Agricultural Productivity, Infrastructures, and the Optimal Timing of Opening Trade*

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Abstract

This study develops a dynamic Ricardian trade model that incorporates productive infrastructures into the manufacturing sector. The costs of building infrastructures are financed by tax. An increase in the tax rate decreases the amount of consumption in the present period but hastens the timing of opening trade. We investigate the relationship between the timing of opening trade and total welfare. We also compare two kinds of total welfare: (i) the total welfare that a country obtains by closing international trade until it has a comparative advantage in manufacturing and then engaging in free trade and (ii) the total welfare that a country obtains by specializing in agriculture according to the law of comparative advantage from the beginning. The main results are as follows: (1) there is the optimal tax rate maximizing the total welfare; (2) an increase in agricultural productivity can accelerate the timing of opening trade, which, however, does not necessarily improve total welfare; and (3) total welfare under specialization in manufacturing can be higher than that under specialization in agriculture depending on the prevailing conditions.

Keywords: productive infrastructure; industrialization; timing of opening trade; agricultural productivity

JEL Classification: F43; F10; O14

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1 Introduction

Infrastructure provided by the government can be a major contributing factor to facilitating industrialization and obtaining a comparative advantage in manufacturing. For example, although the Korean iron and steel industry had been disadvantaged, the Korean government has assisted POSCO (formerly Pohang Iron and Steel Company) greatly by supplying infrastructure and introducing the latest technology. In this regard, the Korean government has played an important role in the firm's dramatic improvement in productivity (Amsden, 1989). Indeed, POSCO has become one of most productive firms in that sector globally, thereby gaining a crucial comparative advantage.

In addition, the importance of agricultural productivity growth for industrialization has long been recognized by economists. For example, Nurkse (1953) states that "[e]veryone knows that the spectacular industrial revolution would not have been possible without the agricultural revolution that preceded it." Rostow (1960) further argues that "revolutionary changes in agricultural productivity are an essential condition for successful take-off."

Of the theoretical research on these issues, the celebrated study by Matsuyama (1992) examines how an increase in agricultural productivity affects industrialization by using a two-sector growth model.[1)] He shows that an increase in agricultural productivity does not lead to industrialization in a small open economy because it promotes a comparative advantage in agriculture at the expense of manufacturing. In addition, he finds that if a developing country under free trade cannot industrialize, then the country begins to specialize in agriculture and consequently has a lower growth rate. He uses the Stone–Geary utility function of non-homothetic preferences, which implies that the income elasticity of demand for agricultural goods is less than unity. The growth engine of the Matsuyama model is thus driven by learning-by-doing in the manufacturing sector.

1) Lewis (1954) is the forerunner of two-sector models of industrialization in developing counties. For extensions of the Lewis model, see also Kirkpatrick and Barrientos (2004), Temple (2005), and Wang and Piesse (2013). Wong and Yip (1999) also incorporate capital accumulation into the two-sector trade model of Matsuyama (1992).

Chang et al. (2006) investigate the relationship between the public provision of infrastructure based on Barro (1990) and industrialization. They introduce the provision of infrastructure into Matsuyama's (1992) model and show that an increase in the public provision of infrastructure promotes a comparative advantage in manufacturing, while an increase in agricultural productivity can promote industrialization.

Despite these attractive results, the model proposed by Chang et al. (2006) suffers from the following two problems. First, it assumes that infrastructures are automatically produced by tax revenues: no production factor is used for the production of infrastructures. Hence, they do not consider the trade-off of labor allocation between sectors: an increase in workers in the public sector decreases workers in the agricultural and/or manufacturing sectors. Second, they focus their attention on the analysis of the GDP growth rate. Therefore, they do not explicitly consider the relationship between the tax rate and welfare.

In this study, we extend the model of Chang et al. (2006) by supposing that the government employs workers in the public sector, whose wages are financed by tax revenue. In addition, our model derives total welfare and investigates its relationship with tax rates. Moreover, our model considers the concept of dynamic comparative advantage proposed by Redding (1999). His model assumes that the home country does not initially have a comparative advantage in the manufacturing sector. Therefore, if the home country engages in free trade at the initial time, then it specializes in the agricultural sector. However, it is assumed that the learning potential, that is, the efficiency of learning-by-doing in the manufacturing sector of the home country, is higher than that of the foreign country. Hence, if the home country continues to be an autarkic economy until it has a comparative advantage in the manufacturing sector, it will eventually be able to industrialize. This type of endogenous comparative advantage is called a "dynamic comparative advantage." Note that neither Matsuyama (1992) nor Chang et al. (2006) directly address the concept of dynamic comparative advantage.[2)]

2) The model of Ortiz (2004) is similar to that of Chang et al. (2006). He incorporates productive public

Redding (1999) suggests that an increase in agricultural productivity would not promote a dynamic comparative advantage in the manufacturing sector because he assumes the Cobb–Douglas utility function of homothetic preferences. He shows that a country can obtain a dynamic comparative advantage in the manufacturing sector as long as it has high learning potential in that industry. However, we believe that this is only applicable to a small number of developing countries that have growth potential from the beginning.

Wong and Yip (2010) construct a two-sector small open economy model similar to the model of Chang et al. (2006) and discuss the dynamic comparative advantage in the manufacturing sector as Redding (1999). Their model assumes that the lump-sum tax and manufacturing sector subsidy attract labor from the agricultural sector to the manufacturing sector. In addition, they show that a certain tax rate maximizes welfare during the autarkic period. By contrast, if the country can specialize in the manufacturing sector, then the government stops imposing a tax burden.

In this study, we compare welfare under specialization in the agricultural sector with welfare under specialization in the manufacturing sector from some point in time. In our model, the home country has a comparative advantage in the agricultural sector at the initial time. However, if the home country delays engaging in trade until it has a comparative advantage in the manufacturing sector, it can industrialize and obtain higher welfare. In addition, we investigate the effect of an increase in agricultural productivity on total welfare and the timing of opening trade.

The results of our analysis are as follows. (1) There is the optimal tax rate maximizing the total welfare. (2) If the subsistence level of agricultural consumption is positive, that is, the preference is non-homothetic, then an increase in agricultural productivity can accelerate the timing of opening trade. In addition, we show that the acceleration of the timing of opening trade does not necessarily improve total welfare. (3) If there is no subsistence level of

expenditure based on the work of Barro (1990) into the two-sector small open economy model of Matsuyama (1992). However, he also does not investigate dynamic comparative advantage à la Redding (1999).

agricultural consumption, that is, the preference is homothetic, then an increase in agricultural productivity delays the timing of opening trade and necessarily decreases total welfare. (4) An increase in the efficiency of the public sector accelerates the timing of opening trade irrespective of the existence of the subsistence level of agricultural consumption. (5) At the very timing of opening trade, instantaneous utility under industrialization is lower than that under specialization in agriculture. However, the growth rate of instantaneous utility under industrialization is higher than that under specialization in agriculture. Hence, depending on the prevailing conditions, total welfare under industrialization can be higher than that under specialization in agriculture.

Our study obtains the following two policy implications. First, minimizing the timing of opening trade does not imply that total welfare is maximized (i.e., it is not necessarily the optimal policy). Our conclusion does not support the permanent protection of the infant industry because we show that there is an optimal timing of opening trade. If the timing of opening trade is slower than the optimal level, total welfare deteriorates.

Second, compared with agricultural goods, if manufactured goods are sufficiently high quality, then an increase in agricultural productivity improves total welfare in the home country for the following reason. The effect of an increase in agricultural productivity on total welfare depends on the size of the subsistence level of agricultural consumption. A large subsistence level means that the income elasticity of consumption for manufactured goods is high. In addition, according to our results, when the subsistence level is high, an increase in agricultural productivity raises total welfare. Therefore, we can say that if an economy produces high-quality sophisticated goods, agricultural productivity growth is good for industrialization, whereas if an economy produces low-quality less sophisticated goods, agricultural productivity growth is not good for industrialization.

The rest of this paper is organized as follows. Section 2 explains our model and investigates it under autarky. Section 3 investigates the model under a small open economy and obtains the timing of opening trade. Section 4, by using both analytical and numerical methods, shows the existence of optimal tax rates maximizing total welfare and investigates the relationship between agricultural productivity and welfare. Section 5 concludes.

2 Model

2.1 Production

In the home country (Home, hereafter), there are two sectors: the agricultural sector (a) and the manufacturing sector (m). Labor is the only production factor and total labor endowment is normalized to unity, $L\,(=1)$. We neglect population growth and migration.[3)]

We assume that labor is perfectly mobile between the two sectors and wage w_t is identical across them both. We specify the production functions as follows:

$$X_{mt} = M_t L_{mt}, \tag{1}$$

$$X_{at} = A L_{at}, \tag{2}$$

where X_{it} denotes total output in sector $i\,(=a, m)$ at time t, A is agricultural productivity and a constant parameter, and M_t is manufacturing productivity at time t.[4)]

Through learning-by-doing and investment in infrastructures, manufacturing productivity M_t continues to increase over time. In this study, G_t is considered as the quantity of

3) When population growth is allowed, international trade can generate an incentive in less developed countries to specialize in the production of unskilled labor-intensive goods (i.e., agricultural goods in our model), thus leading to the expansion of the population and delaying the process of development, as shown by Galor and Mountford (2006, 2008) and Azarnert (2014a).

4) For analytical tractability, we adopt a linear production function. The linear production functions in equations (1) and (2) are different from those used in Matsuyama (1992), who uses decreasing returns to scale production functions. However, this difference does not affect the results of our study. Indeed, when a linear production function is used, trade liberalization immediately causes complete specialization. By contrast, when a decreasing returns to scale production function is used, trade liberalization causes a gradual switch from diversification to complete specialization. The analysis of the transitional dynamics under decreasing returns to scale will be left for future research.

infrastructures provided by the government.[5] Furthermore, the government must employ labor to supply infrastructures. Following Chang et al. (2006), we specify the growth rate of manufacturing productivity as follows:

$$\dot{M}_t = G_t X_{mt} \Longrightarrow \frac{\dot{M}_t}{M_t} = G_t L_{mt}, \tag{3}$$

where $\forall t\ M_t \geq M_0 > 0$.

The infrastructure production function is defined as follows:

$$G_t = \phi L_{gt}, \tag{4}$$

where $\phi > 0$ is the efficiency parameter in the public sector and L_{gt} is the number of workers employed in the public sector.[6]

We assume that the government imposes the same production tax rate $\tau \in (0, 1)$ on both sectors. This setting of the production tax rate follows Ortiz (2004) and Chang et al. (2006). Thus, the revenue of the government is given by

$$\tau(P_t X_{mt} + X_{at}) = \tau w_t, \tag{5}$$

where P_t is the relative price of manufactured goods compared with agricultural goods. Furthermore, from equation (5), the government uses tax revenue to pay the wages of public sector workers:

$$w_t L_{gt} = \tau w_t \iff L_{gt} = \tau. \tag{6}$$

5) We can consider infrastructures as scientific research, transportation systems, education, and so forth.

6) This specification is similar to the specification used in Sasaki (2008), who considers a skills-producing sector and assumes that the acquisition of skills requires labor. His skills-producing sector is similar to infrastructures in our model.

Accordingly, the share of workers in the public sector depends on the tax rate imposed by the government. The tax rate is specified in Sections 3.2 and 4.2. From equations (5) and (6), we obtain $G_t = \phi\tau$.[7)]

The labor market clearing condition is given by

$$L_{at} + L_{mt} + L_{gt} = 1. \tag{7}$$

The profit of each sector is as follows:

$$\pi_{mt} = P_t(1 - \tau)X_{mt} - w_t L_{mt}, \tag{8}$$

$$\pi_{at} = (1 - \tau)X_{at} - w_t L_{at}. \tag{9}$$

From equations (8) and (9), we derive the profit maximization conditions and rewrite them as follows:

$$P_t = \frac{w_t}{(1 - \tau)M_t}, \tag{10}$$

$$1 = \frac{w_t}{(1 - \tau)A}. \tag{11}$$

Hence, wage is given by $w = (1 - \tau)A$ if both goods are produced. From equations (10) and (11), the relative price is given by $P_t = A/M_t$. Thus, if manufacturing productivity increases, then the relative price decreases.

2.2 Consumer behavior

Let us assume that consumers obtain utility from the consumption of manufactured and agricultural goods. All consumers in Home share identical preferences. We adopt the Stone–

7) In our model, $G = 0$ implies $\tau = 0$. From equation (3), if $\tau = 0$, then $\dot{M} = 0$ and $M = M_0 > 0$.

Geary utility function of non-homothetic preferences.[8)] The utility maximization problem is given by

$$\max_{c_{at},c_{mt}} W = \int_0^{\infty} u_t e^{-\rho t}\, dt \quad \text{where } u_t = \beta \ln(c_{at} - \gamma) + \ln c_{mt}, \tag{12}$$

$$\text{s.t. } c_{at} + P_t c_{mt} = w_t, \tag{13}$$

where $\gamma > 0$ denotes the subsistence level of agricultural consumption, $\rho > 0$ is the constant discount rate, c_{it} is the per capita consumption of good $i\,(= a, m)$ at t, and $\beta > 0$ is the preference parameter. In addition, c_{at} must satisfy the condition $c_{at} > \gamma$, that is, the consumption of agricultural goods exceeds the subsistence level.

By solving the above utility maximization problem, we obtain the aggregated utility-maximizing condition as follows:

$$C_{at} = \gamma + \beta P_t C_{mt}, \tag{14}$$

where C_{at} and C_{mt} denote the aggregated consumption of agricultural goods and that of manufactured goods, respectively.

From equations (13) and (14), we derive the demand functions as follows:

$$C_{mt} = \frac{w - \gamma}{P_t(1 + \beta)}, \tag{15}$$

$$C_{at} = \frac{\beta w + \gamma}{1 + \beta}. \tag{16}$$

For C_{mt} to be positive, we assume that $w > \gamma$.

By substituting equations (15) and (16) into equation (12), we obtain the indirect utility

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8) In the context of trade and development, the Stone–Geary utility function is often used: Matsuyama (1992), Spilimbergo (2000), Kikuchi (2004), and Azarnert (2014b).

function at t, $\tilde{u}_t$ as follows.

$$\tilde{u}_t = J + (1+\beta)\ln(w_t - \gamma) - \ln P_t, \tag{17}$$

where $J \equiv \beta \ln \beta - (1+\beta)\ln(1+\beta)$ is constant. A decrease in the relative price increases real income and hence improves indirect utility.

2.3 Autarky

Consider the equilibrium under autarky. The market clearing conditions are given by

$$C_{at} = X_{at}, \tag{18}$$

$$C_{mt} = X_{mt}. \tag{19}$$

From equations (18) and (19), employment in each sector is given by

$$L_A = \left[\frac{\beta(1-\tau)A + \gamma}{A(1+\beta)}\right], \tag{20}$$

$$L_m = \left[\frac{(1-\tau)A - \gamma}{A(1+\beta)}\right], \tag{21}$$

$$L_g = \tau. \tag{22}$$

From equation (21), for L_m to be positive, we need $\tau < (A-\gamma)/A$. For this condition to be meaningful, we assume that $A > \gamma$: agricultural productivity is larger than the subsistence level. From equations (20) and (21), an increase in τ decreases the number of workers in the agricultural and manufacturing sectors.[9)]

By using equations (3), (4), (21), and (22), we obtain the growth rate of manufacturing

9) By partially differentiating equations (20) and (21) with respect to τ, we obtain $\partial L_A/\partial\tau = -\beta/(1+\beta) < 0$ and $\partial L_m/\partial\tau = -1/(1+\beta) < 0$.

productivity under autarky (i.e., in a closed economy) as follows:

$$\frac{\dot{M}_t}{M_t} = \phi L_g L_m = \frac{\phi\tau[(1-\tau)A-\gamma]}{A(1+\beta)} > 0. \tag{23}$$

Note that we have already imposed the condition $(1-\tau)A-\gamma > 0$.

By using equations (10), (11), and (17), we obtain the indirect utility function under autarky $\tilde{u}_t^c$:

$$\tilde{u}_t^c = J + (1+\beta)\ln[(1-\tau)A-\gamma)] - \ln A + \ln M_t. \tag{24}$$

Equation (24) shows that a rise in agricultural productivity increases indirect utility under autarky.[10)]

Finally, we obtain total welfare under autarky during $t \in [0, \infty)$ from equation (24):

$$W^c = \int_0^\infty \tilde{u}_t^c e^{-\rho t}\, dt. \tag{25}$$

3 Small open economy

In this section, we consider the case of a small open economy. In other words, the small home country trades with the large rest of the world (ROW, hereafter). ROW's variables are distinguished from Home's variables by adding an asterisk "$*$." Home takes the world price P^* as given. ROW behaves like Home under autarky. We assume that $A^* > \gamma$ as in Home, that is, the agricultural productivity of ROW is higher than the subsistence level of agricultural consumption. In addition, we assume that $A \leq A^*$: the agricultural productivity of Home is equal to or lower than that of ROW.

10) By partially differentiating $\tilde{u}_t^c$ with respect to A, we obtain

$$\frac{\partial \tilde{u}_t^c}{\partial A} = \frac{\gamma}{A[(1-\tau)A-\gamma]} > 0.$$

For simplicity, we assume that ROW has the same utility function as Home and that ROW does not provide infrastructures. Here, an increase in ROW's manufacturing productivity M_t^*, is given by $\dot{M}_t^* = \delta^* X_{mt}^*$, where $\delta^* > 0$ denotes the efficiency of learning-by-doing, and its growth rate is given by

$$\frac{\dot{M}_t^*}{M_t^*} = \frac{\delta^*(A^* - \gamma)}{A^*(1 + \beta)}. \tag{26}$$

In addition, we assume that Home (ROW) has a comparative advantage in agriculture (manufacturing) at the initial time.

Assumption 1. *The following inequality holds:*

$$\frac{M_0^*}{A^*} > \frac{M_0}{A}. \tag{27}$$

Hence, if Home begins trading at the initial time, it specializes in agriculture according to the law of comparative advantage. Note that ROW produces both goods.

However, the manufacturing productivities of Home and ROW evolve over time, and hence the initial comparative advantage pattern can change. There are three possible specialization patterns in $t \in (0, \infty)$: (a) if $M_t^*/A^* > M_t/A$, Home specializes in agriculture; (b) if $M_t^*/A^* < M_t/A$, Home specializes in manufacturing; and (c) if $M_t^*/A^* = M_t/A$, Home diversifies, that is, it produces both goods.

Moreover, we assume the following rule with regard to industrialization.

Assumption 2. *If Home's government intends to industrialize, then it chooses to close the economy until it has a comparative advantage in manufacturing.*[11)]

11) Note that if, at $t = 0$, Home chooses to continue to be under autarky over time, instantaneous utility under autarky will be larger than that under specialization in agriculture at some point in time. Accordingly, total welfare under autarky can be larger than that under specialization in agriculture depending on the discount rate. However, if Home's government chooses to adopt the industrialization rule, total welfare under this rule necessarily exceeds that under autarky. For this issue, see Figure 4.

We define t_1 as the time when Home will have a comparative advantage in the manufacturing sector. Therefore, Home begins to open the economy at t_1. We now consider the following two cases. In the first case, we consider a situation in which Home specializes in the agricultural sector during $t \in [0, \infty)$. In this case, Home's government does not impose tax, that is, $\tau = 0$. In the second case, we consider a situation in which Home continues to be under autarky during $t \in [0, t_1]$ and engages in trade with ROW during $t \in (t_1, \infty)$.

3.1 Specialization in the agricultural sector

We consider the case in which Home specializes in agriculture: Home continues to specialize in the agricultural sector during $t \in [0, \infty)$. Hence, from the profit maximization condition, we obtain

$$1 = \frac{w_t}{A} \Longleftrightarrow w_t = A. \tag{28}$$

From equation (28) and $P_t^* = A^*/M_t^*$, we obtain the indirect utility function at t, $\tilde{u}_{at}^f$, as follows:

$$\tilde{u}_{at}^f = J + (1 + \beta) \ln(A - \gamma) - \ln A^* + \ln M_t^*. \tag{29}$$

Note that ROW's growth rate of manufacturing productivity $\dot{M}_t^*/M_t^*$ is positive and hence $\tilde{u}_{at}^f$ continues to increase. From equation (29), the total welfare of Home under specialization in agriculture, W_a^f, is given by

$$W_a^f = \int_0^\infty \tilde{u}_{at}^f e^{-\rho t} \, dt. \tag{30}$$

3.2 Industrialization

We consider the case in which Home's government decides to industrialize. For this policy to be feasible, Home must have a comparative advantage in the manufacturing sector at $t_1 \in (0, \infty)$. Hence, the condition for industrialization is given by

$$\frac{M_t}{A} \geq \frac{M_t^*}{A^*} \quad \text{for any } t \text{ such that } t_1 \leq t. \tag{31}$$

Accordingly, the condition $M_{t_1}^*/A^* = M_{t_1}/A$ gives the timing of opening trade. Therefore, t_1 is given by

$$t_1 = \frac{(\ln M_0^* - \ln M_0) - (\ln A^* - \ln A)}{\frac{\phi\tau[(1-\tau)A - \gamma]}{A(1+\beta)} - \frac{\delta^*(A^* - \gamma)}{A^*(1+\beta)}}. \tag{32}$$

The numerator of equation (32) is positive because equation (27) holds. Next, the denominator of equation (32) shows the difference between the growth rate of the manufacturing productivity of Home under autarky and the growth rate of the manufacturing productivity of ROW. Unless this difference is positive, Home cannot industrialize. Hence, Home cannot have a dynamic comparative advantage in the manufacturing sector over time: there is no timing of opening trade. By contrast, if the denominator of equation (32) is positive, then there exists $t_1 \in (0, \infty)$. Accordingly, we obtain the following lemma with regard to the tax rate.

Lemma 1. *For Home to have a comparative advantage in manufacturing at time t_1, Home's government must impose the tax rate that satisfies the following inequality:*

$$\underline{\tau} \equiv \frac{(A-\gamma) - \sqrt{(A-\gamma)^2 - \frac{4\delta^* A^2 (A^*-\gamma)}{\phi A^*}}}{2A} < \tau < \frac{(A-\gamma) + \sqrt{(A-\gamma)^2 - \frac{4\delta^* A^2 (A^*-\gamma)}{\phi A^*}}}{2A} \equiv \overline{\tau}. \tag{33}$$

Proof. For t_1 to be positive, the denominator of the right-hand side of equation (32) must be positive. Note that the denominator is a quadratic function of τ. Accordingly, by solving the condition that the denominator is positive, we obtain Lemma 1. ■

For the inequality $\underline{\tau} < \tau < \overline{\tau}$ to be meaningful, the term in the square root of equation (33) has to be positive:

$$(A - \gamma)^2 > \frac{4\delta^* A^2 (A^* - \gamma)}{\phi A^*}, \tag{34}$$

which leads to a quadratic inequality of γ. This poses restrictions on the range of γ.

Assumption 3. *The following inequalities hold:*

$$\phi > 4\delta^* \textit{ and } 0 < \gamma < A\left[\left(1 - \frac{2\delta^* A}{\phi A^*}\right) - \sqrt{\left(1 - \frac{2\delta^* A}{\phi A^*}\right)^2 - \left(1 - \frac{4\delta^*}{\phi}\right)}\right] \equiv \overline{\gamma}.$$

In other words, the efficiency parameter of the public sector in Home ϕ must be sufficiently larger than the efficiency of learning-by-doing in ROW δ^* and in addition, the subsistence level must be less than the threshold value $\overline{\gamma}$.

When Home specializes in manufacturing, the employment share of the manufacturing sector and that of the public sector are respectively given by

$$L_m = 1 - \tau, \tag{35}$$

$$L_g = \tau. \tag{36}$$

Hence, the growth rate of manufacturing productivity leads to

$$\frac{\dot{M}_t}{M_t} = \phi L_g L_m = \phi\tau(1 - \tau), \quad \text{for } t \in [t_1, \infty). \tag{37}$$

From the above analysis, Home's manufacturing productivities during $t \in [0, t_1]$ and during $t \in [t_1, \infty)$ are respectively given by

$$M_t = \begin{cases} M_0 \exp\left[\dfrac{\phi\tau[(1-\tau)A-\gamma]}{A(1+\beta)} \cdot t\right], & \text{for } t \in [0, t_1], \\ M_{t_1} \exp\left[\phi\tau(1-\tau)\cdot(t-t_1)\right], & \text{for } t \in [t_1, \infty), \end{cases} \tag{38}$$

$$\text{where } M_{t_1} \equiv M_0 \exp\left[\frac{\phi\tau[(1-\tau)A-\gamma]}{A(1+\beta)} \cdot t_1\right].$$

[Figure 1 around here]

Figure 1 shows the time paths of the logarithms of the relative productivities of Home and ROW. The real line corresponds to ROW, while the dashed and dotted lines correspond to Home. Home continues to be under autarky until t_1 and then it continues to engage in free trade: from t_1, Home specializes in the manufacturing sector.

We investigate the relationship between the tax rate and timing of opening trade. The denominator of equation (32) is convex upward with respect to τ. From equation (32), we obtain

$$\frac{dt_1}{d\tau} = 0 \iff \tau = \frac{A-\gamma}{2A} > 0. \tag{39}$$

Hence, there is the tax rate minimizing the autarkic period $t \in [0, t_1]$.

Equation (32) is clearly a decreasing function of ϕ and hence as the efficiency parameter of the public sector ϕ rises, the timing of opening trade t_1 quickens.

Furthermore, the effect of an increase in agricultural productivity on the timing of opening trade is ambiguous because, if we differentiate t_1 with respect to A, we obtain the following expression:

$$\frac{dt_1}{dA} = \frac{\overbrace{\frac{\Omega}{A}}^{\text{ICA effect}} \overbrace{- \ln\left(\frac{A}{M_0} \cdot \frac{M_0^*}{A^*}\right) \cdot \frac{\phi\tau\gamma}{A^2(1+\beta)}}^{\text{LRA effect}}}{\Omega^2}, \quad \text{where } \Omega \equiv \frac{\phi\tau[(1-\tau)A - \gamma]}{A(1+\beta)} - \frac{\delta^*(A^* - \gamma)}{A^*(1+\beta)} > 0. \tag{40}$$

Note that the term $\ln[(A/M_0) \cdot (M_0^*/A^*)] > 0$ because of Assumption 1. The effect of an increase in agricultural productivity on the timing of opening trade is decomposed into two effects. The first term of the numerator on the right-hand side shows the "initial comparative advantage effect" (ICA effect), while the second term of the numerator shows the "labor reallocation effect" (LRA effect). The ICA effect means that a rise in A increases the degree of Home's ICA in agriculture, thus making it more difficult to have a comparative advantage in manufacturing. Accordingly, the ICA effect delays the timing of opening trade. The LRA effect means that an increase in A releases labor from the agricultural sector to the manufacturing sector, thereby intensifying learning-by-doing in manufacturing from equation (3) and increasing manufacturing productivity M_t. Accordingly, the LRA effect accelerates the timing of opening trade. Therefore, if the LRA effect dominates the ICA effect, then an increase in agricultural productivity accelerates the timing of opening trade. By contrast, if the ICA effect dominates the LRA effect, then an increase in agricultural productivity delays the timing of opening trade.

Moreover, if $\gamma = 0$, an increase in agricultural productivity delays the timing of opening trade from equation (40) because the LRA effect vanishes.

Lemma 2. *If $\gamma = 0$, an increase in agricultural productivity necessarily delays the timing of opening trade.*

Lemma 2 is similar to the result of Redding (1999). Therefore, the non-homotheticity of preferences, that is, $\gamma > 0$, generalizes the result of Redding (1999). If agricultural productivity is extremely low, the second term of equation (40) is very large. Note that Matsuyama (1992) and Chang et al. (2006) use the Stone–Geary utility function but do not discuss the timing of opening trade.

[Figures 2 and 3 around here]

Figures 2 and 3 numerically reveal the relationship between the effect of an increase in agricultural productivity and the timing of opening trade. The parameters are set as follows: $A = 1$, $A^* = 1$, $M_0 = 0.5$, $M_0^* = 1$, $\phi = 1$, and $\delta^* = 0.05$.

Many studies introducing the subsistence level of agricultural consumption and conducting numerical simulations set the parameter γ so that the employment share of agriculture in the models can fit the actual values of the employment share of agriculture. In those studies, according to which country is the target of the analysis or which year is a reference standard, various values of γ are used. Our model does not confine our analysis to a specific country. In addition, it is not our objective to fit our model to the actual data on a specific country. Accordingly, it is difficult to specify γ by using calibration. For this reason, we take values for γ from previous studies. For example, Duarte and Restuccia (2010) and Gollin et al. (2004) use 0.11 and 0.393 for γ, respectively. Following these studies, we use $\gamma = 0.1$ and $\gamma = 0.4$ for the numerical simulations.

The above numerical values of the parameters satisfy the conditions given by Assumption 3, that is, $A \leq A^*$ and $\phi > 4\delta^*$. In addition, we have $0 < \gamma < \overline{\gamma} = 0.8$. Hence, $\gamma = 0.1$ and $\gamma = 0.4$ satisfy the existence condition of (33). Moreover, from Equation (33), we have $0.053 < \tau < 0.847$ for $\gamma = 0.1$ and $0.055 < \tau < 0.545$ for $\gamma = 0.4$. From the condition $\tau < (A - \gamma)/A$, we have $0 < \tau < 0.9$ for $\gamma = 0.1$, which is broader than the restriction $0.053 < \tau < 0.847$, and $0 < \tau < 0.6$ for $\gamma = 0.4$, which is broader than the restriction $0.055 < \tau < 0.545$. Accordingly, in our setting, $\underline{\tau} < \tau < \overline{\tau}$ is more restrictive than $0 < \tau < (A - \gamma)/A$.

If $\gamma = 0.1$, an increase in agricultural productivity from $A = 1$ to $A = 1.5$ almost delays the timing of opening trade: the ICA effect is larger than the LRA effect. However, if $\gamma = 0.4$, an increase in agricultural productivity accelerates and delays the timing of opening trade depending on the size of the tax rate: the ICA effect is smaller than the LRA effect.

The relative world price of manufactured goods determines w_t. When Home specializes in the manufacturing sector, the profit maximization condition gives

$$P_t^* = \frac{w_t}{(1-\tau)M_t}. \tag{41}$$

Since $P_t^* = A^*/M_t^*$ under free trade, from equation (41), we obtain

$$w_t = \frac{(1-\tau)A^* M_t}{M_t^*}. \tag{42}$$

By substituting equation (42) and $P_t^* = A^*/M_t^*$ into equation (17), we obtain the instantaneous indirect utility function for $t > t_1$ as follows:

$$\text{for } t \in [t_1, \infty) \quad \tilde{u}_{mt}^f = J + (1+\beta)\ln\left[\frac{(1-\tau)A^* M_t}{M_t^*} - \gamma\right] - \ln A^* + \ln M_t^*. \tag{43}$$

We explain the intuition of equation (43). The effect of M_t^* is decomposed into the following two opposing effects. An increase in M_t^* lowers the relative world price of manufactured goods and hence Home's instantaneous utility improves. By contrast, an increase in M_t^* reduces Home's comparative advantage in the manufacturing sector and hence Home's instantaneous utility declines. In addition, we know that an increase in agricultural productivity does not affect Home's instantaneous utility during $t \in (t_1, \infty)$.

Home's welfare during $[t_1, \infty)$, W_m^f, is given by

$$W_m^f = \int_{t_1}^{\infty} \tilde{u}_{mt}^f e^{-\rho t}\, dt. \tag{44}$$

Hence, Home's total welfare over time, W_m, is given by

$$W_m = W^c + W_m^f = \int_0^{t_1} \tilde{u}_t^c e^{-\rho t}\, dt + \int_{t_1}^{\infty} \tilde{u}_{mt}^f e^{-\rho t}\, dt. \tag{45}$$

4 Welfare analysis

4.1 Welfare

In this section, we compare W_a^f with W_m. Figure 4 shows the time paths of instantaneous utility. In the case of industrialization, Home specializes in the manufacturing sector at t_1. Note that Home's instantaneous utility under specialization in manufacturing during $t \in [t_1, t_2]$ is smaller than that under specialization in agriculture during $t \in [t_1, t_2]$. However, from t_2 on, the growth rate of $\tilde{u}_{mt}^f$ is larger than that of $\tilde{u}_{at}^f$ and, accordingly, the level of $\tilde{u}_{mt}^f$ becomes larger than that of $\tilde{u}_{at}^f$. Hence, depending on the size of the discount rate ρ, W_m can be larger than W_a^f.

Proposition 1. *Total welfare under industrialization can be larger than total welfare under specialization in agriculture depending on the size of the discount rate.*

[Figure 4 around here]

4.2 Optimal tax rate

In this section, we examine the existence of the optimal tax rate maximizing W_m. However, the computation of equation (45) and partial derivative of the resultant expression with respect to τ, $\partial W_m/\partial \tau$, are very complicated; accordingly, we use numerical simulations. The reason for the existence of the optimal tax rate is explained below.

Figures 5 and 6 show the existence of optimal tax rates in the cases of $\gamma = 0.1$ and $\gamma = 0.4$, respectively.

[Figures 5 and 6 around here]

In addition, Figure 5 shows that an increase in agricultural productivity can reduce total welfare, while Figure 6 shows that an increase in agricultural productivity improves total welfare. The reason for this is as follows. As stated above, an increase in agricultural productivity has two effects: the ICA effect and the LRA effect. The ICA effect decreases total welfare because an increase in agricultural productivity directly delays the timing of opening trade. By contrast, the LRA effect shows that an increase in agricultural productivity reallocates labor from the agricultural sector to the manufacturing and public sectors, thereby improving total welfare. Therefore, in Figure 5, the ICA effect dominates the LRA effect, while in Figure 6, the LRA effect dominates the ICA effect.[12)]

Proposition 2. *The optimal tax rate maximizes total welfare under industrialization.*

Proposition 3. *An increase in agricultural productivity increases or decreases total welfare under industrialization depending on the size of the subsistence level of agricultural consumption.*

In our model, instantaneous utility continues to increase over time because manufacturing productivity continues to increase over time. Note that the growth rate of manufacturing productivity is a quadratic function convex upward with respect to the tax rate both before and after industrialization. Accordingly, both too high and too low tax rates decrease the growth rate of manufacturing productivity. In other words, roughly speaking, a tax rate that maximizes the growth rate of manufacturing productivity might correspond to a tax rate that maximizes total welfare over time.

However, this inference is not strict. Note that a tax rate that maximizes the growth rate of manufacturing productivity under autarky, τ_a, is given by differentiating equation (23)

12) Moreover, the LRA effect does not exist in the case of $\gamma = 0$. Then, in this case, an increase in agricultural productivity always reduces total welfare.

with respect to τ:

$$\tau_a = \frac{A-\gamma}{2A}, \tag{46}$$

which is exactly the same as equation (39), and a tax rate that maximizes the growth rate of manufacturing productivity under specialization in manufacturing, τ_m, is given by differentiating equation (37) with respect to τ:

$$\tau_m = \frac{1}{2}. \tag{47}$$

From equations (46) and (47), we find that productivity growth-maximizing tax rates do not depend on the discount rate ρ. By contrast, from equation (45), we find that the optimal tax rate that maximizes total welfare depends on the discount rate. Hence, the welfare-maximizing tax rate is not equal to a productivity growth-maximizing tax rate.

Next, we investigate the relationship between total welfare and the timing of opening trade. From Figures 2, 3, 5, and 6, we know that the optimal tax rate maximizing W_m is different from the tax rate minimizing the timing of opening trade because providing infrastructures sacrifices consumption, which contributes to lowering total welfare. In addition, the loss of consumption under autarky is larger than that under specialization in the manufacturing sector. If the discount rate is large, the loss under autarky is also large and hence the optimal tax rate is smaller than the tax rate minimizing the timing of opening trade.

Lemma 3. *The optimal timing of opening trade maximizes total welfare and this timing is obtained from the optimal tax rate.*

Wong and Yip (2010) also construct a dynamic Ricardian trade model and show that by intentionally delaying the timing of opening trade, an economy succeeds in industrialization. However, while they argue in favor of a dynamic comparative advantage, they do not refer to Redding (1999). They assume that under autarky, the government subsidizes the manufacturing sector to promote industrialization. On the one hand, this policy attracts workers from agriculture to manufacturing, which intensifies learning-by-doing in the manufacturing sector, strengthening its potential comparative advantage. On the other hand, after opening trade (i.e., industrialization), all workers are employed in the manufacturing sector and hence the optimal policy is not to subsidize.

By contrast, in our model, under both autarky and industrialization, when the tax rate is zero, employment in the public sector is zero; hence, the growth rate of the manufacturing sector is zero. Then, the government must impose a positive tax rate to sustain specialization in manufacturing after industrialization. To sum up, in Wong and Yip (2010), there is no government policy after industrialization, whereas in our model there is government policy after industrialization.

5 Conclusion

We constructed a dynamic Ricardian trade model that incorporates the public provision of infrastructures. We showed that an increase in agricultural productivity plays an important role in industrialization. The results are summarized as follows.

First, there is the optimal tax rate maximizing the total welfare. Second, if the level of basic consumption is positive, an increase in agricultural productivity can accelerate the timing of opening trade. Third, if the level of basic consumption is zero, an increase in agricultural productivity delays the timing of opening trade and decreases total welfare. Fourth, an increase in the efficiency of the public provision of infrastructures accelerates the timing of opening trade. This result does not depend on the existence of basic consumption. Fifth, at the timing of opening trade, instantaneous utility under industrialization is lower than that under specialization in agriculture. However, the growth rate of instantaneous utility under industrialization is larger than that under specialization in agriculture. Therefore, depending on the size of the discount rate, total welfare under industrialization can be larger than that under specialization in agriculture.

Some extensions are left for future research. First, capital accumulation could be introduced. In our model, labor is the only factor of production in both agriculture and manufacturing. However, manufactured goods are more capital-intensive than agricultural goods. Accordingly, the introduction of capital accumulation would be an interesting research direction.

Second, the tax rule could be modified. In our model, the tax rule is constant over time, that is, the government imposes a constant fixed tax rate during both autarky and industrialization. However, as a more general tax rule, it is possible that the government imposes different tax rates during autarky and industrialization.

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Figures

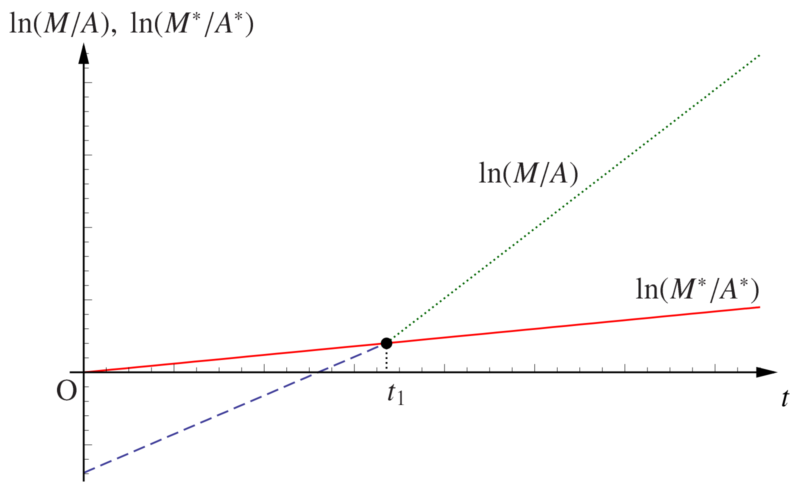

Figure 1: Time paths of relative productivities

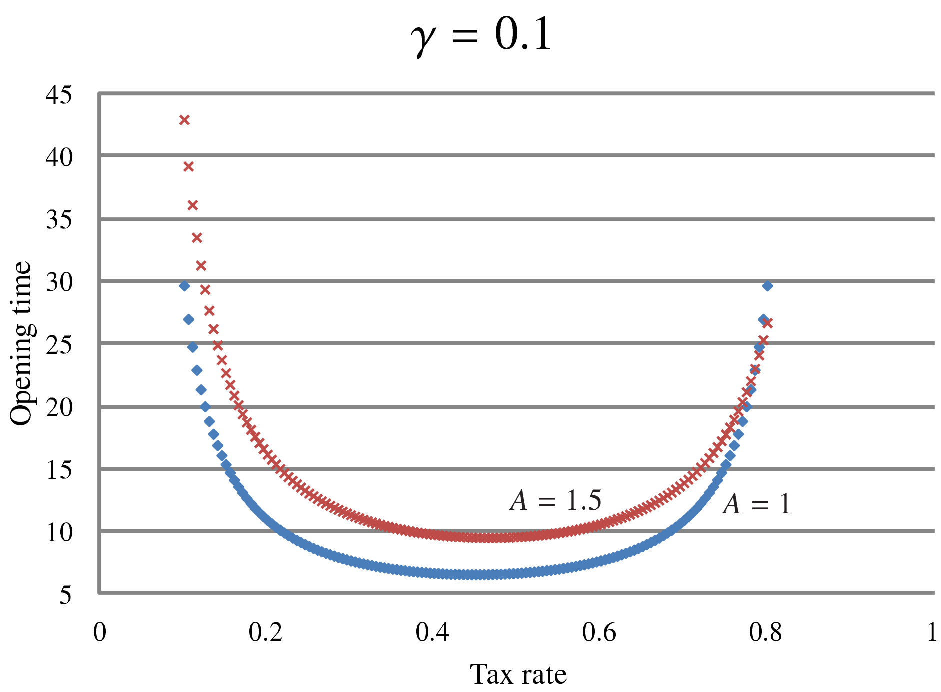

Figure 2: Relationship between agricultural productivity and the timing of opening trade when $\gamma = 0.1$

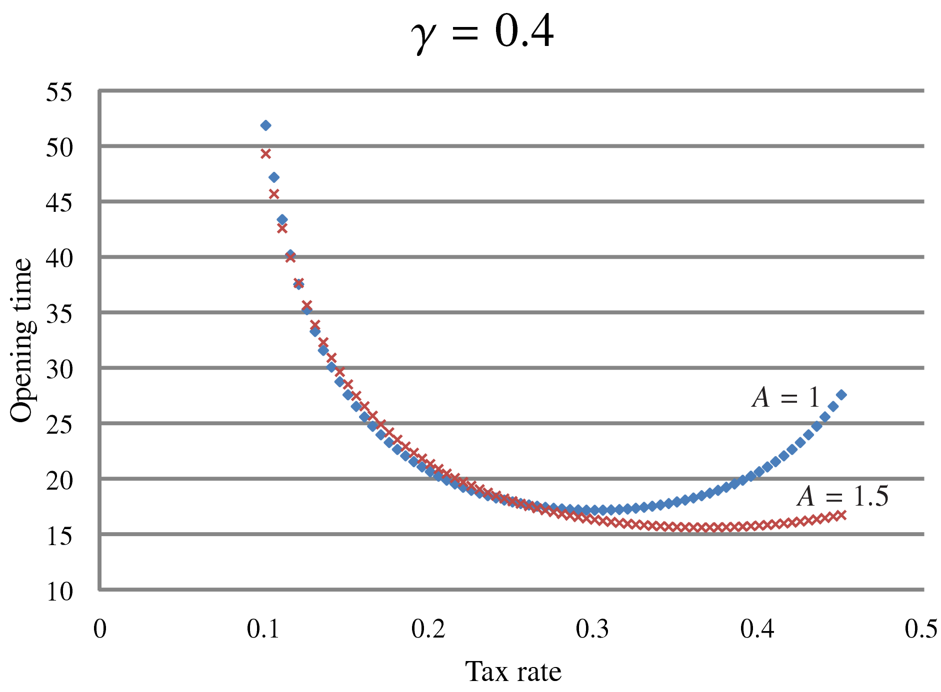

Figure 3: Relationship between agricultural productivity and the timing of opening trade when $\gamma = 0.4$

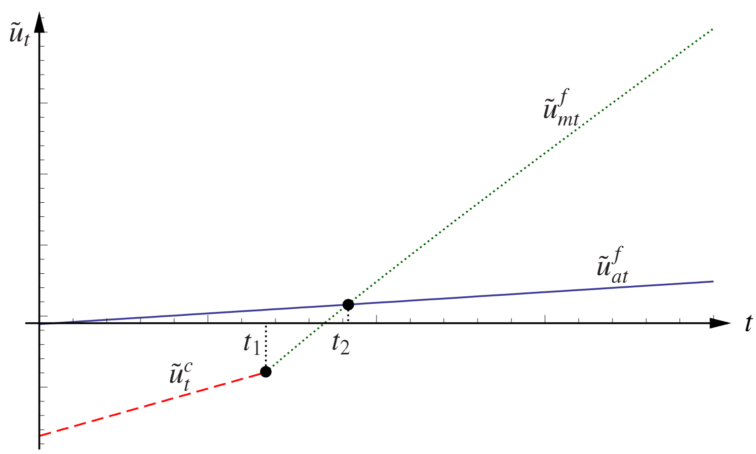

Figure 4: Time paths of instantaneous utilities

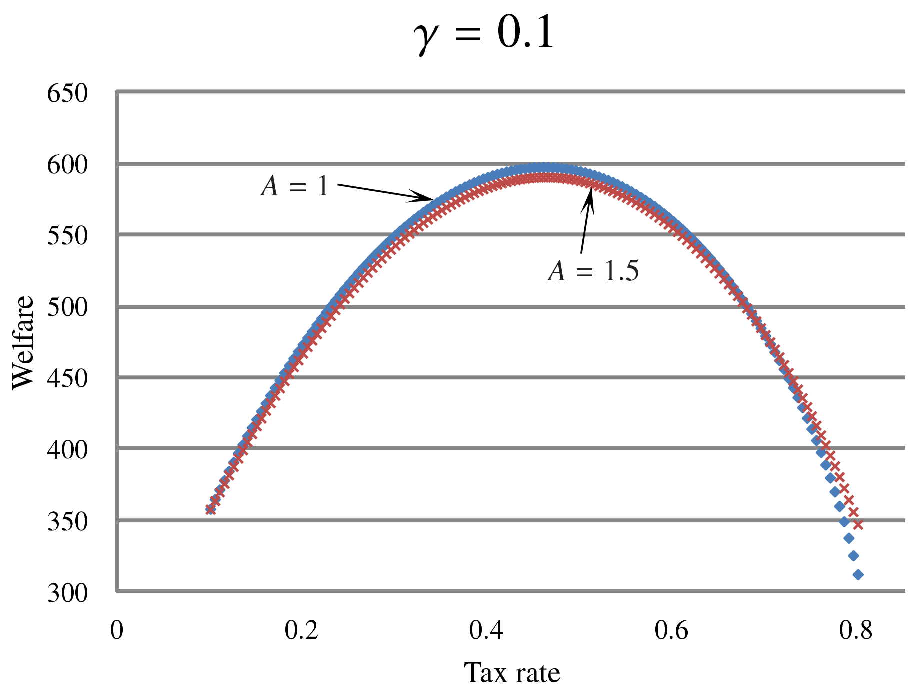

Figure 5: Optimal tax rate, welfare, and agricultural productivity when $\gamma = 0.1$

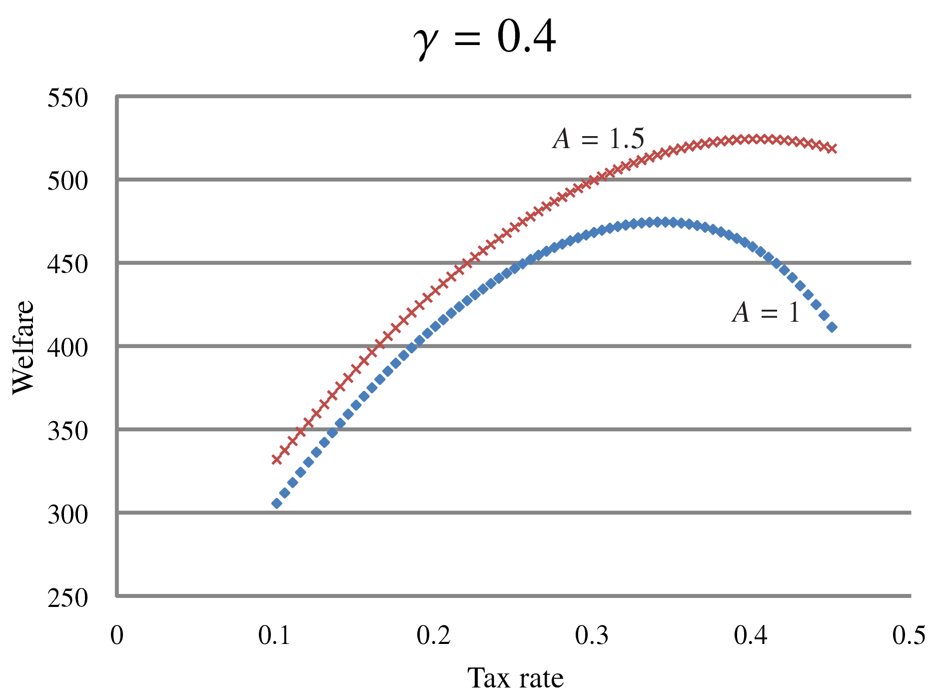

Figure 6: Optimal tax rate, welfare, and agricultural productivity when $\gamma = 0.4$