Firm Selection, Trade Costs, and International Inequalities

Xi Yang* Dao-Zhi Zeng†

March 20, 2014

Abstract

This paper examines the relationship between firm selection in exporting and inequalities across countries of different population size. We develop a fully tractable footloose capital model that captures the home market effect (HME) with firm heterogeneity. Our model studies international inequalities in wage rates and firm shares as well as in welfare. We show that, when trade is liberalized, the international inequality of wage rates is either bell-shaped or monotonically increasing. With firm selection, however, the international inequality of firm shares is always magnified due to a stronger reallocation effect in favor of exporting firms within the smaller country. We also find that trade liberalization does not always generate welfare convergence across different countries since the parameters of firm heterogeneity and fixed exporting costs matter.

Keywords: home market effect, firm heterogeneity, footloose capital, trade

JEL Classification: F12, R12

*Department of International Economics and Business, School of Economics, Xiamen University, 422 Siming South Road, Fujian 361005, China. Email: yangxi0206@gmail.com.

†Graduate School of Information Sciences, Tohoku University, Aoba 6-3-09, Aramaki, Aoba-ku, Sendai 980-8579, Japan. Email: zeng@se.is.tohuku.ac.jp.

1 Introduction

Research on the home market effect (HME) aims to clarify how market size asymmetry and trade costs affect international or regional inequalities in wages, firm locations, trade patterns and welfare. The surprising results of Davis (1998) encourage researchers to restructure models of the HME without the freely-traded homogeneous (agricultural) good, which is extensively assumed since Helpman and Krugman (1985). In this way, labor wages are not necessarily equalized across countries. Both firm share and wage rates endogenously change with variables such as trade costs.

Two approaches can achieve this goal. The first allows for capital mobility, which relaxes the trade balance condition widely used in one-industry one-factor trade models. Takahashi et al. (2013) reveal various international inequalities in wages, firm shares, and trade patterns. The second approach embeds firm heterogeneity into the standard one-industry one-factor trade models (Arkolakis et al., 2008). Felbermayr and Jung (2012) show further that the productivity cutoffs for firm survival change with equilibrium wage rates that are asymmetric between countries. Together with the trade balance condition, HMEs exist in terms of firm share and wage.

These two types of models, however, derive different results regarding the effect of trade liberalization. More specifically, in the two-factor model of Takahashi et al. (2013), the firm share and the wage rate in the larger country are shown to have the same bell-shaped response to trade liberalization. In contrast, the one-factor model of Felbermayr and Jung (2012) concludes that the wage rate declines while the incumbent firm share increases. These results differ because firm share is positively related to the relative wage rate in Takahashi et al. (2013), whereas the trade balance condition in Felbermayr and Jung (2012) always engenders a stronger reallocation effect in favor of the exporting firms in the smaller country.

This paper provides a fully tractable framework of the HME, including both mobile capital and firm selection in exporting. Our general equilibrium model captures wage rate adjustments and firm decisions in location and production resulting from both productivity heterogeneity and mobile capital. As compared with Takahashi et al. (2013) and Felbermayr and Jung (2012), our model examines the interaction between changes in labor demand due to firm relocation and responses to firm exits due to productivity heterogeneity. This enables us to identify how trade liberalization affects international inequalities through forces that work both across countries and within an industry in each country.

Our model yields three main aspects of international inequalities. First, wage inequality represented by the HME in terms of wage is observed. In the two-factor model of Takahashi et al. (2013), the wage rate in the larger country has a bell-shaped response to trade liberalization. In contrast, the one-factor model of Felbermayr and Jung (2012) concludes that the wage rate declines. Unlike these studies, our model yields two possible forms of wage inequality with respect to trade liberalization: either it increases monotonically, or it exhibits a bell-shaped pattern when trade costs decrease. Specifically, if the fixed cost of exporting is sufficiently larger than that of domestic production, or the degree of firm heterogeneity in productivity is small, then exporting is relatively difficult for firms in both countries; further trade liberalization always increases wage inequality. An important fact leading to the contrasting results is that when the share of exporting firms is small, trade liberalization always engenders a greater increase of labor demand in the larger country.

Second, there is inequality in the shares of manufacturing firms, both in the entrant margin and in the incumbent margin after firm selection. The HME in the entrant margin is positively related to the relative wage rate, which reproduces the result of Takahashi et al. (2013). In the incumbent margin, the HME is related to the intensity of firm selection, represented by the difference of productivity cutoffs between countries. The productivity cutoff decreases with country size since, in a large country, the market access advantage dominates the disadvantage of higher local wages. With firm selection, the HME in the share of incumbent firms is, therefore, stronger than that in the share of entrant firms. Moreover, our model indicates two noteworthy features of the effects of trade liberalization. On the one hand, it always magnifies the international inequality in firm shares despite firm relocation possibly generating a weaker HME at the entrant margin. In this case, the within-industry reallocation effect arising from firm heterogeneity becomes the dominating force in shaping changes of the HME in firm share. On the other hand, depending on the parameters of firm heterogeneity and the fixed cost of exporting, the HME in terms of wage may evolve in a different way. Thus it is likely that trade liberalization reduces wage inequality while generating a larger international inequality in firm shares.

Our third result relates to the welfare difference between countries. Arkolakis et al. (2012) show that the gains from trade in a number of existing models, including Krugman (1980), Eaton and Kortum (2002), and Melitz (2003), can be summarized by changes in the share of domestic consumption and the elasticity of trade volume with respect to variable trade costs. These quantitative trade models, however, rest on the trade balance condition, which is not met in our setup since we have mobile capital as another production factor that counteracts the current account imbalance. As emphasized by Arkolakis et al. (2012, P. 116), two-factor models are complicated, and we cannot obtain sufficient statistics representing the gains from trade in each country. Nevertheless, we are able to provide some novel results regarding international inequality in welfare. Specifically, it may either increase monotonically or exhibit a bell-shaped pattern when countries move from autarky to free trade. Trade liberalization, thus, does not always generate welfare convergence across different countries, a result not observed in either Felbermayr and Jung (2012) or Takahashi et al. (2013).

The result of the HME in firm share is indicated by some existing trade models that allow for firm heterogeneity and asymmetric country size (Helpman et al., 2004; Chaney, 2008; Baldwin and Forslid, 2010). These studies start from a standard constant elasticity of substitution (CES) utility setup, including a freely traded agricultural sector. Factor price equalization, however, generates two disadvantages in analyzing how different HMEs behave. First, there is no HME in wages. Second, and more importantly, there is an irrelevance between market size and the productivity cutoff in a country. Melitz and Ottaviano (2008), Zhelobodko et al. (2012), and Behrens et al. (2014) show further that how market size affects the productivity cutoff in selection relies on the utility function, and that market size affects the productivity cutoff if some pro-competitive effects exist. Their models, however, do not include factor mobility, which may affect the existence of the HME and the response to trade liberalization.

Our paper is also related to the recent new economic geography literature that examines the relationship between firm heterogeneity and spatial configuration. Baldwin and Okubo (2006) and Okubo et al. (2010) examine firm sorting issues. Productivity is revealed before a firm decides on relocation across different regions. Unlike these studies, a firm in our model cannot observe its productivity unless it pays the sunk cost of entry. In this sense, our model is more closely related to those that focus on firm selection, such as Ottaviano (2012), Behrens and Robert-Nicoud (2014), and von Ehrlich and Seidel (2013). The first two models are based on Melitz and Ottaviano (2008). It is shown that even though selection generates a higher local productivity in the region with more firms, the evolution of regional agglomeration relies on both the mean and the shape parameter of the productivity distribution. The third model adds firm heterogeneity into the standard footloose entrepreneur model of Forslid and Ottaviano (2003) and shows that firm heterogeneity magnifies regional inequalities. In contrast to these studies, our model, in a fully tractable setup, captures the interaction between wage adjustment and firm selection in exporting.

The rest of the paper is organized as follows. In Section 2, we present the model while Section 3 examines the equilibrium. Section 4 explores the impact of trade liberalization. We also discuss different changes of wages, entrant firm share, incumbent firm share, and various productivity cutoffs there. Section 5 examines welfare. Section 6 is the conclusion.

2 The model

The economy consists of two countries: country 1 and country 2. The countries are identical in production technology and geographical conditions but differ in population size. Assume that country 1 is larger. There is one sector with increasing returns to scale, and firms in the sector are heterogeneous in production technology. Two factors, labor and capital, are used in production. The endowments of labor and capital in the economy are denoted as L and K, respectively. To rule out the Heckscher-Ohlin comparative advantage,[1] we assume that every resident in each country is endowed with one unit of capital, thus, $L = K$ and $L_i = K_i$ $(i = 1, 2)$. Meanwhile, the share of capital endowment in country 1 is identical to the local population share, which we denote by

$$\theta \equiv \frac{K_1}{K} = \frac{L_1}{L} \in \left(\frac{1}{2}, 1\right).$$

2.1 Preference

Individuals in both countries share identical preferences over an aggregate good $U = Q$. For country i, Q_i is expressed by the CES aggregator:

$$Q_i = \left(\sum_{j=1}^{2} \int_0^{n_j} m_{ji}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where $m_{ji}(\omega)$ is the consumption of variety ω produced in country j by an individual located in i, n_j is the number of varieties that are bought from country j, and $\sigma > 1$ represents the elasticity of substitution between any differentiated varieties.

For a firm located in country i producing variety ω, the aggregate demand in country j is given by

$$d_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} \frac{E_j}{P_j^{1-\sigma}}, \tag{1}$$

where p_{ij} is the delivered price from i to j and E_j is the aggregate expenditure in j. In this one-industry setup, E_j is equal to the aggregate income in j. Meanwhile, the price index in country j is defined as

$$P_j = \left[\sum_{i=1}^{2} \int_0^{n_i} p_{ij}(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}. \tag{2}$$

[1] Bernard et al. (2007) examine the interaction between Heckscher-Ohlin comparative advantage and firm heterogeneity. Our model simplifies this issue while allowing for mobile capital and country asymmetry in market size.

2.2 Production

The market structure of the manufacturing industry is monopolistic competition. There is a continuum of firms that enter the industry in each country. Of the two production factors, capital is used in two ways. First, given that the entrant firms cannot know their productivity before entering the industry, they pay the sunk cost of entry to observe their productivity. Assume that f_e units of capital are required to enter into the industry. With rental price r_i in country i, the sunk cost of entry for a firm located in country i is $r_i f_e$. Second, after observing its productivity, the firm decides whether or not to produce. Each incumbent firm that stays in the industry also decides whether to only sell in the domestic market or to export as well. Production for either domestic sale or for export requires fixed costs in the usage of capital.[2] The fixed costs of sale in country i for any incumbent firms in i are

$$r_i f_{ij} = \begin{cases} r_i f_d & \text{for } i = j, \\ r_i f_x & \text{for } i \neq j. \end{cases}$$

Let us assume that $f_x \geq f_d$, i.e., that exporting may incur more fixed costs than would domestic production.

The variable inputs of production are labor. A firm with productivity φ faces marginal cost w_i/φ for producing an extra unit of output where w_i is the wage rate in country i. Assume that productivity φ is a random variable in $[1, \infty)$ and is drawn from a commonly known distribution with density function $g(\varphi)$ and cumulative distribution $G(\varphi)$. Countries do not exhibit any differences in production technologies, i.e., $G(\varphi)$ is identical. Following the literature, we assume a Pareto distribution of firm productivity so that

$$G(\varphi) = 1 - \varphi^{-\kappa} \quad \text{where } \kappa > \sigma - 1. \tag{3}$$

Condition $\kappa > \sigma - 1$ implies that the ex-post productivity index $\tilde{\varphi}_{ij}$ is finite, and that firm revenue has a finite mean. A smaller value of κ indicates a more dispersed firm productivity distribution.

In addition to the fixed costs of exporting, there are iceberg-type trade costs in international trade. To provide one unit of output in country j ($j \neq i$), $\tau_{ij} = \tau > 1$ units of output should be produced and sent from country i. For a firm in country i with productivity φ, the marginal cost of exporting to country j is $w_i \tau/\varphi$. Serving the domestic market, however, does not incur iceberg trade costs, thus $\tau_{ii} = 1$ for $i = 1, 2$. Given market demand in (1), a

[2] This is the same as the footloose capital model of Martin and Rogers (1995).

firm maximizes its profit by choosing prices in both the domestic and the foreign market:

$$\pi_{ij}(\omega,\varphi) = \begin{cases} \max\limits_{p_{ii}(\omega,\varphi)} \left[p_{ii}(\omega,\varphi) - \dfrac{w_i}{\varphi}\right] d_{ii}(\omega) - r_i f_d & \text{for } i=j, \\ \max\limits_{p_{ij}(\omega,\varphi)} \left[p_{ij}(\omega,\varphi) - \dfrac{w_i \tau}{\varphi}\right] d_{ij}(\omega) - r_i f_x & \text{for } i \neq j. \end{cases}$$

Given the constant price elasticity of demand, the optimal prices are a constant mark-up over the delivered marginal cost in different destinations:

$$p^*_{ii}(\omega,\varphi) = \frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}, \quad p^*_{ij}(\omega,\varphi) = \frac{\sigma}{\sigma-1}\frac{w_i\tau}{\varphi} \; (i \neq j),$$

resulting in the firm's net profit

$$\pi_{ij}(\omega,\varphi) = \begin{cases} \dfrac{1}{\sigma}\left(\dfrac{\sigma}{\sigma-1}\dfrac{w_i}{\varphi}\right)^{1-\sigma}\dfrac{E_i}{P_i^{1-\sigma}} - r_i f_d & \text{for} \quad i=j, \\ \dfrac{1}{\sigma}\left(\dfrac{\sigma}{\sigma-1}\dfrac{w_i\tau}{\varphi}\right)^{1-\sigma}\dfrac{E_j}{P_j^{1-\sigma}} - r_i f_x, & \text{for} \quad i \neq j. \end{cases} \tag{4}$$

Equation (4) determines the productivity cutoff φ^*_{ij} in selection:

$$(\varphi^*_{ij})^{\sigma-1} = \begin{cases} \dfrac{\sigma r_i f_d P_i^{1-\sigma}}{E_i}\left(\dfrac{\sigma w_i}{\sigma-1}\right)^{\sigma-1} & \text{for} \quad i=j, \\ \dfrac{\sigma r_i f_x P_j^{1-\sigma}}{E_j}\left(\dfrac{\sigma w_i \tau}{\sigma-1}\right)^{\sigma-1} & \text{for} \quad i \neq j. \end{cases} \tag{5}$$

As shown in Appendix F, $\varphi^*_{11} < \varphi^*_{12}$ is always true, while $\varphi^*_{22} < \varphi^*_{21}$ holds true when θ is not too large (i.e., $\theta \leq \hat{\theta}$). In the following analysis, we assume that condition $\theta \leq \hat{\theta}$ is met. Then a firm located in country i with productivity below φ^*_{ii} exits the market, and a firm does not export if its productivity is below φ^*_{ij}.

With market selection among heterogeneous firms, the ex-post density distribution $\mu_{ij}(\varphi)$ is:

$$\mu_{ij}(\varphi) = \begin{cases} \dfrac{g(\varphi)}{1-G(\varphi^*_{ij})}, & \text{if } \varphi \geq \varphi^*_{ij}, \\ 0, & \text{otherwise.} \end{cases}$$

3 Market equilibrium

3.1 Firm shares and productivity cutoffs

We choose the labor in country 2 as numéraire so that $w_2 = 1$. The wage rate in country 1 is denoted by w, which also represents the relative wage rate in the larger country. With capital endowment K in the whole economy, we denote by n^e the aggregate number of entrant firms. Let the number of entrant firms in country i be n_i^e $(i = 1, 2)$. Then $n^e = n_1^e + n_2^e$. Due to the free mobility of capital, n_i^e adjusts endogenously in each country. Given that firm selection generates different production statuses for the incumbent firms, we can summarize the local price indexes defined in equation (2) as

$$P_1^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{\kappa}{\kappa-(\sigma-1)} \Big[n_1^e(\varphi_{11}^*)^{\sigma-1-\kappa} w^{1-\sigma} + \phi n_2^e(\varphi_{21}^*)^{\sigma-1-\kappa}\Big], \tag{6}$$

$$P_2^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{\kappa}{\kappa-(\sigma-1)} \Big[n_2^e(\varphi_{22}^*)^{\sigma-1-\kappa} + \phi n_1^e(\varphi_{12}^*)^{\sigma-1-\kappa} w^{1-\sigma}\Big]. \tag{7}$$

The free entry condition implies that the ex-ante expected profit of entry in country i is equal to the local sunk cost of entry. For any given rental rate r_i, we have

$$r_i f_e = [1-G(\varphi_{ii}^*)]\left[\int_{\varphi_{ii}^*}^{\infty} \pi_{ii}(\varphi)\mu_{ii}(\varphi)d\varphi + \chi_{ij}\int_{\varphi_{ij}^*}^{\infty} \pi_{ij}(\varphi)\mu_{ij}(\varphi)d\varphi\right], \quad j \neq i, \tag{8}$$

where $1 - G(\varphi_{ii}^*)$ is the ex-ante probability that the entrant's productivity is above φ_{ii}^*, and $\chi_{ij} \equiv [1-G(\varphi_{ij}^*)]/[1-G(\varphi_{ii}^*)] \in (0, 1)$ represents the ex-ante probability that an incumbent firm in country i exports to country j $(j \neq i)$.

At the cutoff productivities, net profit is zero for both the domestic market and the foreign market. Equation (4) then implies that net profit can be written as functions of the productivity cutoffs:

$$\pi_{ii}(\varphi) = \left[\left(\frac{\varphi}{\varphi_{ii}^*}\right)^{\sigma-1} - 1\right] r_i f_d, \qquad \pi_{ij}(\varphi) = \left[\left(\frac{\varphi}{\varphi_{ij}^*}\right)^{\sigma-1} - 1\right] r_i f_x.$$

With the Pareto distribution (3) of φ and the ex-post density distribution $\mu_{ij}(\varphi)$, we obtain the following free entry conditions in each country:

$$f_e = \left[\frac{\sigma-1}{\kappa-(\sigma-1)}\right][(\varphi_{11}^*)^{-\kappa} f_d + (\varphi_{12}^*)^{-\kappa} f_x], \tag{9}$$

$$f_e = \left[\frac{\sigma-1}{\kappa-(\sigma-1)}\right][(\varphi_{22}^*)^{-\kappa} f_d + (\varphi_{21}^*)^{-\kappa} f_x]. \tag{10}$$

Note that the domestic cutoff productivity φ_{ii}^* and exporting cutoff productivity φ_{ij}^* are the only endogenous variables in (9) and (10). In addition, φ_{ii}^* and φ_{ij}^* are negatively related, i.e., a higher φ_{ii}^* accompanies a lower φ_{ij}^*.

Next, consider the capital market equilibrium. Capital in the economy is used in both countries as the sunk cost of entry for entrants as well as the fixed costs of domestic production and exporting for incumbents. Therefore,

$$K = n^e f_e + \sum_{i=1}^{2} n_i^e \{[1 - G(\varphi_{ii}^*)] f_d + [1 - G(\varphi_{ij}^*)] f_x\}. \tag{11}$$

Substituting equations (9) and (10) into (11), the equilibrium number of entrant firms in the economy is

$$n^e = \frac{(\sigma - 1)K}{\kappa f_e}. \tag{12}$$

In an interior equilibrium of firm shares in each country, we obtain $r_i = r_j = \overline{r}$ where $\overline{r}$ is the equilibrium rental rate of capital in the economy. To solve for $\overline{r}$, we note that the ratio between the aggregate payment to workers and to capital is fixed at $\sigma - 1$, which is derived from the CES utility and the linear cost functions (see Appendix A for details). Since the aggregate payment to workers in the economy equals $w\theta L + (1 - \theta) L$, and the aggregate capital payment is $\overline{r}K$, $\overline{r}$ is solved as

$$\overline{r} = \frac{w\theta + (1 - \theta)}{\sigma - 1} \frac{L}{K}. \tag{13}$$

Let λ^e be the share of entrant firms that flow into country 1, i.e., $\lambda^e \equiv n_1^e / n^e$. The CES feature and the capital mobility condition indicate that in the interior equilibrium we have:

$$\frac{wL_1}{L_2} = \frac{n_1^e}{n_2^e}, \qquad \text{thus } \lambda^e = \frac{w\theta}{w\theta + (1 - \theta)}. \tag{14}$$

Thus, the share of entrant firms in country 1 increases with the local relative wage ($\partial \lambda^e / \partial w > 0$). When the larger country obtains a higher equilibrium wage rate, i.e., $\theta > 1/2$ and $w > 1$, there exists a more-than-proportionate share of entrant firms in the larger country:

$$\lambda^e - \theta = \frac{\theta(1 - \theta)(w - 1)}{w\theta + (1 - \theta)} > 0 \quad \text{if } w > 1. \tag{15}$$

The market size advantage benefits the larger country through increasing the local wage rate and producing a more-than-proportionate share of firms entering into that country. In other words, there is an equivalence between the HMEs in terms of wage and in terms of entrant firm share.

Consider next the determination of aggregate income E_i, which consists of both rental income and wage income for residents located in i ($i = 1, 2$):

$$E_1 = \theta(wL + \bar{r}K), \qquad E_2 = (1-\theta)(L + \bar{r}K). \tag{16}$$

We now consider the relationship between different productivity cutoffs. First, since capital mobility equalizes rental rates, the relationship between φ_{ij}^* and φ_{ii}^* is derived from equation (5):

$$\varphi_{12}^* = \varphi_{22}^* \Lambda w, \qquad \varphi_{21}^* = \varphi_{11}^* (\frac{\Lambda}{w}), \tag{17}$$

where $\Lambda \equiv \tau(f_x/f_d)^{1/(\sigma-1)} > 1$. Second, by substituting equation (17) into the free entry conditions of (9) and (10), the ratio of domestic productivity cutoffs satisfies:

$$\left(\frac{\varphi_{11}^*}{\varphi_{22}^*}\right)^\kappa = \frac{1 - w^\kappa \Delta}{1 - w^{-\kappa}\Delta}, \tag{18}$$

where $\Delta \equiv \tau^{-\kappa}(f_x/f_d)^{(\sigma-1-\kappa)/(\sigma-1)} = \phi\Lambda^{\sigma-1-\kappa} \in [0,1]$ can be taken as a measure of trade freeness whose value is higher the lower the variable or fixed trade costs are.

In (18), $\varphi_{11}^*/\varphi_{22}^*$ decreases with w and $\varphi_{11}^* < \varphi_{22}^*$ holds iff $w > 1$. Namely, the productivity cutoff in one country is lower than in the other if and only if the wage rate in that country is higher. Recall that $w > 1$ exhibits the HME in terms of wage, we obtain another equivalent definition for the case of heterogeneous firms: the productivity cutoff in the larger country is lower. A higher wage rate in country 1 increases local income. The larger market demand compensates more than the higher wage costs disadvantage and, thus, increases profitability for firms that enter country 1. Less productive firms are, therefore, more likely to survive. This result differs from the result of a partial equilibrium model in Melitz and Ottaviano (2008), where the income effect is absent and the pro-competitive effect results in a higher cutoff productivity for firm survival.

Plugging equations (6), (7), (12), (13), (16) and (17) into equation (5), the cutoff productivity indexes are reduced to a function of w:

$$(\varphi_{11}^*)^\kappa = \frac{\sigma(\sigma-1)}{\kappa-\sigma+1}\frac{f_d}{f_e}\frac{\theta w + (1-\theta)w^\kappa\Delta}{\theta[w(\sigma-1+\theta)+(1-\theta)]}, \tag{19}$$

$$(\varphi_{22}^*)^\kappa = \frac{\sigma(\sigma-1)}{\kappa-\sigma+1}\frac{f_d}{f_e}\frac{(1-\theta)+\theta w^{1-\kappa}\Delta}{(1-\theta)(\theta w+\sigma-\theta)}. \tag{20}$$

Now we are able to examine the incumbent firm share λ. By using (14) and (18), λ is calculated as

$$\lambda = \frac{n_1^e(\varphi_{11}^*)^{-\kappa}}{n_1^e(\varphi_{11}^*)^{-\kappa} + n_2^e(\varphi_{22}^*)^{-\kappa}} = \frac{w\theta(1-w^{-\kappa}\Delta)}{w\theta(1-w^{-\kappa}\Delta)+(1-\theta)(1-w^{\kappa}\Delta)}. \tag{21}$$

It is easy to show that $\lambda > \theta$ iff $w > 1$.

The above results are summarized in Lemma 1.

Lemma 1 *The following facts are equivalent: (i) Entrant firm share λ^e is larger than θ; (ii) Incumbent firm share λ is larger than θ; (iii) $\varphi_{11}^* < \varphi_{22}^*$; and, (iv) $w > 1$.*

3.2 Wage equation

The equilibrium cutoffs, as well as most equilibrium aggregate variables, can be reduced to functions of w. To close the model, we focus on the equilibrium wage rate. According to (18), (19), and (20), we obtain a wage equation that determines the equilibrium relative wage at any given Δ:

$$\begin{aligned}\mathcal{G}(w) \equiv& \frac{(1-\theta)(\theta w+\sigma-\theta)}{(1-\theta)+\theta w^{1-\kappa}\Delta}\ \frac{\theta w+(1-\theta)w^{\kappa}\Delta}{\theta[w(\sigma-1+\theta)+1-\theta]}(1-\Delta w^{-\kappa})-(1-\Delta w^{\kappa})\\ =&0.\end{aligned} \tag{22}$$

For $w \in (0,\infty)$, function $\mathcal{G}(w)$ is continuous for $w > 0$. Each term of (22) increases with w. Furthermore, we have

$$\mathcal{G}(1) < 0 \quad \text{and} \quad \mathcal{G}(\Delta^{-\frac{1}{\kappa}}) > 0 \text{ if } \Delta \in (0,1).$$

Therefore, (22) has a unique solution of the equilibrium relative wage rate in interval

$$w^* \in (1, \Delta^{-\frac{1}{\kappa}}), \tag{23}$$

for $\Delta \in (0,1)$. In addition, $w^* = 1$ is the only positive solution of (22) when $\Delta = 0, 1$.

It is more convenient to rewrite (22) as

$$\mathcal{F}(w,\Delta) \equiv \mathcal{A}_2(w)\Delta^2 + \mathcal{A}_1(w)\Delta + \mathcal{A}_0(w) = 0, \tag{24}$$

where

$$\begin{aligned}\mathcal{A}_2(w) &= (1-\theta)^2(\theta w+\sigma-\theta) - \theta^2 w[w(\sigma-1+\theta)+1-\theta],\\ \mathcal{A}_1(w) &= \sigma[\theta w^{1-\kappa} - (1-\theta)w^{\kappa}](\theta w+1-\theta),\end{aligned}$$

$$\mathcal{A}_0(w) = \theta(1-\theta)(1-w)(1-\theta+\theta w).$$

Two facts can be derived from (24). First, since $\mathcal{F}(1,\Delta) = (1-\Delta)\Delta(2\theta-1)\sigma$ and $\theta \in (1/2, 1)$, we obtain $w^* = 1$ again for $\Delta = 0$ or $\Delta = 1$. Otherwise, we always observe $w^* > 1$, i.e., the HME in terms of wage. Second, since $w^* \geq 1$, $\mathcal{A}_0(w^*) \leq 0$ holds so that

$$\begin{aligned}
\mathcal{A}_1(w^*)\Delta =& - \mathcal{A}_2(w^*)\Delta^2 - \mathcal{A}_0(w^*) \\
=& - [\mathcal{A}_2(w^*) + \mathcal{A}_0(w^*)]\Delta^2 - \mathcal{A}_0(w^*)(1-\Delta^2) \\
=& \{2\theta - 1 + [(w^*)^2 - 1]\theta^2\}\sigma - \mathcal{A}_0(w^*) > 0.
\end{aligned}$$

Accordingly, we know $\mathcal{A}_1(w^*) > 0$, obtaining an upper bound of w^*:

$$w^* < \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{2\kappa-1}}. \tag{25}$$

The properties of the wage equation are summarized in Lemma 2.

Lemma 2 *(i) There is a unique equilibrium wage rate w^* in $[1, \min\{\Delta^{-\frac{1}{\kappa}}, (\frac{\theta}{1-\theta})^{\frac{1}{2\kappa-1}}\})$.*
(ii) $\partial\mathcal{F}(w,\Delta)/\partial w < 0$ holds at $w = w^*$.
(iii) $\partial w^*/\partial\theta > 0$.

Proof. See Appendix B. ■

Note that, according to Lemma 2, we have $\varphi_{11}^* < \varphi_{22}^*$ from (18) and $\lambda^{e*} > \theta$ from (15).

3.3 Trade pattern

We now examine the implications of firm heterogeneity on the trade pattern between countries. First, there is a negative pattern between the productivity cutoff for exporting and market size: $\varphi_{12}^*/\varphi_{21}^* = w^2(\varphi_{22}^*/\varphi_{11}^*) > 1$ holds according to (17) and Lemma 2. This result is straightforward because firms located in the smaller country benefit from exporting to the larger country, and the lower local wage rate magnifies the foreign market access effect. Second, this result also indicates that $\chi_{12} < \chi_{21}$, so the share of exporting firms is lower in the larger country. This result is also found in Medin (2003). With capital mobility, our model differs, however, in the prediction of trade patterns between countries, which is quite contrastive. Lemma 3 concludes that the larger country is a net exporter of the differentiated varieties and is a net importer of capital due to the balance of payment.

Lemma 3 *In the presence of capital mobility and adjustment in productivity cutoffs under firm selection, the larger country is a net importer of capital and a net exporter of goods, except when $\Delta = 0, 1$.*

Proof. See Appendix C. ■

As in the literature, trade volume is affected by two margins. The intensive margin reflects the weighted average of export volume among exporting firms. At this margin, trade is balanced between countries under the Pareto distribution because the average export value of an exporting firm in country i is identical across countries and equals $r_{ij}(\tilde{\varphi}_{ij}) = \bar{r} f_x \sigma\kappa/(\kappa - \sigma + 1)$. The trade pattern is, thus, dependent on the extensive margin, i.e., the number of exporting firms. Two opposing effects are at work: $\varphi_{12}^* > \varphi_{21}^*$ implies that the share of exporting firms is lower in country 1, and, hence, the higher wage cost is a disadvantage for the larger country; capital mobility, on the other hand, benefits country 1, as represented by a larger measure of entrant firms. This effect outweighs the wage cost disadvantage and results in a larger export volume from country 1 to country 2.

Proposition 1 summarizes our results of equilibrium.

Proposition 1 *For $\Delta \in (0,1)$, we have $w^* > 1$, $\lambda^{e*} > \theta$, $\lambda^* > \theta$ and $\varphi_{11}^* < \varphi_{22}^*$; the larger country is a net importer of capital and a net exporter of goods.*

Proof. This is immediately derived from Lemmas 1, 2, and 3. ■

4 Impact of trade liberalization

4.1 Changes in wages

This subsection examines how w^* is affected by Δ by taking the partial derivatives of (24). According to the implicit function theorem,

$$w'(0) = -\frac{\partial\mathcal{F}/\partial\Delta}{\partial\mathcal{F}/\partial w}\bigg|_{\Delta=0,w=1} = \frac{\sigma(2\theta-1)}{\theta(1-\theta)} > 0, \tag{26}$$

$$w'(1) = -\frac{\partial\mathcal{F}/\partial\Delta}{\partial\mathcal{F}/\partial w}\bigg|_{\Delta=1,w=1} = -\frac{2\theta-1}{\kappa} < 0 \tag{27}$$

hold. Since $\mathcal{F}(w,\Delta)$ is a quadratic function of Δ, wage equation (24) has, at most, two roots of Δ for a given w. In other words, any horizontal line crosses the wage curve $w(\Delta)$ twice at most in the Δ-w plane. Given (26) and (27), wage curve $w(\Delta)$ has a bell shape. Therefore, there exists a threshold level $\widehat{\Delta} \in (0,1)$ at which $w'(\Delta)$ changes its sign from positive to negative. The corresponding wage rate, $\widehat{\Delta}$, satisfies

$$\mathcal{F}\big(w(\widehat{\Delta}),\widehat{\Delta}\big) = 0 \quad \text{and} \quad \mathcal{A}_2(w(\widehat{\Delta}))\widehat{\Delta}^2 - \mathcal{A}_0(w(\widehat{\Delta})) < 0.$$

Since our model features firm selection in exporting, which is related to the fixed trade costs (f_x/f_d) and the firm heterogeneity parameter (κ), we can show how these parameters affect the secondary magnification effect, i.e., the change of HME due to smaller values of τ). If $f_x > f_d$, then $\Delta < 1$ always holds, and firm selection in exporting still occurs at $\phi = 1$. A further decrease of ϕ now generates two opposing forces on labor demand. On the one hand, exporting becomes relatively difficult, reflected by a larger productivity cutoff for exporting. Given the asymmetry of w and P at $\Delta < 1$, firms in the smaller country are more negatively affected by decreased foreign market access and, hence, generate a larger drop in labor demand. This effect is strong when the share of exporting firms is large in the economy, which occurs if f_x/f_d is small, or if firms are more likely to obtain a high productivity (i.e., when κ is small). On the other hand, competition from foreign exporters is attenuated when ϕ is lower. Due to f_x and asymmetry in productivity cutoffs, the smaller country is more protected from weaker market competition and so generates a larger increase in labor demand. This effect is strong when exporting is very difficult or when the initial market competition level is low, i.e., when f_x/f_d or κ is large.[3]

Keeping in mind that the opposing effects of ϕ on labor demand are now related to firm selection of exporting, we are then left with two possible patterns between ϕ and the HME in wages. For large values of f_x/f_d and κ, exporting is difficult and an increase of ϕ always magnifies the HME in wages. In contrast, for small values of f_x/f_d and κ, the share of firm exporting is large and there is a bell-shaped curve between ϕ and the HME in wages.

Our result reveals that the wages in two countries do not necessarily converge, implying that the existing worldwide income inequality does not disappear automatically through trade integration. Such a result does not come from a one-industry one-factor setup. Trade liberalization always results in wage convergence between countries in the CES setup with homogenous firms (Krugman, 1980), in the CES setup with firm heterogeneity (Arkolakis et al., 2008; Felbermayr and Jung, 2012), and also in the variable price elasticity of demand setup with firm heterogeneity (Behrens et al., 2014). Two reasons for the wage convergence seen in these studies are that the number of entrant firms is always fixed in each country, ensuring no changes in labor demand at the entrant margin, and that the smaller country benefits more from the improved foreign market access when trade is liberalized. This is represented by a greater extent of reallocation that benefits exporting firms and generates a larger increase of labor demand in the smaller country.

Since the wage-convergence result is also true in a two-factor model with homogeneous firms (Takahashi et al, 2013), we conclude that our striking result is derived only when both

[3] In contrast, when all firms are able to export when either $f_x = f_d$ or κ is near $\sigma - 1$, the countries are affected symmetrically by weaker market competition.

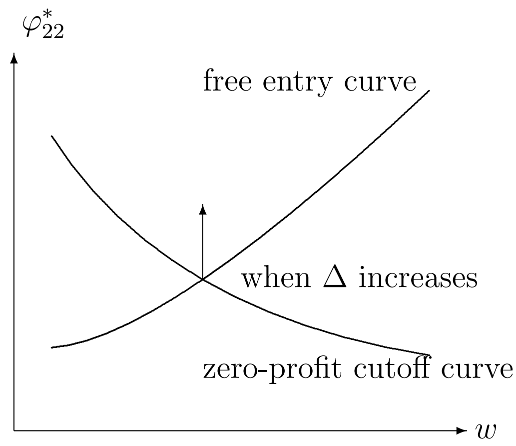

Figure 1: φ_{22}^* increases in Δ

capital mobility and firm heterogeneity are taken into consideration. On the one hand, under the positive relationship between λ^e and w in (14), there is a magnification of labor demand in the larger country, arising from the inflow of capital. As a result, when countries are integrated from autarky, the larger country's labor demand increases more. On the other hand, the changes in labor demand now rely on the parameters of f_x/f_d and κ. In particular, when firm exporting is difficult, trade liberalization, as reflected by smaller values of trade costs, may generate more labor demand in the larger country, which strengthens the HME in terms of wage.

4.2 Changes in productivity cutoffs

4.2.1 Local productivity cutoffs

In this subsection, we consider the effect of Δ on each country's productivity cutoffs for domestic production and exporting. In particular, we show that, regardless of changes in w or λ, an increase of Δ always results in a reallocation in favor of the exporting firms and increases the productivity cutoff for domestic production. Since there is a negative relationship between φ_{ii}^* and φ_{ij}^* ($j \neq i$) because Δ does not enter the free entry conditions of (9) and (10), we know that $\partial\varphi_{ii}^*/\partial\Delta > 0$ and $\partial\varphi_{ij}^*/\partial\Delta < 0$ should always hold.

We prove $\partial\varphi_{ii}^*/\partial\Delta > 0$ through a graphical analysis that links w with φ_{ii}^*. Take country 2, for example, and see how Δ impacts φ_{22}^*. As shown in Figure 1, equilibrium cutoff φ_{22}^* is the intersection of two curves representing the free entry condition and the zero-profit cutoff.

The free entry condition refers to how both w and φ_{22}^* adjust such that the net profit of entry into country 2's manufacturing sector becomes zero. From (10), (18), and the definitions of Λ and Δ, this condition can be written as

$$\frac{\kappa - \sigma + 1}{\sigma - 1}\frac{f_e}{f_d} = (\varphi_{22}^*)^{-\kappa}\Big(\frac{1 - \Delta^2}{1 - w^\kappa \Delta}\Big). \tag{28}$$

The RHS of (28) decreases in φ_{22}^* and increases in w. Therefore, it yields a positively sloped free entry curve that links w with φ_{22}^* in country 2. This is because an increase in country 1's wage rate indicates a higher local income and makes it easier for firms in country 2 to export. The resulting reallocation effect works in favor of the exporting firms in country 2 and gives rise to an increase of φ_{22}^*.

Meanwhile, (20) provides a zero-profit cutoff curve, revealing another relationship between w and φ_{22}^*. The RHS of (20) decreases in w, so the zero-profit cutoff curve is negatively sloped in Figure 1. Intuitively, a higher wage rate in country 1 indicates a cost disadvantage for its exporters. Firms in country 2, thus, face less competition from foreign goods. This weakened market competition allows less productive firms to enter country 2.

It is important to note that an increase of Δ shifts both the free entry curve and the zero-profit cutoff curve upwards. The shift of the free entry curve is due to a larger Δ making it easier for firms in country 2 to export to country 1 for any given w, which lowers φ_{21}^*. Since φ_{21}^* and φ_{22}^* are negatively related by (10), an increase of Δ pushes less productive local firms out of the market in country 2. In contrast, the shift of the zero-profit cutoff curve occurs due to the following: Holding the relative wage constant, country 1's exporting goods are cheaper in country 2 if Δ is larger. As a result, the market in country 2 becomes relatively more competitive, which results in the exit of some less productive firms.

Since equilibrium cutoff φ_{22}^* is always the intersection point of the free entry curve and the zero-profit cutoff curve, we are able to conclude that φ_{22}^* increases in Δ.

4.2.2 Cutoff productivity ratio

Since our model features interaction between country size asymmetry and capital mobility, another important question is raised about the effect of trade liberalization on the ratio of productivity cutoffs. As shown in Figure 2, we use the free entry curve and the balance of payment curve to examine the impact of trade liberalization on $\varphi_{11}^*/\varphi_{22}^*$.

First, the free entry curve is drawn from (18), showing a negatively sloped relationship between $\varphi_{11}^*/\varphi_{22}^*$ and w. An increase of country 1's relative wage enlarges the local market, which works in favor of the less productive firms in country 1 and leads to a smaller value of $\varphi_{11}^*/\varphi_{22}^*$. Since (18) is derived from the free entry conditions of (9) and (10), this curve is called the free entry curve again.

Second, the balance of payment condition is reached with capital mobility: the net export

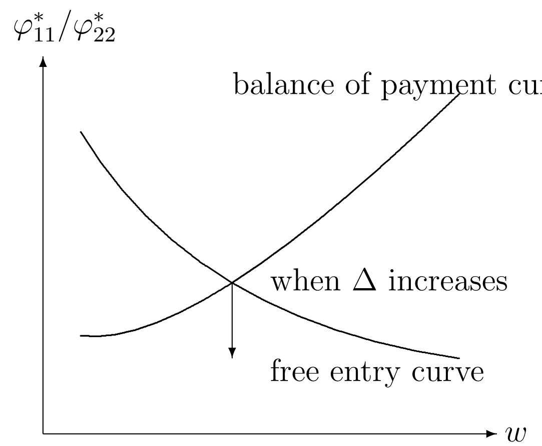

Figure 2: $\varphi^*_{11}/\varphi^*_{22}$ decreases in Δ

volume of manufacturing goods of country equals the amount of net capital inflow, i.e.,

$$X_{12} - X_{21} = \bar{r}(n_1^e f_e + n_{11} f_d + n_{12} f_x) - \bar{r}K_1. \tag{29}$$

The RHS of (29) is the net inflow of capital to country 1, measured by the difference between the demand of capital from local firms and the endowment of capital owned by local workers. As shown in Appendix D, we are able to rewrite (29) in terms of w and $\varphi^*_{11}/\varphi^*_{22}$ as

$$\Delta\Big[\theta w^{1-\kappa} - (1-\theta)w^{\kappa}\Big(\frac{\varphi^*_{11}}{\varphi^*_{22}}\Big)^{-\kappa}\Big] = \theta(w-1)\Big(\frac{1-\theta+\theta w^{1-\kappa}\Delta}{\theta w + \sigma - \theta}\Big). \tag{30}$$

The balance of payment curve in Figure 2 is drawn from (30). Appendix D shows that it exhibits a positive relationship between $\varphi^*_{11}/\varphi^*_{22}$ and w. Intuitively, we note that both $\bar{r}$ and λ_e increase with w, according to (13) and (14). Since the amount of capital flow from country 2 to country 1 is $\bar{r}K(\lambda_e - \theta)$, more capital flows into country 1 and, hence, it should export more when w increases. Responding to this change of trade pattern, the within-industry reallocation effect works to increase $\varphi^*_{11}/\varphi^*_{22}$.

Turning to the effect of Δ on the equilibrium values of $\varphi^*_{11}/\varphi^*_{22}$, a larger Δ always shifts the free entry curve down because the RHS of (18) decreases in Δ. Intuitively, holding $w > 1$ fixed, trade liberalization makes it easy for firms in country 2 to access the larger market in country 1, which lowers φ^*_{21}. This results in a larger φ^*_{22}, which is negatively related to φ^*_{21}, according to (10). Thus, $\varphi^*_{11}/\varphi^*_{22}$ becomes lower. Appendix D proves that the balance of payment curve also moves down when Δ increases. Holding constant the higher labor wage w in country 1, this disadvantage of firms in country 1 is strengthened by trade liberalization. Therefore, holding w constant, the net export volume from country 1 to country 2 is reduced with higher Δ, resulting in a smaller capital inflow from country 2 to country 1. Within-industry reallocation then works to shape a smaller $\varphi_{11}^*/\varphi_{22}^*$.

In sum, as the intersection of the free entry curve and the balance of payment curve, the equilibrium cutoff productivity ratio $\varphi_{11}^*/\varphi_{22}^*$ decreases in Δ. Regardless of the ambiguous HME change in wage, trade liberalization always produces a stronger within-industry reallocation effect in the smaller country. When Δ is low, further trade liberalization increases competition in the smaller country, even though the outflow of capital decreases the local number of firms competing at the entrant margin. When Δ is large, trade liberalization indicates a larger benefit for the exporting firms located in the smaller country. This causes a larger increase of labor demand and reallocation in favor of local exporting firms.

4.3 Changes in firm share

This subsection considers the effect of Δ on the share λ of incumbent firms in each country. Since both the productivity cutoffs and λ^e are related to w, we follow the previous analysis by utilizing another graphical analysis that plots the relationship between w and λ, examining how Δ impacts both variables in equilibrium.

First, (21) provides a positively sloped curve between w and λ, as shown in Appendix E. Namely, a higher w induces more entrant firms in country 1 and generates a lower domestic productivity cutoff level, both of which magnifies country 1's size advantage. Since it is derived from the capital mobility condition, we call it the capital mobility curve.

Second, viewing the balance of payment condition as a function of w and λ, (29) becomes:

$$\Delta\left[\theta w^{1-\kappa} - (1-\theta)w^{\kappa-1}\ \frac{1-\theta}{\theta}\ \frac{\lambda}{1-\lambda}\right] = \theta(w-1)\ \frac{1-\theta+\theta w^{1-\kappa}\Delta}{\theta w+\sigma-\theta}. \tag{31}$$

Appendix E also shows that the balance of payment indicates a negative relationship between λ and w, called the balance of payment curve in Figure 3. To understand the monotonicity, note that the balance of payment curve in Figure 2 reveals that an increase of w increases $\varphi_{11}^*/\varphi_{22}^*$. This within-industry effect is strong enough to dominate the effect of increasing λ^e, leading to a smaller λ.

Appendix E shows that both the capital mobility curve and the balance of payment curve shift up when Δ increases. The intuition is similar to the analysis in Subsection 4.2.2 since, for any given w, λ is negatively related to $\varphi_{11}^*/\varphi_{22}^*$ by (E1).

As the intersection of the capital mobility curve and the balance of payment curve, the share of incumbent firms (λ^*) always increases in Δ. This result differs from the effect

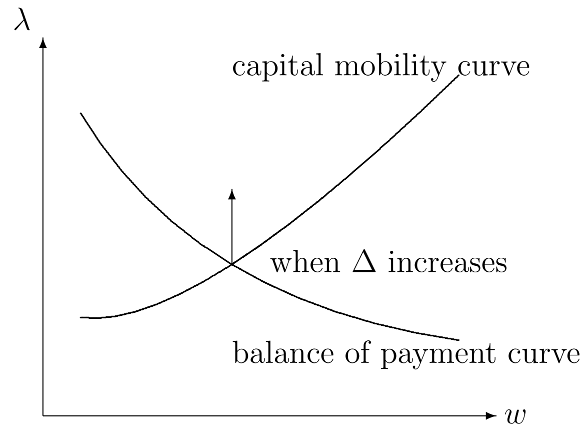

Figure 3: λ increases in Δ

of Δ on the HME at the entrant margin: when Δ is large, any further increase of trade liberalization always generates a larger share of firms that survive after firm selection in the larger country, even though fewer firms enter the local market. In other words, our model shows that the effect of firm selection is strong enough to counteract the downsizing effect of capital mobility on the number of firms when Δ is sufficiently high.

We summarize these results as follows.

Proposition 2 *When trade is liberalized, represented by a smaller Δ, cutoffs φ_{11}^* and φ_{22}^* increase but their ratio $\varphi_{11}^*/\varphi_{22}^*$ falls. Furthermore, the incumbent firm share λ^* increases.*

These results contrast with many findings in the existing literature. First, in Takahashi et al. (2013), all firms are identical and can be treated as if they are assigned the same productivity indexes in both regions. Thus their setup cannot capture how the HME evolves when there are adjustments of aggregate firm productivity. Second, in the two-sector trade models (Helpman et al., 2004; Baldwin and Forslid, 2010), changes of productivity cutoffs are symmetric across countries, and trade liberalization always intensifies the HME in terms of firm share by generating a weaker market-crowding effect. Third, Felbermayr and Jung (2012) show that trade liberalization results in a weaker HME in terms of wage but a stronger HME in terms of firm share. These results, however, are related to the trade balance condition, which is relaxed in our setup. Fourth, in Ottaviano (2012), the strength of the HME also depends on the firm heterogeneity parameters of both the mean and the dispersion values of productivity distribution. His model explores the pro-competitive effect but does not allow for the income effect. In comparison, our model exhibits the general equilibrium effects of capital mobility and firm selection in exporting although we do not allow the markup to differ among heterogeneous firms.

5 Relative welfare

In this section, we examine the welfare differential between countries. The ratio of per capita welfare between country 1 and country 2 is

$$V \equiv \frac{V_1}{V_2} = \Big[\frac{1-\theta+w^*(\sigma-1+\theta)}{\theta w^*+\sigma-\theta}\Big]\Big[\frac{\theta w^*+(w^*)^{\kappa}(1-\theta)\Delta}{\theta(w^*)^{1-\kappa}\Delta+1-\theta}\Big]^{\frac{1}{\sigma-1}}\Big(\frac{\varphi_{22}^*}{\varphi_{11}^*}\Big)^{\frac{\kappa-\sigma+1}{\sigma-1}}. \tag{32}$$

From (25), $V_1/V_2 > 1$ always holds for $\phi \in [0,1]$. The larger country benefits in three aspects: the per capita income is higher, the local price index is lower, and the aggregate productivity is higher.

According to (19) and (20), it holds that

$$\frac{\varphi_{22}^*}{\varphi_{11}^*} = \Big[\frac{\theta}{1-\theta}\,\frac{1-\theta+\theta(w^*)^{1-\kappa}\Delta}{\theta w^*+(1-\theta)(w^*)^{\kappa}\Delta}\,\frac{w^*(\sigma-1+\theta)+1-\theta}{\theta w^*+\sigma-\theta}\Big]^{\frac{1}{\kappa}}.$$

Furthermore, by using (22) and (32), we have

$$V^{\kappa} = \Big(\frac{\theta}{1-\theta}\Big)^{\frac{\kappa-\sigma+1}{\sigma-1}}\Big[\frac{1-\theta+w^*(\sigma-1+\theta)}{\theta w^*+\sigma-\theta}\Big]^{\frac{\sigma\kappa}{\sigma-1}-1}\Big[\frac{\theta w^*+(1-\theta)(w^*)^{\kappa}\Delta}{1-\theta+\theta(w^*)^{1-\kappa}\Delta}\Big]. \tag{33}$$

Note that both the second and third terms of (33) increase with w^*, while the third term decreases in Δ by using (25). Therefore, V decreases in Δ as long as w^* decreases in Δ through (24), which is true when Δ is large.

To show how V^{κ} depends on Δ when Δ is small, we calculate its derivative at $\Delta = 0$ by using $w^*(0) = 1$ as follows:

$$\begin{aligned}\frac{dV^{\kappa}}{d\Delta}\Big|_{\Delta=0,w^*=1} &= \frac{\partial V^{\kappa}}{\partial w}\Big|_{\Delta=0,w^*=1}\frac{dw^*}{d\Delta}\Big|_{\Delta=0,w^*=1} + \frac{\partial V^{\kappa}}{\partial \Delta}\Big|_{\Delta=0,w^*=1} \\ &= \Big(\frac{\kappa\sigma+1}{\sigma}\Big)\Big(\frac{\theta}{1-\theta}\Big)^{\frac{\kappa}{\sigma-1}}\frac{\sigma(2\theta-1)}{\theta(1-\theta)} - \frac{2\theta-1}{(1-\theta)^2}\Big(\frac{\theta}{1-\theta}\Big)^{\frac{\kappa}{\sigma-1}-1} \\ &= \frac{(2\theta-1)\kappa\sigma}{(1-\theta)^2}\Big(\frac{\theta}{1-\theta}\Big)^{\frac{\kappa}{\sigma-1}-1} > 0.\end{aligned}$$

Thus, V increases in Δ when Δ is small.

Our simulations show that relative welfare forms a bell shape with respect to Δ when f_x/f_d and/or κ are small, and monotonically increases when f_x/f_d and/or κ are large. In

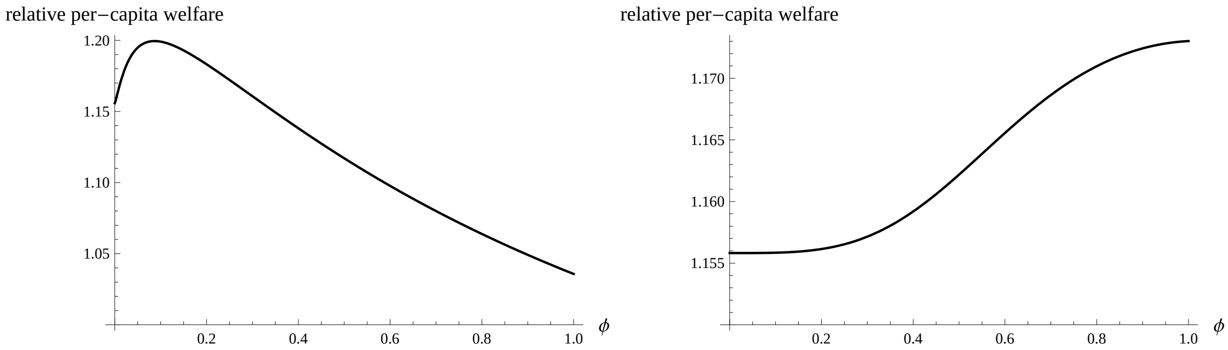

Figure 4: Relative welfare

Figure 4, the simulation parameters are $f_x = 4, f_d = 2, f_e = 0.3, \sigma = 3.8, \kappa = 4, K = 1000$, and $\theta = 0.6$, in the left panel and $f_x = 4, f_d = 1, f_e = 0.3, \sigma = 3.8, \kappa = 10, K = 1000$, and $\theta = 0.6$ in the right panel.

These simulation exercises indicate that, depending on f_x/f_d and κ, trade liberalization may not generate welfare convergence between countries. Since both parameters are related to the share of exporting firms in each country, it is likely that a decrease in trade cost always magnifies the larger country's size advantage if only a small fraction of productive firms export. In this case, even though a larger ϕ promotes convergence of price indexes, the welfare difference across countries increases due to a larger nominal wage inequality. In contrast, a smaller f_x/f_d or κ implies that some less productive firms can now export. Trade liberalization, then, may benefit the smaller country in both nominal and real terms. Note that our result is realized by incorporating both firm heterogeneity and mobile capital, whereas existing studies, having only one of them, all conclude welfare convergence (e.g., the heterogeneous productivity model of Felbermayr and Jung (2012) without relative productivity difference in the two countries, the two-factor CES model of Takahashi et al. (2013), and the non-CES one-factor model of Behrens et al. (2014)).

6 Conclusion

Using a setup with capital mobility and goods trade between countries, this paper examines the impact of trade liberalization on international inequalities including firm selection in exporting. Our model captures changes in the HME in different dimensions. For the HME in wages, we see that it either exhibits a bell-shaped pattern if the fixed cost of exporting is small and firm heterogeneity is high, or that it always gets magnified if the fixed cost of exporting is large and firm heterogeneity is low. For the HME in the share of incumbent firms that survive after selection, we show that it always gets magnified under trade liberalization.

This result occurs when market size generates asymmetric intensity of firm selection in different countries, and less productive firms in the larger country benefit from the local market access. Our model also shows that trade liberalization may not always lead to welfare convergence between countries. This is true when capital mobility and firm selection in exporting magnify the market size advantage of the larger country.

Appendix A: Aggregate payment of capital and labor

The total wage cost in country i equals the payment to workers, who are used as flexible inputs in production among the incumbent firms. Let $l_{ij}(\varphi)$ be the labor input of a firm with productivity φ in country i to produce for country j. Then

$$l_{ij}(\varphi) = l_{ij}(\varphi^*_{ij})\Big(\frac{\varphi}{\varphi^*_{ij}}\Big)^{\sigma-1} = \frac{(\sigma-1)r_i f_{ij}}{w_i}\Big(\frac{\varphi}{\varphi^*_{ij}}\Big)^{\sigma-1}, \tag{A1}$$

where the second equality is known as a feature of the CES setup in the literature.

Since all firms pay identical wages (w_i) in country i, the aggregate wage payment equals

$$\begin{aligned} \text{wage payment in } i &= w_i \sum_j n_i^e[1-G(\varphi^*_{ij})] \int_{\varphi^*_{ij}}^{\infty} l_{ij}(\varphi)\mu_{ij}(\varphi)d\varphi \\ &= \frac{(\sigma-1)\kappa}{\kappa-(\sigma-1)} n_i^e \sum_j [1-G(\varphi^*_{ij})] r_i f_{ij} \\ &= n_i^e \kappa r_i f_e, \end{aligned} \tag{A2}$$

where the second inequality is from (A1), and the last equality is from (9) and (10).

Payment to capital, on the other hand, arises from two sources: the sunk costs paid by all entrant firms and the fixed costs of production by all incumbent firms. We can then write the aggregate cost payment to capital in country i as

$$\text{capital payment in } i = n_i^e r_i\Big[f_e + \sum_j (1-G(\varphi^*_{ij}))f_{ij}\Big] = \frac{1}{\sigma-1} n_i^e \kappa r_i f_e, \tag{A3}$$

where the last equality is again from (9) and (10).

Equations (A2) and (A3) then imply that the ratio between aggregate wage payment and aggregate capital payment is fixed at $\sigma - 1$ for country $i = 1, 2$.

Appendix B: Proof of Lemma 2

(i) This is given by (23) and (25).

(ii) By use of $\mathcal{A}_2\Delta^2 + \mathcal{A}_1\Delta + \mathcal{A}_0 = 0$ at equilibrium,[4] we have

$$\Delta^2 = -\frac{\mathcal{A}_1(w)\Delta + \mathcal{A}_0(w)}{\mathcal{A}_2(w)},$$

where $\mathcal{A}_0(w) < 0$, $\mathcal{A}_1(w) > 0$, and $\mathcal{A}_2(w) < 0$. Accordingly, it holds that

$$\begin{aligned}\frac{\partial}{\partial w}\mathcal{F}(w,\Delta) =& \mathcal{A}_2'(w)\Delta^2 + \mathcal{A}_1'(w)\Delta + \mathcal{A}_0(w)\\ =& -\frac{\mathcal{A}_1(w)\mathcal{A}_2'(w) - \mathcal{A}_1'(w)\mathcal{A}_2(w)}{\mathcal{A}_2(w)}\Delta\\ &-\frac{\mathcal{A}_0(w)\mathcal{A}_2'(w) - \mathcal{A}_0'(w)\mathcal{A}_2(w)}{\mathcal{A}_2(w)}.\end{aligned} \tag{B1}$$

Meanwhile, for $w \in [1, \Delta^{-\frac{1}{\kappa}})$,

$$\begin{aligned}\mathcal{A}_2' =& -\theta[3\theta - 1 + 2(w-1)\theta^2 + 2\theta w(\sigma - 1)] < 0,\\ \mathcal{A}_1' =& -w^{-\kappa}\sigma\{(1-\theta)[w^{2\kappa-1}(1-\theta)\kappa\\ &+ \theta(\kappa-1)] + w^{2\kappa}\theta(1-\theta)(\kappa+1) + w\theta^2(\kappa-2)\},\\ \mathcal{A}_0' =& -\theta(1-\theta)[1 + 2(w-1)\theta] < 0,\end{aligned}$$

which lead to

$$\begin{aligned}\mathcal{A}_0(w)\mathcal{A}_2'(w) - \mathcal{A}_0'(w)\mathcal{A}_2(w) =& -(1-\theta)\theta(2\theta-1)[(w-1)\theta+1]\sigma < 0,\\ \mathcal{A}_1(w)\mathcal{A}_2'(w) - \mathcal{A}_1'(w)\mathcal{A}_2(w) +& \mathcal{A}_0(w)\mathcal{A}_2'(w) - \mathcal{A}_0'(w)\mathcal{A}_2(w)\\ &= w^{-\kappa-1}[1+(w-1)\theta]^2\sigma\mathcal{B},\end{aligned}$$

where

$$\begin{aligned}\mathcal{B} =& w^{2\kappa}(1-\theta)^2(\sigma-\theta) - w^{2\kappa}(1-\theta)(\kappa-1)[(w-1)\theta(\sigma-1+\theta) + (2\theta-1)\sigma]\\ &- w^{1+\kappa}(1-\theta)\theta(2\theta-1) - (w-1)^2\theta^2\kappa(\sigma-1+\theta)\\ &- (w-1)\theta\{(\kappa-1)[\theta(\sigma-1+\theta) + (2\theta-1)\sigma] + 2\theta(\sigma-1+\theta)\}\\ &- \theta[(1-\theta)(\sigma-\theta) + (2\theta-1)\kappa\sigma]\\ \leq& w\theta^2(\sigma-\theta) - w^{2\kappa}(1-\theta)(\kappa-1)[(w-1)\theta(\sigma-1+\theta) + (2\theta-1)\sigma]\end{aligned}$$

[4] For simplicity, we omit the equilibrium notation “$*$” hereafter.

$$
\begin{aligned}
&- w^{1+\kappa}(1-\theta)\theta(2\theta-1) - (w-1)^2\theta^2\kappa(\sigma-1+\theta) \\
&- (w-1)\theta\{(\kappa-1)[\theta(\sigma-1+\theta) + (2\theta-1)\sigma] + 2\theta(\sigma-1+\theta)\} \\
&- \theta[(1-\theta)(\sigma-\theta) + (2\theta-1)\kappa\sigma] \\
=&- (w^{2\kappa}-1)(1-\theta)(\kappa-1)[(w-1)\theta(\sigma-1+\theta) + (2\theta-1)\sigma] \\
&- (w^{1+\kappa}-1)(1-\theta)\theta(2\theta-1) - (w-1)^2\theta^2\kappa(\sigma-1+\theta) \\
&- (w-1)\theta\{(\kappa-1)[(2\theta-1)(\sigma-1+\theta) + (2\theta-1)\sigma] + \theta[(\sigma-1) + (2\theta-1) + \theta)]\} \\
&- (2\theta-1)[(\kappa-1)\sigma + \theta) \\
<&0
\end{aligned}
$$

and where the first inequality is from (25). Since $\Delta \in [0,1]$, (B1) implies that $\partial\mathcal{F}/\partial w < 0$.

(iii) The partial derivative is calculated as

$$
\begin{aligned}
\frac{\partial \mathcal{F}}{\partial \theta} =&(w-1)[2\theta-1+(w-1)(3\theta-2)\theta] \\
&+ [2w^{2\kappa-1}(1-\theta) + 2\theta(w-1) + 1 + w^{2\kappa}(2\theta-1)]w^{1-\kappa}\Delta\sigma \\
&- [(w-1)(2\theta-1) + (w-1)^2(3\theta-2)\theta + 2\sigma(1-\theta+w^2\theta)]\Delta^2.
\end{aligned}
\tag{B2}
$$

From $\mathcal{F}(w,\phi) = 0$,

$$
\Delta^2 = -\frac{\mathcal{A}_1(w)\Delta + \mathcal{A}_0(w)}{\mathcal{A}_2(w)}. \tag{B3}
$$

Substituting (B3) into (B2) gives

$$
\left.\frac{\partial \mathcal{F}}{\partial \theta}\right|_{\mathcal{F}(w,\phi)=0} = \frac{[1+(w(\phi)-1)\theta]\sigma w(\phi)^{1-\kappa}}{(w(\phi)-1)\theta^2 + (2\theta-1)\sigma + \theta(w(\phi)-1)(\sigma-1)}\mathcal{H}(w(\phi)), \tag{B4}
$$

where

$$
\begin{aligned}
\mathcal{H}(w) =&w^{\kappa-1}(1-\theta)^2(w-1) + (w-1)w^{2\kappa-1}\Delta[\theta^2-(1-\theta)^2] + (w-1)\Delta(1-\theta^2) \\
&+ [w(w^{2\kappa-2}-1) + (w-1)w^{2\kappa-1}(1-\theta^2)]\Delta + (w-1)w^{\kappa}\theta^2 \\
&+ (w^{2\kappa}-1)\Delta(\sigma-1).
\end{aligned}
$$

Since $w(\phi) \geq 1$, $\mathcal{H}(w(\phi)) \geq 0$ holds. Meanwhile, the fraction on the RHS of (B4) is evidently positive; therefore, we obtain the non-negativeness of (B4).

Appendix C: Proof of Lemma 3

We first provide an inequality, which will be applied later. Note that $\Delta \in (0,1)$ implies $w^* > 1$.[5] We have

$$\begin{aligned}
&\Big(\frac{\theta}{1-\theta}w^{1-2\kappa}\frac{1-\Delta w^{\kappa}}{1-\Delta w^{-\kappa}}-1\Big)\Big[\frac{\theta w^{1-\kappa}\Delta+(1-\theta)\Delta w^{-\kappa}}{1-\theta+\theta w^{1-\kappa}\Delta}\Big] \\
&\qquad = \frac{\theta}{1-\theta}w^{1-2\kappa}\frac{1-\Delta w^{\kappa}}{1-\Delta w^{-\kappa}} - \frac{\theta w^{1-2\kappa}+(1-\theta)w^{-\kappa}\Delta}{1-\theta+\theta w^{1-\kappa}\Delta} \\
&\qquad = \frac{\theta w^{1-2\kappa}+(1-\theta)w^{-\kappa}\Delta}{1-\theta+\theta w^{1-\kappa}\Delta}\Big[\frac{\theta w^2+w(\sigma-\theta)}{w(\sigma-1+\theta)+1-\theta}-1\Big] \\
&\qquad = \frac{\theta w^{1-2\kappa}+(1-\theta)w^{-\kappa}\Delta}{1-\theta+\theta w^{1-\kappa}\Delta}\;\frac{(w-1)(1-\theta+w\theta)}{w(\sigma-1+\theta)+1-\theta} > 0,
\end{aligned}$$

where the second equality is from (22), and the inequality is because $w > 1$. Thus,

$$\frac{\theta}{1-\theta}w^{1-2\kappa}\frac{1-\Delta w^{\kappa}}{1-\Delta w^{-\kappa}} > 1. \tag{C1}$$

The export of a firm in country i to country j with productivity φ is

$$r_{ij}(\varphi) = r_{ij}(\varphi_{ij}^*)\Big(\frac{\varphi}{\varphi_{ij}^*}\Big)^{\sigma-1} = \sigma\bar{r}f_x\Big(\frac{\varphi}{\varphi_{ij}^*}\Big)^{\sigma-1}.$$

Let X_{ij} be the total export from country i to country j, or

$$\begin{aligned}
X_{12} &= n_{12}\int_{\varphi_{12}^*}^{\infty} r_{12}(\varphi)\,\mu_{12}(\varphi)\,d\varphi \\
&= n^e\lambda^e\left(\varphi_{12}^*\right)^{-\kappa}\bar{r}f_x\frac{\sigma\kappa}{\kappa-(\sigma-1)}.
\end{aligned} \tag{C2}$$

Similarly, total export from country 2 to country 1 is

$$X_{21} = n^e(1-\lambda^e)(\varphi_{21}^*)^{-\kappa}\frac{\sigma\kappa}{\kappa-(\sigma-1)}\bar{r}f_x. \tag{C3}$$

The relative aggregate export is expressed as

$$\frac{X_{12}}{X_{21}} = \Big(\frac{w\theta}{1-\theta}\Big)\Big(w^2\frac{\varphi_{22}^*}{\varphi_{11}^*}\Big)^{-\kappa} = \Big(\frac{\theta}{1-\theta}\Big)w^{1-2\kappa}\Big(\frac{\varphi_{11}^*}{\varphi_{22}^*}\Big)^{\kappa} = \Big(\frac{\theta}{1-\theta}\Big)w^{1-2\kappa}\frac{1-w^{\kappa}\Delta}{1-w^{-\kappa}\Delta} > 1,$$

where the inequality is due to (C1).

[5] For simplicity, we omit the equilibrium notation "*" hereafter.

Appendix D: Relationship between w and $\varphi_{11}^*/\varphi_{22}^*$

Derivation of (30)

According to (C2) and (C3), the LHS of (29) is

$$\begin{aligned} X_{12} - X_{21} &= n^e \frac{\sigma\kappa}{\kappa-\sigma+1} \bar{r} f_x [\lambda^e (\varphi_{12}^*)^{-\kappa} - (1-\lambda^e)(\varphi_{21}^*)^{-\kappa}] \\ &= n^e \frac{\sigma\kappa}{\kappa-\sigma+1} \bar{r} f_x \left[\lambda^e (\Lambda w)^{-\kappa} (\varphi_{22}^*)^{-\kappa} - (1-\lambda^e)\left(\frac{\Lambda}{w}\right)^{-\kappa} (\varphi_{11}^*)^{-\kappa}\right], \end{aligned} \tag{D1}$$

where the second equality is from (17).

Meanwhile, the RHS of (29) is

$$n^e \lambda^e \bar{r} [f_e + (\varphi_{11}^*)^{-\kappa} f_d + (\varphi_{12}^*)^{-\kappa} f_x] - \bar{r} K_1 = n^e \lambda^e \frac{\kappa}{\sigma-1} \bar{r} f_e - \bar{r}\theta K, \tag{D2}$$

according to (9).

Substituting (12) and (14) into (D1) and (D2), we obtain another form of (29) as

$$\frac{\sigma(\sigma-1)}{\kappa-\sigma+1} \frac{f_x}{f_e} (\varphi_{22}^*)^\kappa \left[\theta w (\Lambda w)^{-\kappa} - (1-\theta)\left(\frac{\Lambda}{w}\right)^{-\kappa} \left(\frac{\varphi_{11}^*}{\varphi_{22}^*}\right)^{-\kappa}\right] = \theta(1-\theta)(w-1).$$

Substituting (20) into this equation gives the balance of payment condition (30).

The balance of payment curve in Figure 2

Let $\Phi = \varphi_{11}^*/\varphi_{22}^*$. Equation (30) is equivalent to $\mathcal{I}(w, \Phi_{11}, \Delta) = 0$, where

$$\mathcal{I}(w, \Phi, \Delta) = \Delta[\theta w^{1-\kappa} - (1-\theta) w^\kappa \Phi^{-\kappa}] - \theta(w-1)\left(\frac{1-\theta+\theta w^{1-\kappa}\Delta}{\theta w + 1 - \theta}\right),$$

which has the following derivatives at the balance of payment curve:

$$\begin{aligned} \frac{\partial \mathcal{I}(w,\Phi,\Delta)}{\partial w} &= -\Delta(1-\theta)\kappa w^{\kappa-1}\Phi^{-\kappa} - \frac{\Delta\theta w^{-\kappa}\sigma(\kappa-1)}{\theta w + \sigma - \theta} - \frac{\theta\sigma(1-\theta+\theta\Delta w^{1-\kappa})}{(\theta w + \sigma - \theta)^2} \\ &< 0, \\ \frac{\partial \mathcal{I}(w,\Phi,\Delta)}{\partial \Phi} &= \Delta(1-\theta) w^\kappa \kappa \Phi^{-\kappa-1} > 0, \\ \frac{\partial \mathcal{I}(w,\Phi,\Delta)}{\partial \Delta} &= \theta w^{1-\kappa} - (1-\theta) w^\kappa \Phi^{-\kappa} - \theta(w-1)\frac{\theta w^{1-\kappa}}{\theta w + 1 - \theta} = \frac{\theta(w-1)(1-\theta)}{\Delta(\theta w + 1 - \theta)} \\ &> 0, \end{aligned} \tag{D3}$$

where the last equality is because $\mathcal{I}(w,\phi,\Delta)=0$ holds at the balance of payment curve. Therefore, implicit function $\Phi(w,\Delta)$, defined by $\mathcal{I}(w,\Phi,\Delta)=0$, has the property of

$$\frac{\partial\Phi}{\partial w}=-\frac{\partial\mathcal{I}/\partial w}{\partial\mathcal{I}/\partial\Phi}>0,\quad \frac{\partial\Phi}{\partial\Delta}=-\frac{\partial\mathcal{I}/\partial\Delta}{\partial\mathcal{I}/\partial\Phi}<0,$$

showing that the balance of payment curve in Figure 2 increases in w and decreases in Δ.

Appendix E: The curves of Figure 3

The capital mobility curve

We calculate the derivatives of (21) as follows.

$$\begin{aligned}\frac{\partial\lambda}{\partial w}=&\frac{\theta(1-\theta)w^{-\kappa}\{(\kappa-1)\Delta(1+w^{-2\kappa})+w^{-\kappa}[1+(1-2\kappa)\Delta^2]\}}{[\Delta+\theta w^{1-2\kappa}\Delta-\theta\Delta-w^{-\kappa}(1-\theta+\theta w)]^2}\\ \geq&\frac{\theta(1-\theta)w^{-\kappa}\{2(\kappa-1)\Delta w^{-\kappa}+w^{-\kappa}[1+(1-2\kappa)\Delta^2]\}}{[\Delta+\theta w^{1-2\kappa}\Delta-\theta\Delta-w^{-\kappa}(1-\theta+\theta w)]^2}\\ =&\frac{\theta(1-\theta)w^{-\kappa}\{2(\kappa-1)\Delta w^{-\kappa}(1-\Delta)+w^{-\kappa}(1-\Delta^2)\}}{[\Delta+\theta w^{1-2\kappa}\Delta-\theta\Delta-w^{-\kappa}(1-\theta+\theta w)]^2}\\ \geq&0,\\ \frac{\partial\lambda}{\partial\Delta}=&\frac{\theta(1-\theta)w^{\kappa+1}(w^{2\kappa}-1)}{\left[w^{2\kappa}(\theta-1)\Delta+w^{\kappa}(1+\theta(w-1))-w\Delta\theta\right]^2}>0.\end{aligned}$$

The second inequality holds strictly as long as $\Delta<1$. These inequalities show that the capital mobility curve increases in both w and Δ.

The balance of payment curve

By using (14) and (21), we have

$$\left(\frac{\varphi^*_{11}}{\varphi^*_{22}}\right)^{-\kappa}=\frac{1-\theta}{w\theta}\;\frac{\lambda}{1-\lambda},\tag{E1}$$

so we can rewrite (31) as $\mathcal{J}(w,\lambda,\Delta)=0$, where

$$\mathcal{J}(w,\lambda,\Delta)\equiv\Delta\Big[\theta w^{1-\kappa}-w^{\kappa-1}\frac{(1-\theta)^2}{\theta}\frac{\lambda}{1-\lambda}\Big]-\theta(w-1)\frac{1-\theta+\theta w^{1-\kappa}\Delta}{\theta w+1-\theta}.$$

It follows immediately that

$$\begin{aligned}\frac{\partial \mathcal{J}(w,\lambda,\Delta)}{\partial \lambda} =& -\Delta w^{\kappa-1}(\theta w+\sigma-\theta)\frac{(1-\theta)^2}{\theta(1-\lambda)^2}<0,\\ \frac{\partial \mathcal{J}(w,\lambda,\Delta)}{\partial \Delta} >& 0\end{aligned}$$

where the second inequality is exactly the same as (D3). Meanwhile,

$$\begin{aligned}\frac{\partial}{\partial w}\mathcal{J}(w,\lambda,\Delta) =& \Delta\theta(1-\kappa)w^{-\kappa}[\theta w+\sigma-\theta-\theta(w-1)]\\ &-\Delta(k-1)w^{\kappa-2}(\theta w+\sigma-\theta)\frac{(1-\theta)^2\lambda}{\theta(1-\lambda)}\\ &-\theta(1-\theta+\theta w^{1-\kappa}\Delta)+\Delta\theta\Big[\theta w^{1-\kappa}-w^{\kappa-1}\frac{(1-\theta)^2\lambda}{\theta(1-\lambda)}\Big].\end{aligned}$$

The first and second terms are negative. Because $\mathcal{F}(w,\lambda)=0$, the last two terms can be rewritten as

$$\begin{aligned}-\theta[(1-\theta)+& \theta\Delta w^{1-\kappa}]+\theta^2(w-1)\frac{1-\theta+\theta\Delta w^{1-\kappa}}{\theta w+\sigma-\theta}\\ =& \theta(1-\theta+\theta\Delta w^{1-\kappa})\Big[-1+\frac{\theta(w-1)}{\theta w+\sigma-\theta}\Big]\\ =& -(1-\theta+\theta\Delta w^{1-\kappa})\frac{\theta\sigma}{\theta w+\sigma-\theta}<0.\end{aligned}$$

Therefore, as an implicit function $\lambda(w,\Delta)$ determined by $\mathcal{J}(w,\lambda,\Delta)=0$, it holds that

$$\frac{\partial\lambda}{\partial w}=-\frac{\partial\mathcal{J}/\partial w}{\partial\mathcal{J}/\partial\lambda}<0,\quad\text{and}\quad\frac{\partial\lambda}{\partial\Delta}=-\frac{\partial\mathcal{J}/\partial\Delta}{\partial\mathcal{J}/\partial\lambda}>0.$$

Appendix F: The relationship between θ and productivity cutoffs

For the larger country, exporting always requires a higher cutoff productivity than for the domestic production, i.e., $\varphi_{11}^*<\varphi_{12}^*$ holds. For the smaller country, however, when market size asymmetry is sufficiently large, the cutoff productivity level for exporting may be lower than that for domestic production, i.e., $\varphi_{22}^*>\varphi_{21}^*$. From (17) and (18),

$$\Big(\frac{\varphi_{22}^*}{\varphi_{21}^*}\Big)^{\kappa}=\frac{1-w^{-\kappa}\Delta}{1-w^{\kappa}\Delta}w^{\kappa}\Big(\frac{\Delta}{\phi}\Big)^{\frac{\kappa}{\kappa-\sigma+1}}. \tag{F1}$$

The RHS of (F1) increases in w because

$$\frac{d}{dw}\Big(\frac{1-w^{-\kappa}\Delta}{1-w^{\kappa}\Delta}\Big) = \frac{\kappa w^{\kappa-1}(1-\Delta^2)}{(1-w^{\kappa}\Delta)^2} \geq 0.$$

It also increases in θ by Lemma 2 (iii). Therefore, $\varphi^*_{22} > \varphi^*_{21}$ holds when $w(\theta)$ is above the threshold level of $\hat{w}(\hat{\theta})$, which is solved by

$$1 = \left[\frac{1-\hat{w}(\hat{\theta})^{-\kappa}\Delta}{1-\hat{w}(\hat{\theta})^{\kappa}\Delta}\right]\hat{w}(\hat{\theta})^{\kappa}\left(\frac{\Delta}{\phi}\right)^{\frac{\kappa}{\kappa-\sigma+1}}. \tag{F2}$$

In summary, (F2) and (24) determine the threshold level of $(\hat{\theta}, \hat{w}(\theta))$ above which $\varphi^*_{22} > \varphi^*_{21}$ holds.[6] When $\theta > \hat{\theta}$, the share of incumbent firms in country 1, λ^*_1, then becomes

$$\lambda^*_1|_{\theta>\hat{\theta}} = \frac{w^*\theta(\varphi^*_{11})^{-\kappa}}{w^*\theta(\varphi^*_{11})^{-\kappa} + (1-\theta)(\varphi^*_{21})^{-\kappa}},$$

where w^*, φ^*_{11}, and φ^*_{21} are derived from (19), (20), and (24).

References

Arkolakis, C., Costinot, A., Rodríguez-Clare, A., 2012. New trade models, same old gains? American Economic Review 102, 94-130.

Arkolakis, C., Demidova, S., Klenow, P.J., Rodríguez-Clare, A., 2008. Endogenous variety and the gains from trade. American Economic Review 98, 444-450.

Baldwin, R.E., Forslid, R., 2010. Trade liberalization with heterogeneous firms. Review of Development Economics 14, 161-176.

Baldwin, R.E., Okubo, T., 2006. Heterogeneous firms, agglomeration and economic geography: spatial selection and sorting. Journal of Economic Geography 6, 323-346.

Bernard, A.B., Redding, S.J., Schott, P.K., 2007. Comparative advantage and heterogeneous firms. Review of Economic Studies 74, 31-66

Behrens, K., Mion, G., Murata, Y., Südekum, J., 2014. Trade, wages, and productivity. International Economic Review, forthcoming.

[6] Note that changes in θ affect neither the wage equation nor the free entry conditions.

Behrens, K., Robert-Nicoud, F., 2014. Survival of the fittest in cities: Urbanisation and inequality. Economic Journal, forthcoming.

Chaney, T., 2008. Distorted gravity: The intensive and extensive margins of international trade. American Economic Review 98, 1707-1721.

Davis, D.R., 1998. The home market effect, trade and industrial structure. American Economic Review 88, 1264-1276.

Eaton, J., Kortum, S., 2002. Technology, geography, and trade. Econometrica 70(5), 1741-1779.

Felbermayr, G., Jung, B., 2012. The home market effect, regional inequality, and intra-industry reallocations. University of Tuebingen Working Papers in Economics and Finance.

Forslid, R., Ottaviano, G.I.P., 2003. An analytically solvable core-periphery model. Journal of Economic Geography 3 (3), 229-240.

Helpman, E., Krugman, P., 1985. Market Structure and Foreign Trade. MIT Press, Cambridge, MA.

Helpman, E., Melitz, M.J., Yeaple, S.R., 2004. Export versus FDI with heterogeneous firms. American Economic Review 94, 300-316.

Krugman, P., 1980. Scale economies, product differentiation, and the pattern of trade. American Economic Review 70, 950-959.

Martin, P., Rogers, C.A., 1995. Industrial location and public infrastructure. Journal of International Economics 39, 335-351.

Medin, H., 2003. Firms' export decisions—fixed trade costs and the size of the export market. Journal of International Economics 61, 225-241.

Melitz, M.J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica 71, 1695-1725.

Melitz, M.J., Ottaviano, G.I.P., 2008. Market size, trade, and productivity. Review of Economic Studies 75, 295-316.

Okubo, T., Picard, P.M., Thisse, J.-F., 2010. The spatial selection of heterogeneous firms. Journal of International Economics 82, 230-237.

Ottaviano, G.I.P., 2012. Agglomeration, trade and selection. Regional Science and Urban Economics 42, 987-997.

Takahashi, T., Takatsuka, H., Zeng, D.-Z., 2013. Spatial inequality, globalization, and footloose capital. Economic Theory 53, 213-238.

von Ehrlich, M., Seidel, T., 2013. More similar firms — more similar regions? On the role of firm heterogeneity for agglomeration. Regional Science and Urban Economics 43, 539-548.

Zhelobodko, E., Kokovin, S., Parenti, M., Thisse, J.-F., 2012. Monopolistic competition: beyond the CES. Econometrica 80, 2765-2784.