technology choice, does not intend to explain the EKC, or, precisely, the right tail of the EKC.

There is a strand of literature upon the optimal control of pollution pioneered by Keeler et al. (1972). Some work in the field relates to economic development; see van der Ploeg and Withagen (1991) for an excellent analysis of the problem in the Ramsey framework. These models, like many others in this literature, focus on the case of a single sector in closed economy. This study extends the prevailing one-sector models to a two-sector model with international trade. Furthermore, most work in this strand of literature characterizes the optimal pollution tax in the form of differential equation. This study provides, by utilizing the transversality condition, an expression of the optimal pollution tax in the integral form. This helps reveal its economic meaning—a dynamic version of the Pigouvian tax. Recently, van der Ploeg and Withagen (2014) and Golosov et al. (2014) have provided similar integral (summation) expressions of social damage. Differing from their models, in this study the environment can recover by itself, resulting in a new discount term in the expression.

The rest of the paper is organized as follows. Section 2 presents the basic model, in which government is absent (laissez faire). Section 3 examines the basic model in a closed economy. Section 4 introduces trade in a small open economy (SOE). Section 5 extends the basic model and characterizes the optimal policy in autarky. Section 6 examines the optimal policy in SOE. Section 7 discusses some extensions. The last section concludes.

2 The Basic Model

There are two intermediate goods, one each from agriculture sector and manufacturing sector, and a single final good. Both intermediate goods are tradable, and produced using capital—the only factor of production in the model. The production of intermediate goods yields pollution as the by-product, which, as in Copeland and Taylor (1999), reduces the quality of the environment and consequently the productivity in agriculture sector. The
final good is assembled using two intermediate goods, and either consumed or invested. Households behave in a Ramsey fashion. Throughout the paper, assume the final good to be the numeraire.

### 2.1 Households

There are a large number of identical households, who own capital, $K$, and take their income, $X$, as given. The representative household receives income and behaves in a Ramsey fashion—choosing between consumption, $C$, and investment to maximize their discounted lifetime utility. Formally, the representative household maximizes $\int_0^{\infty} \ln C e^{-\rho t} dt$ subject to

$$\dot{K} = X - C - \delta K, \quad (1)$$

where $\delta$ is the depreciation rate of capital. To save notations, here and in what follows I omit the time index whenever there is no ambiguity.

### 2.2 Firms

The single final good is assembled using two intermediate goods:

$$Y = \frac{X_a^b X_m^{1-b}}{B}, \quad (2)$$

where $X_a$ and $X_m$ denote the agricultural and manufacturing inputs; $Y$ denotes the final output; $B \equiv b^b (1 - b)^{1-b}$ is introduced to simplify subsequent notations. Given the prices of intermediate goods, the choice of numeraire good (final good) implies, under perfect competition,

$$p_a^b p_m^{1-b} = 1. \quad (3)$$

Let $p$ denote the relative price:

$$p \equiv \frac{p_m}{p_a}, \quad (4)$$

then the prices of intermediate goods can be written as

$$p_a = p_m^{b-1}, \quad p_m = p_m^b. \quad (5)$$

Both intermediate goods are produced using capital:

$$Y_i = q_i K_i, \quad (6)$$

where $K_i (i = a, m)$ is the amount of capital employed in intermediate sector $i$; $Y_i$ and $q_i$ denote sector $i$'s output and productivity. Following Copeland and Taylor (1999), assume that manufacturing productivity $q_m$ is given but agriculture productivity $q_a$ satisfies

$$q_a = q_a (V), \quad q_a (0) = 0, \quad q_a (V) > 0, \quad q_a'' (V) < 0, \quad (7)$$
where $V$ denotes the quality of the environment. Under perfect competition, firms maximize their profits by taking environmental quality $V$ and rental rate $r$ as given.\(^3\) Since capital is freely and instantaneously mobile across sectors, the first-order condition gives $p_i q_i = r$ as long as sector $i$ is active ($K_i > 0$).

2.3 The Environment

Pollution arises from the use of capital as the by-product:

$$Z_i = \omega_i K_i,$$

where $Z_i$ denotes the flow of pollution from sector $i$; $\omega_i > 0$ is the sector-specific pollution intensity. If $\omega_a < \omega_m$, a unit of capital in manufacturing causes more pollution than in agriculture, so we say manufacturing is relatively dirty (compared to agriculture). Similarly, if $\omega_a > \omega_m$, we say agriculture is relatively dirty. Here, I assume that agriculture is polluting to capture the fact that some industries including agriculture are sensitive to environmental changes, but on the other hand, are contributing significant pollution and wastes to the environment.\(^4\) The total flow of pollution is then

$$Z = Z_a + Z_m = \omega_a K_a + \omega_m K_m.$$ (9)

The environment is local in that a country’s environment is affected only by the pollution that arises within that country, and only affects agriculture of that country. The quality of the environment evolves according to, as in Copeland and Taylor (1999),

$$\dot{V} = g (\bar{V} - V) - Z,$$ (10)

where $g$ and $\bar{V}$ are the environment’s recovery rate and carrying capacity, and $g (\bar{V} - V)$ can be interpreted as the natural growth of the environment.

3 Laissez Faire: Autarky

Because of the existence of pollution, clearly the social optimum cannot be achieved under laissez faire. In the paper, I analyze both laissez faire and optimal policy. In this section, I focus on the analysis of a closed economy under laissez faire.

Under laissez faire, the income of households comes only from the rental of capital: $X = rK$. Capital accumulation (1) becomes

$$\dot{K} = rK - C - \delta K$$ (11)
The current value Hamiltonian can be written as $H = \ln C + \gamma (rK - C - \delta K)$, where $\gamma$ is a costate variable of $K$. The first-order condition yields the Euler equation\(^5\)

$$\frac{\dot{C}}{C} = r - \delta - \rho, \quad \forall t \in [0, \infty). \tag{12}$$

The transversality condition $\lim_{t \to \infty} \gamma Ke^{-\rho t} = 0$ is required to ensure optimization.

Three dynamic equations—environmental changes (10), capital accumulation (11), Euler equation (12)—characterize the dynamics of the economy. The optimal transition path follows by solving the three differential equations with the initial condition $(K_0, V_0, C_0)$. Note that the initial stocks $(K_0, V_0)$ are given but the initial consumption $C_0$ need to be pinned down by applying the transversality condition.

In general, an analytical expression of $C_0$ is not available. But in the paper, as shown in Appendix, the logarithmic instantaneous utility implies $C_0 = \rho K_0$, and further a simple form of consumption function along the optimal transition path:

$$C = \rho K, \tag{13}$$

which gives the saving rate $1 - \rho / r$. That is, higher rental rate, the more households invest.\(^6\)

Substituting (13) into (1) yields

$$\frac{\dot{K}}{K} = r - \delta - \rho. \tag{14}$$

Therefore, with the consumption function (13) in hand, the dynamics under laissez faire can be characterized by two equations, (10) and (14), instead of by three. This simplifies the analysis of transition dynamics.

In what follows, to close the dynamic system (10) and (14), I express rental rate $r$ and pollution flow $Z$ in terms of capital stock $K$ and environmental quality $V$. I then analyze the steady state and transition dynamics.

### 3.1 Dynamic System

Under laissez faire, the marginal rate of transformation (MRT) from manufacturing to agriculture goods is

$$T = \frac{q_a(V)}{q_m}. \tag{15}$$

Since both intermediate goods are essential, both are produced in autarky equilibrium and the relative price equals the MRT: $p = T$. The intermediate prices can be written as, by (5),

$$p_a = T^{b-1}, \quad p_m = T^b. \tag{16}$$

---

\(^5\)Households do know the consequence of capital accumulation on capital rental rate through affecting the environment. But there are so many households who compete with each other in investment. The competition leads to the Nash equilibrium in which households make investment decisions only according to the current level of rental rate, thus behaving as if they do not care about the consequence of capital accumulation.

\(^6\)In steady state, $r = \rho + \delta$ and the saving rate is $\delta / (\rho + \delta)$. 

---
It follows immediately that
\[ r = p_a q_a (V) = q_a (V)^b q_m^{1-b}. \] (17)

The production technologies (2) and (6) imply in autarky equilibrium
\[ K_a = bK, \quad K_m = (1 - b) K. \] (18)

The flow of pollution is, by (8) and (18),
\[ Z = (b\omega_a + (1 - b) \omega_m) K \equiv \Omega K. \] (19)

Substitute these results into (10) and (14) to obtain the complete description of the dynamics in autarky under laissez faire:
\[ \dot{K} = q_a (V)^b q_m^{1-b} - \delta - \rho, \] (20)
\[ \dot{V} = g (\bar{V} - V) - \Omega K. \] (21)

### 3.2 Steady State and Transition Dynamics

Let \( \dot{K} = 0 \) and \( \dot{V} = 0 \) in (20) and (21) to solve for the steady state in autarky:
\[ K^A = \frac{g}{\Omega} \left( \bar{V} - V^A \right), \quad V^A = q_a^{-1} \left( \frac{\rho + \delta}{q_m^{1-b}} \right)^{\frac{1}{b}}, \] (22)

where the superscript \( A \) denotes the values in autarky steady state, and \( q_a^{-1} (\cdot) \) is the inverse function of \( q_a (\cdot) \). The autarky steady-state consumption is simply, by (13),
\[ C^A = \rho K^A. \] (23)

Focus on the case that \( K^A > 0 \), assuming
\[ q_a (\bar{V})^b q_m^{1-b} - \delta - \rho > 0. \] (24)

That is, the environmental capacity is large enough so that a positive stock of capital is sustainable. It is easy to check that

**Proposition 1.** In autarky, there exists a unique, locally (saddle) stable steady state satisfying (22) and (23). The environment in steady state is independent of environmental endowments, \( \bar{V} \) and \( g \), and pollution intensities, \( \omega_i \).

If there is no capital accumulation (fixed \( K \)), the environment in steady state will vary with the values of \( \bar{V} \), \( g \), and \( \omega_i \). This highlights the crucial role of endogenous investment as a channel through which households exploits richer environmental endowments (higher \( \bar{V} \) or \( g \)) or cleaner technologies (smaller \( \omega_i \)). Specifically, households invest according to the level of rental rate, which depends upon environmental quality, \( V \), and manufacturing
productivity, \( q_m \), but not upon \( \bar{V}, g, \) and \( \omega_i \). Since the steady state requires rental rate equal to \( \rho + \delta \), this implies that the steady-state environmental quality, \( V^A \), also has nothing to do with \( \bar{V}, g, \) and \( \omega_i \). Though not better environment, households still enjoy the advantage of higher \( \bar{V} \) or \( g \), or smaller \( \omega_i \) through higher capital stock and thus consumption in steady state.

On the other hand, environmental quality in steady state increases with depreciation rate, \( \delta \), and time preference, \( \rho \), and as mentioned, declines with manufacturing productivity, \( q_m \). The intuition is as follows. First, a lower depreciation rate means that capital is more durable, whereas a higher manufacturing productivity drives a higher rental rate, both making investment more attractive. Second, a lower time preference means that households care more about the future, thus consuming less and investing more at present. These lead to more capital but worse environment in steady state.

Transition dynamics in autarky under laissez faire: A diagrammatical exposition To illustrate the dynamics in autarky intuitively, Figure 1 depicts the vector field in the \((K, V)\) plane using (20) and (21). To be specific, the figure is generated numerically using Table 1 and letting \( \omega_a = 0.025, \omega_m = 0.1 \).

| \( \rho \) | Time preference | 0.05 |
| \( \delta \) | Capital depreciation rate | 0.1 |
| \( b \) | Share of agriculture good | 0.4 |
| \( \bar{V} \) | Environmental carrying capacity | 2 |
| \( g \) | Environmental recovery rate | 0.07 |
| \( q_a (V) \) | Agriculture productivity | 0.375V |
| \( q_m \) | Manufacturing productivity | 0.25 |

Table 1: Numerical specification

The streamlines in Figure 1 illustrate the trajectories of \((K, V)\) given various initial conditions. As emphasized, although the original dynamic system consists of three variables (two stock variables \( K \) and \( V \), and a control variable \( C \)), it is sufficient to focus on \( K \) and \( V \) since the optimal consumption has been considered in (20).

There are two key lines: the \( \dot{K} = 0 \) line and the \( \dot{V} = 0 \) line. The \( \dot{K} = 0 \) line is horizontal, above (below) which capital stock increases (decreases) over time because rental rate is higher (lower) than \( \rho + \delta \). The \( \dot{V} = 0 \) line is downward-sloping, above (below) which environmental quality decreases (increases) over time because pollution flow is greater (smaller) than the natural growth of the environment.

The two lines divide the plane into four regions. The dynamics in each region is as follows. Starting from a point in region I, say \((K_0, V_0)\), environmental quality is relatively high.
Notes: $\omega_a = 0.025$, $\omega_m = 0.1$, $(K_0, V_0) = (0.1, 2)$, and other parameters in Table 1.

and, by (17), rental rate is high enough to attract sufficient investment for capital accumulation. On the other hand, by (19), capital accumulation raises pollution flow and in turn reduces environmental quality. Thus, the trajectory of $(K, V)$ takes the lower-right direction in region I. The trajectory may cross the horizontal $\dot{K} = 0$ line or approach the steady state $(K^A, V^A)$, depending upon the nature of the dynamic system and the position of the trajectory. If the former is the case, the trajectory enters region II, as in Figure 1.

In region II, pollution flow remains higher than the natural growth of the environment, and so environmental quality $V$ continues to fall. On the other hand, rental rate becomes lower than $\rho + \delta$, so households reduce the investment below the maintenance level $\delta K$, resulting in a decline in capital stock. The decline in capital stock reduces further pollution flow and consequently the speed of environmental deterioration. Eventually, the trajectory of $(K, V)$ leaves region II and enters region III.

In region III, investment remains low such that capital stock continues to fall. However, pollution flow is relatively low and so environmental quality starts to recover, causing an increase in rental rate. Again, the economy may enter region IV or converge the steady state. Figure 1 illustrates the former case.

\footnote{For example, around the steady state, $(K, V)$ converges to $(K^A, V^A)$ along a straight line if $\Delta > 0$, and converges spirally if $\Delta < 0$, where $\Delta \equiv g - 4b (\rho + \delta) (V - V^A) q'_a (V^A) / q_a (V^A)$ is the discriminant of the characteristic equation of the dynamic system that is obtained by linearizing (20) and (21) at $(K^A, V^A)$.}
In region IV, rental rate becomes higher than $\rho + \delta$. Capital accumulates again. On the other hand, pollution flow remains relatively low, so environmental quality continues to increase over time. Eventually, the economy enters region I once again.

4 Laissez Faire: Small Open Economy

Trade liberalization breaks down the correlation between domestic demand and supply, and changes how capital accumulation interacts with the environment. In this section, I still focus on laissez faire but consider a small open economy (SOE) to examine how trade, capital accumulation, and the environment interact with one another. Note that, however, under laissez faire the consumption function (13) remains true in free trade.

4.1 Trade Patterns and SOE Dynamic System

We first examine trade patterns since it affects heavily how rental rate and pollution flow are determined in an SOE. Let $P$ denote the world relative price of manufacturing good to agriculture good. The comparative advantage is revealed by comparing the MRT, $T = \frac{q_a (V)}{q_m}$, with the world relative price, $P$. If the MRT is higher (lower) than the world relative price, the economy has a comparative advantage in agriculture (manufacturing). In free trade, the economy completely specialize in agriculture (manufacturing) owing to the short-run Ricardian structure of the model. It is convenient to define $V^W$ as

$$\frac{q_a (V^W)}{q_m} = P,$$

which can be interpreted as a measure of the environment in the rest of the world.

The horizontal $V = V^W$ line divides the $(K, V)$ plane into two regions. Above is the agriculture regime, where $T > P$ and the SOE has a comparative advantage in agriculture and thus completely specializes in it. Note that in SOE $p = P$ and, using (5),

$$p_a = P^{b-1}, \quad p_m = P^b.$$

Therefore, in agriculture regime $(V > V^W)$, rental rate is determined only by agriculture productivity: $r = p_a q_a (V) = P^{b-1} q_a (V)$. Pollution flow is simply $Z = \omega_a K$.

Below the $V = V^W$ line is the manufacturing regime, where $q_a (V) / q_m < P$ and the economy completely specializes in manufacturing. Rental rate and pollution flow are, respectively, $r = p_m q_m = P^b q_m$ and $Z = \omega_m K$.

On the $V = V^W$ line, $r = P^{b-1} q_a (V^W) = P^b q_m$. This implies the indeterminacy of capital allocation since there is no difference for capital to be employed in whichever sector. To get around the difficulty, I simply assume no trade arising (thus $Z = \Omega K$) in this knife-edge
situation of $V = V^W$. To summarize, rental rate and pollution flow in an SOE are

$$r = \begin{cases} 
   p^b q_m & \text{if } V = V^W, \\
   p^b q_a(V) & \text{if } V > V^W, \\
   p^b q_m & \text{if } V < V^W, 
\end{cases} \quad (27)$$

$$Z = \begin{cases} 
   \omega_a K & \text{if } V > V^W, \\
   \Omega K & \text{if } V = V^W, \\
   \omega_m K & \text{if } V < V^W. 
\end{cases} \quad (28)$$

Since the consumption function (13) still holds, the dynamics in SOE is also governed by (10) and (14). Substituting (27) and (28) into the two equations for $r$ and $Z$ gives the complete description of the motion of $K$ and $V$ in SOE.

### 4.2 SOEs under Laissez Faire: Two Cases

Unlike autarky, rental rate in SOE does not necessarily depends upon environmental quality. When environmental quality is low, the SOE specializes in manufacturing and, consequently, rental rate becomes a constant. So, is there still a steady state in SOE or, instead, a growth path arising? Is trade good or bad to the environment? Does the economy gain or lose from trade? To answer these questions, it proves useful to consider two cases:

**Case 1.** (PRCA-M) Pre-trade comparative advantage in manufacturing: $P > P^A$,

**Case 2.** (PRCA-A) Pre-trade comparative advantage in agriculture: $P < P^A$,

where $P^A \equiv p^A_m/p^A_a$ denotes the relative price of manufacturing good to agriculture good in autarky steady state, which is parametrically determined. The following lemma shows why the two cases matter.

**Lemma 2.** In an SOE satisfying $P > P^A$, $\dot{K} > 0$ always holds. In an SOE satisfying $P < P^A$, $\dot{K} \geq 0$ for $V \geq V^S$, where $V^S \in (V^W, \bar{V})$.

The lemma suggests that steady state does not exist in an SOE with PRCA-M ($P > P^A$). In the rest of the section, I analyze both cases one by one. But before that, I give some intuition of the lemma as follows.

The intuition comes from noting that rental rate is higher when the economy specializes in agriculture than in manufacturing, i.e., $p^b q_a(V) > p^b q_m$ for $V > V^W$. If the world relative price of manufacturing good is high ($P > P^A$), rental rate is high such that $\dot{K} > 0$ when specializing in manufacturing ($r = p^b q_m > \rho + \delta$). Since rental rate is even higher when specializing in agriculture, $\dot{K} > 0$ always holds. In contrast, if the world relative price

---

8The knife-edge case $P = P^A$ is not of special interest and thus neglected.
is low \((P < P^A)\), \(K < 0\) when specializing in manufacturing. However, when environmental quality is high such that the economy specializes in agriculture \((V > V^W)\), better is the environment, higher agriculture productivity and thus rental rate. Note that, by assumption (24), \(\dot{K} > 0\) when \(V = \bar{V}\). Environmental quality satisfying \(\dot{K} = 0\) lies between \(V^W\) and \(\bar{V}\).

### 4.3 PRCA-M SOE: A Growth Path

Consider a small economy with pre-trade comparative advantage in manufacturing (PRCA-M). By Lemma 2, \(\dot{K} > 0\) always holds when the economy opens to free trade. This has two consequences. First, by (28), pollution flow grows over time, causing environmental destruction. Second, by (13), consumption also grows over time, implying welfare gains from trade. The following proposition summarizes what happen in an SOE with PRCA-M:

**Proposition 3.** An SOE with PRCA-M \((P > P^A)\) adopts the following characteristics:

(i) There is no steady state.

(ii) At every point in time, the economy may specialize in either sector depending upon the environment at the moment; but in the long run, the economy specializes in manufacturing.

(iii) After trade liberalization, the environment may initially improve or degrade depending upon both capital stock and environmental quality at the openness; but in the long run, the environment goes to destruction.

(iv) Capital and thus consumption grows over time and there are welfare gains from trade.

**Transition dynamics in SOE with PRCA-M under laissez faire: A diagrammatical exposition**  
Again, a diagram is useful to illustrate the dynamics intuitively. Figure 2 draws the vector field in an SOE with PRCA-M and with a relatively dirty manufacturing sector \((\omega_a < \omega_m)\). The figure is generated numerically by using Table 1. The \(V = V^W\) line divides the \((K, V)\) plane into two regions, above (below) which the economy specializes in agriculture (manufacturing), and the \(\dot{V} = 0\) line has a slope of \(-\omega_a/g\) \((-\omega_m/g)\). Since manufacturing is relatively dirty, the latter is steeper. Moreover, according to Lemma 2, there is no \(\dot{K} = 0\) line.

The arrowed streamlines in Figure 2 illustrate the trajectories of \((K, V)\) given various initial conditions. For example, if the SOE starts from \((K_0, V_0)\) in the figure, the environment is relatively good and the SOE specializes in agriculture. At the same time, the natural growth of the environment is too slow to surpass the flow of pollution, resulting in environmental degradation over time. The trajectory then goes in the lower-right direction (noting that \(\dot{K} > 0\) always holds), and sooner or later crosses the \(V = V^W\) line from above. After that, the SOE loses its comparative advantage in agriculture and specializes in manufacturing, where rental rate becomes constant, but remaining high enough (because of \(P > P^A\)) to sustain capital accumulation. This leads to further environmental degradation.
Figure 2: Transition dynamics in SOE with PRCA-M: $\omega_a < \omega_m$

Notes: $P = 2.25$, $\omega_a = 0.025$, $\omega_m = 0.1$, $(K_0, V_0) = (0.1, 2)$, and other parameters in Table 1.

Figure 3: Transition dynamics in SOE with PRCA-M: $\omega_a > \omega_m$

Notes: $P = 2.25$, $\omega_a = 0.1$, $\omega_m = 0.05$, $(K_0, V_0) = (0.1, 2)$, and other parameters in Table 1.
The dynamics in an SOE with PRCA-M but with a relatively dirty agriculture ($\omega_a > \omega_m$) is similar. Figure 3 illustrated such a case. There are, however, two differences worth emphasizing. First, now the $\dot{V} = 0$ line above the $V = V_W$ line, instead of below, is steeper. As a result, the trajectory of $(K, V)$ around a segment of the $V = V_W$ line (in-between the $\dot{V} = 0$ line) experiences a sliding mode. That is, trajectories are attracted into and slide right along that segment of the $V = V_W$ line. Second, the autarky steady state, $(K^A, V^A)$, lies below the $\dot{V} = 0$ line, instead of above. Thus, starting from the autarky steady state, the environment improves short after trade liberalization.

### 4.4 PRCA-A SOE: A Steady State

Now consider a small economy with pre-trade comparative advantage in agriculture (PRCA-A), namely $P < P^A$. By Lemma 2, there exists steady state where $V = V^S$. Let $\dot{K} = \dot{V} = 0$ in the SOE dynamic system and solve for $(K, V)$, noting that in steady state the economy specializes in agriculture since $V^S \in (V_W, \bar{V})$, we obtain

$$K^S = \frac{\bar{g}}{\omega_a}(\bar{V} - V^S), \quad V^S = q_a^{-1}\left(\frac{\rho + \delta}{Pb - 1}\right).$$

(29)

The consumption in trade steady state is, using (13),

$$C^S = \rho K^S.$$  

(30)

The following proposition summarizes what happen to an SOE with PRCA-A:

**Proposition 4.** An SOE with PRCA-A ($P < P^A$) adopts the following characteristics:

(i) There is a unique, locally (saddle) stable steady state satisfying (29) and (30); moreover, there may exist a periodic orbit when manufacturing is relatively dirty.

(ii) At every point in time, the economy may specialize in either sector depending upon the environment at the moment; but in steady state, the economy specializes in agriculture.

(iii) After trade liberalization, the environment may initially improve or degrade depending upon both capital stock and environmental quality at the openness; but in the long run, the environment degrades.

(iv) Starting from autarky steady state, consumption increases initially, implying welfare gains if households are myopic enough; in steady state, consumption increases if manufacturing is relatively dirty (but depends upon parameters if agriculture is relatively dirty), implying welfare gains if households are patient enough.

The proposition suggests that if agriculture is relatively dirty, capital stock (and thus consumption) in trade steady state may be lower than that in autarky. The intuition is as follows. On one hand, trade liberalization raises rental rate and stimulates capital accumulation (with others remaining the same). How much trade can raise rental rate depends upon the size of pre-trade comparative advantage. On the other hand, the economy specializes in
agriculture in trade steady state. If agriculture is relatively dirty ($\omega_a > \omega_m$), the capability of hosting capital decreases (with others remaining the same). The two forces work in opposite directions and the outcome depends upon which one dominates. If pre-trade comparative advantage is small, the second force tends to dominate such that $K_S < K_A$ (thus $C_S < C_A$). In contrast, if pollution intensities are similar, the first force tends to dominate and results in $K_S > K_A$ (thus $C_S > C_A$).

Transition dynamics in SOE with PRCA-A under laissez faire: A diagrammatical exposition

To understand the dynamics, the diagram is useful. Figure 4 illustrates the dynamics in an SOE with PRCA-A and with relatively dirty manufacturing ($\omega_a < \omega_m$). The state plane is divided into four quadrants by the horizontal $\dot{K} = 0$ line and the downward-sloping $\dot{V} = 0$ line. The trajectory of $(K, V)$ goes toward the lower-right, lower-left, upper-left, and upper-right directions in quadrant I, II, III, and IV, respectively. Since $V_S > V_W$, the $V = V_W$ line lies in quadrants II and III.

Starting from a point in quadrant I, say $(K_0, V_0)$ in Figure 4, since $V_0$ is relatively high, the economy has a comparative advantage in agriculture and specializes in it. Rental rate is high enough to sustain capital accumulation. On the other hand, the natural growth of the environment is too slow to surpass pollution flow. So the trajectory of $(K, V)$ goes in the lower-right direction in quadrant I. Depending on the distance to $(K_S, V_S)$ and parameters,
the trajectory may converge to \((K^S, V^S)\) or enter quadrant II by crossing the \(\dot{K} = 0\) line. If the latter is the case, as illustrated in the figure, the economy continues to specialize in agriculture, but by now environmental deterioration cancels out some trade premium of rental rate such that it becomes too low to attract sufficient investment to sustain capital accumulation. Capital stock starts to decline and the trajectory moves in the lower-left direction. Again, depending on the position, the trajectory might cross the \(V = V^W\) line or the \(\dot{V} = 0\) line. If it crosses the \(V = V^W\) line, as illustrated in the figure, the economy specializes in manufacturing and the trajectory continues to move in the lower-left direction, and eventually crosses the \(\dot{V} = 0\) line and enters quadrant III. Thereafter, the environment improves over time and the trajectory crosses the \(V = V^W\) line and, consequently, the \(\dot{K} = 0\) line, and enters quadrant IV. In quadrant IV, capital stock and environmental quality increase over time, and the trajectory moves in the upper-right direction until it enters quadrant I.

If the economy starts from autarky steady state \((K^A, V^A)\) when opened to trade, initially it has a pre-trade comparative advantage in agriculture and specializes in it. Because agriculture is relatively clean, this reduces pollution flow and enhances the environment right after trade liberalization. On the other hand, trade increases rental rate and stimulates capital accumulation, and the trajectory of \((K, V)\) moves in upper-right direction and eventually enters quadrant I. Thereafter, it proceeds similar to the manner as above.

Note that, depending on parameters, there may exist a periodic orbit to which the trajec-
tory of \((K, V)\) can converge. Figure 12 in Appendix provides an example.

An SOE with PRCA-A but with relatively dirty agriculture \((\omega_a > \omega_m)\) has similar dynamics. Figure 5 illustrates such case. Two differences are worth mentioning. First, the \(\dot{V} = 0\) line has a steeper segment above the \(V = V^W\) line. Second, starting from the autarky steady state, the environment degrades when opened to trade.

5 Optimal Policy: Autarky

Laissez faire clearly does not yield social optimum since there are two sources of externalities. First, intermediate firms maximize the profit by taking environmental quality as given, implying overproduction in the relatively dirty sector. Second, households make investment decisions by taking rental rate as given, leading to excessive investment. To achieve the social optimum in autarky, in this section I consider the social planner problem and see how to replicate the planner’s decision in a market-based economy in autarky.

5.1 Social Planner and Dynamic System

The social planner chooses consumption \(C\), investment \(I\), and capital allocation \(K_i \geq 0\) \((i = a, m)\) to maximize \(\int_0^\infty \ln C e^{-\rho t} dt\), subject to capital accumulation \(\dot{K} = I - \delta K\), environmental changes (10), pollution flow (9), production technologies (2) and (6), and material constraints \(C + I \leq Y, X_i \leq Y_i \((i = a, m)\), and \(K_a + K_m \leq K\). Since consumption is always valuable, \(C + I = Y\) and \(X_i = Y_i\) in optimum. Substitute the former into \(\dot{K} = I - \delta K\) and obtain

\[ \dot{K} = Y - C - \delta K. \]  

(31)

The Hamiltonian can be written as

\[
H = \ln C + \gamma (Y - C - \delta K) + \lambda \left(g (\bar{V} - V) - Z\right) \\
- \tau \gamma (\omega_a K_a + \omega_m K_m - Z) - r \gamma (K_a + K_m - K) - p_a \gamma \left(Y - \frac{X_a Y_a^{1-b}}{B}\right) \\
- p_a \gamma (Y_a - q_a (V) K_a) - p_a \gamma (X_a - Y_a) - p_m \gamma (Y_m - q_m K_m) - p_m \gamma (X_m - Y_m),
\]

(32)

where costate variables \(\gamma\) and \(\lambda\) measure marginal values (in terms of utility) of a unit of increase in capital stock \(K\) and environmental quality \(V\); Lagrange multipliers \(\tau, r, p, p_a, p_a, p_m, p_m\) measure the shadow prices of pollution flow \(Z\), capital service \(K\), final good \(Y\), agriculture supply \(Y_a\) and demand \(X_a\), and manufacturing supply \(Y_m\) and demand \(X_m\). They are multiplied by \(\gamma\) such that \(p = 1\) and other shadow prices are measured in terms of the final good. In doing so, the final good plays the role of the numeraire.

The autarky economy ruled by the social planner can be characterized by capital accumulation (31), environmental change (10), the Euler equation for capital change (12), and the
Euler equation for environmental change

\[ \dot{\tau} = \left( \rho + g - \frac{\dot{\gamma}}{\gamma} \right) \tau - p_a q_a'(V) K_a, \]  

(33)

which is not present under laissez faire.\(^9\) Using \( Y = rK + \tau Z \) and \( \dot{\gamma} / \gamma = \rho + \delta - r \) (the Euler equation for \( K \)), the dynamic system can be summarized as\(^10\)

\[ \dot{K} = rK + \tau Z - C - \delta K, \]  

(34)

\[ \dot{V} = \dot{\gamma} (\dot{V} - V) - Z, \]  

(35)

\[ \frac{\dot{\gamma}}{\gamma} = r - \delta - \rho, \]  

(36)

\[ \dot{\tau} = (g + r - \delta) \tau - p_a q_a'(V) K_a. \]  

(37)

Note that rental rate \( r \), intermediate prices \( p_a \) and \( p_m \), capital allocation \( K_a \) and \( K_m \), and pollution flow \( Z \) are determined by \( \tau, V, \) and \( K \) in remaining first-order conditions. The following lemma shows how they are related with each other, though analytical expressions are not available,

**Lemma 5.** An increase in \( \tau \) reduces rental rate \( r \), whereas an increase in \( V \) has the opposite effect:

\[ \frac{\partial r}{\partial \tau} < 0, \quad \frac{\partial r}{\partial V} > 0. \]  

(38)

An increase in \( \tau \) raises (reduces) the price of relatively dirty (clean) good, whereas an increase in \( V \) reduces (raises) the price of agriculture (manufacturing) good:

\[ \frac{\partial p_i}{\partial \tau} < 0 < \frac{\partial p_i}{\partial V} \text{ if } \omega_i > \omega_j, \quad \frac{\partial p_a}{\partial V} < 0 < \frac{\partial p_m}{\partial V}. \]  

(39)

An increase in \( \tau \) shifts capital to the relatively clean sector, whereas an increase in \( V \) has the opposite effect:

\[ \frac{\partial l}{\partial \tau} \geq 0 \text{ and } \frac{\partial l}{\partial V} \leq 0 \text{ if } \omega_a \leq \omega_m, \]  

(40)

where \( l \) is capital allocation share in agriculture, i.e., \( K_a = lK, \) \( K_m = (1 - l)K. \) An increase in \( \tau \) reduces pollution flow, whereas an increase in \( V \) has the opposite effect:

\[ \frac{\partial Z}{\partial \tau} < 0 \text{ and } \frac{\partial Z}{\partial V} > 0 \text{ if } \omega_a \neq \omega_m. \]  

(41)

\(^9\)Precisely, the Euler equation for environmental change is \( \lambda = (\rho + g) - p_a \gamma q_a'(V) K_a. \) Substituting \( \lambda = \tau \gamma \) (from \( \partial H/\partial Z = 0 \)) for \( \lambda \) yields (33).

\(^10\)The first-order condition includes, using the Hamiltonian (32), \( \gamma = 1/C \) (from \( \partial H/\partial C = 0 \)), \( p = 1 \) (from \( \partial H/\partial Y = 0 \)), \( \lambda = \tau \gamma \) (from \( \partial H/\partial Z = 0 \)), \( p_a = p_a' = bX_a^{b-1}X_m^{-b}/B \) (from \( \partial H/\partial Y_a = 0 \) and \( \partial H/\partial X_a = 0 \)), \( p_m = p_m' = (1 - b) X_a^{b}X_m^{-b}/B \) (from \( \partial H/\partial Y_m = 0 \) and \( \partial H/\partial X_m = 0 \)), \( p_a q_a'(V) = r + \tau \omega_a \) (from \( \partial H/\partial K_a = 0 \)), \( p_m q_m = r + \tau \omega_m \) (from \( \partial H/\partial K_m = 0 \)), the Kuhn–Tucker condition \( r \geq 0, \) \( K_a + K_m - K \leq 0, \) \( r (K_a + K_m - K) = 0 \), and \( \dot{\gamma} = (\rho + \delta - r) \gamma \) (from the Euler equation \( \partial H/\partial K = \rho \gamma - \dot{\gamma} \)). The non-negative constraint of \( K_i \) (i = a, m) is not explicitly considered since both intermediate goods are essential and thus \( K_i > 0 \) holds in autarky optimum. Again, the transversality conditions \( \lim_{t \to \infty} \gamma K e^{-\rho t} = 0 \) and \( \lim_{t \to \infty} \lambda V e^{-\rho t} = 0 \) are required to pin down the optimal path.
5.2 Optimal Policy: A Dynamic Pigouvian Tax

The most straightforward way to replicate the social optimum in a market-based economy is to (i) impose a tax on pollution at the rate equal to the shadow price of pollution, and (ii) redistribute the tax revenue to households in a lump-sum fashion. The former provides the market proper information about the cost of pollution, and the latter is to clear the market.\(^\text{11}\)

The dynamic equation (37) characterizes the optimal pollution tax \(\tau^*\) (the asterisk indicates the social optimum), but the following lemma goes further.

**Lemma 6.** The shadow price of pollution can be expressed as

\[
\tau^* (t) = \frac{1}{\gamma (t)} \int_t^\infty \gamma (s) p_a q'_a (V (s)) K_a (s) e^{-(\rho+g)(s-t)} ds, \quad (42)
\]

where time indices are introduced to prevent confusion.

Lemma 6 has significant economic meaning: the shadow price of pollution is equal to the social damage of an additional unit of pollution. That is, Pigou’s idea of internalizing externalities remains valid in this dynamic framework, though the Pigouvian tax now adopts an integral form in (42).

To see how (42) is related to the social damage of pollution, note that an additional unit of pollution at time \(t\) causes, by (10), a unit of reduction in environmental quality. This harms agriculture productivity not only at time \(t\), but also all subsequent periods. If the environment does not recover by itself, agriculture productivity declines by \(q'_a (V (s))\) at time \(s\), which implies \(p_a (s) q'_a (V (s)) K_a (s)\) units of income loss at time \(s\). However, the environment does recover itself by converging to the capacity level \(\bar{V}\) at speed \(g\). This implies that the damage of a unit of pollution at time \(t\) disperses over time at the same speed and causes \(p_a (s) q'_a (V (s)) K_a (s) e^{-g(s-t)}\) units of income loss at time \(s\). Note the damage at time \(s\) are measured in the current value. To measure it in the present (time \(t\)) value, three steps are required. First, multiply \(p_a (s) q'_a (V (s)) K_a (s) e^{-g(s-t)}\) by \(\gamma (s)\) to convert the unit from income to utility. Second, multiply the result from the first step by \(e^{-\rho(s-t)}\) to obtain the present value in utility unit. Third, divide the result by \(\gamma (t)\) to convert it back to income unit and obtain

\[
d (t, s) = \frac{\gamma (s)}{\gamma (t)} p_a (s) q'_a (V (s)) K_a (s) e^{-(\rho+g)(s-t)}, \quad (43)
\]

which measures the present (time \(t\)) value of the income loss that takes place at time \(s\) \((s > t)\) while caused by a unit of pollution at time \(t\). The social damage of a unit of pollution at time \(t\) can be obtained by integrating \(d (t, s)\) from \(t\) to infinity with respect to \(s:\)

\[
D (t) = \int_t^\infty d (t, s) ds, \quad (44)
\]

\(^{11}\)If households receive the tax revenue \(\tau Z\), their income becomes \(rK + \tau Z\). On the other hand, the first-order condition \(p_a = (r + \omega_a \tau) / q_a (V)\) and \(p_m = (r + \omega_m \tau) / q_m\) imply that \(Y = p_a Y_a + p_m Y_m = rK + \tau Z\).
which would give exactly the right-hand side of (42). Therefore, \( \tau^* (t) = D(t) \), that is, the shadow price of pollution (also the optimal rate of pollution tax) should be equalized to the social damage of a unit of pollution.

The following proposition summarizes these results.

**Proposition 7.** In autarky, the social optimum can be achieved in a market-based economy by imposing a dynamic Pigouvian tax (42) on pollution, with lump-sum transfers of tax revenue to households.

### 5.3 Steady State

In steady state, the optimal pollution tax (42) becomes\(^\text{12}\)

\[
\tau^{*A} = \frac{p_a^* q_a (V^{*A}) K^{*A}}{\rho + g}, \tag{45}
\]

where the superscript \( *A \) denotes the social optimum in autarky steady state. On the other hand, letting \( \dot{K} = 0 \) and \( \dot{C} = 0 \) in (34) and (36), we have

\[
C^{*A} = \rho K^{*A} + \tau^{*A} Z^{*A} = \left( \rho + \tau^{*A} \Psi^{*A} \right) K^{*A}, \tag{46}
\]

where \( \Psi \equiv \omega_a l + \omega_m (1 - l) \). The saving rate (in steady state) is \( \delta / (\rho + \delta + \tau^{*A} \Psi^{*A}) \), which is lower than that under laissez faire \( \delta / (\rho + \delta) \). Note that in general (46) does not hold along the transition dynamics, though it may serve as an approximation around the steady state. The following proposition summarizes the characteristics in autarky under optimal policy.

**Proposition 8.** In autarky under optimal policy, there exists a unique and locally (saddle) stable steady state, where the pollution tax and consumption can be expressed as (45) and (46), and the environment is better than that under laissez faire.

**Steady state in autarky under optimal policy: A diagrammatical exposition**  Again, a diagram helps illustrate the economy under optimal policy intuitively. Differing from laissez faire, however, now we have four differential equations governing two stock variables (\( K \) and \( V \)) and two control variables (\( \tau \) and \( C \)). Furthermore, none of them can be eliminated since a simple form of consumption function is not available (due to the complication caused by income transfer). Therefore, a complete description of the economy requires a four-dimensional diagram, which is impossible to draw. To get around the difficulty, we focus on the steady state. For such purpose, the \((\tau, V)\) plane turns out to be sufficiently informative, as illustrated in Figure 6.

Figure 6 is obtained as follows. First, let \( \dot{V} = 0 \) in (35) and obtain

\[
g (\bar{V} - V) = Z, \tag{47}
\]

---

\(^{12}\)Letting \( \dot{r} = \dot{\gamma} = 0 \) in (33) yields the same result.
which defines a plane in the \((\tau, V, K)\) space. Given \(K\) (as \(K^A\) or \(K^{*A}\) in Figure 6), by Lemma 5, equation (47) represents a upward-sloping locus on the \((\tau, V)\) plane, above (below) which \(\dot{V} < 0\) (\(\dot{V} > 0\)). Note that the vertical axis \((\tau = 0)\) corresponds with the case of laissez faire. Letting \(K = K^A\), (47) intersects the vertical axis at \(V = V^A\).

Second, let \(\dot{\tau} = 0\) in (37) and obtain
\[
(g + r - \delta) \tau = p_a q_a' (V) lK,
\]
which defines another plane in the \((\tau, V, K)\) space. Again, for each given \(K\), it gives a locus on the \((\tau, V)\) plane, above (below) which \(\dot{\tau} > 0\) (\(\dot{\tau} < 0\)). Moreover, \(q''_a (V) \leq 0\) implies that the locus is U-shaped, as illustrated in the figure.

Third, changing \(K\) continuously, the intersection points of the two loci constitute the \(\dot{V} = \dot{\tau} = 0\) curve, whose expression follows by combining (47) and (48) to eliminate \(K\):
\[
(g + r - \delta) \tau \Psi = p_a q_a' (V) l g (\bar{V} - V).
\]

Note that \(K\) changes as moving along the \(\dot{V} = \dot{\tau} = 0\) curve.

Finally, let \(\dot{C} = 0\) in (36) and obtain
\[
r = \rho + \delta,
\]
By Lemma 5, this gives a upward-sloping locus, above (below) which \(\dot{C} > 0\) (\(\dot{C} < 0\)).
The $\dot{C} = 0$ curve and the $\dot{V} = \dot{\tau} = 0$ curve intersect and pin down the steady state, as $(\tau^*A, V^*A)$ in Figure 6. The uniqueness of $(\tau^*A, V^*A)$ is ensured by the opposite signs in the slopes of the two loci. Substituting $(\tau^*A, V^*A)$ into (47) or (48) yields $K^*A$, which can be plugged, together with $(\tau^*A, V^*A)$, into (46) to obtain $C^*A$. The saddle stability of $(\tau^*A, V^*A, K^*A, C^*A)$ in the dynamic system of (34)-(37) can be readily verified. Also, note that the upward-sloping $\dot{C} = 0$ curve passes through $(0, V^A)$, implying that $V^*A > V^A$.

5.4 Transition Dynamics: A Numerical Example

Differing from laissez faire, there is no closed-form consumption function under the optimal policy, so the transition dynamics is not straightforward. To compare the transition dynamics observed under the optimal policy and under laissez faire, I consider a simple numerical example specified by Table 1 and letting $\omega_a = 0.025$, $\omega_m = 0.1$. The results are illustrated in Figure 7.

Figure 7a depicts the trajectory of $(K, V)$ under the optimal policy and that under laissez faire, both starting from $(K_0, V_0)$. Figure 7b illustrates the corresponding optimal pollution tax and consumption level during the transition period. It shows that the optimal pollution tax is low at the starting point but increases gradually. Intuitively, this is because initially the environment is good ($V_0 = \bar{V} = 2$) and the stock of capital is relatively small ($K_0 = 0.1$). This implies a relatively small $p_a$ and $K_a$, and thus, given that $q'_a(V)$ is constant in our numerical example, a relatively small social damage in the short run by (43). Because the damage taking place more recently weights more in the total social damage—the optimal pollution tax, this relatively small social damage in the short run is likely to yield a relatively low pollution tax. As the environment degrades and capital accumulates over time, which, the optimal level of pollution tax tends to, for the similar reason, increase as well.

The steady-state consumption under the optimal policy could be higher or lower, depending upon parameters and functional forms. Our numerical example indicates a lower steady-state consumption. By the definition of optimal policy, welfare under the optimal policy is necessarily higher. To see this, note that consumption under the optimal policy is slightly higher in the early period of the time span, as prominently shown in Figure 7b. It is this initially small difference that leads to a higher lifetime discounted utility.

The steady-state consumption under the optimal policy could be higher or lower, depending upon parameters and functional forms. Our numerical example indicates a lower steady-state consumption. By the definition of optimal policy, welfare under the optimal policy is necessarily higher. To see this, note that consumption under the optimal policy is slightly higher in the early period of the time span, as prominently shown in Figure 7b. It is this initially small difference that leads to a higher lifetime discounted utility.

Also, although Figure 7a shows that $K^*A < K^A$, this is not necessarily the case. There are two forces working in opposite directions. On one hand, a pollution tax shifts out some capital from the dirtier sector, and thereby reduces, on average, per unit capital emission. Thus, more capital can be brought in without pulling down the quality of the environment and hence capital rental rate. On the other hand, from (88), a pollution tax directly reduces rental rate, thereby discouraging investment. The final outcome will depend upon which force dominates.
Figure 7: Transition dynamics in autarky: Comparison
6 Optimal Policy: Small Open Economy

In this section, I consider the optimal policy in an SOE. Given the world relative price \( P \), the social planner determines the volume of trade \( M_i \) \((i = a, m)\), as well as those variables in autarky.

6.1 Social Planner and Optimal Policy in SOE

With trade, the material constraint for intermediate goods is \( X_i + M_i \leq Y_i \) \((i = a, m)\). The positive sign of \( M_i \) denotes exports and the negative denotes imports. Assume the balance of trade, \( M_a + PM_m = 0 \), at every point in time. Also, as in autarky, \( C + I \leq Y \) and \( X_i + M_i \leq Y_i \) bind in optimum. The Hamiltonian can be then written as

\[
H = \ln C + \gamma (Y - C - \delta K) + \lambda (g (\bar{V} - V) - Z) \\
- \tau a\gamma (\omega_a K_a + \omega_m K_m - Z) - r a\gamma (K_a + K_m - K) - p a\gamma \left( Y - \frac{X_a X_{1-a}}{B} \right) \\
- p a\gamma (Y_a - q_a (V) K_a) - p a\gamma (X_a - PM_m - Y_a) \\
- p m\gamma (Y_m - q_m K_m) - p m\gamma (X_m + M_m - Y_m). \tag{51}
\]

The first-order condition is similar to that in autarky given in Appendix, except for the following two. First, as discussed in details in the next subsection, the non-negative constraint of \( K_i \) should be considered because the economy could completely specializes. Second, there is a new first-order condition, \( p_m / p_a = P \), which simply says that the relative (shadow) price should be equalized to the world relative price in optimum. But the basic scheme of the optimal pollution tax in autarky remains valid here, and the new condition implies that no tariff is needed to achieve the social optimum in a market-based SOE, as long as the pollution tax has already internalized all externalities. Thus, we have the following lemma:

**Lemma 9.** In the SOE, zero tariff and a tax-transfer system as in autarky achieve the social optimum.

6.2 SOE Dynamic System under Optimal Policy

Trade introduces the possibility of specialization, so we need to consider the non-negative constraint of \( K_i \) \((i = a, m)\). This implies the following lemma:

**Lemma 10.** In SOE, trade patterns are determined by the comparative advantage at every point in time, which is revealed by comparing the MRT and the world relative price at the moment.

Lemma 10 suggests that rental rate and pollution flow can be expressed as piecewise functions of \((\tau, V)\). Specifically, if \( T > P \), the economy completely specializes in agriculture, implying \( r = p a q_a (V) - \omega_a \tau = p a^{-1} q_a (V) - \omega_a \tau \) and \( Z = \omega_a K \). If \( T < P \), the economy completely specializes in manufacturing, yielding \( r = p m q_m - \omega_m \tau = p m q_m - \omega_m \tau \) and
\( Z = \omega_m K \). If \( T = P \), assume as under laissez faire no trade to eliminate indeterminacy, then \( r = P^b q_m - \omega_m \tau \) and \( Z = l \omega_a K + (1 - l) \omega_m K \equiv \Psi K \).

There remains a question about how to calculate the MRT in an SOE. It is somewhat puzzling because rental rate and the MRT are dependent of one another. As shown above, rental rate is determined in different manners when the MRT crosses the world price. Moreover, by (89), the MRT depends upon rental rate as long as \( \omega_a \neq \omega_m \). The following lemma provides an answer:

**Lemma 11.** Define the locus on the \((\tau, V)\) plane as

\[
\frac{q_a(V)(r + \omega_m \tau)}{q_m(r + \omega_a \tau)} = P, \quad (52)
\]

where \( r \) is determined as in autarky. Then in SOE, the MRT satisfies \( T = P \) on the locus, and \( T > P \) (\( T < P \)) above (below) it.

With Lemma 10 and Lemma 11 in hand, rental rate and pollution flow can be written as

\[
r = \begin{cases} 
  P^b q_a(V) - \omega_a \tau & \text{if } T > P, \\
  P^b q_m - \omega_m \tau & \text{if } T = P, \\
  P^b q_m - \omega_m \tau & \text{if } T < P,
\end{cases} \quad (53)
\]

\[
Z = \begin{cases} 
  \omega_a K & \text{if } T > P, \\
  l \omega_a K + (1 - l) \omega_m K & \text{if } T = P, \\
  \omega_m K & \text{if } T < P.
\end{cases} \quad (54)
\]

Substituting these results into the autarky dynamic system (34)-(37) for \( r \) and \( Z \), we can obtain the dynamic system in SOE under optimal policy as shown in Appendix.

### 6.3 SOEs under Optimal Policy: Three Cases

To characterize the dynamics and the steady state under optimal policy, it turns out to be useful to categorize SOEs into the following three types:

**Case 1.** (SPRCA-M) Strong pre-trade comparative advantage in manufacturing: \( P > \bar{P} \).

**Case 2.** (WPRCA-M) Weak pre-trade comparative advantage in manufacturing: \( P \in (P^A, \bar{P}] \).

**Case 3.** (PRCA-A) Pre-trade comparative advantage in agriculture: \( P < P^A \).

Note that the categorization PRCA-A (\( P < P^A \)) under laissez faire remain valid here. But PRCA-M (\( P > P^A \)) is further divided into SPRCA-M and WPRCA-M according to whether the world relative price \( P \) is greater than \( \bar{P} \) or not. The threshold value \( \bar{P} \) will be explained later on.
First, note that the following lemma holds in an SOE with pre-trade comparative advantage in agriculture (PRCA-A, \( P < P^A \)):

**Lemma 12.** Under optimal policy, an SOE with PRCA-A cannot remain specializing in manufacturing.

On the other hand, there exists a steady state where the economy can specialize in agriculture. The optimal pollution tax in steady state is

\[
\tau^* = \frac{BP^{b-1}q_a'(V^*) K^*}{\rho + g}, \tag{55}
\]

where the superscript \( *S \) denotes the SOE steady-state value in the social optimum. Using equations (99) to (102), we can solve for \( K^* \), \( V^* \) and \( C^* \). Again, a diagram on the \((\tau, V)\) plane provides an intuitive exposition for pinning down the steady state.

Figure 8 depicts the phase diagram in an SOE with relatively dirty manufacturing \((\omega_a < \omega_m)\). First, the \( T = P \) curve divides the \((\tau, V)\) plane into two regimes, above (below) which the economy specializes in agriculture (manufacturing). Note that the \( T = P \) curve starts from \((0, V^W)\) and, by (90), goes in the lower-right direction.

The \( \dot{C} = 0 \) curve is defined by (101). Lemma 2 suggests that there exists \( V^S \in (V^W, V) \) such that \( r = P^{b-1}q_a(V^S) = \rho + \delta \) at \((0, V^S)\), so the \( \dot{C} = 0 \) curve (on which \( r = \rho + \delta \) starts
from \((0, V^S)\), and goes in the upper-right direction.

To pin down the steady state, we also need the \(V = \dot{\tau} = 0\) curve, whose expression can be obtained by combining (100) and (102):

\[
\begin{cases}
(g + \rho b q_m - \omega_m \tau - \delta) \tau \omega_a = p_b - 1 q_a (V) g (\dot{V} - V) & \text{if } T > \bar{w}, \\
(g + \rho b q_m - \omega_m \tau - \delta) \tau \Psi = p_b - 1 q_a (V) l g (\dot{V} - V) & \text{if } T = \bar{w}, \\
(g + \rho b q_m - \omega_m \tau - \delta) \tau = 0 & \text{if } T < \bar{w}.
\end{cases}
\]

Expression (56) suggests that the \(V = \dot{\tau} = 0\) curve has three parts, as labeled by \(N_1N_2, N_3N_4,\) and \(N_5N_6\) in the figure. The locus \(N_1N_2\) lies in agriculture regime, corresponding with the first equality in (56). The loci \(N_3N_4\) and \(N_5N_6\) are in manufacturing regime, in which \(N_3N_4\) corresponds with \(\tau = 0\), and \(N_5N_6\) corresponds with \(g + Bp_b q_m - \omega_m \tau - \delta = 0\). Since the \(\dot{C} = 0\) curve lies above the \(T = \bar{w}\) curve, its intersection with the locus \(N_1N_2\) pins down the steady state \((\bar{\tau}^S, V^S)\), as shown in the figure. Moreover, Appendix proves that

**Lemma 13.** Under optimal policy, the steady state is unique in an SOE with PRCA-A.

The case of relatively dirty agriculture is similar, except that the \(T = \bar{w}\) curve is upward-sloping.\(^{13}\) Thus, we have the following proposition:

**Proposition 14.** Under optimal policy, there is a unique steady state in an SOE with PRCA-A \((P < P^A)\), in which the economy specializes in agriculture.

### 6.5 WPRCA-M SOE under Optimal Policy: A Steady State or A Growth Path

The threshold value \(\bar{P}\) satisfies that when \(P = \bar{P}\), the \(T = \bar{w}\) curve, the \(V = \dot{\tau} = 0\) curve, and the \(\dot{C} = 0\) curve pass through the same point.\(^{14}\) Note that \(\bar{P} > P^*\). To see this, suppose that \(P = P^*\). This implies that the \(T = \bar{w}\) curve will pass through \((\tau^*, V^*)\), like the \(\dot{C} = 0\) curve in autarky. From (101), the \(\dot{C} = 0\) curve in an SOE also passes through \((\tau^*, V^*)\), from where it goes in the upper-right direction in agriculture regime and becomes a vertical line in the manufacturing regime. In contrast, a comparison of (49) with (100) and (102) shows that the \(V = \dot{\tau} = 0\) curve in an SOE lies above \((\tau^*, V^*)\). Thus, the \(\dot{C} = 0\) curve and the \(\dot{V} = \dot{\tau} = 0\) curve must intersect somewhere in agriculture regime. This holds true when \(P\) is slightly higher than \(P^*\), as illustrated in Figure 9. This means directly \(\bar{P} > P^*\). Figure

\(^{13}\)In this case, both the \(\dot{C} = 0\) curve and the \(T = \bar{w}\) curve go upper-right. But the \(\dot{C} = 0\) curve will not intersect the \(T = \bar{w}\) curve from above. To see this, assume toward a contradiction that the former intersects the latter at \((\tau', V')\). By the definition of the \(\dot{C} = 0\) curve, this requires that rental rate \(r = \rho + \delta\) at \((\tau', V')\). On the other hand, \(r = \rho b q_m - \omega_m \tau\) along the \(T = \bar{w}\) curve. But noting that \(\rho b q_m < \rho + \delta\) by the definition PRCA-A, \(r = \rho b q_m - \omega_m V' < \rho + \delta\) at \((\tau', V')\). This leads to a contradiction.

\(^{14}\)Formally, \(\bar{P}\) (together with \(\tau\) and \(V\)) can be solved from the following three equations: \((\rho + g) \tau \omega_a = p_b - 1 q_a (V) g (\dot{V} - V), \rho + \delta = \rho b - 1 q_a (V) - \omega_a \tau,\) and \(\rho + \delta = \rho b q_m - \omega_m \tau.\)
Figure 9: SOE with WPRCA-M under optimal policy ($\omega_a < \omega_m$)

9 corresponds with the case of relatively dirty manufacturing ($\omega_a < \omega_m$). Again, the figure for the case of relatively dirty agriculture is similar except that the slope of the $T = P$ curve is positive.

Note that $P > P^A$ by the definition of WPRCA-M. This implies that the SOE with WPRCA-M can remain specializing in manufacturing, where the optimal pollution tax is zero as under laissez faire. If this is the case, the consumption function (13) holds and the transition dynamics becomes the same as under laissez faire. Second, an SOE with WPRCA-M is also featured by the existence of the steady state, as clearly shown in Figure 9. Whether the optimal path converges to the steady state or to the growth path depends upon the starting point of the economy. To summarize,

**Proposition 15.** Under the optimal policy, an SOE with WPRCA-M may specialize in agriculture in steady state or specialize in manufacturing along a growth path, depending upon the initial condition.

Further comments can be addressed. First, if the SOE starts from agriculture regime but finally specializes in manufacturing, the trajectory of $(\tau^*, V^*)$ necessarily passes through $(0, V^W)$. This is because, from (42), the optimal pollution tax is positive as long as the economy still enters agriculture regime, otherwise it becomes zero. On the other hand, since the optimal pollution tax $\tau^*$ does not jump, the only way to enter manufacturing regime from agriculture regime is to pass through $(0, V^W)$.

Second, unlike under laissez faire, where trade always harms the environment in the
long run (compared to the autarky steady state), under the optimal policy, trade does not necessarily harm the environment in the long run because of the existence of pollution tax. In Figure 9, the environment actually becomes better after trade liberalization ($V^* > V^* A$).

6.6 SPRCA-M SOE under Optimal Policy: A Growth Path

If $P$ is large enough ($P > \bar{P}$), the $\dot{C} = 0$ curve in agriculture regime could start from some point above the $\dot{V} = \dot{\tau} = 0$ curve, and so the $\dot{C} = 0$ curve cannot intersect the $\dot{V} = \dot{\tau} = 0$ curve. As a result, there is no steady state. This is what happens in an SOE with SPRCA-M. Figure 10 illustrates the case of relatively dirty manufacturing. Again, the figure for the case of relatively dirty agriculture is similar, except for the slope of the $T = P$ curve. We have the following proposition:

**Proposition 16.** Under the optimal policy, an SOE with SPRCA-M can specialize in manufacturing along a growth path, where the optimal pollution tax is zero, and capital and consumption grow at a constant rate.

7 Extensions

In this section, I consider some extensions to the model and their possible outcomes.
7.1 Consumption Externalities

In addition to production externalities, another crucial aspect of environment degradation is its damage upon utility through affecting, say, human health and environmental amenities. Our model can incorporate this aspect of consumption externalities straightforward by letting households maximize, instead of \( \int_{0}^{\infty} \ln C(t) e^{-\rho t} dt \),

\[
\int_{0}^{\infty} (\ln C(t) + h(V)) e^{-\rho t} dt, \tag{57}
\]

where \( h(\cdot) \) satisfying \( h'(\cdot) > 0 \) represents the contribution of the environment. Two observations follow immediately.

First, under laissez faire, atomic households make the same investment decisions by taking environmental quality as given. Thus the same results prevail about specialization patterns and environmental impacts. On the other hand, welfare loss from trade becomes more likely. Depending upon the specific form of \( h(\cdot) \), a growth path could be a disaster rather than a bless as the environment goes to destruction.

Second, under optimal policy, the optimal pollution tax should take into account the damage upon utility. Specifically, some algebra yields

\[
\tau^*(t) = \frac{1}{\gamma(t)} \int_{s}^{\infty} (\gamma(s) p_a q_a(V) K_a + h'(V)) e^{-(\rho + g)(s-t)} ds, \tag{58}
\]

where \( h'(V) \) is the new term that captures the damage of environmental degradation upon utility.

7.2 Large Open Economy

In an SOE, there may exist a growth path along which the economy specializes in manufacturing. Such scenario, however, has a logic difficulty that the economy cannot stay “small” if the rest of world does not grow as fast. This is actually the case given that the rest of world faces limited environmental capacity and that agriculture is not pure clean. To get around this potential inconsistency, a possible way is to drop SOE setup and assume that the world demand for manufacturing good decreases with the relative price. That is,

\[
\frac{M_m}{M_m(P)} = \frac{M_m(P)}{\tilde{P}}, \quad M'_m(P) < 0, \quad M_m(\tilde{P}) = 0. \tag{59}
\]

Let \( \varepsilon \) denote the elasticity of the world demand for manufacturing good to its relative price:

\[
\varepsilon \equiv \frac{-PM'_m(P)}{M_m(P)}, \tag{60}
\]

which is positive (negative) when the economy exports (imports) manufacturing good. Assume that, if exporting manufacturing good,

\[
\varepsilon > 1. \tag{61}
\]
Under laissez faire \((\tau = 0)\), the large economy completely specializes in manufacturing if \(P \geq T (V) = q_a (V) / q_m\) (comparative advantage condition) and \(M_m (P) = q_m K\) (complete specialization condition). Another situation arises if \(P = T (V)\) and \(M_m (P) \in (0, q_m K)\), in which the economy exports manufacturing good without complete specialization. The margin in-between is obtained by substituting \(P = T (V)\) into \(M_m (P) = q_m K\), yielding

\[
M_m \left( \frac{q_a (V)}{q_m} \right) = q_m K, \tag{62}
\]

which represents a downward-sloping locus that starts from \((0, \tilde{V})\) as illustrated in Figure 11. Here \(\tilde{V}\) satisfies \(T (\tilde{V}) = \tilde{P}\). Similarly, the economy completely specializes in agriculture if \(P \leq T (V) = q_a (V) / q_m\) and \(-PM_m (P) = q_a (V) K\), where agriculture export is given by \(-PM_m (P)\) because of the balance of trade. Also, the economy exports agriculture good without complete specialization if \(P = T (V)\) and \(-PM_m (P) \in (0, q_a (V) K)\). The margin in-between follows then by plugging \(P = T (V)\) into \(-PM_m (P) = q_a (V) K\) to obtain

\[
M_m \left( \frac{q_a (V)}{q_m} \right) = -q_m K, \tag{63}
\]

which is a upward-sloping locus as shown in Figure 11. The \((K,V)\) plane is divided into four regimes using (62), (63), and the \(V = \tilde{V}\) line. Each regime corresponds with a pattern of specialization and export, as illustrated in Figure 11.
The analysis above implies that the relative world price is determined as follows. When incompletely specializing, \( P = T(V) = \frac{q_a(V)}{q_m} \); when completely specializing in manufacturing, \( P = M_m^{-1}(q_m K) \); when completely specializing in agriculture, \( P = M_m^{-1}(-q_m K) \). Rental rate is therefore

\[
P = \begin{cases} 
M_m^{-1}(-q_m K)^{b-1}q_a(V) & \text{if } M_m \left( \frac{q_a(V)}{q_m} \right) \leq -q_m K, \\
q_a(V)^b q_m^{1-b} & \text{if } M_m \left( \frac{q_a(V)}{q_m} \right) \in (-q_m K, q_m K), \\
M_m^{-1}(q_m K)^b q_m & \text{if } M_m \left( \frac{q_a(V)}{q_m} \right) \geq q_m K.
\end{cases}
\] (64)

Unlike in an SOE, rental rate in a large economy depends also upon the stock of capital when the economy completely specializes, and the locus along which \( r = \rho + \delta \) (thus \( \dot{K} = 0 \)) exists in both PRCA-A and PRCA-M cases. In the former case, the \( \dot{K} = 0 \) curve goes upward in regime I and becomes horizontal in regime II, as illustrated in Figure 11. In the latter case, it is vertical in regime IV and horizontal in regime III, as shown in the figure. In both cases, there exists a steady state pinned down by the intersection between the \( \dot{K} = 0 \) curve and the \( \dot{V} = 0 \) curve (though the latter is not given in the figure to make the diagram clearer). Therefore, the position of the steady state matters for environmental impacts of trade. If the economy completely specializes in steady state, trade harms the environment in the long run, as in an SOE. But if this the economy remains diversified in steady state, the environment will be the same as in autarky. This suggests that the diversity of domestic industries could be important for preserving the environment.

Under optimal policy, specialization and export patterns can be analyzed similarly. A detailed exposition is beyond the scope of the paper. It is worth mentioning that the optimal tariff in the large open economy is not zero so as to exploit the power as a large economy. The optimal tariff should be imposed such that the following relationship holds between the domestic relative price, measured by \( T \) as given in (89), and the world relative price \( P \):

\[
P \left( 1 - \frac{1}{\varepsilon} \right) = T. \] (65)

### 7.3 Decreasing Returns to Scale

In the model, capital is the single factor of production. By seeing it as the composition of all factors, it seems plausible to assume constant returns to scale in terms of capital. But it is surely arguable to consider, instead of (7),

\[
q_a = q_a(V, K_a), \quad q_m = q_m(K_m), \quad \frac{\partial q_i}{\partial K_i} < 0. \] (66)

That is, although firms under perfect competition take the productivity as given, the sector as a whole faces deceasing returns to scale. Three implications are worth mentioning.

---

15Here the definition of PRCA-A and PRCA-M is extended and becomes \( P^A > \tilde{P} \) and \( P^A < \tilde{P} \).
First, rental rate depends also upon the stock of capital. For example, under laissez faire \((τ = 0)\) in autarky, we have, noting that (18) still holds,

\[
r = q_a (V, bK)^b q_m ((1 - b) K)^{1-b}.
\]  

(67)

This implies a upward-sloping locus along which \(r = ρ + δ\) (thus \(\dot{K} = 0\)) on the \((K, V)\) plane. The environment in autarky steady state then depends upon the position of the \(V = 0\) curve, and consequently depends upon environmental endowments, \(\bar{V}\) and \(g\), and pollution intensities, \(ω_i\). So Proposition 1 fails to hold.

Second, the economy may export without specializing, as in the case of large open economy. To see this, note that the MRT can be written as, instead of (89),

\[
T = \frac{q_a (V, K_a) (r + ω_m τ)}{q_m (K_m) (r + ω_a τ)}.
\]  

(68)

Then the point can be clearly made by considering the extreme case of \(\lim_{K_i \to 0} \partial q_i / \partial K_i = \infty\), where complete specialization is excluded. Therefore, unlike the model with constant returns to scale, here \(K_i\) can be uniquely determined when \(T = P\). So we do not have to assume that there is no trade in such situation just for the purpose to eliminate the indeterminacy of capital allocation.

Third, trade does not necessarily harm the environment in the long run even under laissez faire. The intuition is straightforward. With decreasing returns to scale, rental rate declines as capital accumulates. This weakens the scale effect such that it may not be able to dominate the composition effect in the long run. Although not presented here, we can conduct the similar diagrammatical analysis as in Figure 11 to show the point intuitively.

### 7.4 Two Factors

In the analysis of trade and economic growth, many models are based on two-factor Heckscher–Ohlin framework. Along this line, assume, instead of (6) and (8),

\[
Y_i = q_i F_i (K_i, L_i),
\]  

(69)

\[
Z_i = ω_i F_i (K_i, L_i),
\]  

(70)

where \(F_i (K_i, L_i) (i = a, m)\) is a neoclassical production function measuring the scale of production activity in sector \(i\); \(L_i\) is the labor employed in sector \(i\). Given that labor endowment is fixed, the Euler equations remain unchanged. For example, under laissez faire \((τ = 0)\), capital accumulation can be written as \(\dot{K} = rK + wL - C - δK\), which leads to the same Euler equation (12) corresponding with capital accumulation. But note that, because the wage contributes to the income, there is no simple form of consumption function as in (13). Further comments follow immediately.

First, together with demand side, rental rate and wage rate can be written as functions of \(K, V,\) and (if any) \(τ\): \(r = r (τ, V, K)\) and \(w = w (τ, V, K)\). So the locus along which \(r = ρ + δ\) (thus \(C = 0\)) is not a horizontal line under laissez faire: Proposition 1 does not hold, too.
Second, as in the case of decreasing returns to scale, the economy may export without specializing, where \( K_i \) is uniquely determined. To see this, first write down the MRT as

\[
T = \frac{q_a (V) (c_m (r, w) + \omega_m \tau)}{q_m (c_a (r, w) + \omega_a \tau)},
\]

where \( c_i (r, w) \) is the minimized cost for \( F_i (K_i, L_i) \) given \( r \) and \( w \). For each \( V \), the model under laissez faire degenerates to standard Heckscher–Ohlin model. There is a range of capital (diversification cone), within which the economy exports but remains diversified.

Finally, under laissez faire, trade does not necessarily harm the environment in the long run. The intuition is similar with that in the case of decreasing returns to scale.

### 7.5 Abatement

To save costs, firms facing pollution tax may want to use resources to abate pollution. The model can be extended to allow such substitution between factor inputs and pollution. For this purpose, it is convenient to treat pollution as an input. That is, following Copeland and Taylor (1994) and others,

\[
Y_i = \begin{cases} 
q_i Z_i^\alpha_i K_i^{1-\alpha_i} & \text{if } \frac{Z_i}{K_i} \leq \omega_i, \\
q_i K_i & \text{if } \frac{Z_i}{K_i} = \omega_i,
\end{cases}
\]

where \( \alpha_i (i = a, m) \) is a parameter. Note that abatement happens only if, given \( r \) and \( \tau \),

\[
\frac{Z_i}{K_i} = \frac{\alpha_i r}{(1-\alpha_i) \tau} \leq \omega_i,
\]

which can be rewritten into \( \tau \geq \alpha_i r / (1-\alpha_i) \omega_i \). Two comments follow immediately. First, under laissez faire (\( \tau = 0 \)), firms have no incentive to conduct abatement and thus the same results prevail. Second, under optimal policy, firms may not abate pollution as well if the optimal pollution tax is not high enough.

### 7.6 Distinct Production Function for Investment Good

For simplicity, our model implicitly assume that both consumption and investment goods are produced by using the same technology. One can relax it by assuming firms under perfect competition produce the investment good with different technology:

\[
Q = f (X_{ak}, X_{mk}),
\]

where \( Q \) is the output of investment good; \( f (X_{ak}, X_{mk}) \) is linearly homogenous; \( x_{ak} \) and \( x_{mk} \) denote the inputs of agriculture and manufacturing goods. Two comments follow then.

First, under laissez faire (\( \tau = 0 \)), there exists a steady state in autarky, where the environment remains independent of environmental endowments, \( \bar{V} \) and \( g \), and pollution intensities, \( \omega_i \). To see this, note that capital accumulation under laissez faire can be written as \( \dot{K} = (rK - C) / c_k (p_a, p_m) - \delta K \), where \( c_k (p_a, p_m) \) measures the minimized cost (thus
price) of the investment good given intermediate prices. Using the Hamiltonian \( H = \ln C + \gamma \left( (rK - C) / c_k(p_a, p_m) - \delta K \right) \), we can obtain the Euler equation
\[
\frac{\dot{\gamma}}{\gamma} = \rho + \delta - \frac{r}{c_k(p_a, p_m)},
\tag{75}
\]
where \( \gamma = c_k(p_a, p_m) / C \) by the first-order condition. Note that (75) alone is enough to pin down the environment in steady state since \( r, p_a, \) and \( p_m \) are still given by (16) and (17). This also implies that the environment in steady state is independent of the position of the \( \dot{V} = 0 \) curve and has nothing to do with environmental endowments and pollution intensities. Furthermore, with the similar arguments as in Section 4, it can be shown that under laissez faire trade necessarily harms the environment in the long run.

Second, the result above fails to hold if the production of investment good requires only manufacturing good. To see this, assume
\[
Q = q_kX_{mk},
\tag{76}
\]
where \( q_k \) is a parameter. Using (75), the transversality condition, and the similar steps used in the derivation of (13), we can show that \( C / c_k(p_a, p_m) = \rho K \) holds along the transition path. Plugging it back to (75) yields
\[
\frac{\dot{K}}{K} = \frac{r}{c_k(p_a, p_m)} - \delta - \rho, \tag{77}
\]
Now we can use (76) to obtain \( c_k(p_a, p_m) = p_m / q_k \), so (77) turns into, using (17),
\[
\frac{\dot{K}}{K} = q_kq_m - \rho + \delta. \tag{78}
\]
As long as \( q_kq_m \neq \rho + \delta \), there exists no steady state. If \( q_kq_m > \rho + \delta \), capital will keep accumulating and the environment eventually goes to destruction even in autarky.

### 7.7 Pure Clean Agriculture Sector

As argued, I assume away a pure clean agriculture sector to better reflect the reality. But for comparison with other models (e.g., Copeland and Taylor, 1999), it is worthwhile to see what if \( \omega_a = 0 \) and \( \omega_m > 0 \). Three comments follow immediately. First, the results about autarky will not change much since both intermediate goods are essential and manufacturing is still pollutive. Second, contrast to the case of \( \omega_a > 0 \), trade does not necessarily harms the environment in the long run, even under laissez faire. Specifically, if the economy specializes in agriculture, the economy takes on a growth path along which the environment approaches to the capacity level \( \bar{V} \). Third, this also holds under optimal policy. It is worth emphasizing that, although there is no pollution by specializing in agriculture, the optimal pollution tax is positive according to (42).
8 Conclusion

Featuring the Ramsey style investment and agricultural production externalities, the two-sector dynamic model helps understand the close nexus between trade, economic development, and the environment. I show that trade has a scale effect through capital accumulation, and under laissez faire, this scale effect necessarily dominates the composition effect in the long run and harms the environment. The policy implication is significant. The government should be cautious about trade liberalization, which can be catastrophic if there is lack of appropriate environmental regulations. I also show that the social optimum can be achieved through a pollution tax with a lump-sum transfer to households. The optimal pollution tax can be interpreted as a dynamic version of the Pigouvian tax. Furthermore, I show that although the specialization pattern is endogenously determined at every point in time, the long-run specialization pattern basically can be predicted from pre-trade comparative advantage, which is determined by the parameters.

In the model, pollution is local. The transboundary type of pollution, such as greenhouse gases, is also crucial. Moreover, technical changes towards cleaner technology are not considered, which is another significant aspect in reality. It will be interesting to incorporate these aspects into this framework in future research.

A Appendix

A.1 The Consumption Function under Laissez Faire

First, solve the accumulation of capital (1) to obtain

$$K(t) = e^{\int_0^t (r(\sigma) - \delta) d\sigma} \left( K_0 - \int_0^t C(s) e^{\int_0^s (r(\sigma) - \delta) d\sigma} ds \right),$$

(79)

and solve the Euler equation (12) to obtain

$$C(t) = C_0 e^{\int_0^t (r(\sigma) - \rho - \delta) d\sigma},$$

(80)

where $t, \sigma$, and $s$ are time indices to prevent ambiguity. Second, let $t = s$ in (80) to obtain the expression of $C(s)$ and substitute it into (79) to obtain

$$K(t) = e^{\int_0^t (r(\sigma) - \delta) d\sigma} \left( K_0 - C_0 \int_0^t e^{-\rho s} ds \right).$$

(81)

Third, use the first-order condition $\partial H / \partial C = 0$ to obtain $\gamma(t) = 1/C(t)$. Finally, substitute previous results into the transversality condition $\lim_{t \to \infty} \gamma(t) K(t) e^{-\rho t} = 0$ to obtain

$$\lim_{t \to \infty} \left( K_0 / C_0 - \int_0^t e^{-\rho s} ds \right) = 0,$$

(82)
which implies $C_0 = \rho K_0$. Plug it back into (81) to obtain

$$K(t) = K_0 e^{\int_0^t (r(\sigma) - \rho - \delta) d\sigma}. \quad (83)$$

Comparing (80) and (83) gives $C(t) = \rho K(t)$.

### A.2 Proof of Proposition 1

The uniqueness of $(K^A, V^A)$ comes by noting that the first equation in (22) represents a downward-sloping line on the $(K, V)$ plane, whereas the second represents a horizontal one. The two lines have a unique intersection, which pins down $(K^A, V^A)$.

The stability of $(K^A, V^A)$ is verified by calculating the Jacobian of (20) and (21) at $(K^A, V^A)$:

$$J^A = \begin{bmatrix} 0 & J^A_{12} \\ -\Omega & -g \end{bmatrix}, \quad (84)$$

where $J^A_{12} = bq_a(V^A) q_a(V^A)^{b-1} q_m^{-b} K^A > 0$. The local stability of $(K^A, V^A)$ follows from $\det J^A = \Omega J^A_{12} > 0$ and $\text{tr} J^A = -g < 0$.

Finally, by (22), $\bar{V}, g$, and $\omega_t$ do not enter the expression of $V^A$.

### A.3 Proof of Lemma 2

Note that $P - P^A$ and $P^b q_m - \delta - \rho$ have the same sign. To see this, use (22) to obtain $(P^A)^b q_m = \rho + \delta$, which gives $P^b q_m - \delta - \rho = P^b q_m - (P^A)^b q_m = (P^b - (P^A)^b) q_m$.

Given that $P > P^A$, $\dot{K} = P^b q_m - \delta - \rho > 0$ when $V < V^W$ or $\bar{V} = V^W$ by (27). When $V > V^W$, $P^{b - 1} q_a(V) > P^b q_m > \rho + \delta$ and thus $\dot{K} = P^{b - 1} q_a(V) - \delta - \rho > 0$.

Given that $P < P^A$, $P^{b - 1} q_a(V^W) = P^b q_m < \rho + \delta$. On the other hand, $P^{b - 1} q_a(\bar{V}) > \rho + \delta$. To see this, note that $P^{b - 1} q_a(\bar{V}) > P^{b - 1} q_a(V) q_a(V^A)^{1 - b} = P^{b - 1} q_a(V) (P^A q_m)^{1 - b}$. Since $P < P^A$, $P^{b - 1} q_a(V^W) (P^A q_m)^{1 - b} > P^{b - 1} q_a(\bar{V}) (P q_m)^{1 - b} = q_a(\bar{V}) q_m^{-b} > \rho + \delta$, where the last inequality follows from (24). Now we have $P^{b - 1} q_a(V^W) < \rho + \delta$ and $P^{b - 1} q_a(\bar{V}) > \rho + \delta$, the continuity of $q_a(V)$ implies that there exists $V^S \in (V^W, \bar{V})$ such that $P^{b - 1} q_a(V^S) = \rho + \delta$, i.e., $\dot{K} = 0$.

### A.4 Proof of Proposition 3

Proof of (i). By Lemma 2, $\dot{K} > 0$ always holds and thus there is no steady state in an SOE with PRCA-M.

---

\[\text{The stability is quite robust. Consider more generally } \dot{V} = E(V, Z) \text{ instead of (10). The local stability holds as long as } \partial E / \partial V < 0 \text{ and } \partial E / \partial Z < 0 \text{ around the steady state. The global stability, however, does not necessarily hold. The possibility cannot be excluded of the existence of a limit cycle where the pair } (K, V) \text{ repeats the same pattern of evolution. The discussion on global stability and limit cycle goes beyond the scope of the paper. Note that } (K^A, V^A, C^A) \text{ is locally saddle stable in the dynamic system (1), (12), and (10).}\]
Proof of (ii). At every point in time, the SOE specializes in agriculture (manufacturing) if $V > V^W$ ($V < V^W$) at the moment. However, since $\dot{K} > 0$ always holds, there exists certain point in time, say $T_1$, such that $\dot{V} < 0$ for $t > T_1$. This further implies that there exists another point in time, say $T_2$, such that $V < V^W$ for $t > T_2$. That is, the economy eventually specializes in manufacturing.

Proof of (iii). Since there exists $T_1$ such that $\dot{V} < 0$ for $t > T_1$ according to the proof of (ii), the environment goes to destruction.

Proof of (iv). Since $\dot{K} > 0$, it follows from the consumption function (13) that $\dot{C} > 0$.

A.5 Proof of Proposition 4

Proof of (i). The uniqueness of $(K^S, V^S)$ follows directly from the fact that the first equation in (29) represents a downward-sloping line, whereas the second represents a horizontal line on the $(K, V)$ plane.

The stability of $(K^S, V^S)$ is verified by calculating the Jacobian of the SOE dynamic system at $(K^S, V^S)$:

$$J^S = \begin{bmatrix} 0 & J_{12}^S \\ -\omega_a & -g \end{bmatrix},$$

where $J_{12}^S \equiv P^{b-1}q_a(V^S)K^S > 0$. The local stability follows directly from $\det J^S = \omega_a J_{12}^S > 0$ and $\text{tr} J^S = -g < 0$. Moreover, $(K^S, V^S, C^S)$ is saddle stable in the original dynamic system.

The existence of a periodic orbit can be illustrated using a numerical example as shown in Figure 12. In the example, starting from $(K_0, V_0)$ or $(K^A, V^A)$, the economy converges to the periodic orbit. Note that, if the starting point is close enough to $(K^S, V^S)$, the local stability ensures that the economy converges to the steady state instead of to the periodic orbit.

Proof of (ii). By Lemma 2, $V^S \in (V^W, \bar{V})$, which simply says that the economy specializes in agriculture in steady state.

Proof of (iii). Use (22) and (25) to obtain an alternative expression for $V^S$:

$$q_a(V^S) = q_a(V^A)^b q_a(V^W)^{1-b},$$

which, together with $V^A > V^W$ (by $P < P^A$), implies $V^S \in (V^W, V^A)$. That is, compared to autarky, trade liberalization causes environmental degradation in steady state.

Proof of (iv). First, prove that consumption increases right after trade liberalization. Right after the openness, environmental quality remains $V^A$. Given that $P < P^A$, $V^A > V^W$ and thus the economy specializes in agriculture. By (27), rental rate is $P^{b-1}q_a(V^A) > (P^A)^{b-1}q_a(V^A) = q_a(V^A)^b q_m^{1-b} = \rho + \delta$. So, $\dot{K} > 0$ and thus, by (13), $\dot{C} > 0$ right after the openness (if starting from autarky steady state).
Figure 12: Dynamics in SOE with PRCA-A ($\omega_a < \omega_m$): Periodic orbit case  

Notes: $P = 0.75, \omega_a = 0.025, \omega_m = 0.1, (K_0, V_0) = (0.1, 2)$, and other parameters in Table 1.

Second, compare consumption in SOE steady state with that in autarky steady state. Note that $C^S - C^A = \rho (K^S - K^A)$, whose sign depends upon, using (22) and (29),

$$\Omega (V^A - V^S) + (1 - b) (\omega_m - \omega_a) \left( \bar{V} - V^A \right),$$

where $\Omega \equiv b \omega_a + (1 - b) \omega_m$. Since $V^A - V^S > 0$ In an SOE with PRCA-A, (87) and thus $C^S - C^A$ is positive if $\omega_a < \omega_m$.

Finally, discuss welfare gains. If households are myopic, they enjoy the increase, even temporary, in consumption and there are gains from trade. If households are patient, they focus more on the long run and there are gains if the steady-state consumption increases, which is the case if manufacturing is relatively dirty.

### A.6 Proof of Lemma 5

First, rental rate is determined as follows. As in a planned economy, perfect competition yields $p_a = \left( r + \omega_a \tau \right) / q_a (V)$ and $p_m = \left( r + \omega_m \tau \right) / q_m$, which when plugged into (3) gives

$$\left( \frac{r + \omega_a \tau}{q_a (V)} \right)^b \left( \frac{r + \omega_m \tau}{q_m} \right)^{1-b} = 1.$$  

With some algebra, we can obtain (38).
Second, the prices of intermediate goods are determined as follows. Note that, with pollution tax \( \tau \), the MRT is now, instead of (15),

\[
T = \frac{q_a(V) (r + \omega_m \tau)}{q_m (r + \omega_a \tau)}.
\]

By (38), we have

\[
\frac{\partial T}{\partial \tau} \geq 0 \text{ if } \omega_a \leq \omega_m, \quad \frac{\partial T}{\partial V} > 0.
\]

Since \( T = p_m / p_a \) in autarky, (16) holds and (90) implies (39).

Third, let \( l \) denote the share of capital allocated in agriculture, i.e., \( K_a = lK, K_m = (1 - l) K \). It follows from (6), (16), and \( p_a X_a / p_m X_m = b / (1 - b) \) that

\[
l = \frac{b (r + \omega_m \tau)}{b (r + \omega_m \tau) + (1 - b) (r + \omega_a \tau)},
\]

which together with (38) implies (40).

Finally, note that \( Z = \omega_a lK + \omega_m (1 - l) K \), which together with (40) implies (41).

### A.7 Proof of Lemma 6

Integrating (33) yields

\[
\tau(t) = \frac{\gamma(0)}{\gamma(t)} e^{(\rho + \gamma)t} \left( \tau(0) - \int_0^t \frac{\gamma(s)}{\gamma(0)} p_a q_a'(V) K_a e^{-(\rho + \gamma)s} ds \right),
\]

where time indices are introduced explicitly to prevent confusion. Plug (92) into the transversality condition \( \lim_{t \to \infty} \lambda V e^{-\rho t} = 0 \) and use again \( \lambda = \tau \gamma \) to obtain the initial value of \( \tau \) as follows:

\[
\tau^*(0) = \int_0^\infty \frac{\gamma(s)}{\gamma(0)} p_a q_a'(V) K_a e^{-(\rho + \gamma)s} ds,
\]

which can be plugged back into (92) to substitute for \( \tau(0) \) and derive (42).

### A.8 Proof of Lemma 10

The Kuhn–Tucker condition for the non-negative constraint of \( K_i \) \((i = a, m)\) includes \( K_a \geq 0, \partial H / \partial K_a \leq 0, K_a \partial H / \partial K_a = 0, \) and \( K_m \geq 0, \partial H / \partial K_m \leq 0, K_m \partial H / \partial K_m = 0. \) So one of the following four necessarily holds: (i) \( \partial H / \partial K_a < 0 \) and \( \partial H / \partial K_m < 0, \) (ii) \( \partial H / \partial K_a = 0 \) and \( \partial H / \partial K_m < 0, \) (iii) \( \partial H / \partial K_a < 0 \) and \( \partial H / \partial K_m = 0, \) and (iv) \( \partial H / \partial K_a = \partial H / \partial K_m = 0. \) The first gives \( K_a = K_m = 0, \) which is clearly not optimal. The second gives \( K_m = 0, \) which implies that \( p_a = (r + \omega_a \tau) / q_a(V) \) and \( p_m < (r + \omega_m \tau) / q_m, \) and thus, by (89), \( T > p_m / p_a = P. \)

The third gives \( K_a = 0, \) which implies that \( p_a < (r + \omega_a \tau) / q_a(V) \) and \( p_m = (r + \omega_m \tau) / q_m, \) and thus \( T < p_m / p_a = P. \) The fourth gives \( T = p_m / p_a = P, \) which implies that there is no difference in how to allocate capital between sectors. Therefore, in social optimum, the SOE should specialize in agriculture (manufacturing) when the MRT \( T \) is greater (less) than the world relative price \( P. \)
A.9 Proof of Lemma 11

Suppose that the economy completely specializes in agriculture, then rental rate \( r = \frac{p^b q_a (V) - \omega_a \tau}{q_m} \). Using (89), we obtain the MRT

\[
T_1 = T_1 (\tau, V) \equiv \frac{q_a (V) \ p^{b-1} q_a (V) + (\omega_m - \omega_a) \tau}{p^{b-1} q_m (V)} = \frac{p^{b-1} q_a (V) + (\omega_m - \omega_a) \tau}{p^{b-1} q_m (V)}. \tag{94}
\]

On the other hand, suppose that the economy completely specializes in manufacturing, then \( r = \frac{p^b q_m - \omega_m \tau}{q_m} \). The MRT is

\[
T_2 = T_2 (\tau, V) \equiv \frac{q_a (V) \ p^b q_m}{q_m (p^b q_m + (\omega_a - \omega_m) \tau)} = \frac{p^b q_a (V)}{p^b q_m (\omega_a - \omega_m) \tau}. \tag{95}
\]

Let \((\tau', V')\) denote the value(s) of \((\tau, V)\) such that there is no difference in rental rate between specializing in agriculture and specializing in manufacturing, that is, \((\tau', V')\) satisfies

\[
p^{b-1} q_a (V') - \omega_a \tau' = p^b q_m - \omega_m \tau'. \tag{96}
\]

Then it can be verified that

\[
T_1 (\tau', V') = T_2 (\tau', V') = T (\tau', V') = P, \tag{97}
\]

where

\[
T (\tau, V) \equiv \frac{q_a (V) \ (r + \omega_m \tau)}{q_m (r + \omega_a \tau)}
\]

is the MRT in autarky given in (89). Note that \( T_1 (\tau, V) \) and \( T_2 (\tau, V) \) change in the same way as \( T (\tau, V) \) does in (90), that is, increases with \( V \) and increases (decreases) with \( \tau \) if agriculture is relatively clean (dirty). Therefore, by (97), for any \((\tau, V)\) satisfying \( T (\tau, V) > P \), both \( T_1 (\tau, V) > P \) and \( T_2 (\tau, V) > P \) hold, which excludes the possibility of specializing in manufacturing since that requires \( T_2 (\tau, V) < P \) and indicates the complete specialization in agriculture. Similarly, if \( T (\tau, V) < P \), both \( T_1 (\tau, V) < P \) and \( T_2 (\tau, V) < P \) hold, but only the complete specialization in manufacturing is consistent. To summarize, the MRT in SOE satisfies

\[
T \equiv \begin{cases} 
T_1 (\tau, V) > P & \text{if } T (\tau, V) > P, \\
P & \text{if } T (\tau, V) = P, \\
T_2 (\tau, V) < P & \text{if } T (\tau, V) < P.
\end{cases} \tag{98}
\]
A.10 SOE Dynamic System under Optimal Policy

Substitute (53) and (54) into (34)-(37) for r and Z. Moreover, in (37), substitute $p^{b-1}$ for $p_a$ and letting $l = 1$ and $l = 0$ for $T > P$ and $T < P$. This gives the dynamic system in SOE:

$$
\dot{K} = \begin{cases} 
  p^{b-1}q_a (V) K - C - \delta K & \text{if } T > P, \\
  p^bq_m K - C - \delta K & \text{if } T = P, \\
  p^bq_m K - C - \delta K & \text{if } T < P,
\end{cases}
$$

$$
\dot{V} = \begin{cases} 
  g (\dot{V} - V) - \omega_a K & \text{if } T > P, \\
  g (\dot{V} - V) - \Psi K & \text{if } T = P, \\
  g (\dot{V} - V) - \omega_m K & \text{if } T < P,
\end{cases}
$$

$$
\frac{\dot{C}}{C} = \begin{cases} 
  p^{b-1}q_a (V) - \omega_a \tau - \delta - \rho & \text{if } T > P, \\
  p^bq_m - \omega_m \tau - \delta - \rho & \text{if } T = P, \\
  p^bq_m - \omega_m \tau - \delta - \rho & \text{if } T < P,
\end{cases}
$$

$$
\dot{\tau} = \begin{cases} 
  (g + p^{b-1}q_a (V) - \omega_a \tau - \delta) \tau - p^{b-1}q_a' (V) K & \text{if } T > P, \\
  (g + p^bq_m - \omega_m \tau - \delta) \tau - p^{b-1}q_a' (V) lK & \text{if } T = P, \\
  (g + p^bq_m - \omega_m \tau - \delta) \tau & \text{if } T < P.
\end{cases}
$$

A.11 Proof of Lemma 12

Toward a contradiction, assume that the economy remains specializing in manufacturing from certain point in time. According to (42), the optimal pollution tax is zero from then on, as under laissez faire. This implies that the consumption function under laissez faire, $C = \rho K$, holds as well. Substitute it into the SOE dynamic system to obtain the growth rate of capital: $\dot{K}/K = p^bq_m - \delta - \rho$, which is a negative constant by the definition of PRCA-A. Therefore, the stock of capital decreases over time and approaches zero, so does the flow of pollution. As a result, the environment improves over time and approaches $\bar{V}$. But note that $\dot{V} > V^W$, so $V$ will eventually become greater than $V^W$. Then the economy should specialize in agriculture given zero pollution tax. This leads to a contradiction.

A.12 Proof of Lemma 13

Note that $r = \rho + \delta$ holds along the $\dot{C} = 0$ curve, and $(g + p^{b-1}q_a (V) - \omega_a \tau - \delta) = (g + r - \delta) \tau \omega_a = p^{b-1}q_a' (V) g (\dot{V} - V)$ holds along the $\dot{V} = \dot{\tau} = 0$ curve. The steady state $(\tau^*, V^*)$ can be solved from the two. Substituting the former into the latter gives $(g + \rho) \tau \omega_a = p^{b-1}q_a' (V) g (\dot{V} - V)$, which represents a locus on the $(\tau, V)$ plane starting from $(0, \bar{V})$ with negative slope. On the other hand, in an SOE with PRCA-A, the $\dot{C} = 0$
curve is a locus on the \((\tau, V)\) plane starting from \((0, V^S)\) with a positive slope. The two loci can only intersect with each other once, implying the uniqueness of \((\tau^*, V^*)\).

References


