

# The endogenous decisions of unionization and international trade in general oligopolistic equilibrium

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## **Abstract**

This paper develops a multi-sector general oligopolistic equilibrium trade model in which unionized and non-unionized sectors interact.

We assume that the proportion of unionized sectors as an endogenous parameter. We show that the proportion of unionized sectors depends on such exogenous parameters as population and union cost. As a result, the increase in population size raises the proportion of unionized sectors and lowers the competitive wage, whereas the increase in the number of firms and the increase in the union cost lower the proportion of unionized sectors and raise the competitive wage.

We also show that trade openness between symmetric countries raises the competitive wage and lowers the proportion of unionized sectors, whereas the effect on the welfare is ambiguous.

Keywords: labor union, international trade, general oligopolistic equilibrium

JEL classification F15, F16, L13

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## 1. Introduction

The rates of labor union participation are different among countries. For example, rates of labor union participation are low in U.S., Japan, and Korea, while those are high in Iceland, Sweden, and Denmark.<sup>1</sup> We can also observe that the rates of labor union participation are different among industries. For example, the rates of labor union participation are high in utilities industry and financial and insurance industry, while those are low in agriculture, real estate, and service sectors, in Japan.<sup>2</sup> The rates of those are high in utilities and transportation industry in U.S.<sup>3</sup> Our purpose in this paper is to point out factors which bring about those international and inter-industry differences of rates of labor union participation.

We develop the general oligopolistic equilibrium model (GOLE) (Neary 2009) in which there are several industries. In each industry, there are firms which are unionized or non-unionized. In our model, the unionized firms rates to the total firms are endogenously determined. The labor unions bargain with firms and raise the wage of workers. We assume that when the difference between the union wage and non-union wage exceeds union costs shared by workers, union is organized. We study factors, (like union costs, population, and etc.) which influence on these rates. In addition, we introduce heterogeneous productivity across industry following Melitz (2003). We study the effects of productivity on unionized firms rates in the industry.

Our model shows that several factors prompt the organization of labor union. First factor is low union costs. Union costs involves not only an activity fund but also efforts for bargaining over wages. Second factor is the large rents for firms. If firm does not have the large rents, a high wage claim brings about negative profits for firms. Consequently, the large rents for firms are necessary for the union activity. The large rents are generated by the high productivity of firms. Thus, the labor union tend to be organized in the high productivity firms. In fact, a positive correlation is observed between labor union participant rates and labor productivities in Japan. The industries with high rates of labor union participation, like utilities, financial, and insurance have relatively high labor productivities while the sector with low rates of those, agriculture and service sectors have relatively low labor productivity.<sup>4</sup> A lot of literatures pointed out the positive effect by labor unions on productivities (Addison and Hirsch 1989, Booth 1995). This paper theoretically presents the converse causality relationship, namely, that high productivity attracts the labor union.

Next, we analyze an effect of globalization on labor unions. In Japan, the number of labor union members has been little changed, but the percentage of participation in union has notably decreased. In half century, the participation rate dropped to less than about 20% from 50%. In the whole world

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<sup>1</sup> OECD (2014).

<sup>2</sup> Ministry of Health, Labour and Welfare Japan 2014

<sup>3</sup> U.S. Bureau of Labor Statistics 2014 Economic News Release

<sup>4</sup> Cabinet Office, Government of Japan 2014

also, this tendency of decrease in the rates of labor union participation can be observed (OECD 2014). Generally, globalization brings about more intense competition and decreases rents of firm. Accordingly, globalization would induce deunionization. A lot of literatures study the effect of globalization on labor union, and this paper would be one of the theoretical research which study the effect of globalization on labor union participation rates.

This issue of International trade and labor union has been studied in several literatures with an international oligopoly model (Mezzetti and Dinopoulos 1991; Naylor 1998, 1999; Lommerud et al 2003). They constructed partial equilibrium models, and the competitive wages are exogenously given. We extend those researches to study the relationship between international trade and labor unions with a general equilibrium model.

We follow the general oligopolistic equilibrium model (GOLE) (Neary 2009). There are several literatures using this model (Bastos and Kreickemeier 2009; Egger and Etzel 2012, 2014). These papers analyze unionized labor market in general equilibrium models, but the rate of unionized firms to the total number of firms is an exogenous parameter. Accordingly, these papers do not present the mechanism of the decrease in the rates of unionized firms. So, treating the rates of unionized sectors as an endogenous parameter, we analyze the decreasing trend of organization of unionized firms.

## 2. The model

### 2.1 Preference and consumer demand

We assume that the representative utility function is an additively separable over a continuum of different goods with each subutility function quadratic,

$$U[\{x(z)\}] = \int_0^1 \left[ ax(z) - \frac{1}{2} bx^2(z) \right] dz \quad (1)$$

where  $x(z)$  denotes consumption of good  $z$ . The budget constraint of the representative consumer is given by

$$\int_0^1 p(z)x(z)dz \leq I \quad (2)$$

where  $p(z)$  denotes price of good  $z$ , and  $I$  is aggregate income.

Maximizing Eq. (1) subject to budget constraint for each good, gives the inverse demand function for good  $z$ :

$$P(z) = \frac{1}{\lambda} (a - bx(z)), \quad x(z) = \frac{a - \lambda p(z)}{b}, \quad (3)$$

$$\lambda[\{p(z)\}, I] = \frac{a\mu_p - bI}{\sigma_p^2}$$

where  $\lambda$  is marginal utility of income, and  $\mu_p$  and  $\sigma_p$  are the first and second moment of prices, respectively. The  $\mu_p$  and  $\sigma_p$  are given by

$$\mu_p \equiv \int_0^1 p(z)dz, \quad \sigma_p^2 \equiv \int_0^1 p^2(z)dz.$$

Furthermore, substituting  $x(z)$  into Eq. (1), we can derive the indirect utility function as follow:

$$\tilde{U} = \frac{a^2 - \lambda^2 \sigma_p^2}{2b}. \quad (4)$$

Hence, consumer welfare is decreasing in the second moment of prices.

## 2.2 Technology and production

We choose consumer's marginal utility of income as numéraire and set  $\lambda$  equal to one. This is the standard procedure in the GOLE literatures (see Neary 2009, Bastos and Kreickemeier 2009, and Egger and Etzel 2012, 2014). Therefore, here after, wages, prices, union utility, and profits are weighted by the marginal utility of income.

In a country, there is continuum  $[0,1]$  of industries and each industry produces a differentiated good and has  $n$  symmetric firms. Hence, firms relatively are large in their industry but are infinitesimal in the economy as a whole. Firms use labor to produce a homogenous output and compete in quantity in their industry. Output is linear in the labor input:  $y = l/\alpha(z)$  where  $\alpha(z)$  denotes the labor input coefficient in industry  $z$ . In an industry, firm's productivities are identical. We assume that  $\alpha(z)$  is an increasing function of  $z$ . Firms compete as Cournot competition in each industry. Thus, the profit function of firm  $j$  is given by

$$\pi_j(z) = \left[ a - b \sum_{i=1}^n y_i(z) - c_j(z) \right] y_j(z), \quad (5)$$

$$c_j(z) = \alpha(z)w_j(z).$$

Maximizing the profit, outputs of firm  $j$  is given by

$$y_j(z) = \frac{a + (n-1)\alpha(z)w_i(z) - n\alpha(z)w_j(z)}{b(n+1)}. \quad (6)$$

Hence, the employment per firm is given by

$$l_j(z) = \alpha(z)y_j(z) = \alpha(z) \frac{a + (n-1)\alpha(z)w_i(z) - n\alpha(z)w_j(z)}{b(n+1)}.$$

### 2.3 Labor union

The workers are identical in all respects, but their wages depend on features of their industry. The workers receive union wage which their labor union sets, when labor unions are present in their industry. But, workers receive the competitive wage that is common wage of non-unionized sectors, when labor union is not organized in their industry.

We assume that union is firm level and workers can organize a union in each firm. When the union is organized in a firm, all workers of their firm belong to this union. Namely, in the unionized sectors, all workers of the sectors are unionized.

We introduce a Stone-Geary function to represent union's preference. We assume that to organize an union requires a fixed cost  $f$ . Hence, the union utility can be written as

$$V_j(z) = (w_j(z) - w^c)l_j(z) - f \quad (7)$$

where  $w$  is the union wage and  $w^c$  is the competitive wage (non-union wage). In other word, each union sets the union wage that maximizes the union utility. The fixed union cost is interpreted as a maintenance cost of union or a bargaining cost for unilaterally setting wages. All workers equally share the fixed union cost as the union due in their firm.

## 3. Solving the equilibrium in the closed economy

### 3.1 Game structure

The proportion of unionized sectors and the competitive wage are determined by the outcome of a three stages game in equilibrium. In the 1st stage, workers decide whether to organize labor union. They organize a labor union when the net union wage (the union wage minus the union due) is higher than the competitive wage. In the 2nd stage, each union sets unilaterally the union wage  $w$  in unionized sectors taking  $w^c$  as given. In non-unionized sector, all workers receive the same competitive wage  $w^c$ . Unionized and non-unionized workers are identical, but their wages depend on their sectors. We assume that there are not any unemployment. In the 3rd stage, each firm decides output taking wages and competitor's outputs as given. We solve a subgame perfect Nash equilibrium (SPNE) by backward induction.

From Eq. (6), outputs of firm  $j$  is

$$y_j(z) = \frac{a + (n-1)\alpha(z)w_i(z) - n\alpha(z)w_j(z)}{b(n+1)}.$$

Substituting outputs to Eq. (7), and maximizing union utility function, the union wage of firm  $j$  is given by

$$w_j(z) = \frac{a + n\alpha(z)w^c}{(n+1)\alpha(z)} \equiv w(z). \quad (8)$$

The workers decide whether to organize a labor union by comparing the competitive wage and the net union wage  $w^f$ . Since the amount of production demand does not depend on the income, a change of the wages does not influence firm's outputs and the outcome of union wage setting. The following equation is an arbitrage condition of the union organization:

$$w^f \equiv w(z) - \frac{f}{l(z)} = w^c.$$

The condition for the organization of the union is as follow:

$$w^f > w^c.$$

We substitute Eqs. (6) and (8) into the above equation. Hence, the condition of organization of labor union is given by

$$w^c < \frac{a - B}{\alpha(z)}, \quad B \equiv \sqrt{\frac{b}{n}(n+1)^3 f}$$

The workers have an incentive to organize a labor union in sector  $z$  when this condition is satisfied. Hence, the threshold  $\tilde{z}$  that divides unionized and non-unionized sector is determined endogenously by this condition. Therefore, the arbitrage condition of organizing a labor union is given by

$$w^c = \frac{a - B}{\alpha(z)}. \quad (9)$$

Firms in the same industry are identical and produce the same amount of outputs,  $y_j(z) = y_i(z)$ . Hence, the wage is the same in the same industry,  $w_j(z) = w_i(z)$ .

Then, we substitute Eq. (8) into Eq. (6) to derive output per unionized and non-unionized firm. We assume that  $a - \alpha(z)w^c > 0$  because the outputs cannot be a negative value. Outputs are given by

$$y^U = \frac{n(a - \alpha(z)w^c)}{b(n+1)^2}, \quad y^{NU} = \frac{a - \alpha(z)w^c}{b(n+1)}. \quad (10)$$

The presence of union lowers the employment level of a firm because  $y^U = y^{NU} \frac{n}{n+1}$  holds.

Differentiating Eq. (9) with  $z$ , the arbitrage condition of organizing a labor union is a decreasing function of  $z$ ;  $dw^c/dz < 0$ . This means that the workers have a large incentive to

organize a labor union in high productivity industry, but the workers do not have an incentive to organize a union in low productivity industry when the competitive wage is too large. Hence,  $z \in [0, \tilde{z})$  sectors are unionized, and  $z \in [\tilde{z}, 1]$  sectors are non-unionized (see Fig. 1).

### Proposition 1

$z \in [0, \tilde{z})$  sectors are unionized, and  $z \in [\tilde{z}, 1]$  sectors are non-unionized.

### 3.2 Labor market

In a country, there are  $L$  workers and we assume that there are not any unemployments. Therefore, the labor market clearing condition is given by

$$L \equiv \int_0^1 nl(z)dz = \int_0^{\tilde{z}} n\alpha(z)y^U(z)dz + \int_{\tilde{z}}^1 n\alpha(z)y^{NU}(z)dz.$$

We substitute the number of worker of each unionized and non-unionized sector to the labor market clearing condition. Solving this condition, the competitive wage is given by,

$$w^c = \frac{na\mu_1 + a \int_{\tilde{z}}^1 \alpha(z)dz - \frac{b}{n}(n+1)^2 L}{\mu_2 n + \int_{\tilde{z}}^1 \alpha^2(z)dz}, \quad (13)$$

$$\mu_1 \equiv \int_0^1 \alpha(z)dz, \quad \mu_2 \equiv \int_0^1 \alpha^2(z)dz.$$

The Eq. (13) is a decreasing function of  $\tilde{z}$ ;  $dw^c/d\tilde{z} < 0$ .

### 3.3 Equilibrium

Consider the equilibrium  $\tilde{z}^*$  and  $w^{c*}$ . The equilibrium needs to satisfy labor market clearing condition. When net union wage is larger than the competitive wage (i.e.  $w^c < (a - B)/\alpha(z)$  holds), workers organize a labor union.

We can divide equilibrium into the four cases<sup>5</sup> by the positional relationship of Eqs. (9) and (13). First, we divide the cases whether Eqs. (9) and (13) intersect or not. Second, we divide the cases by the vertical position relationship of Eqs. (9) and (13).

We define that the case of intersection is that Eqs. (9) and (13) have an intersection point in  $\tilde{z} \in$

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<sup>5</sup> There is another case with two intersections of Eq. (9) and Eq. (13), but we omit it here because the range of the case is very small. For details, see Appendix.

$(0, 1)$ , and we can derive equilibrium  $(\tilde{z}^*, w^{c*})$ . We also define that the case of non-intersection is that Eqs. (9) and (13) do not have an intersection point in  $\tilde{z} \in (0, 1)$ . So, in the case of non-intersection, the equilibrium is either of corner equilibrium  $(0, w^{c*})$  or  $(1, w^{c*})$ .

There are two cases of intersection as follow. The one case is that the slope of Eq. (13) is larger than the slope of Eq. (9) on the intersection point of Eqs. (9) and (13). Figure 2 represents this case. The other case is that slope of Eq. (13) is smaller than the slope of Eq. (9) on the intersection point. Figure 3 represents this case.

In the case of Figure 2, equilibrium  $(\tilde{z}^*, w^{c*})$  is the intersection point  $E$ . This point is stable because workers do not have an incentive to organize the union in the sector  $z \in (\tilde{z}^*, 1]$ . If workers organize unions in the sector  $\tilde{z}^* + \Delta$  ( $\Delta$  is sufficient small and positive), their net union wages is lower than the competitive wage. Therefore, labor union is not organized in the sector  $\tilde{z}^* + \Delta$ . Similarly, workers do not have an incentive to dissolve the union in the sector  $\tilde{z}^* - \Delta$  because their net union wages are larger than the competitive wage. As a result, the point  $E$  at which  $d(w^f - w^c)/d\tilde{z} < 0$  holds is an interior equilibrium  $(\tilde{z}^*, w^{c*})$  and this equilibrium is stable.

In the case of Figure 3, the intersection point  $E'$  is unstable because at the point  $E'$   $d(w^f - w^c)/d\tilde{z} > 0$  holds. The corner point  $E$  and  $E''$  are stable because at those points  $d(w^f - w^c)/d\tilde{z} < 0$  holds. Hence,  $E$  and  $E''$  are stable corner equilibria  $(0, w^{c*})$  and  $(1, w^{c*})$ .

There are two cases of non-intersection as follow. The one case is that Eq. (9) is always higher than Eq. (13) in  $\tilde{z} \in [0, 1]$  which represents the case of Figure 4. The other case is that Eq. (9) is always lower than Eq. (13) in  $\tilde{z} \in [0, 1]$  which represents the case of Figure 5.

In the case of figure 4, the point  $E$  is stable because at this point  $d(w^f - w^c)/d\tilde{z} < 0$  holds. Hence, equilibrium  $(\tilde{z}^*, w^{c*})$  is the corner equilibrium  $(1, w^{c*})$ .

Similarly, in the case of Figure 5, the point  $E$  is stable because at this point  $d(w^f - w^c)/d\tilde{z} < 0$  holds. Hence, equilibrium  $(\tilde{z}^*, w^{c*})$  is the corner equilibrium  $(0, w^{c*})$ .

### 3.4 Analysis of pattern of equilibrium

We divide the cases among Fig. 2 to Fig. 5. For simplicity, we specify that the input labor coefficient  $\alpha(z)$  equals  $z + 1$ . We divide the case of intersection and non-intersection. We derive  $w^c$  of the intersection Eqs. (9) and (13) with  $\tilde{z} = 0$  and  $\tilde{z} = 1$ . We compare those values as follows:

$$w^{c(Eq. 9)}|_{\tilde{z}=0} > w^{c(Eq. 13)}|_{\tilde{z}=0} \Rightarrow L > \frac{7}{3}(n+1)\sqrt{H}\sqrt{f} - \frac{5}{6}aH, \quad (14)$$

$$w^{c(Eq. 9)}|_{\bar{z}=0} < w^{c(Eq. 13)}|_{\bar{z}=0} \Rightarrow L < \frac{7}{3}(n+1)\sqrt{H}\sqrt{f} - \frac{5}{6}aH, \quad (15)$$

$$w^{c(Eq. 9)}|_{\bar{z}=1} > w^{c(Eq. 13)}|_{\bar{z}=1} \Rightarrow L > \frac{7}{6}n\sqrt{H}\sqrt{f} + \frac{an}{3(n+1)}H, \quad (16)$$

$$w^{c(Eq. 9)}|_{\bar{z}=1} < w^{c(Eq. 13)}|_{\bar{z}=1} \Rightarrow L < \frac{7}{6}n\sqrt{H}\sqrt{f} + \frac{an}{3(n+1)}H, \quad (17)$$

$$H \equiv \frac{n}{b(n+1)}.$$

where  $w^{c(Eq. 9)}$  and  $w^{c(Eq. 13)}$  are the competitive wage of Eqs. (9) and (13), respectively. We summarize the above discussion as follows:

Eqs. (14) and (17) hold  $\Rightarrow$  Region A; the case of that interior equilibrium (Fig.2),

Eqs. (15) and (17) hold  $\Rightarrow$  Region B; the case of that all sector is unionized or non-unionized (Fig.3),

Eqs. (14) and (16) hold  $\Rightarrow$  Region C; the case of that all sector is unionized (Fig.4),

Eqs. (15) and (17) hold  $\Rightarrow$  Region D; the case of that all sector is non-unionized (Fig.5).

We can depict the graph of  $L$  and  $\sqrt{f}$  (Fig.6).

### 3.5 Comparative statics

We can analyze graphically the impact of exogenous parameters such as  $f$ ,  $L$ , and  $n$  on the equilibrium values of  $(\bar{z}^*, w^c)$  at the case of interior equilibrium (Fig.2).<sup>6</sup>

Eq. (9) is a decreasing function of the fixed cost of union  $f$ . Hence, the threshold value of  $z$  between unionized sectors and non-unionizes sectors decreases and the competitive wage increases as the fixed cost of union  $f$  increases (Fig.7). When the union cost becomes costly, the incentive of unionized workers to maintain their labor union weakens and the threshold  $\bar{z}^*$  lowers. The decrease in the threshold causes the increase in the labor demand in non-unionized sectors, because non-unionized sectors absorbs larger employments than unionized sectors. Therefore, non-unionized wage; the competitive wage increase when  $f$  increases.

Eq. (13) is a decreasing function of the size of population  $L$ . Hence, the threshold value of  $\bar{z}^*$  between unionized sectors and non-unionizes sectors increases and the competitive wage decreases as the population size  $L$  increases. The increase in the population size raises the labor supply and, consequentially, lowers the competitive wage. The decrease in the competitive wage widens a difference of the competitive wage and the union wage, and therefore, raises the incentive of the workers to organize a labor union.

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<sup>6</sup> We can also analyze the impact of  $f$ ,  $L$ , and  $n$  with the implicit function theorem. See Appendix.

Eq. (9) is a decreasing function of the number of firms  $n$  and Eq. (13) is an increasing function of that. Hence, the threshold of  $\tilde{z}^*$  between unionized sectors and non-unionized sectors decreases and the competitive wage increases as the number of firms increases. The increase in firms causes an intense competition in each industry and lowers rents of each firm. The labor union cannot claim a high wage to firm. Therefore, the increase in firms weakens the incentive of the workers to organize a labor union and lowers the proportion of unionized sectors. For the labor market, the increase in firms raises the labor demand. Therefore, the competitive wage is raised.

### Proposition 2

*The increase in population size raises the proportion of unionized sectors and lowers the competitive wage, whereas the increase in the number of firms and the increase in the union cost lower the proportion of unionized sectors and raise the competitive wage.*

### 3.6 Welfare

Since the welfare depends on the second moment of prices (Eq. (4)), we substitute each price to  $\sigma_p^2$ .

Hence,  $\sigma_p^2$  is represented by

$$\begin{aligned}\sigma_p^2 &= \int_0^1 p^2(z) dz = \int_0^{\tilde{z}} (p^U(z))^2 dz + \int_{\tilde{z}}^1 (p^{NU}(z))^2 dz \\ &= \frac{1}{(n+1)^4} \{a^2 n (3n+2)\tilde{z} + a^2 (n+1)^2 + 2a(2n+1)n^2 w^c \mu_1 + n^4 w^{c2} \mu_2 \\ &\quad + 2anw^c(-n^2 + n + 1) \int_{\tilde{z}}^1 \alpha(z) dz + n^2 w^{c2} (2n+1) \int_{\tilde{z}}^1 \alpha^2(z) dz\}.\end{aligned}$$

In  $\tilde{z} \in [0, 1]$ , differentiation  $\sigma_p^2$  in  $w^c$  and  $\tilde{z}$  are strictly positive:

$$\frac{d\sigma_p^2}{dw^c} > 0, \quad \frac{d\sigma_p^2}{d\tilde{z}} > 0. \quad (18)$$

Therefore, impact of change of fixed union cost is given by

$$\frac{d\tilde{U}}{df} = \frac{d\tilde{U}}{d\sigma_p^2} \frac{d\sigma_p^2}{df} = -\frac{1}{2b} \left( \frac{\partial \sigma_p^2}{\partial w^c} \frac{dw^c}{df} + \frac{\partial \sigma_p^2}{\partial \tilde{z}} \frac{d\tilde{z}}{df} \right)$$

We cannot get explicit values of  $\tilde{z}$  and  $w^c$ . So, later, we analyze the welfare using numerical examples.

### 4. Open economy

We consider international trade between two symmetric countries. We assume that international trade incurs no-trade cost and goods markets are fully integrated, while labor markets are separated and workers are immobile between two countries. Foreign and home firm's outputs and wages are the same, because two countries are symmetric in all respects.

Reaction function of firms  $j$  taking wages, competitor's outputs, and foreign firm's outputs as given is represented by

$$y_{1j}^t(z) = \frac{2a + 2(n-1)\alpha(z)w_{1i}^t(z) - 4n\alpha(z)w_{1j}^t(z) + 2n\alpha(z)w_2^t(z)}{b(2n+1)}$$

where  $y_k$  and  $w_k$  are firm's outputs and union wage in country  $k \in \{1,2\}$ , respectively.

Union wage is given by

$$w_j^t(z) = \frac{a + 2n\alpha(z)w^{ct}}{(2n+1)\alpha(z)} \equiv w^t(z)$$

Output of union sector and non-union sectors are given by

$$y^{Ut}(z) = \frac{4n(a - \alpha(z)w^{ct})}{b(2n+1)^2}, \quad y^{NUT}(z) = \frac{2(a - \alpha(z)w^{ct})}{b(2n+1)}.$$

Arbitrage condition of organizing a labor union is given by

$$w^{tc} = \frac{a - \sqrt{\frac{b}{4n}(2n+1)^3 f}}{\alpha(z)}. \quad (9t)$$

We solve competitive wage from labor market clearing condition:

$$w^{tc} = \frac{2na\mu_1 + a \int_{\tilde{z}}^1 \alpha(z)dz - \frac{b}{2n}(2n+1)^2 L}{2\mu_2 n + \int_{\tilde{z}}^1 \alpha^2(z)dz}. \quad (13t)$$

When  $n > 2$ , Eq. (9t) is located over Eq. (9); therefore, trade openness moves Eq. (9) down. And, Eq. (13t) is located below Eq. (13); therefore, trade openness moves Eq. (13) up. Consequently, in equilibrium  $(\tilde{z}^*, w^c)$ , trade openness decreases  $\tilde{z}^*$  and increases  $w^{c*}$  (Fig. 8).

This result is similar to the case of the increase in the number of firms.

In the other cases of corner equilibrium, trade openness also increases  $w^c$  and does not increase  $\tilde{z}^*$ .

### Proposition 3

*Trade openness raises the competitive wage and lowers the proportion of unionized sectors.*

#### 4.1 Numerical examples

Eventually, we cannot derive the equilibrium values  $\tilde{z}^*$  and  $w^{c*}$ . Then, we analyze the effect of trade openness and a change of parameters on welfare using numerical examples. Our example show the effects of changes in parameters  $f$ ,  $L$ , and  $n$ , but  $a(= 60)$  and  $b(= 1)$  are fixed in the case of the interior equilibrium; Table. 2. As a benchmark with Case 1, we present the increase in  $f$ : Case 2, the increase in  $L$ : Case 3, and the decrease in  $n$ : Case 4.

First note that (see Table. 1), the trade openness raises the welfare in the all cases 1-4. From Eq. (4), welfare is strictly decreasing in the second order moment of prices:  $\sigma_p^2$ . Since trade openness raises the competitive wage and lowers the proportion of unionized sectors, trade openness has the two opposite effects on  $\sigma_p^2$  from Eq. (18). Here, the effect of decrease in  $\tilde{z}^*$  is larger than the effect of increase in  $w^{c*}$ . The increase in  $\tilde{z}^*$  raises the proportion of the non-unionized workers which receive the same competitive wage. Since the variance of firm's production cost decreases, which lowers the variance of price,  $\sigma_p^2$  decreases. Therefore, trade openness lowers the level of income inequality (wage variance).

Here, since the effects of changes in parameters  $f$ ,  $L$ , and  $n$ , on  $\tilde{z}^*$  is larger than on  $w^{c*}$ , the increase in  $f$  and  $L$  raise welfare, and the decrease in  $n$  raises welfare.

## 5. Conclusion

This paper develops a multi-sector general oligopolistic equilibrium trade model for analyzing the effects of trade openness and changes of parameters on the proportion of unionized sector treating the proportion of unionized sectors as an endogenous parameter.

We show that the worker in high productivity sectors have a large incentive to organize a labor union. For this reason, the firms in high productivity sectors have large rents and can accept a high wage requirement by unionized workers. If the firms do not have sufficient rents, the unionized workers cannot claim a high wage to firm. So, in low productivity sectors the workers have a small incentive to organize a labor union. We also show that trade openness decreases the proportion of unionized sectors. Trade openness leads to intense competition to firms and reduces firm's rents. Therefore, when the globalization is progressed, the incentive of workers to organize a labor union becomes small.

The paper can be extended by introducing asymmetries between countries and a union bargaining power.

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## Appendix

### The case of multiple intersections

There are two intersection points of Eqs. (9) and (13) in the case (Fig. 9). For the stability, the points  $E$  and  $E''$  are stable since at these points  $d(w^f - w^c)/d\tilde{z} < 0$  holds. However, the point  $E'$  is unstable since at this point  $d(w^f - w^c)/d\tilde{z} > 0$  holds. As the result, the equilibria are  $E$  and  $E''$ . The equilibrium of  $E$  is the same to the case of interior equilibrium, and  $E''$  is the same to the case of corner equilibrium.

### Comparative statics

We can also analyze the impact of the parameters at the case of interior equilibrium Fig.2, using implicit function theorem.

From Eqs. (9) and (13), we define function  $F$  as follow:

$$F(\tilde{z}, f) = \frac{n\mu_1 + a \int_{\tilde{z}}^1 \alpha(z) dz - \frac{b}{n} (n+1)^2 L}{\mu_2 n + \int_{\tilde{z}}^1 \alpha^2(z) dz} - \frac{a - B}{\alpha(\tilde{z})} = 0.$$

Using implicit function theorem we can get

$$\begin{aligned} \frac{d\tilde{z}(f)}{df} &= -\frac{F_f}{F_{\tilde{z}}} \\ &= -\frac{\sqrt{\frac{b(n+1)^3}{nf}}}{\frac{\alpha(\tilde{z})}{n\mu_2 + \int_{\tilde{z}}^1 \alpha^2(z) dz} (\alpha(\tilde{z})w^c - a) + \frac{a - B}{\alpha^2(\tilde{z})} \frac{d\alpha(\tilde{z})}{d\tilde{z}}} \\ &= -\frac{F_f}{\frac{dw^c}{d\tilde{z}} \stackrel{(Eq.13)}{=} -\frac{dw^c}{d\tilde{z}} \stackrel{(Eq.9)}{=}} \end{aligned}$$

where  $\frac{dw^c}{d\tilde{z}}^{(Eq.9)}$  and  $\frac{dw^c}{d\tilde{z}}^{(Eq.13)}$  are slope of Eqs. (9) and (13), respectively.

Since at the case of Fig.2  $\frac{dw^c}{d\tilde{z}}^{(Eq.13)} > \frac{dw^c}{d\tilde{z}}^{(Eq.9)}$  is satisfied in the equilibrium  $E$ , the following equations hold:

$$\frac{dw^c}{d\tilde{z}}^{(Eq.13)} > \frac{dw^c}{d\tilde{z}}^{(Eq.9)} \Rightarrow \frac{d\tilde{z}^*(f)}{df} < 0, \frac{d\tilde{z}^*(L)}{dL} > 0, \frac{d\tilde{z}^*(n)}{dn} < 0.$$

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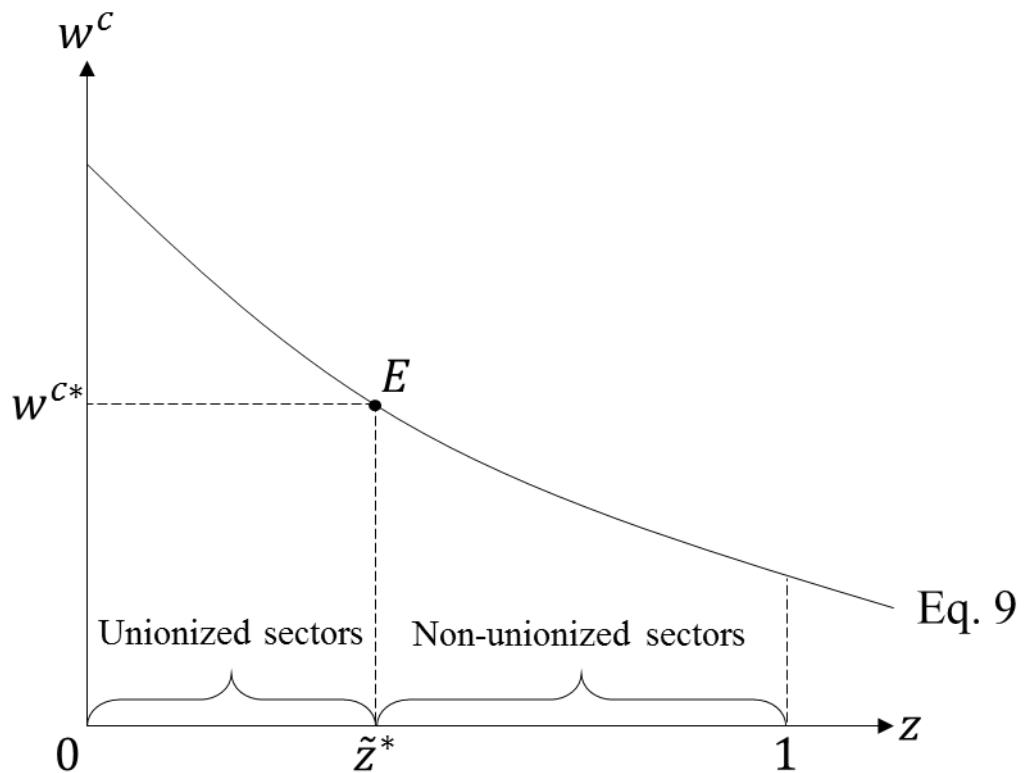


Figure 1 arbitrage condition of organizing a labor union

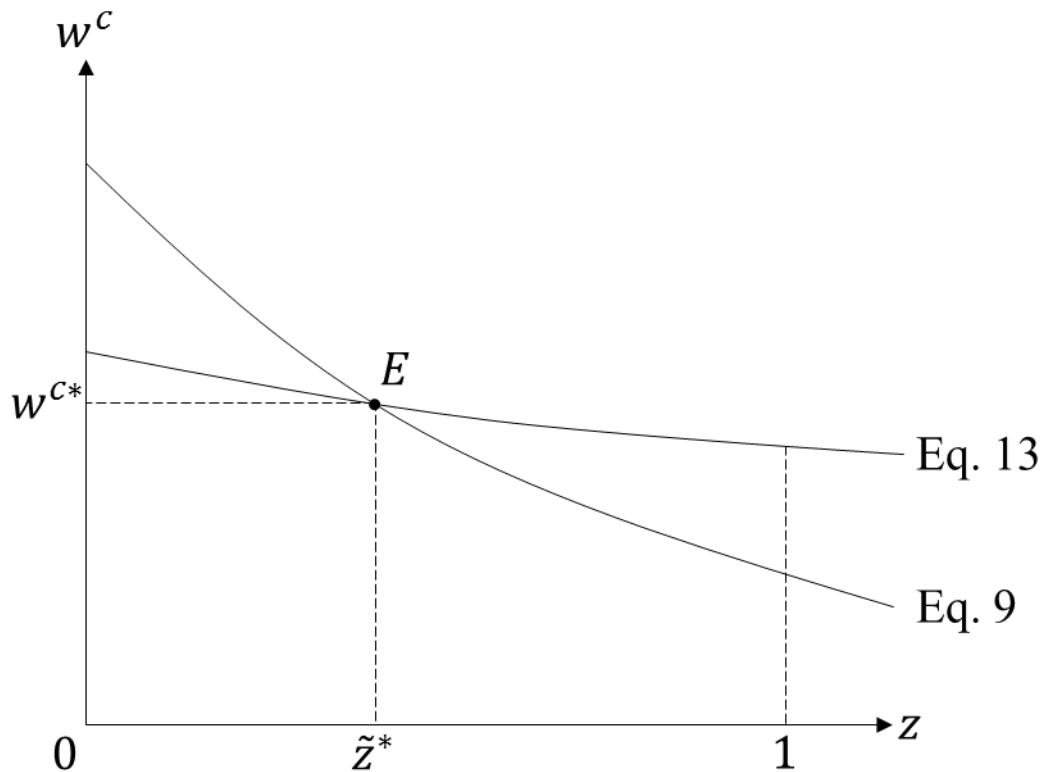
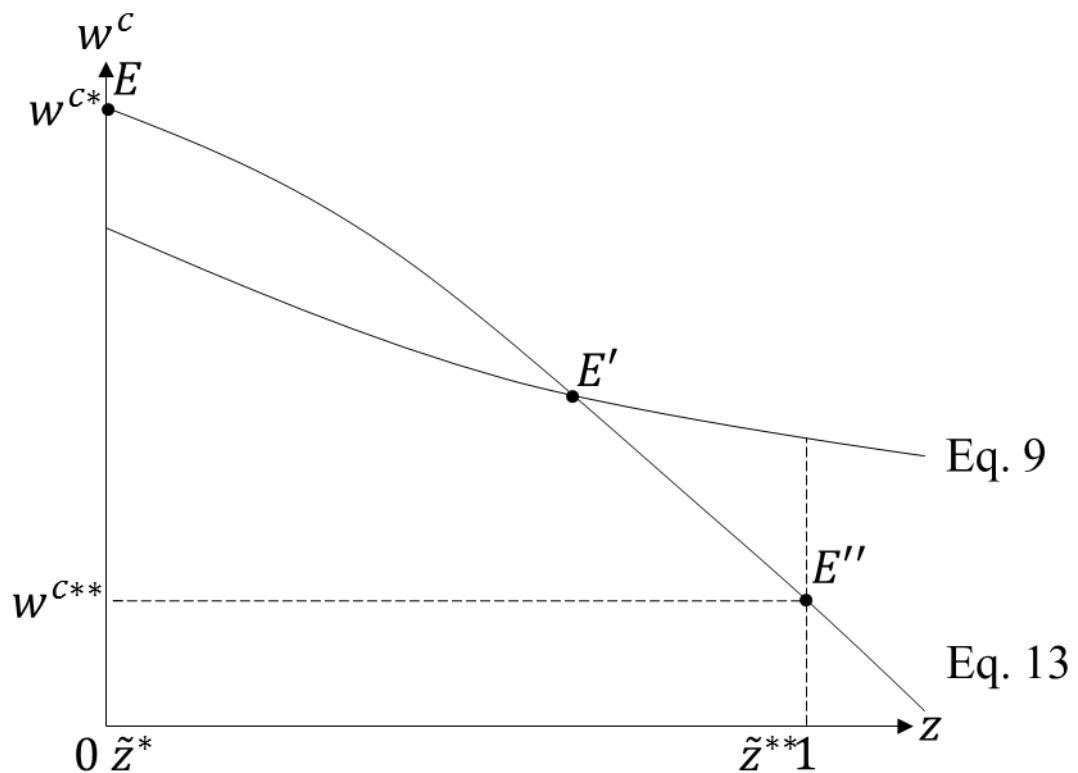
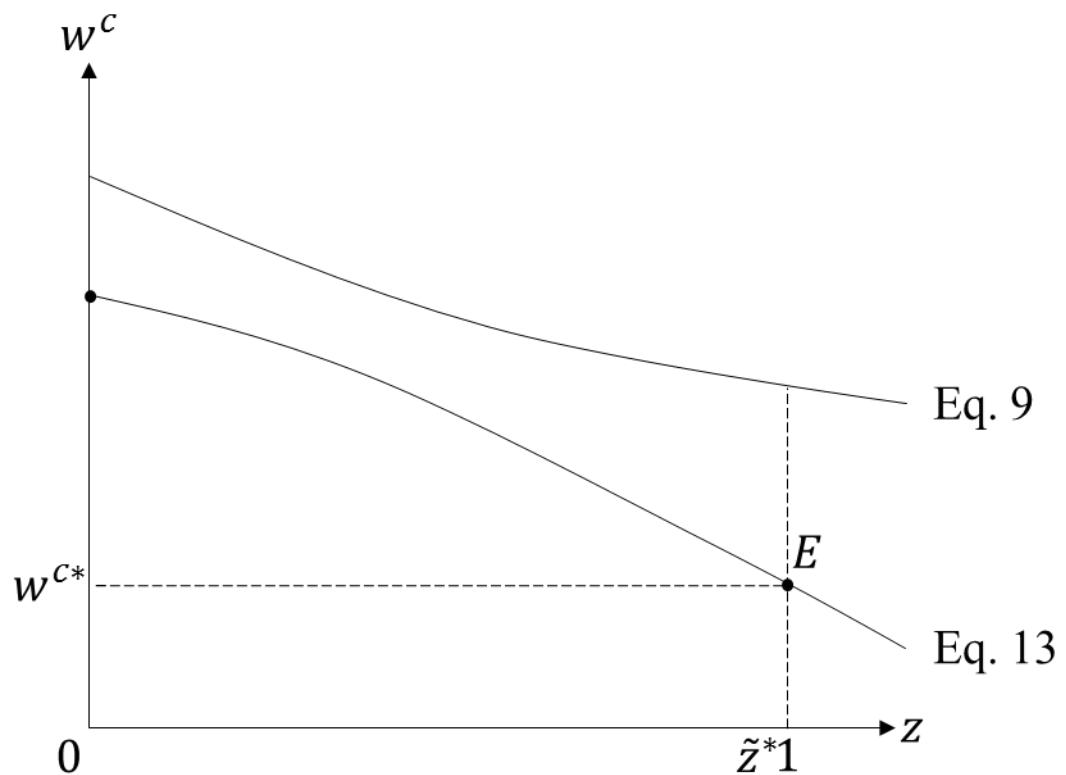


Figure 2 The case of interior equilibrium



**Figure 3** The case that all sector is unionized or non-unionized



**Figure 4** The case that all sector is unionized

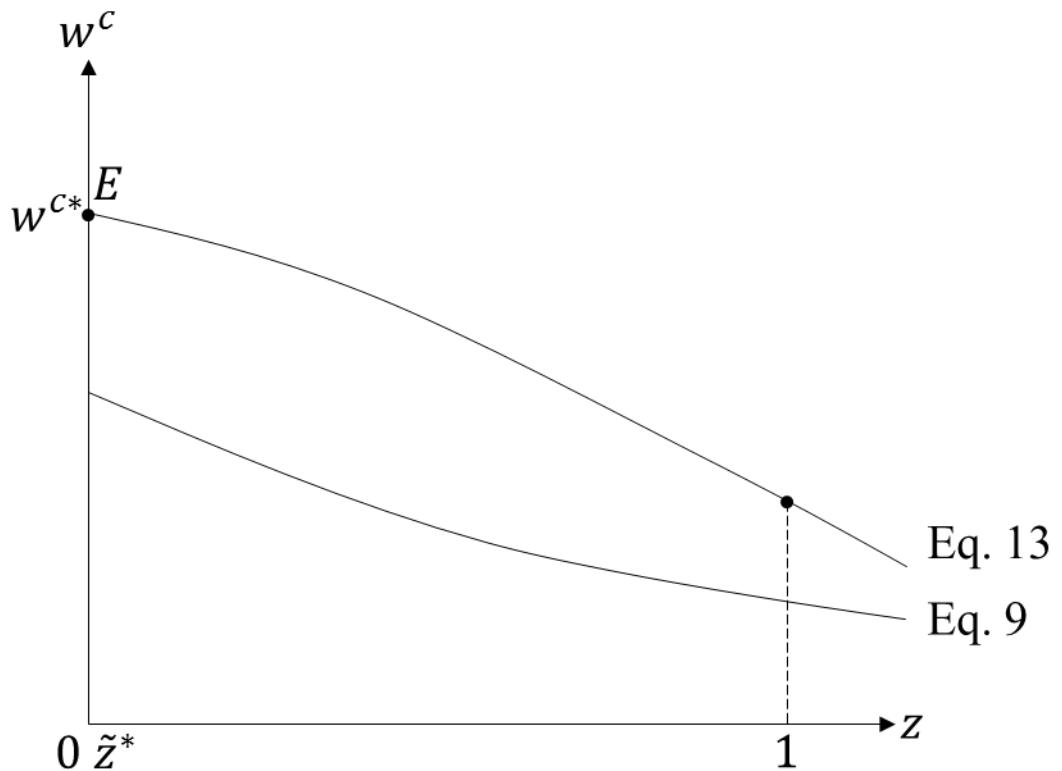


Figure 5 The case that all sector is non-unionized

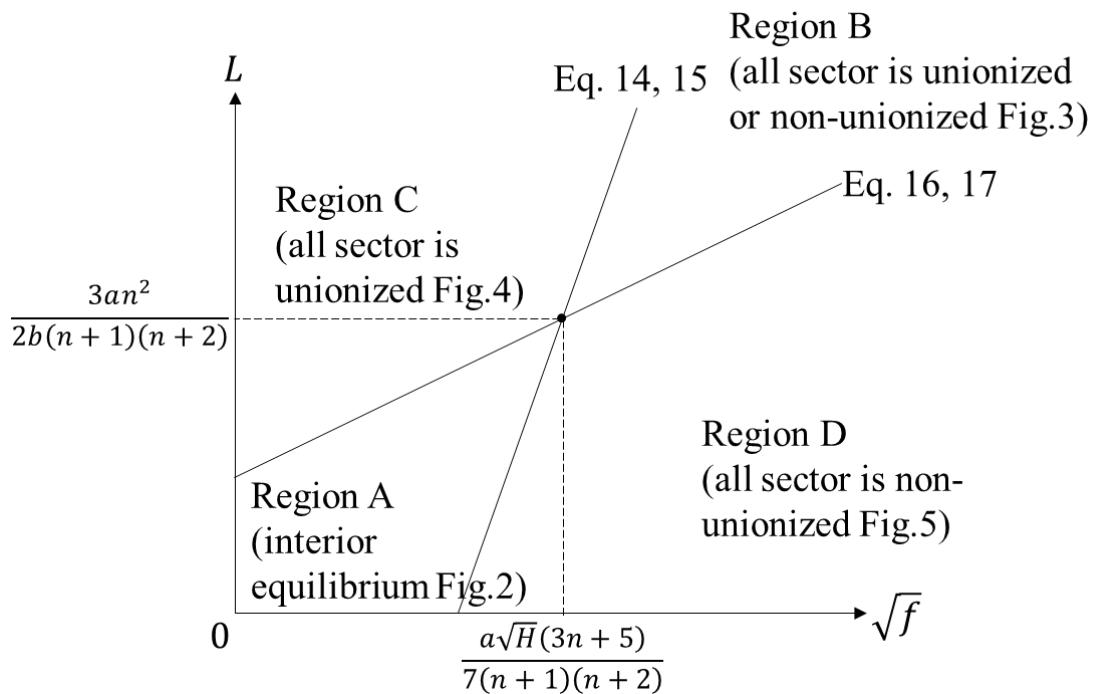


Figure 6 The division cases of equilibrium

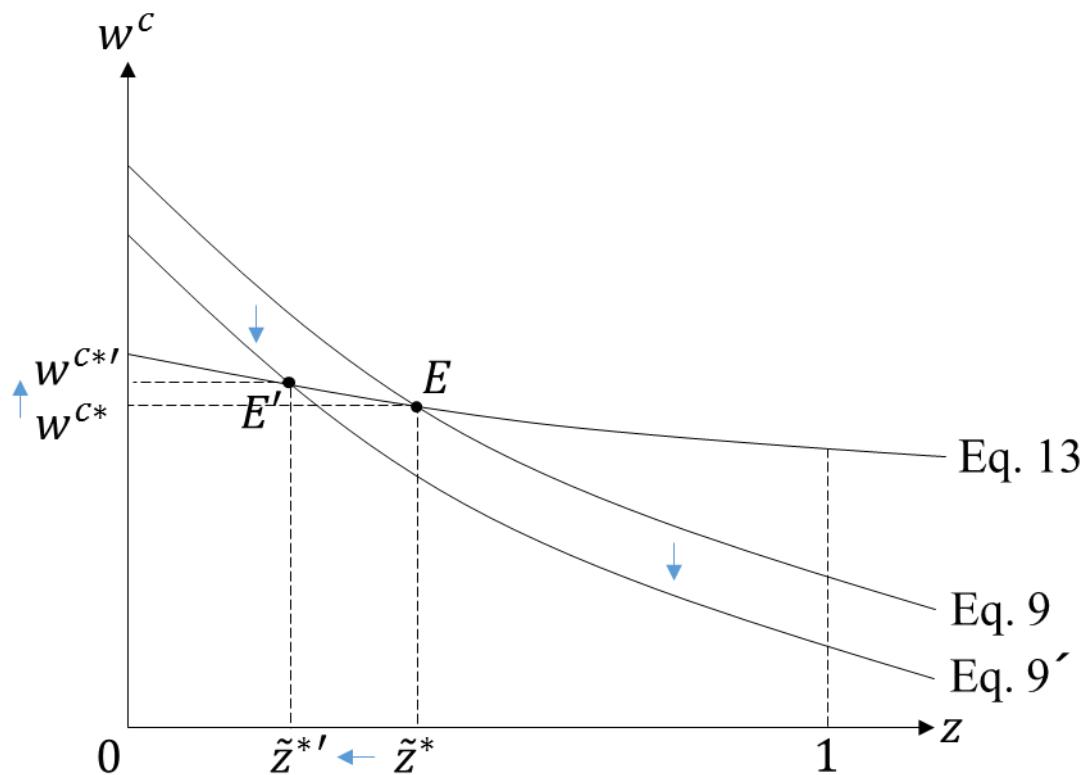


Figure 7 A change of union cost  $f$

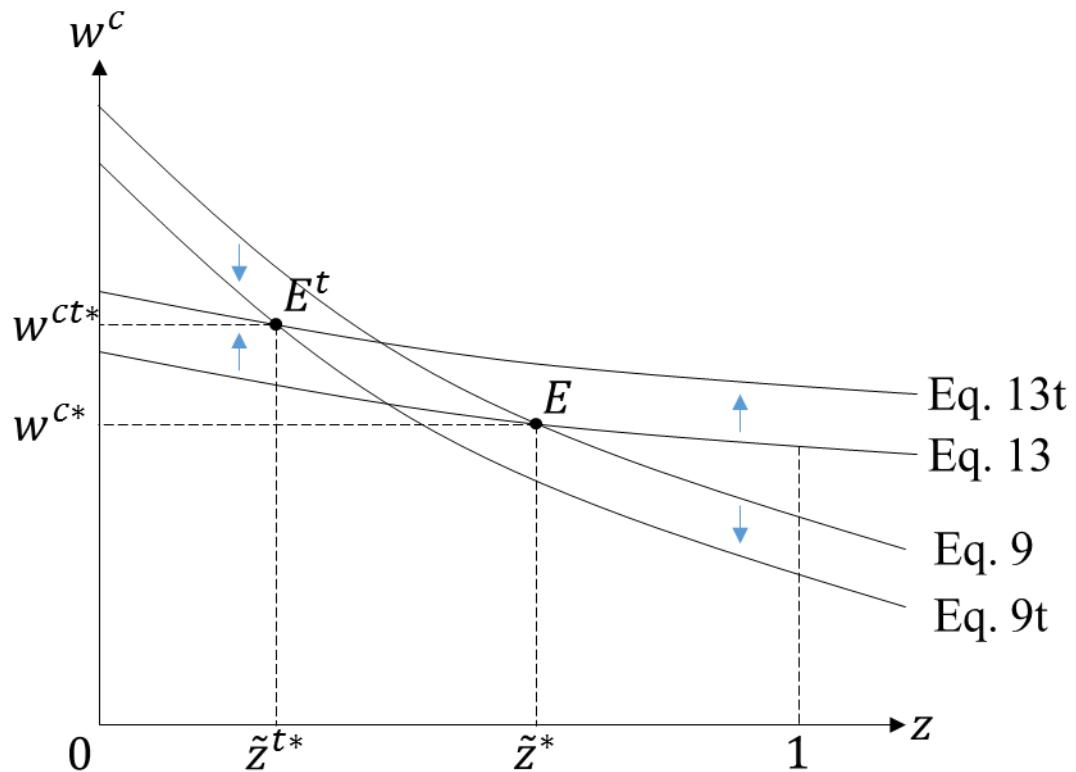
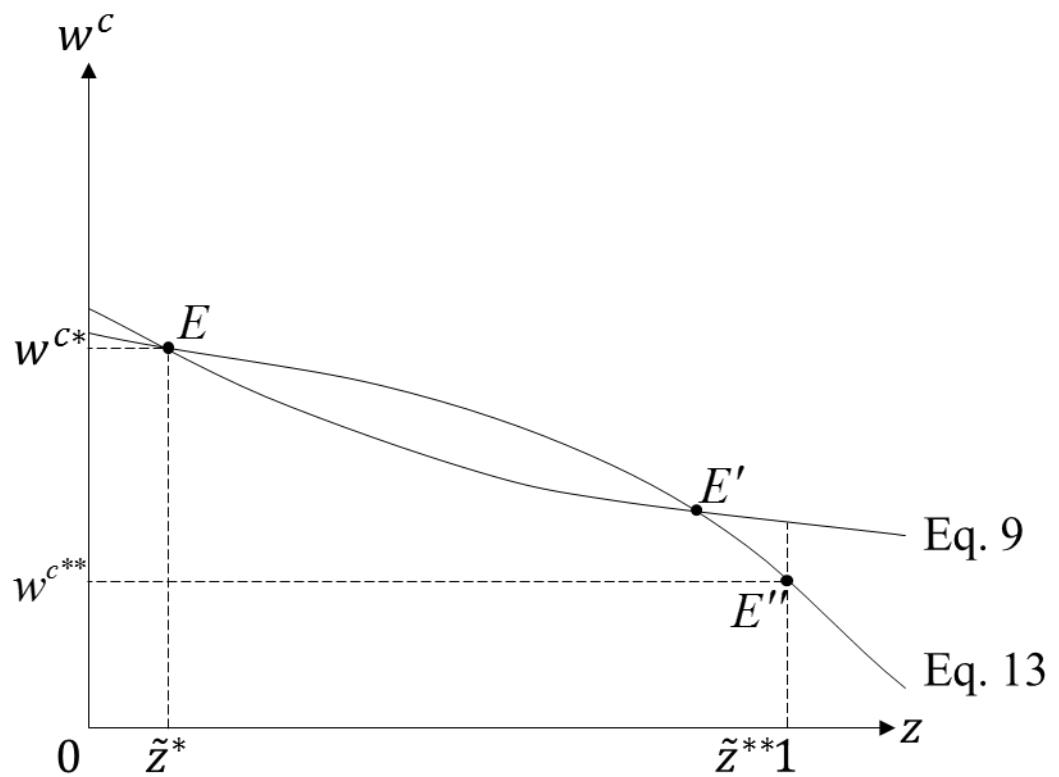


Figure 8 Effect of shift to open economy



**Figure 9** The case of multiple intersections

	Case 1	Case 2	Case 3	Case 4
$f$	15	20	15	15
$L$	15	15	20	15
$n$	3	3	3	2
Autarky	Case 1	Case 2	Case 3	Case 4
Threshold $\tilde{z}^*$	0.48	0.37	0.73	0.84
Competitive wage $w^{c*}$	28.45	28.81	24.39	24.93
$\sigma_p^2$	2440.67	2438.85	2133.35	2458.11
Welfare	579.67	580.58	733.33	570.95
Wage Variance	0.96	0.83	3.51	0.96
Trade openness	Case 1	Case 2	Case 3	Case 4
Threshold $\tilde{z}^*$	0.29	0.17	0.41	0.53
Competitive wage $w^{c*}$	30.58	30.77	27.77	29.25
$\sigma_p^2$	2419.69	2417.94	2111.33	2430.46
Welfare	590.15	591.03	744.33	584.77
Wage Variance	0.14	0.15	2.28	0.83

**Table 1** numerical examples