Mixed Duopoly and Strategic Trade Policy

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Abstract
This paper examines a three-person international mixed model. First, a social-welfare-maximizing domestic government announces a tariff level per unit of output imposed on a profit-maximizing foreign private firm. Second, the foreign private firm decides whether or not to enter the market. Third, a social-welfare-maximizing domestic public firm decides whether or not to adopt either a lifetime employment contract or a wage-rise contract as a strategic commitment. Fourth, if the foreign private firm enters, each firm independently chooses its actual output, while if the foreign private firm does not enter, the public firm prevails as a monopoly. The paper shows the equilibrium of the mixed model.

Keywords: Mixed model, Tariff, Foreign private firm, Lifetime employment contract, Wage-rise contract

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1. Introduction


However, these studies are mixed models with domestic private firms and do not include foreign private firms. Some more recent studies include foreign private firms. Fjell and Pal (1996) extend the analysis to an international context by considering a mixed model where a state-owned public firm competes with both domestic and foreign private firms, and show that the public firm reduces its output if a domestic private firm enters, whereas it increases its output if a foreign private firm enters. Fjell and Heywood (2002) consider a mixed oligopoly in which a public Stackelberg leader competes with both domestic and foreign

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³ Malerba (1993) reports that in Italy, during the 1960s through to the 1980s, there were two public firms in the top R&D investors.
private firms and show that the welfare maximizing leader always produces less than under previous Cournot conjectures. Furthermore, Matsumura (2003) examines a Stackelberg mixed duopoly where a public firm competes with a foreign private firm and find that, in contrast to Pal (1998) discussing a case of domestic competitors, the public firm should be the leader.

We study the behaviours of a social-welfare-maximizing domestic government, a social-welfare-maximizing domestic public firm and a profit-maximizing foreign private firm in a mixed market. We consider the following situation. In the first stage, the government announces a tariff level per unit of output imposed on the foreign private firm to the firms. In the second stage, the foreign private firm decides whether or not to enter the market. If the foreign private firm enters the market, then it bears the tariff. The foreign private firm enters the market if and only if its post-entry profit is positive. In the third stage, the public firm decides whether or not to adopt either a lifetime-employment-contract policy (LECP) or a wage-rise-contract policy (WRCP). In the fourth stage, if the foreign private firm enters, each firm independently chooses its actual output, while if the foreign private firm does not enter, the public firm prevails as a monopoly.

The public firm can adopt either LECP or WRCP. If the firm adopts LECP, then it chooses an output level and enters into a lifetime employment contract with the number of employees necessary to achieve the output level. Hence, the firm’s wage cost changes from a variable cost to a fixed cost. Furthermore, WRCP is a promise by the firm that it will announce a certain output level and a wage premium rate, and if it actually produces more than the announced output level, then it will pay each employee a wage premium uniformly.

We discuss the equilibrium of the quantity-setting mixed model in which the government and the domestic public firm compete with the foreign private firm.

This paper is organized as follows. In Section 2, we describe the mixed model. Section 3 gives supplementary explanations of the mixed model. Section 4 discusses the equilibrium of the mixed model. Section 5 concludes the paper.

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4 LECP and WRCP are proposed and discussed in Ohnishi (2001, 2003), respectively.
2. The Model

Let us consider a mixed market, where the domestic public firm is designated as firm 1 and the foreign private firm as firm 2. For the remainder of this paper, when \( i \) and \( j \) are used to refer to firms in an expression, they should be understood to refer to 1 and 2 with \( i \neq j \). The market price is determined by the inverse demand function \( p(Q) \), where \( Q = \sum_{i=1}^{2} q_i \) denotes the aggregate quantity. We assume that \( p' < 0 \) and \( p'' < 0 \). The four stages of the model run as follows. In the first stage, the domestic government chooses and announces a tariff level \( t > 0 \) per unit of output imposed on firm 2 to the firms. The government’s objective is to set \( t \) optimally to maximize domestic social welfare. In the second stage, firm 2 decides whether or not to enter the market. If firm 2 enters the market, then it bears \( t q_2 \). Firm 2 enters the market if and only if its post-entry profit is positive. In the third stage, if firm 2 enters, then firm 1 can adopt a countermeasure against firm 2. In the fourth stage, if firm 2 enters, each firm independently chooses its actual output \( q_i \), while if firm 2 does not enter, firm 1 prevails as a monopoly.

Firm 1’s countermeasure in the third stage is either LECP or WRCP, which are described in the following section. Of course, we can consider the case in which firm 1 adopts both LECP and WRCP. However, since each has opposing effects on firm 1’s marginal cost and thus counteracts the effect of the other, we do not consider the case in which firm 1 adopts both LECP and WRCP.

Firm 1’s profit is

\[
\pi_1 = p(Q)q_1 - c_1 ,
\]

where \( c_1 \) is firm 1’s cost.

Firm 2’s profit is

\[
\pi_2 = p(Q)q_2 - c_2 - t q_2 ,
\]

where \( c_2 \) is firm 2’s cost.

Social welfare \( (W) \) is

\[
W = \int_0^Q p(x)dx - c_1 - pq q_2 + t q_2 .
\]

The government and firm 1 aim to maximize social welfare, and firm 2 aims to maximize its own profit. The first-order condition for firm 1 is

\[
p - c_1' - p' q_2 = 0 .
\]

We assume that the following second-order condition is satisfied:
\[ p^* - c_1'' - p'' q_2 < 0. \] 

Our equilibrium concept is the subgame perfect equilibrium, which is solved by backward induction.

### 3. Supplementary Explanations

In this section, we will give supplementary explanations of the model described in the preceding section. First, we briefly describe LECP. We assume that firm \( i \) employs and dismisses its employees according to the amount of output. If firm \( i \) increases its output, the employees are increased accordingly. On the other hand, if firm \( i \) decreases its output, the employees are decreased accordingly. That is, the wages of the employees of firm \( i \) are originally its variable cost. In the third stage, if firm 1 adopts LECP, then it chooses an output level \( q_1^* \) and enters into a lifetime employment contract with the number of employees necessary to achieve \( q_1^* \). Therefore, the wages of employees employed up to \( q_1^* \) are sunk as firm 1’s fixed cost, and the wages of the employees employed exceeding \( q_1^* \) are firm 1’s variable cost. Hence, firm 1’s cost becomes as follows:

\[
c_1^L(q_1, q_1^*) = \begin{cases} 
(v_1 - r_i)q_1 + r_i q_1^* & \text{if } q_i \leq q_1^*, \\
v_1 q_1 & \text{if } q_i \geq q_1^*, 
\end{cases}
\]  

(6)

where \( v_1 > 0 \) is firm 1’s total cost for each unit of output and \( r_i \in (0, v_1] \) is firm 1’s wage cost for each unit of output. Firm 1’s marginal cost exhibits a discontinuity at \( q_1 = q_1^* \).

On the other hand, firm 2’s cost is

\[ c_2(q_2) = v_2 q_2, \]  

(7)

where \( v_2 > 0 \) is firm 2’s total cost for each unit of output. We assume that firm 1 is less efficient than firm 2, i.e., \( v_1 > v_2 \).

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This assumption is justified in Gunderson (1979), Cremer, Marchand, and Thisse (1989), and Nett (1993, 1994) and is often used in literature studying mixed markets. See, for instance, Sertel (1988), George and La Manna (1996), Kenneth and Manfredi (1996), Mujumdar and Pal (1998), Pal (1998), Nishimori and Ogawa (2002), and Matsumura (2003). If firm 1 is more efficient than or equally as efficient as firm 2, then firm 1 supplies the entire market, resulting in a social-welfare-maximizing public monopoly. This assumption is made to eliminate such a trivial solution.
Second, we briefly describe WRCP. In the third stage, if firm 1 adopts WRCP, then it chooses an output level \( q^*_1 \) and a wage premium rate \( w_i > 0 \), and agrees to pay each employee a wage premium uniformly if it actually produces more than \( q^*_1 \). Hence, firm 1’s cost becomes as follows:

\[
e_{i}^{w}(q_{1}, q^*_1, w_i) = \begin{cases} 
    v_i q_1 & \text{if } q_1 \leq q^*_1, \\
    v_i q_1 + (q_1 - q^*_1)w_i & \text{if } q_1 \geq q^*_1.
\end{cases}
\]  

(8)

Firm 1’s marginal cost exhibits a discontinuity at \( q_1 = q^*_1 \). LECP specifies a lower marginal cost for output, while WRCP specifies a higher marginal cost for output.

4. Equilibrium

In this section, we will arrive at the equilibrium of the mixed model. First of all, we compare social welfare when entry is prevented with that when entry is allowed. The government’s objective is to set a tariff level optimally to maximize social welfare. Social welfare is the sum of consumer surplus, firm 1’s profit and tariff revenue. It is thought that the government will allow the entry of firm 2 if social welfare is higher when entry is allowed than when entry is prevented, whereas it will deter the entry of firm 2 if social welfare is higher when entry is prevented than when entry is allowed. If firm 2 enters the market, then the government gets the tariff revenue of \( t q_2 \). On the other hand, if firm 2 does not enter the market, then tariff revenue is zero. We will consider the following proposition:

**Proposition 1.** \( W^M \) denotes social welfare when firm 1 is the monopolist, and \( W^D \) denotes social welfare when firm 2 enters the market. Then \( W^M < W^D \).

Proof. See Appendix A.

Now, we discuss the equilibrium of the mixed model. Firm 2’s objective is to maximize its own profit. If firm 2 does not enter the market, it cannot get any profit. Therefore, firm 2 enters the market except that \( t \geq p - v_2 \). Proposition 1 means that in equilibrium firm 2 exists in the market.

The government’s objective is to set a tariff level optimally to maximize social welfare. Tariff revenue is decided by \( t \) and \( q_2 \). Since firm 2’s profit is (2), if \( t \) is zero, firm 2
receives the profit of $(p - v_2)q_2$. At this time, the government can increase tariff revenue by raising $t$, and social welfare also increases.

Firm 1’s objective is to maximize social welfare. From (6), we know that LECP specifies a lower marginal cost for output. Hence, it is thought that firm 1 increases its output if it offers LECP. If firm 2 exists in the market, increasing firm 1’s output decreases firm 2’s output because of strategic substitutes, and in $t > 0$, tariff revenue decreases.

From (8), we know that WRCP specifies a higher marginal cost for output. Hence, it is thought that firm 1 decreases its output if it offers WRCP. If firm 2 exists in the market, decreasing firm 1’s output increases firm 2’s output because of strategic substitutes, and in $t > 0$, tariff revenue increases.

The main result of this study is described by the following proposition:

**Proposition 2.** In the mixed model, there exists an equilibrium in which (i) the government chooses $t$ that is just a little smaller than $p - v_2$, (ii) firm 2 enters the market, and (iii) firm 1 does not adopt LECP but WRCP. Furthermore, the equilibrium coincides with the Stackelberg solution where firm 1 is the leader.

**Proof.** See Appendix B.

**5. Conclusion**

We have examined a three-person international mixed model in which a domestic government and a domestic public firm compete with a foreign private firm and have shown that there exists an equilibrium where the government chooses a tariff level that is just a little smaller than the market price minus the foreign private firm’s marginal cost, the foreign private firm enters the market, and the domestic public firm adopts WRCP. Furthermore, we have shown that the equilibrium coincides with the Stackelberg solution where the domestic public firm is the leader. We have found that eliminating a foreign competitor does not improve domestic social welfare. There are many studies dealing with mixed market models that incorporate social-welfare-maximizing public firms. We will pursue further research on these studies in the future.
Appendix A

Proof of Proposition 1

If the government deters the entry of firm 2 sets the level of \( t \) on \( t \geq p - v_2 \), then the entry of firm 2 is prevented and firm 1 supplies monopolistically in the market. Since the market demand curve is downward sloping, firm 1 has a strong incentive to expand its output in order to reduce the market price. That is, if entry is prevented, then firm 1 produces an output such that the market price equals \( v_1 \), and its profit and tariff revenue are zero. Therefore, social welfare is

\[
W^M = \int_0^{Q^M} p(x)dx - v_1 Q^M.
\]  (9)

If the market price is smaller than \( v_1 \), then social welfare is

\[
W^{M(p^c)} = \int_0^{Q^{M(p^c)}} p(x)dx - v_1 Q^{M(p^c)}.
\]  (10)

From \( p' < 0 \) and \( Q^v < Q^{M(p^c)} \), we have \( W^M > W^{M(p^c)} \). Hence, if entry is prevented, then firm 1 produces an output such that the market price equals \( v_1 \).

On the other hand, if the government sets the level of \( t \) on \( t < p - v_2 \), then firm 2 enters the market and bears \( tq_2 \). That is, if entry is allowed, then the government gets the tariff revenue of \( tq_2 \). Firm 1 can decide whether or not to adopt either LECP or WRCP. If firm 1 adopts neither LECP nor WRCP, then firm 1 produces an output such that the market price equals \( v_1 \), and its profit is zero. Therefore, social welfare is

\[
W^D = \int_0^{Q^D} p(x)dx - v_1 q_1^v - v_1 q_2^v + tq_2^v
\]

\[= \int_0^{Q^D} p(x)dx - v_1 Q^v + tq_2^v.\]  (11)

In \( p = v_1 > v_2 \), the government can set the level of \( t > 0 \) on \( t < p - v_2 \). Hence, we see that social welfare is larger when entry is allowed than when entry is prevented. Furthermore, firm 1 adopts either LECP or WRCP if and only if social welfare is increased. Our equilibrium concept is the subgame perfect equilibrium and all information in the model is common knowledge. Thus, social welfare is higher when entry is allowed than when entry is prevented. Q.E.D.
Appendix B

First of all, we will present the next three supplementary lemmas.

**Lemma 1.** Firm 1’s social-welfare-maximizing output is larger when it adopts LECP than when it does not.

Proof. From (6), we see that lifetime employment will never increase the marginal cost of firm 1. The first-order condition for firm 1 when its marginal cost is \( v_i - r_i \) is

\[
p - v_i + r_i - p'q_2 = 0, \tag{12}
\]

where \( r_i \) is positive. To satisfy the first-order condition, \( p - v_i - p'q_2 \) must be negative. Thus, firm 1’s optimum output is larger when its marginal cost is \( v_i - r_i \) than when its marginal cost is \( v_i \). Q.E.D.

**Lemma 2.** Firm 1’s social-welfare-maximizing output is smaller when it adopts WRCP than when it does not.

Proof. From (8), we see that WRCP will never decrease the marginal cost of firm 1. The first-order condition for firm 1 when its marginal cost is \( v_i + w_i \) is

\[
p - v_i - w_i - p'q_2 = 0, \tag{13}
\]

where \( w_i \) is positive. To satisfy the first-order condition, \( p - v_i - p'q_2 \) must be positive. Thus, firm 1’s optimum output is smaller when its marginal cost is \( v_i + w_i \) than when its marginal cost is \( v_i \). Q.E.D.

**Lemma 3.** If firm 1 adopts WRCP, then in equilibrium \( q_i = q_i^* \).

Proof. First, consider the possibility that \( q_i > q_i^* \) in equilibrium when firm 1 adopts WRCP. Social welfare is \( \int_0^q p(x)dx - v_iq_i - (q_i - q_i^*)w_i - pq_2 + tq_2 \). Then, firm 1 must pay its employees wage premiums \( (q_i - q_i^*)w_i \). That is, firm 1 can improve social welfare by rising \( q_i^* \), and the equilibrium point does not change in \( q_i \geq q_i^* \). Hence, \( q_i > q_i^* \) does not result in
an equilibrium.

Second, consider the possibility that $q_1 < q_1^*$ in equilibrium. In this case, firm 1’s marginal cost is $v_1$. It is impossible for firm 1 to change its output in equilibrium because such a strategy is not credible. That is, WRCP does not function as a strategic commitment. Q.E.D.

Now, we will prove Proposition 2.

**Proof of Proposition 2**

First, (ii) follows from Proposition 1.

Second, consider the possibility that firm 1 adopts LECP. Firm 1’s objective is to maximize social welfare. Lemma 1 states that firm 1’s social-welfare-maximizing output is larger when it adopts LECP than when it does not. Furthermore, Lemma 4 states that if firm 1 adopts LECP, then in equilibrium $q_1 = q_1^*$.

From (2) and (7), the first-order condition for firm 2 is

$$p'q_2 + p - v_2 - t = 0.$$  \hspace{1cm} (14)

Furthermore, we have

$$R_2'(q_1) = -\frac{p''q_2 + p'}{p''q_2 + 2p'}.$$  \hspace{1cm} (15)

We consider firm 1’s Stackelberg leader output when each firm’s marginal cost is constantly equal to $v_1$. Firm 1 selects $q_1$, and firm 2 selects $q_2$ after observing $q_1$. If firm 1 is the Stackelberg leader, then it maximizes social welfare $W(q_1, R_2(q_1))$ with respect to $q_1$. Therefore, firm 1’s Stackelberg leader output satisfies the first order condition:

$$p - v_1 - p'q_2 - p'q_2' R_2' = 0.$$  \hspace{1cm} (16)

From $p', R_2' < 0$, to satisfy the first-order condition, $p - v_1 - p'q_2$ must be positive. Hence, firm 1’s Stackelberg leader output is smaller than its Cournot output. The further the point on $R_2$ gets from the Stackelberg point, the more social welfare decreases. Thus, firm 1 does not adopt LECP.

Third, we show that firm 1 adopts WRCP. Lemma 2 states that firm 1’s social-welfare-maximizing output is smaller when it adopts WRCP than when it does not. Furthermore, Lemma 1 states that if firm 1 adopts WRCP, then in equilibrium $q_1 = q_1^*$.

From (15), we see that the slope of $R_2(q_1)$ is larger than $-1$, and further that it is smaller..
than zero. Hence, a decrease in firm 1’s output is larger than an increase in firm 2’s output.

From (13) and (15), we see that a decrease in firm 1’s output is decided by the value of \( w_1 \).

Let \( w_1 \) be a variable that can take any value of zero and over. Thus, firm 1 adopts WRCP, and the equilibrium coincides with the Stackelberg solution where firm 1 is the leader.

Fourth, we prove (i). The government’s objective is to set \( t \) optimally to maximize social welfare. Social welfare is the sum of consumer surplus, firm 1’s profit and tariff revenue. Firm 2 enters the market if and only if its post-entry profit is positive. Tariff revenue is decided by \( t \) and \( q_2 \). Since firm 2’s profit is (2), if \( t \) is zero, firm 2 gets \( (p - v_2)q_2 \).

Tariff revenue increases to the level in which \( t \) is just a little smaller than \( p - v_2 \), and social welfare also increases. Our equilibrium concept is the subgame perfect equilibrium and all information in the model is common knowledge. Thus, the proposition follows.

Q.E.D.
References


Gunderson, M. (1979), Earnings differentials between the public and private sectors,


Nishimori, A. and Ogawa, H. (2002), Public monopoly, mixed oligopoly and productive
efficiency, Australian Economic Papers 41, 185-190.