Consumer's Enthusiasm and Trade-triggered Short-run Price Increasing Competition of Horizontally Differentiated Goods

Megumi Nishimura

nisimegu@seiryo-u.ac.jp*

September 26, 2015

1 Introduction

What do Hello Kitty, Mickey mouse, Rolling stones, Lady Gaga, Lois Vuitton, and CHANEL have in common? Of course, all of them are well-established brand. In addition, what is characteristic about these brands is that their customers show different enthusiasm for their products. Some fans literally live in a Hello Kitty world and their rooms are covered with Hello Kitty from floor to ceiling, while other fans are satisfied with just one plush. Such a link between a consumer's enthusiasm and his/her consumption behavior has been paid little attention in the field of industrial organization and international trade. Most existing studies on differentiated goods put a unit-demand assumption, where every consumer buys no more than one unit of a differentiated good regardless of his/her enthusiastic level, and therefore, the difference in consumers' enthusiastic levels has no effect on their consumption. In these existing studies, effects of trade liberalization on horizontally differentiated goods have been discussed since Lancaster (1966,1975) and Chamberlain (1933) created theoretical

*This paper is based on the first chapter of my Ph.D Dissertation.

models to analyze differentiated goods. Although models used in these existing studies are different, the heterogeneity of consumers' enthusiasm levels has been ignored, and the same non-surprising conclusion has been reached: trade liberalization spurs competition among horizontally differentiated goods, and therefore their prices will have no chance to increase either in the short-run or in the long-run. However, it is the combination of the unit-demand assumption and homogeneity of consumers' enthusiasm levels that derives the result. These assumptions are not appropriate for goods, whose consumers show different enthusiasm levels, and the difference on enthusiasm levels lead to the difference in consumption levels. This paper relaxes these assumptions and allows consumers to buy different amount of goods depending on their enthusiastic levels. Specifically, this paper incorporates the heterogeneity of consumer enthusiasm for differentiated goods into the Helpman model (1981), a well-known the Lancaster's ideal-variety approach model and shows that trade liberalization can lead to a price-increasing competition at least in the short-run. That can happen because the trade liberalization changes not only the number of rival goods but also the composition of consumers each differentiated good firm sell its product to. On one hand, the trade liberalization allows foreign rival goods flow in the domestic market, which spurs the competition and puts downward pressures on price. On the other hand, the trade liberalization makes each firm sell its product to narrower range of consumers, who show more enthusiastic support for its product and less price elastic, which can give firms a chance to increase their prices. Specifically, the trade liberalization induces firms to raise their prices if (1) less enthusiastic consumers are much more price-elastic than weighted-average consumers are, and/or (2) the enthusiasm function curve $e(v)$ evaluated at the location of marginal consumers is less elastic but sufficiently convex. This paper also shows that the price-increasing competition can also be a result under the imperfect trade liberalization with tariff. Under a separated market setting, the imposition of tariff decreases the producer price of imports, but increases consumer prices of imports. If the consumers' enthusiasm function becomes more elastic and/or less convex with consumers' enthusiasm level, domestic firms have a strategic complementary relationship with foreign counterparts, and they also raise their price in home market.

This paper is not the first paper to predict the price-increasing competition after the trade liberalization. Chen and Riordan (2008) derive conditions that the price increasing effect "the price sensitivity effect" dominates the price decreasing effect "the market share effect", and the competition does increase price in their unique model with special model settings. The contribution of this paper is to show that the price-increasing competition will be the result in a more general model setting.

The simplified Helpman model is presented in section 2.[1] Using the Helpman model, this

[1] A purpose of the Helpman model is to generalize the Heckscher-Ohlin theorem and shows conditions

paper analyzes a situation, where a country close its market for foreign differentiated goods (ex film industry) at the beginning, however, it eventually opens its market to foreign goods perfectly and imperfectly.

2 The Simplified Helpman Model (1981)

2.1 Closed Economy

Consumers: Lancaster's Ideal Variety Preference and Two-Stage Maximization Problem

In one closed economy, every resident is endowed with one unit of labor and supplies it inelastically. Without inequality of wealth distribution, all residents earn the same income and they spend their income in two types of goods, horizontally differentiated goods X and the composite of homogeneous goods Y. All L residents(consumers) in a country are identical except their taste for horizontally differentiated goods X. Every consumer has his/her own ideal product x_i, which is shown as a point on the circumference of the production possibility circle in figure 1.[the Lancaster's ideal-variety preference]

Consumers' ideal products are assumed to be uniformly distributed over the circumference of the production possibility circle ($2\pi r = 2\beta$), and accordingly, the density of consumers at each point of the circumference of the circle becomes $n = \frac{L}{2\beta}$. The circumference of the production possibility circle shows goods a country can potentially produce. Despite an infinite varieties of goods on the production possibility circle, as shown later, the variety of goods actually produced in a country results in a finite variety due to the zero-profit condition. Therefore, some lucky consumers can buy exactly their ideal products, while other consumers have no choice but to buy substitutes x_j for their ideal products x_i. The distance v between the consumer i's ideal product x_i and its substitute x_j determines the consumer's enthusiasm for the substitute good x_j. The closer the distance v, the more enthusiastic the consumer becomes for the product. Such consumer's enthusiasm is represented by the enthusiasm function $e(v)$. [2] The consumer enthusiasm function works as if it discounts the consumption of x_j (v) by $e(v)$. i.e $x_j = e(v)x_i$. The consumer enthusiasm function $e(v)$ is a function of the distance between a consumer's ideal good x_i and the substitute good x_j and

that the theorem still holds even if goods are differentiated, are produced with an increasing returns to scale technology. Therefore, his model uses two factors of production, labor and capital. However, a purpose of this paper is to discuss the impact of the international trade on a particular type of differentiated goods, whose consumers change their demand depending on their enthusiastic level. Thus, this model simplifies the Helpman model by using only one factor of production, labor.

[2] $e(v)$ is the inverse of the Lancaster's compensation function $h(v)$

satisfies the following properties.

$$e'(v) < 0 \quad \forall \quad v > 0 \tag{1a}$$
$$e(0) = 1 \quad if\ v = 0 \tag{1b}$$
$$0 < e(v) < 1 \quad if\ v > 0 \tag{1c}$$

(1a) shows that the longer the distance from his/her ideal product, the less enthusiastic a consumer becomes for the good $x_j(v)$. This enthusiastic function $e(v)$ works as a converter, which converts the price and the quantity of every good into those of each consumer's ideal good. One unit of the v-distant good worths $e(v)$ units of his/her ideal product, and one unit of his/her ideal product worths $\frac{1}{e(v)}$ units of the v-distant good. Thus, the price of a v-distant good p_j means that the good j offers a consumer one unit of his/her ideal good at an effective price of $\frac{p_j}{e(v)}$. As a result,the effective price $\frac{p_j}{e(v)}$ increases with the distance from the ideal good.

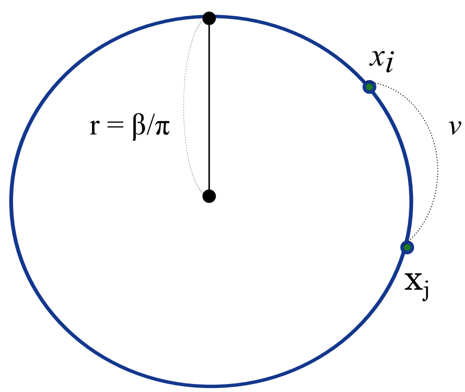

Figure 1: The Lancaster's Ideal Variety Preference

Then, consumers derive utility from the consumption of differentiated goods and homogeneous good Y. All consumers are assumed to share the same utility function.

$$U(\hat{x}_i(v), y)\ \forall i \tag{2}$$

where $\hat{x}^i \equiv x^i_j(v)e(v)$ denotes the effective consumption of the differentiated good j converted into the consumer's ideal products, and y denotes the consumption of homogeneous goods. As discussed later, the best strategy for consumers is to buy one variety of differentiated goods, whose effective price is the lowest. The common utility function is assumed to be strictly concave, increased by each argument, and homothetic. With this utility function, every consumer solves the two-stage maximization problem. In the first stage, each consumer

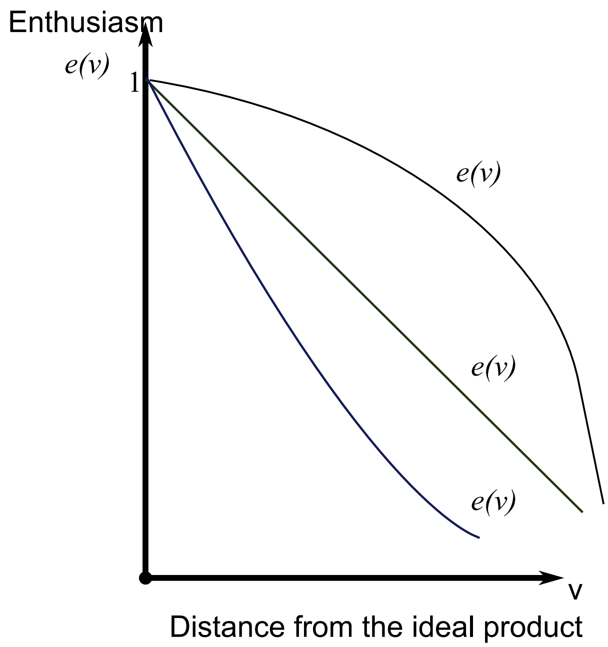

Figure 2: Enthusiasm of a Consumer

allocates his/her income between the differentiated good $\hat{x}(v)$ and homogeneous goods y to maximize his/her utility.

$$\max U(y, \hat{x})$$

$$\text{s.t. } y + \tfrac{p_x}{e(v)}\hat{x} = I$$

where $\frac{p_x}{e(v)}$ the effective price of the v-distant good j converted into consumer's ideal product price. With the homothetic utility function, the individual demand functions for the differentiated good $x(v)$ and homogeneous goods y become as follows.

$$\hat{x}(v) = x(v)e(v) = \alpha_x\left(\frac{p_x}{e(v)}, p_y\right) I \tag{3a}$$

$$y = \alpha_y\left(p_y, \frac{p_x}{e(v)}\right) I \tag{3b}$$

$\alpha_x\left(\frac{p_x}{e(v)}, p_y\right)$ and $\alpha_y\left(p_y, \frac{p_x}{e(v)}\right)$ denote the share of income spent on ideal differentiated goods and homogeneous goods respectively. These functions decrease with its own price and increase with the price of the other goods.[3] Given the income allocation, each consumer

[3] α_x and α_y satisfy the following properties:(1)They decrease with its own price $\alpha_{x1} = \frac{\partial \alpha_x\left[\frac{p_x}{e(v)}, p_y\right]}{\partial p_x} < 0$, $\alpha_{y1} = \frac{\partial \alpha_y\left(p_y, \frac{p_x}{e(v)}\right)}{\partial p_y} < 0$ (2) They increase with the other good's price $\alpha_{x2} = \frac{\partial \alpha_x\left[\frac{p_x}{e(v)}, p_y\right]}{\partial p_y} > 0$, $\alpha_{y2} =$

chooses the best differentiated good, whose effective price is the lowest $\frac{p_x}{e(v)} \equiv \frac{p_j}{e(v)}$ in the second stage.

As a result of the two stage maximization problem and (3a), individual demand functions for any variety of differentiated goods are derived as follows.

$$x_j^i\left(\frac{p_j}{e\left(v_j^i\right)}, p_y\right) = \begin{cases} \frac{\alpha_x\left(\frac{p_j}{e\left(v_j^i\right)}, p_y\right) I}{e(v_j^i)} & \text{if } \frac{p_j}{e\left(v_j^i\right)} = \min\{\frac{p_k}{e\left(v_k^i\right)}\} \quad \forall k \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where v_j^i is the distance between the consumer i's ideal product x^i and the good x_j.

The comparative statics shows that producers of differentiated goods face the downward-sloping individual demand curve: it decreases with the price p_j or the distance v_j^i. [4]

$$\frac{\partial x_j^i}{\partial p_j} = \underbrace{\alpha_{x1}\left(\frac{p_j}{e\left(v_j^i\right)}, p_y\right)}_{<0} \frac{I}{\left(e(v_j^i)\right)^2} \; < \; 0 \tag{5}$$

The individual demand also decreases with the distance v_j^i iff the differentiated good is a luxury good and its price elasticity of demand is greater than 1.

$$\frac{\partial x_j^i}{\partial v_j^i} = \underbrace{\left(\frac{-e'(v_j^i)}{e(v_j^i)}\right)}_{>0} \left(1 - \epsilon_j^i\right) x_j^i \; < 0 \qquad iff \;\, \epsilon_j^i > 1 \tag{6}$$

where $\epsilon_j^i \equiv \left(-\frac{\partial x_j^i}{\partial p_j}\right)\left(\frac{p_j}{x_j^i}\right) = \frac{-\alpha_{x1}\left(\frac{p_j}{e(v)}, p_y\right)}{\alpha_x\left(\frac{p_j}{e(v)}, p_y\right)}\left(\frac{p_j}{e(v)}\right)$ is the price elasticity of an individual demand.

The price elasticity of an individual demand varies with the distance from his/her good v_j^i. The further consumers locate, the more price elastic they become iff their individual demand curves are not so convex.

($\frac{\partial \epsilon_j^i}{\partial v_j^i} > 0$ iff the convexity of the individual demand $\gamma_j^i < 1 + \epsilon_j^i$)

$\frac{\partial \alpha_y\left(p_y, \frac{p_x}{e(v)}\right)}{\partial p_x} > 0$, and (3) The sum of these income shares multiplied by their own prices must be equal to 1. $\alpha_x\left[\frac{p_x}{e(v)}, p_y\right]\frac{p_x}{e(v)} + \alpha_y\left(p_y, \frac{p_x}{e(v)}\right) p_y \equiv 1$ It should be noted that the original Helpman model assumes homogeneous degree -1 for α_x and α_y. However, results of this paper holds without this assumption, thus this paper does not set this assumption.

[4] $e'(v_j^i) < 0$ (the further the distance from his/her ideal good, the less enthusiastic the consumer becomes.)

$$\frac{\partial \epsilon_j^i}{\partial v_j^i} = \frac{-\alpha_{x1}\left(\frac{p_j}{e\left(v_i^j\right)}, p_y\right)\left\{\frac{-p_j e'(v_i^j)}{\left(e\left(v_i^j\right)\right)^2}\right\}}{\alpha_x\left(\frac{p_j}{e\left(v_i^j\right)}, p_y\right)}\left[1+\epsilon_j^i-\gamma_j^i\right] \tag{7}$$

where $\gamma_j^i \equiv \frac{-\alpha_{x11}\left(\frac{p_j}{e\left(v_i^j\right)}, p_y\right)\left(\frac{p_j}{e(v)}\right)}{\alpha_{x1}\left(\frac{p_j}{e\left(v_i^j\right)}, p_y\right)}$ measures the convexity (curvature) of the individual demand curve.[5]

Eq (7) shows that the price elasticity of the individual demand ϵ_j^i increases with the distance v_j^i if the individual demand curve is not so convex and/or the price elasticity of the demand ϵ is large enough. In other words, the less(more) enthusiastic a consumer becomes, the more(less) price elastic he/she becomes if each consumer is enough price elastic, and his/her individual demand curve is not so convex. Thus, unless consumers are less price elastic or they decrease their consumption at a drastically decreasing rate with price, firms face more(less) price elastic consumers as the range of consumers they sell their products becomes wider(narrower).

Producers: 1 Factor $\times$ 2 Industries Model

Given differences in consumers' tastes and enthusiasm for differentiated goods, two industries produce two types of goods in a closed economy. Both horizontally-differentiated goods industy (X) and the composite of homogeneous goods industry (Y) use labor as an only factor of production. Homogeneous goods (Y) are produced with the constant returns to scale technology, where one unit of labor produces one unit of a homogeneous good. Thus, the wage is fixed at 1 ($w = 1$), and the equilibrium price of a homogeneous good is 1 (numeraire good). The labor demand of homogeneous goods sector can be written as follows.

$$L_Y^d = Y \tag{8}$$

In the other industry, horizontally-differentiated goods industry (X), all producers have the same increasing returns to scale technology. Their common cost function is expressed as follows.

$$C_j(w, Q_j) = c(w)Q_j + f \quad \forall j \in X \tag{9}$$

where $c(w)$ is the marginal cost, f is the fixed cost, and Q_j is the aggregate demand for

[5] γ_j^i is also known as the elasticity of the slope of the demand curve.

the firm j.

Using Shepherd's lemma, the labor demand of each firm in differentiated goods sector is derived as follows.

$$L_{xj}^d = c'(w)Q_j \quad \forall j \in X$$

The aggregate labor demand from sector X can be written as

$$L_x^d = \sum_{j \,\in\, X} c'(w)Q_j \tag{10}$$

Every firm in the differentiated goods industry X faces the same two-stage game. In the first stage, a firm decides one variety of differentiated good it will produce, which determines the location of its product on the circumference of the production possibility circle (v_{j-1}). [6] After deciding the differentiated goods it will produce, in the second stage, every firm joins the Bertrand competition:it chooses its prices simultaneously given prices of other firms.

Solving backwardly, in the second stage, each producer of differentiated goods chooses its price to maximize its profit.

$$\pi_j = (p_j - c(w))\, Q_j(p_j, v_{j-1}|p_{j-1}, p_{j+1}, p_y, D_j, nI) - f$$

The aggregate demand for each differentiated goods Q_j is determined by the individual demand (4) and the range of consumers it sells its product to. Consumers in the distance between $\underline{d}$ and $\bar{d}$ are covered by the firm j. $\underline{d}$-distant and $\bar{d}$-distant consumers are marginal consumers, who are offered the same effective prices by two firms, and therefore are indifferent between the good x_j and goods of the closest rival firms x_{j-1} (x_{j+1}). These marginal consumers locate at the remotest point from the good x_j,are the least enthusiastic for the good x_j and purchase the least amount of it. The location of marginal consumers, $\underline{d}, \bar{d}$ are determined by the indifferent effective prices they face.

$$\frac{p_j}{e(\underline{d})} = \frac{p_{j-1}}{e(v_{j-1} - \underline{d})} \tag{11}$$

$$\frac{p_j}{e(\bar{d})} = \frac{p_{j+1}}{e(v_{j+1} - \bar{d})} \tag{12}$$

where v_{j-1} and v_{j+1} are the distance between the good x_j and its closest rival goods x_{j-1} and x_j respectively.

[6] With the increasing returns to scale technology, it is the best choice for firms to produce only one variety of differentiated goods.

The above equations imply that the market share of the good x_j $\left[\underline{\mathrm{d}}, \bar{d}\right]$ is determined by its own price p_j, prices of rival goods p_{j-1}, p_{j+1}, and the distance from rival goods v_{j-1}, v_{j+1}.

$$\underline{d} = \underline{\delta}\left(p_j, v_{j-1} | p_{j-1}, p_{j+1}, D_j\right)$$

$$\bar{d} = \bar{\delta}\left(p_j, v_{j+1} | p_{j+1}, p_{j-1}, D_j\right)$$

where $D_j = v_{j-1} + v_{j+1}$ is the exogenously given distance between two closest rival goods.

As shown below, the market share, defined by $\underline{d}, \bar{d}$, is determined by the location of marginal consumers shrinks as the price of the good j increases.

$$\frac{\partial \underline{\delta}}{\partial p_j} = \frac{e(v_{j-1} - \underline{d})}{p_{j-1} e'(\underline{d}) + p_j e'(v_{j-1} - \underline{d})} < 0 \tag{13a}$$

$$\frac{\partial \bar{\delta}}{\partial p_j} = \frac{e(v_{j+1} - \bar{d})}{p_j e'(v_{j+1} - \bar{d}) + p_{j+1} e'(\bar{d})} < 0 \tag{13b}$$

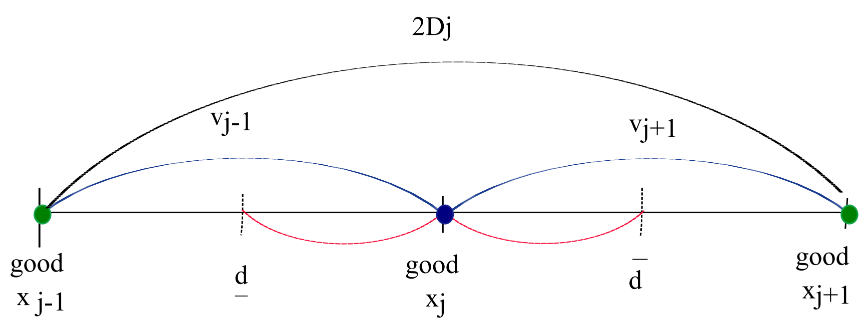

Figure 3: Market Share of the Firm j

Given the location of marginal consumers, the aggregate demand for the good j is given by

$$Q_j = n \left\{ \int_0^{\underline{\delta}(p_j, v_{j-1} | p_{j-1}, p_{j+1}, D_j)} \frac{\alpha_x \left(\frac{p_j}{e(v)}, p_y\right) I}{e(v)} \, dv \; + \; \int_0^{\bar{\delta}(p_j, v_{j-1} | p_{j+1}, p_{j-1}, D_j)} \frac{\alpha_x \left(\frac{p_j}{e(v)}, p_y\right) I}{e(v)} \, dv \right\} \tag{14}$$

$n = \frac{L}{2\beta}$ is the density of consumers at each point of the circumference of the circle.

Using (14), the profit function for each producers of differentiated goods can be rewritten as follows.

$$\pi_j = (p_j - c(w))\, n \left\{ \int_0^{\underline{\delta}(p_j, v_{j-1} | p_{j-1}, p_{j+1}, D_j)} \frac{\alpha_x\left(\frac{p_j}{e(v)}, p_y\right) I}{e(v)}\, dv \right.$$
$$\left. + \int_0^{\bar{\delta}(p_j, v_{j-1} | p_{j+1}, p_{j-1}, D_j)} \frac{\alpha_x\left(\frac{p_j}{e(v)}, p_y\right) I}{e(v)}\, dv \right\} - f \tag{15}$$

First order conditions with respect to the price p_j and the location v_{j-1} solve the second-stage and the first-stage maximization problem respectively.

$$\frac{\partial \pi_j}{\partial p_j} = 0 \quad \Rightarrow \quad p_j = \left(\frac{\epsilon_j}{\epsilon_j - 1}\right) c(w) \tag{16}$$

where ϵ_j denotes the price elasticity of the aggregate demand for the good j. Since the price is nonnegative, (16) requires that the aggregate demand is price elastic enough to satisfy $\epsilon > 1$. If this condition is satisfied, the price under the Bertrand competition is greater than the marginal cost $c(w)$ by $\left(\frac{\epsilon}{\epsilon-1}\right)$

The price elasticity of the aggregate demand is defined as follows.

$$\begin{aligned}
\epsilon_j &= \frac{-\partial Q_j}{\partial p_j} \cdot \frac{p_j}{Q_j} \\
&= -n \int_0^{\underline{\delta}} \frac{\alpha_x\left(\frac{p_j}{e(v)}, p_y\right) I}{\{e(v)\}^2} \cdot \left(\frac{p_j}{Q_j}\right) dv - n \int_0^{\bar{\delta}} \frac{\alpha_x\left(\frac{p_j}{e(v)}, p_y\right) I}{\{e(v)\}^2} \cdot \left(\frac{p_j}{Q_j}\right) dv \\
&\quad - n \left(\frac{\alpha_x\left(\frac{p_j}{e(\underline{\delta})}, p_y\right) I}{e(\underline{\delta})}\right) \left(\frac{p_j}{Q_j}\right) \left(\frac{\partial \underline{\delta}}{\partial p_j}\right) - n \left(\frac{\alpha_x\left(\frac{p_j}{e(\bar{\delta})}, p_y\right) I}{e(\bar{\delta})}\right) \left(\frac{p_j}{Q_j}\right) \left(\frac{\partial \bar{\delta}}{\partial p_j}\right)
\end{aligned}$$

Using the definition of ϵ_j^i(the price elasticity of an individual demand), eq(13a) and eq(13b), the price elasticity of the aggregate demand can simply be rewritten as follows.

$$\begin{aligned}
\epsilon_j &= n \int_0^{\underline{\delta}} \left(\frac{x_j^i(v)}{Q_j}\right) \cdot \epsilon_j^i(v) dv + n \int_0^{\bar{\delta}} \left(\frac{x_j^i(v)}{Q_j}\right) \cdot \epsilon_j^i(v) dv \\
&\quad - n \left(\frac{x_j^i(\underline{\delta})}{Q_j}\right) \left(\frac{p_j e(v_{j-1} - \underline{\delta})}{p_{j-1} e'(\underline{\delta}) + p_j e'(v_{j-1} - \underline{\delta})}\right) - n \left(\frac{x_j^i(\bar{\delta})}{Q_j}\right) \left(\frac{p_j e(v_{j+1} - \bar{\delta})}{p_j e'(v_{j+1} - \bar{\delta}) + p_{j+1} e'(\bar{\delta})}\right)
\end{aligned} \tag{17}$$

Eq(17) shows that the price elasticity of the aggregate demand ϵ_j is determined by the weighted average of the individual price elasticity ϵ_j^i and the enthusiasm of marginal consumers. If the enthusiasm of marginal consumers is not so (distance) elastic that the slope of the enthusiasm function $e(v)$ evaluated at the location of marginal consumers approaches to zero,$\min\{e'(\underline{\delta}), e'(v_{j-1} - \underline{\delta}), e'(\bar{\delta}), e'(v_{j+1} - \bar{\delta})\} \to 0$, the price elasticity of the aggregate demand for the good j, ϵ_j becomes infinity, and the price of the good p_j is driven down to its marginal cost. On the contrary, if the enthusiasm level of marginal consumers is enough (distance) elastic, the first two terms in eq (17), the weighted sum of the price elasticity of individual demand, determine the price elasticity of the aggregate demand, and marginal consumers have little impact on the aggregate price elasticity.

Given the first order condition in the second stage, the first order condition with respect to the location of the product in the first stage is derived as follows.

$$\frac{\partial \pi_j}{\partial v_{j-1}} = 0 \quad \Rightarrow \quad (p_j - c(w))\frac{\partial Q_j}{\partial v_{j-1}} = 0 \tag{18}$$

As (16) implies, the optimal price is set above the marginal cost, (18) shows that the firm j determines the location of its product where a small change in its product has no effect on the aggregate demand. $\left(\frac{\partial Q_j}{\partial v_{j-1}} = 0\right)$

Closed Economy Symmetric Equilibrium

Closed economy equilibrium is characterized as follows. First, labor market is clear, and labor demand from two sectors (8) and (10) equals labor supply L.

$$L = Y + \sum_{j \,\in\, X} c'(w) Q_j \tag{19}$$

Second,in a closed economy equilibrium, the homogeneous good market Y and the differentiated good market X are clear respectively.

$$Y = \sum_{i \in L} \alpha_y \left(p_y, \frac{p_x}{e(v^i)}\right) I \tag{20}$$

$$Q_j = \sum_{i \in L} \alpha_x \left(\frac{p_j}{e(v_j^i)}, p_y\right) I \quad \forall j \in X \tag{21}$$

With the constant returns to scale technology, the equilibrium price of homogeneous good becomes its marginal cost 1.

$$p_y = 1$$

In the horizontally differentiated goods industry, all firms have the same increasing returns to scale technology and face the same two-stage maximization problem. As a result, they choose the same optimal price $p_j = p_x \; \forall j$. As a result, all firms gain the same market share and covers the equal range of consumers. $[0, \underline{\delta}]$ and $[0, \bar{\delta}]$. Marginal consumers of each firm locate the half distance from its rival firms. (i.e. $\underline{\delta} = \bar{\delta} = \frac{D_j}{2} = (\frac{2\beta}{N})\frac{1}{2} = \frac{\beta}{N}$ where 2β is the circumference of the product possibility circle and N is the number of firms in the differentiated goods industry.)

With $p_j = p_{j-1} = p_{j+1} = p_x$, $\delta\left(p_j, v_{j-1} | p_{j-1}, p_{j+1}, D_j\right) \equiv \underline{\delta} = \bar{\delta} = \frac{\beta}{N}$, the aggregate demand Q_j for every good (14) is rewritten as follows.

$$Q_j = 2n \int_0^{\frac{\beta}{N}} \frac{\alpha_x\left(\frac{p_j}{e(v)}, p_y\right) I}{e(v)} \, dv \quad \forall j \tag{14*}$$

Combining the first order condition (16) with (14), each firm earns the following profit.

$$\pi_j = \left(\frac{c(w)}{\epsilon_{j-1}}\right) Q_j - f \quad \forall j \tag{15*}$$

The first order condition (16) determines the symmetric price of every differentiated good.

$$p_j = \left(\frac{\epsilon_j}{\epsilon_j - 1}\right) c(w) \quad \forall j \tag{16*}$$

The price elasticity of the aggregate demand at a symmetric equilibrium ϵ_j becomes

$$\epsilon_j = 2n \int_0^{\frac{\beta}{N}} \left(\frac{x_j^i(v)}{Q_j}\right) \cdot \epsilon_j^i(v) dv - n \left(\frac{x_j^i(\delta)}{Q_j}\right) \left(\frac{e(\delta)}{e'(\delta)}\right) \quad \forall j \tag{17*}$$

Finally, if free entry and exit are allowed, the zero profit condition determines the number of differentiated goods produced in a closed economy.

$$\pi_j = \left(\frac{c(w)}{\epsilon_{j-1}}\right) Q_j - f = 0 \tag{22}$$

2.2 Trade Liberalization

Suppose that two countries (home and foreign) are in the closed economy equilibrium described in the last section 2.1. These two countries are assume to be symmetric except the variety of differentiated goods produced in each country. Industries in two countries have the same structure: the horizontally differentiated goods sector X and the homogeneous goods sector Y use labor and produce its goods respectively. Moreover, producers in two countries share the same production technologies. Producers of differentiated goods X have the common increasing returns to scale technology, while producers of homogeneous goods Y produce goods with the common constant returns to scale technology. As a result, every consumer in two countries earn the same labor income(wage). In addition, every consumer in two countries is assumed to have the same utility function, the same enthusiasm function, and face the same two-stage budgeting problem described in the former section 2. Every consumer has an ideal differentiated good, and divides his/her income between homogeneous goods and the best differentiated goods with the lowest effective price $p_j/e(v_j^i) = \min p_k/e(v_k^i) \forall\ k\ \in\ N$ Their preference in horizontally differentiated goods is equally distributed on the circumference of the production possibility circle. Since both country has the same population size$L = L^*$, the density of consumers at each point on the circumference of the product possibility circle becomes $n = n^* = L/2\beta = L^*/2\beta$. In these circumstances, one day, governments of two countries suddenly agree to liberalize trade in horizontally differentiated goods.Once trade begins,foreign (home) goods enter home (foreign) market with or without tariff. Despite of trade liberalization, **horizontally differentiated goods markets in two countries assumed to be separated**. (due to language differences, technological differences, etc.) Without arbitrage opportunity, the same product can be sold at different prices in home and foreign market. As shown in Figure 4, trade liberalization brings domestic (foreign) consumers new variety of goods N^* (N) from the foreign country and brings domestic(foreign) producers new foreign consumers L^* (L). Facing new foreign rival firms and foreign consumers, existing firms redo the two-stage game in home and foreign market. At the first stage, domestic and foreign firms adjust the location of their goods on the circumference of production possibility circle in home and foreign country.

It is assumed that the location of the product under autarky continues to play an important role after trade liberalization. The location of a product under autarky shows the character of the product, which had been developed through history, and is a reason why its products are recognized as a differentiated brand by consumers. Thus, this paper assumes that each firm must incur a huge fixed cost to drastically change the character of the product by moving a long distance from its autarky location. However, each firm changes the character and the location of the product without any cost as long as it the change is a slight change. Specifically, this paper assumes the relocation cost f_j^* as follows.

$$f_j^* = \begin{cases} 0 & \text{if } |v_{j-1}^* - v_{j-1}| \leq 2D_j \quad \forall j \\ f & |v_{j-1}^* - v_{j-1}| > 2D_j \end{cases} \tag{23}$$

where $|v_{j-1}^* - v_{j-1}|$ denotes the distance between the new distance from the closest rival firm on its left side. v_{j-1}^* and the old distance from it under autarky v_{j-1}. The relocation cost is equal to an entry cost f if the firm j change its product so drastically that its product is treated as a new product and compete with different rival firms in a different segment in the product space.

Given assumptions about the segmented market, the preservation of the autarkic location of products, the symmetry of the population size, the trade liberalization brings foreign rival goods into home market as shown in Figure 4. After the trade liberalization, two closest rival firms for every domestic firm j change from two domestic firms $j-1$ and $j+1$ to two foreign firms j^*-1 and j^*+1.

Facing these changes, all producers redo the Bertrand competition and re-choose their prices in home and foreign market simultaneously in the second stage.

In the following sections, two cases are analyzed. First, a case of perfect trade liberalization, free trade is discussed, and then, a case of partial trade liberalization with tariff is discussed. It should be noted that the following sections analyze short-run effects after the trade liberazilation. **Short-run means the period during which no new firm enters the market**, and thus only existing domestic and foreign firms participate in the competition.

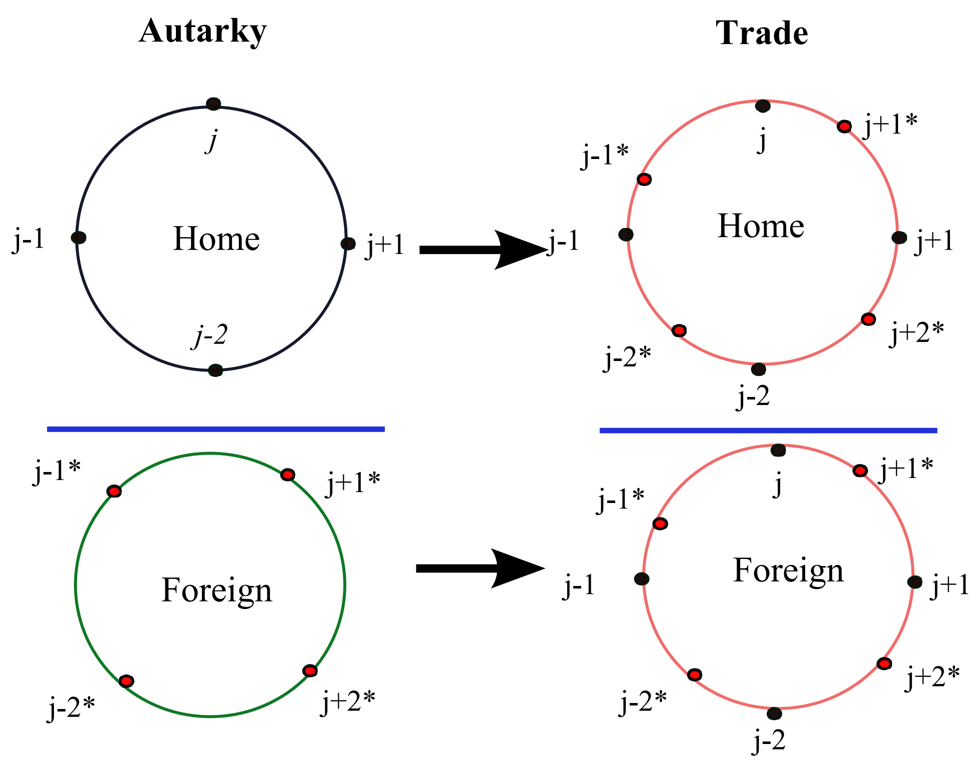

Figure 4: Trade Liberalization

2.2.1 Perfect Trade Liberalization: Free Trade

If trade is perfectly liberalized, and free trade is realized, foreign goods and domestic goods are treated equally in home and foreign market. As a result, free trade just doubles the size of the market, and the post-trade variables(demand, price, profit and the price elasticity of demand) are expressed in the same way as pre-trade variables. (14)*,(15)*, (16)*, (17)*. Only changes from the autarky equilibrium are the shrinkage of each firm's market share ($\delta = \frac{\beta}{N} \rightarrow \delta = \frac{\beta}{N+N^*}$) and the change in two closest rival firms (From two domestic rival firms $j-1$ and $j+1 \rightarrow$ To two foreign rival firms j^*-1 and j^*+1).

Since consumers have different tastes, enthusiastic levels and price elasticity, the shrinkage in the range of consumers covered by each firm changes the composition of consumers and the price elasticity of demand each firm faces. As shown in the following comparative statics Eq (24), the price elasticity of aggregate demand can either, increase, decrease, or stay the same depending on the shape of the enthusiastic function $e(v)$ and the difference between marginal consumers' price elasticity and average consumers' one.

$$\begin{aligned} \frac{\partial \epsilon_j}{\partial \delta} &= \frac{2nQ_j\epsilon_j^i(\delta)x_j^i(\delta) - nQ_j\left(\frac{e(\delta)}{e'(\delta)}\right)\left(\frac{\partial x_j^i(\delta)}{\partial \delta}\right) - nQ_jx_j^i(\delta)\left(\frac{(e'(\delta))^2 - e(\delta)e"(\delta)}{(e'(\delta))^2}\right)}{(Q_j)^2} \\ &= \frac{nx_j^i(\delta)}{Q_j}\left\{\left(\epsilon_j^i(\delta) - 2\epsilon_j\right) + \left(\frac{e(\delta)}{-e'(\delta)\cdot\delta}\right)\left(\frac{e"(\delta)\cdot\delta}{-e'(\delta)}\right)\right\} \\ &= \frac{nx_j^i(\delta)}{Q_j}\left\{\left(\epsilon_j^i(\delta) - 2\epsilon_j\right) + \frac{\mu(\delta)}{\eta(\delta)}\right\} \end{aligned} \tag{24}$$

where $\eta \equiv \left(\frac{-e'(\delta)\cdot\delta}{e(\delta)}\right)$ denote the (distance) elasticity of the enghusiasm function $e(v)$ and $\mu \equiv \left(\frac{e"(\delta)\cdot\delta}{-e'(\delta)}\right)$ measuresthe curvature(convexity) of $e(v)$.[7]

As (24) suggests, the shrinkage in the range of consumers lowers the price elasticity of aggregate demand ($\frac{\partial \epsilon_j}{\partial \delta} > 0$), if (1) the remotest marginal consumers is much more price elastic than weighted-average consumers are and (2) the enthusiasm function $e(v)$ evaluated at the location of marginal consumers shows a strong convexity and/or a low elasticity if $e(v)$ is a convex curve (large $\mu(\delta) > 0$ & small $\eta(\delta)$).[8] If these two conditions are met, **trade liberalization followed by a shrinkage in the market width makes firms concentrate their sales on more enthusiastic closer consumers, lowers the aggregate**

[7]Eq(24) is derived by using (7) $\frac{\partial x_j^i(\delta)}{\partial \delta} = \frac{\partial x_j^i(v)}{\partial v}|_{v=\delta} = x_j^i(\delta)\left(\frac{-e'(\delta)}{e(\delta)}\right)\left(1-\epsilon_j^i\right)$ and $\frac{\partial Q_j}{\partial \delta} = 2nx_j^i(\delta)$

[8]If $e(v)$ is a concave curve, $e(v)$ evaluated at the location of marginal consumers shows a weak concavity and/or a high price elasticity ($\mu(\delta) < 0$, small $|\mu(\delta)|$ & large $\eta(\delta)$).

price elasticity, and gives firms a chance to raise their prices.

In addition to price, profit will also change if prices and the aggregate demand are changed by an increase in the number of consumers ($n \uparrow$) [the market expansion effect] and the number of differentiated products ($N \uparrow$) [the foreign competition effect]. Post-trade profit will change as follows.

$$d\pi_j = \underbrace{\frac{c}{\epsilon_j - 1}\left(\frac{dQ_j}{d\delta}\right)\left(\frac{d\delta}{dN}\right)dN}_{(-)} + \underbrace{\frac{-cQ_j}{(\epsilon_j - 1)^2}\left(\frac{d\epsilon_j}{d\delta}\right)\left(\frac{d\delta}{dN}\right)dN}_{(+/-)} + \underbrace{\frac{cQ_j}{n(\epsilon_j - 1)}dn}_{(+)}$$

Using $\frac{d\delta}{dN} = \frac{-\beta}{N^2} = \frac{-\delta}{N}$, $\frac{dQ_j}{d\delta} = 2nx_j^i(\delta)$,$\frac{dQ_j}{dn} = \frac{Q_j}{n}$, the above equation is simplified as follows.

$$d\pi_j = \left(\frac{c}{\epsilon_j - 1}\right)\left\{-\left(\frac{2n\delta x_j^i(\delta)}{N}\right)dN + \left(\frac{\delta Q_j}{N(\epsilon_j - 1)}\right)\left(\frac{d\epsilon_j}{d\delta}\right)dN + \left(\frac{Q_j}{n}\right)dn\right\} \tag{25}$$

The first term and the second term in the curly bracket show the effect of losing remote consumers on the aggregate demand and the price respectively. The first term is negative since losing remote consumers decreases the aggregate demand Q_j. However,as shown in eq (24), the second term in the curly bracket, the effect of stop selling products to remote consumers on price, can be either positive, negative, or have no effect on the price depending on the price elasticity of demand and the shape of the enthusiastic function. Losing remote consumers increases(decreases) price and profit if (1) the remotest marginal consumers is much price elastic than weighted-average consumers are and (2) the enthusiasm function $e(v)$ evaluated at the location of marginal consumers shows a combination of strong convexity and low elasticity (large $\mu(\delta) > 0$ & small $\eta(\delta)$ if $e(v)$ is a convex curve) [9]. The last term in the curly bracket, the effect of the increase in foreign consumers has always a positive effect on aggregate demand and profit. To summarize, trade liberalization, followed by the entry of foreign rival firms into the domestic market,causes two effects into the market. First, the entry of foreign firms intensifies competition, which puts downward pressure on prices. **However, the entry of foreign firms with the shrinkage in the market coverage gives producers a chance to concentrate their sales on closer, more enthusiastic, and thus less price elastic consumers, which can be a trigger for the price increase.** The latter effect, a change in the composition of consumers effect

[9] If $e(v)$ is a concave, a combination of weak concavity and high elasticity ($\mu(\delta) < 0$, small $|\mu(\delta)|$ & large $\eta(\delta)$ is needed)

dominates the former effect, an increase in competition, and raises both prices and profits if the remotest marginal consumers are much more price-elastic than average consumers, and the enthusiasm function evaluated at the location of marginal consumers shows a combination of strong convexity and low elasticity or a combination of weak concavity and high elasticity.

2.2.2 Imperfect Trade Liberalization with Tariff

This section analyzes a case where two countries liberalize trade on horizontally differentiated goods, but each of them has a discretion to impose *ad valorem* tariff on imports. As discussed in the last section, if free trade is realized, goods produced in home and foreign are treated equally in both markets. However, once tariff is introduced, domestic goods and imports will not be treated equally any longer. Since the population size, and thus the number of differentiated goods produced under autarky are the same, each domestic firm faces a situation where its two closest rival firms are foreign firms, the home government imposes an*ad valorem* tariff only on its two foreign rival firms.

Once *ad valorem* tariff is imposed on imports, foreign firms are forced to change their producer prices again. The profit-maximizing producer price under conditional trade liberalization can be expressed as the same way as that under autarky (16). Differentiating the first order condition (16) implicitly, the following equation shows how tariff drives foreign producers to change their prices in home market. [the following eqution (26) is formally derived in Appendix]

$$\frac{d\ p_j^*}{d\ t_j} = -\left(\frac{p_j^*}{1+t_j}\right) + \left(\frac{c}{1+t_j}\right)\left\{\frac{a+b+g+h}{(2a+2b+d+2e)+(2g+2h+i+2j)+f+k}\right\} \tag{26}$$

where

$$a \equiv \int_0^{\underline{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j} dv \qquad b \equiv x_j^i\left(\frac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)\left(\frac{\partial \underline{\delta}}{\partial p_j^*}\right)$$

$$d \equiv (p_j^* - c)\int_0^{\underline{\delta}} \frac{\partial^2 x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j^{*2}} dv \quad e \equiv (p_j^* - c)\left(\frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)}{\partial p_j^*}\right)\left(\frac{\partial \underline{\delta}}{\partial p_j^*}\right)$$

$$g \equiv \int_0^{\bar{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j} dv \qquad h \equiv x_j^i\left(\frac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)\left(\frac{\partial \bar{\delta}}{\partial p_j^*}\right)$$

$$i \equiv (p_j^* - c)\int_0^{\bar{\delta}} \frac{\partial^2 x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j^{*2}} dv \quad j \equiv \left(\frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)}{\partial p_j^*}\right)\left(\frac{\partial \bar{\delta}}{\partial p_j^*}\right)$$

$$f \equiv (p_j^* - c)x_j^i\left(\frac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)\left(\frac{\partial^2 \underline{\delta}}{\partial p_j^{*2}}\right) \quad k \equiv x_j^i\left(\frac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)\left(\frac{\partial^2 \bar{\delta}}{\partial p_j^{*2}}\right)$$

The sign of the Numerator of (26) is always negative.

$$a+b+g+h \quad \equiv \underbrace{\int_0^{\underline{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j} dv}_{<0} + \underbrace{\int_0^{\bar{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j} dv}_{<0} + \underbrace{x_j^i\left(\frac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)\left(\frac{\partial \underline{\delta}}{\partial p_j^*}\right)}_{<0} + \underbrace{x_j^i\left(\frac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)\left(\frac{\partial \bar{\delta}}{\partial p_j^*}\right)}_{<0}$$

Denominator of (26) is decomposed as follows.

$$2a+2b+d+2e \equiv \underbrace{\int_0^{\underline{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j}}_{<0} \left\{2 - \gamma^i(v)\left(\frac{p_j^* - c}{p_j^*}\right)\right\} dv + 2x_j^i(\underline{\delta})\underbrace{\left(\frac{\partial \underline{\delta}}{\partial p_j^*}\right)}_{<0}\left\{1 - \left(\frac{p_j^* - c}{p_j^*}\right)\epsilon^i(\underline{\delta})\right\}$$

$$2g+2h+i+2j \equiv \underbrace{\int_0^{\bar{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j}}_{<0} \left\{2 - \gamma^i(v)\left(\frac{p_j^* - c}{p_j^*}\right)\right\} dv + 2x_j^i(\bar{\delta})\underbrace{\left(\frac{\partial \bar{\delta}}{\partial p_j^*}\right)}_{<0}\left\{1 - \left(\frac{p_j^* - c}{p_j^*}\right)\epsilon^i(\bar{\delta})\right\}$$

where

$\gamma^i(v) \equiv \left(\frac{p_j^* \cdot {x_j}''(p_j^*)}{-{x_j}'(p_j^*)}\right)$ denotes the convexity of the demand curve, and

$\epsilon^i \equiv \left(\frac{-p_j^* \cdot {x_j}'(p_j^*)}{x_j(p_j^*)}\right)$ denotes the price elasticity of demand.

$$f+k \equiv (p_j^*-c)\{x_j^i(\underline{\delta})\underbrace{\left(\frac{\partial^2\underline{\delta}}{\partial p_j^{*2}}\right)}_{>0}+x_j^i(\bar{\delta})\underbrace{\left(\frac{\partial^2\bar{\delta}}{\partial p_j^{*2}}\right)}_{>0}\}$$

The sign of the last term in the denominator $(f+k)$ is always positive, while the sign of the first two terms ($2a+2b+d+2e$ and $2g+2h+i+2j$)can be either positive or negative depending on the convexity of demand curve, the amount of markup, and the price elasticity of marginal consumers. The sign of the denominator becomes positive if (i) the demand curve is convex enough($\gamma\uparrow$)[10] and/or (ii) the markup $(p-c/p)$ is large enough, and/or (iii) marginal consumers are price-elastic enough($\epsilon(\underline{\delta})$,$\epsilon(\bar{\delta})$). Since the sign of the numerator is unambiguously negative, if these three conditions are met, the sign of the denomenator becomes positive, and therefore the sign of the whole equation eqrefeq:pricechange becomes negative. It means that with the convex demand curve, a high mark-up, and/or high price-elastic marginal consumers,the imposition of tariff $(dt_j>0)$ induces foreign producers to cut their producer price in the home market. Expressing (26) differently, the following expression shows how much a producer cuts its producer price in a home market($\frac{d\ p_j^*}{d\ t_j}<0$).

$$\begin{aligned}\left(\frac{t_j}{p_j^*}\right)\left(-\frac{d\ p_j^*}{d\ t_j}\right) \quad = \quad & \underbrace{\left(\frac{t_j}{1+t_j}\right)}_{<1} \\ & +\underbrace{\left(\frac{t_j}{1+t_j}\right)}_{<1}\underbrace{\left(\frac{c}{p_j^*}\right)}_{<1}\underbrace{\left\{\frac{-(a+f+g+b)}{(a+f+g+b)+(a+d+f+i+b+2e+2j+m+p)}\right\}}_{>0,<1} \quad (27)\end{aligned}$$

Suppose that the above-mentioned three conditions (i)-(iii) are met, and the sign of (26) is negative. (27) shows how much each foreign producer cuts its producer price if the tariff is increased by 1%. (27) suggists that an increase in tariff by 1% induces a producer to cut its producer price less than 1% iff the aggregate demand curve is so convex, and/or marginal consumers are so price elastic that the denominator of the last term in (27) becomes large enough and the last term becomes less than 1. If (27) is less than 1, it means that in spite of a cut in the producer price, the consumer price of imports $\hat{p}_j^*\equiv(1+t_j)p_j^*$ increases after the imposition of tariff on imports.

Facing an increase in consumer prices of imports, domestic producers re-adjust their producer(=consumer) prices in a home market. Differentiating the first order condition (16) implicitly, the following partial derivatives show the strategic relationship between home and foreign producers.

[10] However,γ is low enough to satisfy (7), thus $\frac{2}{\left(\frac{p_j-c}{p_j}\right)}<\gamma_j<1+\epsilon_j$

$$\frac{dp_j}{d\hat{p^*_{j-1}}} = \left(\frac{e\left(\underline{\delta}\right)}{(1+t_j)e\left(v_{j-1}-\underline{\delta}\right)}\right)\left\{\frac{B+E+F+(p_j-c)\left(\frac{e'(\underline{\delta})}{e(\underline{\delta})}+\frac{e'(v_{j-1}-\underline{\delta})}{e(v_{j-1}-\underline{\delta})}\right)\left(\frac{\partial\underline{\delta}}{\partial p_j}\right)B}{2A+2B+D+2E+2G+2H+I+2J+F+K}\right\} \tag{28}$$

where

$$A \equiv \int_0^{\underline{\delta}} \frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(v)},\cdot\right)}{\partial p_j}dv \qquad B \equiv x_j^i\left(\frac{p_j(1+t_j)}{e(\underline{\delta})}\right)\left(\frac{\partial\underline{\delta}}{\partial p_j}\right)$$

$$D \equiv (p_j-c)\int_0^{\underline{\delta}} \frac{\partial^2 x_j^i\left(\frac{p_j(1+t_j)}{e(v)},\cdot\right)}{\partial p_j^2}dv \quad E \equiv (p_j-c)\left(\frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(\underline{\delta})},\cdot\right)}{\partial p_j}\right)\left(\frac{\partial\underline{\delta}}{\partial p_j}\right)$$

$$F \equiv (p_j-c)x_j^i\left(\frac{p_j(1+t_j)}{e(\underline{\delta})}\right)\left(\frac{\partial^2\underline{\delta}}{\partial p_j^2}\right) \qquad G \equiv \int_0^{\bar{\delta}} \frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(v)}\right)}{\partial p_j}dv$$

$$H \equiv x_j^i\left(\frac{p_j(1+t_j)}{e(\bar{\delta})}\right)\left(\frac{\partial\bar{\delta}}{\partial p_j}\right) \qquad I \equiv (p_j-c)\int_0^{\bar{\delta}} \frac{\partial^2 x_j^i\left(\frac{p_j(1+t_j)}{e(v)}\right)}{\partial p_j^2}dv$$

$$J \equiv (p_j-c)\left(\frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(\bar{\delta})}\right)}{\partial p_j}\right)\left(\frac{\partial\bar{\delta}}{\partial p_j}\right) \quad K \equiv (p_j-c)x_j^i\left(\frac{p_j(1+t_j)}{e(\bar{\delta})}\right)\left(\frac{\partial^2\bar{\delta}}{\partial p_j^2}\right)$$

This equation(28) shows the strategic relationship between the domestic firm j and the foreign rival firm $j-1$.[11] [12]

The numerator of the above equation (28) can be expressed as follows.

$$B+E+F+(p_j-c)\left(\frac{e'(\underline{\delta})}{e(\underline{\delta})}+\frac{e'(v_{j-1}-\underline{\delta})}{e(v_{j-1}-\underline{\delta})}\right)\left(\frac{\partial\underline{\delta}}{\partial p_j}\right)B$$

$$= x_j^i\left(\frac{p_j(1+t_j)}{e(\underline{\delta})}\right)\underbrace{\left(\frac{\partial\underline{\delta}}{\partial p_j}\right)}_{<0}\underbrace{\left\{1-(\frac{p_j-c}{p_j})\ \epsilon_j^i(\underline{\delta})\right\}}_{<0}+(p_j-c)x_j^i\left(\frac{p_j(1+t_j)}{e(\underline{\delta})}\right)\left(\frac{\partial\underline{\delta}}{\partial p_j}\right)^2\underbrace{\left\{\frac{\eta\left(v_{j-1}-\underline{\delta}\right)}{v_{j-1}-\underline{\delta}}-\frac{\eta\left(\underline{\delta}\right)}{\underline{\delta}}\right\}}_{\text{assymmetry in elasticity of e(v)}}$$

$$+\underbrace{\frac{(p_j-c)x_j^i\left(\frac{p_j(1+t_j)}{e(\underline{\delta})}\right)\left(\frac{\partial\underline{\delta}}{\partial p_j}\right)^2}{-\left\{p^*_{j-1}e'(\underline{\delta})+(1+t_j)p_je'(v_{j-1}-\underline{\delta})\right\}}}_{>0}\underbrace{\left\{\left(\frac{\left(1+t_{j-1}\right)p^*_{j-1}e'\left(\underline{\delta}\right)}{\underline{\delta}}\right)\mu\left(\underline{\delta}\right)-\left(\frac{\left(1+t_j\right)p_je'\left(v_{j-1}-\underline{\delta}\right)}{v_{j-1}-\underline{\delta}}\right)\mu\left(v_{j-1}-\underline{\delta}\right)\right\}}_{\text{assymmetry in convexity of e(v)}}$$

[11] Formal deriveration of (28) is shown in Appendix.

[12] Similarly, the strategic relationship between the domestic firm j and $j+1$ is also derived in Appendix.

$$\frac{dp_j}{d\hat{p^*_{j-1}}} = \left(\frac{e\left(\bar{\delta}\right)}{(1+t_j)e\left(v_{j+1}-\bar{\delta}\right)}\right)\left\{\frac{H+J+K+(p_j-c)\left(\frac{e'(\bar{\delta})}{e(\bar{\delta})}+\frac{e'(v_{j+1}-\bar{\delta})}{e(v_{j+1}-\bar{\delta})}\right)\left(\frac{\partial\bar{\delta}}{\partial p_j}\right)H}{2A+2B+D+2E+2G+2H+I+2J+F+K}\right\}$$

The first term is always positive, while the second and the third term are nonzero iff the consumer price of domestic firms and foreign ones are different and, the range of consumers covered by the domestic firm j and the foreign firm $j-1$ is asymmetric. If the consumer price of the foreign firm $j-1$ is raised more than that of the domestic firm j, the range of consumers covered by the foreign firm $v_{j-1}-\underline{\delta}$ is narrower than that by the domestic form $\underline{\delta}$. Given the difference in the range of consumers covered by domestic and foreign firms, the second term and the third term are positive iff [I] the elasticity of the enthusiasm function $e(v)$ decreases with distance, and [II] the shape of the enthusiasm function becomes more convex with the distance. [13]

The denominator of (28) is arranged as follows.

$$
\begin{aligned}
&2A+2B+D+2E+2G+2H+I+2J+F+K \\
&\equiv \int_0^{\underline{\delta}} \underbrace{\frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(v)},\cdot\right)}{\partial p_j}}_{<0} \left\{2-\left(\frac{p_j-c}{p_j}\right)\gamma\left(\frac{p_j(1+t_j)}{e(v)},\cdot\right)\right\} dv + 2x_j^i(\underline{\delta}) \underbrace{\left\{1-\left(\frac{p_j-c}{p_j}\right)\epsilon(\underline{\delta})\right\}}_{<0} \underbrace{\left(\frac{\partial \underline{\delta}}{\partial p_j}\right)}_{<0} \\
&+(p_j-c)\underbrace{\left\{x_j^i\left(\frac{p_j(1+t_j)}{e(\underline{\delta})},\cdot\right)\left(\frac{\partial^2\underline{\delta}}{\partial p_j^2}\right)+x_j^i\left(\frac{p_j(1+t_j)}{e(\bar{\delta})},\cdot\right)\left(\frac{\partial^2\bar{\delta}}{\partial p_j^2}\right)\right\}}_{>0}
\end{aligned}
$$

The denominator is positive [III] if the demand curve is convex enough to make the first term of the above equation positive. [14]

To summerize,(28) is positive, and the domestic firm j has a strategic complementary relationship with teh foreign firm $j-1$ if [I] the distance elasticity of the enthusiasm function $e(v)$ decreases with the distance, [II] the shape of the $e(v)$ becomes more convex with the distance, and [III] the demand curve x_j^i is convex enough. If these conditions are met, an additional increase in the consumer price of the foreign good $j-1$ triggered by the imposition of tariff also induces the domestic producer of the good j to raise its price.

3 Conclusion

This paper focuses horizontally differentiated goods, consumers show different levels of enthusiasm. This paper modifies the Helpman model to build the model, where every consumer can buy different amount of a differentiated good depending on his/her enthusiastic level. Under Helpman's the "ideal-variety" approach model, the distance v between the good j and each consumer's ideal good determines his/her enthusiasm level for the good. Under

[13] If two conditions [I][II] are met, $\eta\left(v_{j-1}-\underline{\delta}\right)>\eta\left(\underline{\delta}\right)$ and $\mu\left(\underline{\delta}\right)>\mu\left(v_{j-1}-\underline{\delta}\right)$

[14] Again, to satisfy (7), γ should be in the following range. $\frac{2}{\left(\frac{p_j-c}{p_j}\right)}<\gamma_j<1+\epsilon_j$

such a circumstance, more enthusiastic (closer) consumers buy more but less price-elastic than less enthusiastic (remoter) consumers do if their demand curve is highly price elastic but mildly convex. With this model, this paper analyzes short-run effects of trade liberalization on horizontally differentiated goods. Even if no new firm enters the market in the short run, the trade liberalization intensifies the competition between existing domestic and foreign firms and puts a downward pressure on price, which is suggested by most former studies. [a increase in competition effect] However, this paper shows that trade liberalization can lead us to the opposite ending. After foreign firms enter home market, the range of consumers covered by each firm shrinks. This shrinkage gives every producers a chance to concentrate its sales on closer, more enthusiastic, and thus less price elastic consumers, which creates an upward pressure on price.[a change in the composition of consumers effect] The latter price-increasing effect can dominate the former price-decreasing effect if (1) less enthusiastic consumers are much more price-elastic than weighted-average consumers are, and/or (2) the enthusiasm function curve $e(v)$ evaluated at the location of marginal consumers is less elastic but sufficiently convex.

This paper also analyzes short-run effects of imperfect trade liberalization with tariff under the assumption of separated markets. If the government imposes a tariff on imports, foreign producers cut their producer prices. However, an increase in tariff by 1 % induces foreign producers to cut their producer prices by less than 1 % if the aggregate demand curve is sufficiently convex and marginal consumers are price-elastic enough. If these conditions are met,the consumer price of imports increase under imperfect trade liberalization. Facing a consumer price increase in home market, domestic producers, which have a strategic complementary relationship with foreign counterparts will also raise their prices if [I] the distance elasticity of the enthusiasm function $e(v)$ decreases with the distance, [II] the shape of the $e(v)$ becomes more convex with the distance, and [III] the demand curve x_j^i is convex enough.

Although this paper is not the first paper to show that the trade liberalization leads a price increase, the contribution of this paper is to show that the price-increasing competition can be the result in a more general model setting.

References

[1] Chamberlin, E. (1933). *The Theory of Monopolistic Competition.* Boston: Harvard University Press.

[2] Carroll, C. D. and M.S. Kimball. (1996). "On the Concavity of the Consumption Function." *Econometrica* 64, pp981-992.

[3] Chen, Yongmin. and Michael H. Riordan. "Price-increasing Competition." *RAND Journal of Economics* 39, No.4 Winter 2008, pp 1042-1058.

[4] d'Aspremont, C.,J. Gabszewicz, and J.F. Thisse. (1979). "On Hotelling's Stability in Competition." *Econometrica* 17, pp 1145-1171.

[5] Green, H.A.J (1964). *Aggregation in Economic Analysis; An Introductory Survey.* Princeton: Princeton University Press.

[6] Hummels, David. and Peter J. Klenow.(2005) "The Variety and Quality of a Nation's Exports." *The American Economic Review*, 95, pp 704-723.

[7] Helpman, E. (1981). "International Trade in the Presence of Product Differentiation, Economies of Scale and Monopolistic Competition A Chamberlin-Heckscher-Ohlin Approach." *Journal of International Economics.* 11, pp 305-340.

[8] Hotelling, H. (1929). "Stability in Competition." *Economic Journal* 39, pp 41-57.

[9] Krugman, P. (1979). "Increasing Returns, Monopolistic Competition, and International Trade." *Journal of International Economics.* 9, pp469-479.

[10] Lancaster, K.J. (1966)."Model of Consumer Reaction to Product Characteristics." *Journal of Political Economy.* 74, pp132-157.

[11] Lancaster, K.J. (1975). "Socially Optimal Product Differentiation." *The American Economic Review.* 65, pp567-585.

[12] Salop, S.C. (1979). "Monopolistic Competition with Outside Goods." *Bell Journal of Economics*, 10, pp141-156

[13] Seade, J. (1980). "On the Effects of Entry." *Econometrica.* 48, pp 479-489.

A Appendix

A.1 Deriveration of (26)

First in this Appendix (26) is derived as follows.

$$
\begin{aligned}
&\left\{\int_0^{\underline{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j^*}dv + x_j^i\left(\tfrac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)\left(\frac{\partial \underline{\delta}}{\partial p_j^*}\right)\right\}dp_j^* \\
&+\left\{\int_0^{\underline{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial t_j}dv + x_j^i\left(\tfrac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)\left(\frac{\partial \underline{\delta}}{\partial t_j}\right)\right\}dt_j \\
&+\left[\int_0^{\underline{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j^*}dv + (p_j^*-c)\left\{\int_0^{\underline{\delta}} \frac{\partial^2 x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j^{*2}}dv + \left(\frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)}{\partial p_j^*}\right)\left(\frac{\partial \underline{\delta}}{\partial p_j^*}\right)\right\}\right]dp_j^* \\
&+(p_j^*-c)\left\{\int_0^{\underline{\delta}} \frac{\partial^2 x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\ \cdot\right)}{\partial p_j^*\partial t_j}dv + \left(\frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)}{\partial t_j}\right)\left(\frac{\partial \underline{\delta}}{\partial t_j}\right)\right\}dt_j \\
&+\left[x_j^i\left(\tfrac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)\left(\frac{\partial \underline{\delta}}{\partial p_j^*}\right) + (p_j^*-c)\left\{\left(\frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)}{\partial p_j^*}\right)\left(\frac{\partial \underline{\delta}}{\partial p_j^*}\right) + x_j^i\left(\tfrac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)\left(\frac{\partial^2 \underline{\delta}}{\partial p_j^{*2}}\right)\right\}\right]dp_j^* \\
&+(p_j^*-c)\left\{\left(\frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(\underline{\delta})},\cdot\right)}{\partial t_j}\right)\left(\frac{\partial \underline{\delta}}{\partial p_j}\right) + x_j^i\left(\underline{\delta}\right)\left(\frac{\partial^2 \underline{\delta}}{\partial p_j^*\partial t_j}\right)\right\}dt_j \\
&+\left\{\int_0^{\bar{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j^*}dv + x_j^i\left(\tfrac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)\left(\frac{\partial \bar{\delta}}{\partial p_j^*}\right)\right\}dp_j^* \\
&+\left\{\int_0^{\bar{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial t_j}dv + x_j^i\left(\tfrac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)\left(\frac{\partial \bar{\delta}}{\partial t_j}\right)\right\}dt_j \\
&+\left[\int_0^{\bar{\delta}} \frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j^*}dv + (p_j^*-c)\left\{\int_0^{\bar{\delta}} \frac{\partial^2 x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j^{*2}}dv + \left(\frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)}{\partial p_j^*}\right)\left(\frac{\partial \bar{\delta}}{\partial p_j^*}\right)\right\}\right]dp_j^* \\
&+(p_j^*-c)\left\{\int_0^{\bar{\delta}} \frac{\partial^2 x_j^i\left(\frac{(1+t_j)p_j^*}{e(v)},\cdot\right)}{\partial p_j^*\partial t_j}dv + \left(\frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)}{\partial p_j^*}\right)\left(\frac{\partial \bar{\delta}}{\partial t_j}\right)\right\}dt_j \\
&+\left[x_j^i\left(\tfrac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)\left(\frac{\partial \bar{\delta}}{\partial p_j^*}\right) + (p_j^*-c)\left\{\left(\frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)}{\partial p_j^*}\right)\left(\frac{\partial \bar{\delta}}{\partial p_j^*}\right) + x_j^i\left(\tfrac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)\left(\frac{\partial^2 \bar{\delta}}{\partial p_j^{*2}}\right)\right\}\right]dp_j^* \\
&+(p_j^*-c)\left\{\left(\frac{\partial x_j^i\left(\frac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)}{\partial p_j^*}\right)\left(\frac{\partial \bar{\delta}}{\partial t_j}\right) + x_j^i\left(\tfrac{(1+t_j)p_j^*}{e(\bar{\delta})},\cdot\right)\left(\frac{\partial^2 \bar{\delta}}{\partial p_j^*\partial t_j}\right)\right\}dt_j = 0 \qquad \text{(A.29)}
\end{aligned}
$$

Using $\frac{\partial x_j(\frac{(1+t_j)p_j^*}{e(v)})}{\partial t_j} \equiv (\frac{p_j^*}{1+t_j})\left(\frac{\partial x_j(\frac{(1+t_j)p_j^*}{e(v)})}{\partial p_j}\right)$, $\frac{\partial \underline{\delta}}{\partial t_j} \equiv (\frac{p_j^*}{1+t_j})(\frac{\partial \underline{\delta}}{\partial p_j})$, $\frac{\partial \bar{\delta}}{\partial t_j} \equiv (\frac{p_j^*}{1+t_j})(\frac{\partial \bar{\delta}}{\partial p_j})$,
$\frac{\partial^2 x_j(\frac{(1+t_j)p_j^*}{e(v)})}{\partial p_j^* \partial t_j} \equiv (\frac{p_j^*}{1+t_j})\left(\frac{\partial^2 x_j(\frac{(1+t_j)p_j^*}{e(v)})}{\partial p_j^{*2}}\right) + (\frac{1}{1+t_j})\left(\frac{\partial x_j(\frac{(1+t_j)p_j^*}{e(v)})}{\partial p_j^*}\right)$
$\frac{\partial^2 \underline{\delta}}{\partial p_j^* \partial t_j} \equiv (\frac{p_j^*}{1+t_j})\left(\frac{\partial^2 \underline{\delta}}{\partial p_j^{*2}}\right) + (\frac{1}{1+t_j})\left(\frac{\partial \underline{\delta}}{\partial p_j^*}\right)$, and $\frac{\partial^2 \bar{\delta}}{\partial p_j^* \partial t_j} \equiv (\frac{p_j^*}{1+t_j})\left(\frac{\partial^2 \bar{\delta}}{\partial p_j^{*2}}\right) + (\frac{1}{1+t_j})\left(\frac{\partial \bar{\delta}}{\partial p_j^*}\right)$ we reach equation (26).

A.2 Deriveration of (28)

$$
\begin{aligned}
&\left\{\int_0^{\underline{\delta}} \frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(v)},\cdot\right)}{\partial p_j} dv + x_j^i\left(\tfrac{p_j(1+t_j)}{e(\underline{\delta})}\right)\left(\frac{\partial \underline{\delta}}{\partial p_j}\right)\right\} dp_j + \left\{x_j^i\left(\tfrac{p_j(1+t_j)}{e(\underline{\delta})},\cdot\right)\left(\frac{\partial \underline{\delta}}{\partial p_{j-1}^*}\right)\right\} dp_{j-1}^* \\
&+\left\{\int_0^{\underline{\delta}} \frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(v)},\cdot\right)}{\partial p_j} dv + (p_j - c)\int_0^{\underline{\delta}} \frac{\partial^2 x_j^i\left(\frac{p_j(1+t_j)}{e(v)},\cdot\right)}{\partial p_j^2} dv + (p_j - c)\left(\frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(\underline{\delta})},\cdot\right)}{\partial p_j}\right)\left(\frac{\partial \underline{\delta}}{\partial p_j}\right)\right\} dp_j \\
&+\left\{(p_j - c)\left(\frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(\underline{\delta})},\cdot\right)}{\partial p_j}\right)\left(\frac{\partial \underline{\delta}}{\partial p_{j-1}^*}\right)\right\} dp_{j-1}^* \\
&+\left\{\int_0^{\bar{\delta}} \frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(v)}\right)}{\partial p_j} dv + x_j^i\left(\tfrac{p_j(1+t_j)}{e(\bar{\delta})}\right)\left(\frac{\partial \bar{\delta}}{\partial p_j}\right)\right\} dp_j + \left\{x_j^i\left(\tfrac{p_j(1+t_j)}{e(\bar{\delta})}\right)\left(\frac{\partial \bar{\delta}}{\partial p_{j+1}^*}\right)\right\} dp_{j+1}^* \\
&+\left\{\int_0^{\bar{\delta}} \frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(v)}\right)}{\partial p_j} dv + (p_j - c)\int_0^{\bar{\delta}} \frac{\partial^2 x_j^i\left(\frac{p_j(1+t_j)}{e(v)}\right)}{\partial p_j^2} dv + (p_j - c)\left(\frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(\bar{\delta})}\right)}{\partial p_j}\right)\left(\frac{\partial \bar{\delta}}{\partial p_j}\right)\right\} dp_j \\
&+\left\{(p_j - c)\left(\frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(\bar{\delta})}\right)}{\partial p_j}\right)\left(\frac{\partial \bar{\delta}}{\partial p_{j+1}^*}\right)\right\} dp_{j+1}^* \\
&+\left\{x_j^i\left(\tfrac{p_j(1+t_j)}{e(\underline{\delta})}\right)\left(\frac{\partial \underline{\delta}}{\partial p_j}\right) + (p_j - c)\left(\frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(\underline{\delta})}\right)}{\partial p_j}\right)\left(\frac{\partial \underline{\delta}}{\partial p_j}\right) + (p_j - c)x_j^i\left(\tfrac{p_j(1+t_j)}{e(\underline{\delta})}\right)\left(\frac{\partial^2 \underline{\delta}}{\partial p_j^2}\right)\right\} dp_j \\
&+(p_j - c)x_j^i\left(\tfrac{p_j(1+t_j)}{e(\underline{\delta})},\cdot\right)\left(\frac{\partial^2 \underline{\delta}}{\partial p_j \partial p_{j-1}^*}\right) dp_{j-1}^* \\
&+\left\{x_j^i\left(\tfrac{p_j(1+t_j)}{e(\bar{\delta})}\right)\left(\frac{\partial \bar{\delta}}{\partial p_j}\right) + (p_j - c)\left(\frac{\partial x_j^i\left(\frac{p_j(1+t_j)}{e(\bar{\delta})}\right)}{\partial p_j}\right)\left(\frac{\partial \bar{\delta}}{\partial p_j}\right) + (p_j - c)x_j^i\left(\tfrac{p_j(1+t_j)}{e(\bar{\delta})}\right)\left(\frac{\partial^2 \bar{\delta}}{\partial p_j^2}\right)\right\} dp_j \\
&+(p_j - c)x_j^i\left(\tfrac{p_j(1+t_j)}{e(\bar{\delta})},\cdot\right)\left(\frac{\partial^2 \bar{\delta}}{\partial p_j \partial p_{j+1}^*}\right) dp_{j+1}^* = 0 \qquad \text{(A.30)}
\end{aligned}
$$

Using $\frac{\partial \underline{\delta}}{\partial p_{j-1}^*} \equiv (\frac{-e(\underline{\delta})}{(1+t_j)e(v_{j-1}-\underline{\delta})})$, $\frac{\partial \bar{\delta}}{\partial p_{j+1}^*} \equiv (\frac{-e(\bar{\delta})}{(1+t_j)e(v_{j+1}-\bar{\delta})})$,

$\frac{\partial\underline{\delta}}{\partial p_j \partial p^* j-1} \equiv \frac{-e(\underline{\delta})}{(1+t_j)e(v_{j-1}-\underline{\delta})} \left\{ (\frac{e\prime(\underline{\delta})}{e(\underline{\delta})} + \frac{e\prime(v_{j-1}-\underline{\delta})}{e(v_{j-1}\underline{\delta})})(\frac{\partial\underline{\delta}}{\partial p_j})^2 + (\frac{\partial^2\underline{\delta}}{\partial p_j^2}) \right\}$, and
$\frac{\partial\bar{\delta}}{\partial p_j \partial p^* j+1} \equiv \frac{-e(\bar{\delta})}{(1+t_j)e(v_{j+1}-\bar{\delta})} \left\{ (\frac{e\prime(\bar{\delta})}{e(\bar{\delta})} + \frac{e\prime(v_{j+1}-\bar{\delta})}{e(v_{j+1}-\bar{\delta})})(\frac{\partial\bar{\delta}}{\partial p_j})^2 + (\frac{\partial^2\bar{\delta}}{\partial p_j^2}) \right\}$
The above equation can be simplified as follows.

$$
\begin{aligned}
&(2A + 2B + D + 2E + 2G + 2H + I + 2J + F + K)\, dp_j \\
&+ \left(\frac{-e\left(\underline{\delta}\right)}{(1+t_j)e\left(v_{j-1}-\underline{\delta}\right)} \right) \left\{ B + E + F + (p_j - c) \left(\frac{e'\left(\underline{\delta}\right)}{e\left(\underline{\delta}\right)} + \frac{e'\left(v_{j-1}-\underline{\delta}\right)}{e\left(v_{j-1}-\underline{\delta}\right)} \right) \left(\frac{\partial \underline{\delta}}{\partial p_j} \right) B \right\} d\hat{p^*}_{j-1} \\
&+ \left(\frac{-e\left(\bar{\delta}\right)}{(1+t_j)e\left(v_{j+1}-\bar{\delta}\right)} \right) \left\{ H + J + K + (p_j - c) \left(\frac{e'\left(\bar{\delta}\right)}{e\left(\bar{\delta}\right)} + \frac{e'\left(v_{j+1}-\bar{\delta}\right)}{e\left(v_{j+1}-\bar{\delta}\right)} \right) \left(\frac{\partial \bar{\delta}}{\partial p_j} \right) H \right\} d\hat{p^*}_{j+1} = 0
\end{aligned}
\tag{A.31}
$$

where

$$
\begin{aligned}
A &\equiv \int_0^{\underline{\delta}} \frac{\partial x_j^i \left(\frac{p_j(1+t_j)}{e(v)}, \cdot \right)}{\partial p_j} dv & B &\equiv x_j^i \left(\frac{p_j(1+t_j)}{e(\underline{\delta})} \right) \left(\frac{\partial \underline{\delta}}{\partial p_j} \right) \\
D &\equiv (p_j - c) \int_0^{\underline{\delta}} \frac{\partial^2 x_j^i \left(\frac{p_j(1+t_j)}{e(v)}, \cdot \right)}{\partial p_j^2} dv & E &\equiv (p_j - c) \left(\frac{\partial x_j^i \left(\frac{p_j(1+t_j)}{e(\underline{\delta})}, \cdot \right)}{\partial p_j} \right) \left(\frac{\partial \underline{\delta}}{\partial p_j} \right) \\
F &\equiv (p_j - c) x_j^i \left(\frac{p_j(1+t_j)}{e(\underline{\delta})} \right) \left(\frac{\partial^2 \underline{\delta}}{\partial p_j^2} \right) & G &\equiv \int_0^{\bar{\delta}} \frac{\partial x_j^i \left(\frac{p_j(1+t_j)}{e(v)} \right)}{\partial p_j} dv \\
H &\equiv x_j^i \left(\frac{p_j(1+t_j)}{e(\bar{\delta})} \right) \left(\frac{\partial \bar{\delta}}{\partial p_j} \right) & I &\equiv (p_j - c) \int_0^{\bar{\delta}} \frac{\partial^2 x_j^i \left(\frac{p_j(1+t_j)}{e(v)} \right)}{\partial p_j^2} dv \\
J &\equiv (p_j - c) \left(\frac{\partial x_j^i \left(\frac{p_j(1+t_j)}{e(\bar{\delta})} \right)}{\partial p_j} \right) \left(\frac{\partial \bar{\delta}}{\partial p_j} \right) & K &\equiv (p_j - c) x_j^i \left(\frac{p_j(1+t_j)}{e(\bar{\delta})} \right) \left(\frac{\partial^2 \bar{\delta}}{\partial p_j^2} \right)
\end{aligned}
$$