A Monopolistic Competition Model of International Trade with External Economies of Scale

Nobuhito Suga[*]

Graduate School of Economics and Business Administration, Hokkaido University

Abstract

This paper presents a two-country model of monopolistic competition in which differentiated products are produced subject to external economies of scale and two countries differ only in size measured by factor endowment. It is shown that under free trade, the larger country has positive net exports of differentiated products, which results in its gains from trade, whereas the smaller county may lose from trade. Noteworthy is that under trade in differentiated products, the industrial agglomeration is possibly harmful to both countries when the taste for product diversity is sufficiently strong.

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[*] Corresponding address: Graduate School of Economics and Business Administration, Hokkaido University, Kita-9 Nishi-7, Kita-ku, Sapporo, 060-0809, Japan. E-mail address: suga@econ.hokudai.ac.jp.

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1. Introduction

Over the past several decades, a huge literature has been developed to explore the implications of increasing returns to scale for international trade. Early studies such as Jones (1968), Melvin (1969), and Kemp and Herberg (1969) considered the phenomenon of increasing returns which arises from production externalities and therefore is compatible with perfect competition. Ethier (1982) is undoubtedly one of the most influential studies that investigated the role of such *external* economies of scale in international trade. Constructing a two-country model in which the countries differ only in size measured by factor endowment, Ethier (1982) demonstrated that the larger country exports the increasing-returns good, which results in its gains from trade, whereas the smaller country may lose from trade.

On the other hand, the implications of scale economies operating at the firm level, namely *internal* economies of scale, have been extensively studied in a monopolistically competitive framework. Since the seminal work of Krugman (1979), the monopolistic competition models of trade have succeeded in explaining the emergence of intra-industry trade. As is well-known, a standard monopolistic competition model assumes no cross-country technological differences; all monopolistically competitive firms in both countries incur the same marginal and fixed costs. In contrast, Kikuchi (2004) and Suga (2005) incorporated the cross-country technical heterogeneity into the model and examined the Ricadian comparative advantages in a monopolistically competitive framework. These analyses revealed that in the presence of the cross-country technical heterogeneity, the differentiated-product industry tends to concentrate in a country with lower marginal and fixed costs, and therefore, that the emergence of intra-industry trade is very unlikely.

The purpose of this paper is to explore what can be fond if external economies of scale are incorporated into a standard monopolistic competition model with two countries, two (one differentiated-product and one homogeneous good) industries, and one factor of production. In this paper, external economies of scale are modeled by assuming that a marginal cost and a fixed cost decline with an increase in the number of monopolistically competitive firms. In view of the conclusions in Ethier (1982), Kikuchi (2004) and Suga (2005), it is expected that the larger country will have positive net exports of differentiated products and necessarily gain from free trade. Indeed, this anticipation will be confirmed in this paper. Another issue we will investigate is the normative implications of the trade-induced industrial agglomeration. The concentration of a differentiated-product industry in one country implies a reduction in the diversity of products supplied by the other country, which adversely affects welfare in both countries under trade in differentiated products. This paper demonstrates that such industrial agglomeration is possibly harmful to both countries when the taste for variety is sufficiently strong.

This study is closely related to Kikuchi (2002), which explores the role of a decreasing fixed cost as a determination of trade patterns. In Kikuchi (2002), the differentiated product is assumed to be the network good that is produced by making use of the communications network. In the model, the payment of a fixed fee required to get on the network is the source of the declining property of a fixed cost.[1] In the present paper, we assume that the marginal and fixed costs decrease due to technological externalities arising between firms. This paper also differs from Kikuchi (2002) in a manner of formulating a dynamic adjustment process. Kikuchi (2002) assumes the entry-exit process in which firms enter (exit) if profits are positive (negative). On the other hand, this paper employs the labor reallocation process in which labor sluggishly moves to the industry that offers a higher wage rate. Our treatment on adjustment process enhances the tractability of the model and makes it easy to apply a geometrical technique for determining trade patterns that was proposed by Ethier (1979, 1982).

The rest of the paper is organized as follows. The next section presents the basic model. Section 3 gives a brief explanation concerning the autarky equilibrium. Section 4 clarifies the patterns of specialization attained in the free trade equilibrium. Section 5 examines the welfare implications of trade and industrial agglomeration. Section 6 is devoted to concluding remarks.

2. The model

The economy comprises two countries, the home country and the foreign country. The home (foreign) country is endowed with L ($L*$) units of labor, which is the only primary factor of production. The two countries are identical in all respects except the

[1] In Kikuchi (2002), the communications network (the construction of which involves a large fixed cost) is provided by a public monopoly, and a fixed fee paid to utilize it is determined by average cost pricing. As a consequence, a fixed cost of the representative firm is decreasing in the number of monopolistically competitive firms.

size of their labor endowments. This economy have two sectors: the competitive sector which produces a homogeneous good, and the monopolistically competitive sector which produces a large variety of differentiated products.

2.1. The consumption side

We assume that all consumers share the same Cobb-Douglas preferences. Then, the home country's social utility function can be expressed as

$$U = AC_1^{\gamma}C_2^{1-\gamma}, \quad A = \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)}, \quad 0 < \gamma < 1, \tag{1}$$

where C_1 represents the quantity index for differentiated products and C_2 the consumption level of a homogeneous good. The quantity index takes the form

$$C_1 = \left[\int_0^n d(i)^{(\sigma-1)/\sigma} di + \int_0^{n^*} d(i^*)^{(\sigma-1)/\sigma} di^*\right]^{\sigma/(\sigma-1)}, \quad \sigma > 1,$$

where n (n^*) is the number of products produced in the home (foreign) country, $d(i)$ ($d(i^*)$) is the quantity of product i (i^*), and σ is the elasticity of substitution between every pairs of products. The foreign country's preferences are expressed by the same equation with the corresponding foreign variables.

By solving the consumer's utility maximization problem, the home country's demand functions for products i and i^* are obtained as

$$d(i) = \gamma p(i)^{-\sigma} G^{\sigma-1} I, \quad d(i^*) = \gamma p(i^*)^{-\sigma} G^{\sigma-1} I,$$

where $p(i)$ ($p(i^*)$) is the price of product i (i^*), I is the home country's national income, and G is the price index for differentiated products, which takes the form

$$G = \left[\int_0^n p(i)^{1-\sigma} di + \int_0^{n^*} p(i^*)^{1-\sigma} di^*\right]^{1/(1-\sigma)}. \tag{2}$$

In a similar manner, we can derive the foreign country's demand functions. Adding up both countries' demands for products i and $i*$ yields

$$D(i) = \gamma p(i)^{-\sigma} G^{\sigma-1}(I + I*) , \tag{3}$$

$$D(i*) = \gamma p(i*)^{-\sigma} G^{\sigma-1}(I + I*) , \tag{4}$$

where $D(i)$ and $D(i*)$ are the world demands for products i and $i*$.

2.2. The production side

In both countries, the technology in the competitive sector is such that one unit of output requires one unit of labor. Each variety of the monopolistic competitive sector in the home (foreign) country is produced with the same technology such that $x(i)$ ($x(i*)$) units of outputs require $l(i)$ ($l(i*)$) units of labor given by

$$l(i) = Bx(i) + F \quad (l(i*) = B*x(i*) + F*),$$

where B ($B*$) and F ($F*$) are, respectively, the marginal and fixed labor requirements in the home (foreign) country. In the monopolistically competitive sector, the number of available varieties equals that of active firms in the sector because each firm produces a single variety.

In this study, a monopolistically competitive firm creates positive externalities for other firms neighboring to it, so that the marginal and fixed costs ultimately falls as the number of firms in the sector increases. We now assume that such externalities for B ($B*$) and F ($F*$) are expressed as

$$B = b / n^{\alpha} \quad (B* = b / n*^{\alpha}), \quad 0 \le \alpha ,$$

$$F = f / n^{\beta} \quad (F* = f / n*^{\beta}) , \quad 0 \le \beta < 1,$$

where b and f are positive constants and common to both countries' firms. In these specifications, the terms n^{α} ($n*^{\alpha}$) and n^{β} ($n*^{\beta}$) capture the external effects of an increase in the number of firms in the home (foreign) country.

With the number of firms being very large, it can be assumed that each firm takes the price index G as given. Moreover, without loss of generality, units of parameters b and f can be chosen such that $b = 1 - 1/\sigma$ and $f = 1/\sigma$. Therefore, each firm's profit maximization implies that the price of product i (i^*) is

$$p(i) = w_1 / n^{\alpha} \quad (p(i^*) = w_1^* / n^{*\alpha}), \tag{5}$$

where w_1 (w_1^*) is the wage rate in the monopolistically competitive sector in the home (foreign) country. There is assumed to be no barrier to entry or exit. Hence, the zero-profit condition, together with (5), implies that the equilibrium output of any active firm in the home (foreign) country is

$$x(i) = n^{\alpha-\beta} \quad (x(i^*) = n^{*\alpha-\beta}), \tag{6}$$

and the associated equilibrium labor input in the home (foreign) country is

$$l(i) = 1/n^{\beta} \quad (l(i^*) = 1/n^{*\beta}). \tag{7}$$

Now let L_1 (L_1^*) denote the amount of labor forces employed in the monopolistically competitive sector in the home (foreign) country. Then, the number of monopolistically competitive firms in the home (foreign) country is given by

$$n = L_1^{1/(1-\beta)} \quad (n^* = L_1^{*1/(1-\beta)}), \tag{8}$$

where use has been made of (7). Inserting (8) into (6) yields

$$x(i) = L_1^{(\alpha-\beta)/(1-\beta)} \quad (x(i^*) = L_1^{*(\alpha-\beta)/(1-\beta)}). \tag{9}$$

2.3. The dynamic adjustment process

In this study, we assume that within each country, labor sluggishly moves from the sector with a lower wage rate to the sector with a higher wage rate.[2] Letting w_2 and w_2* be the home and foreign country's wage rates in the competitive sectors, the adjustment process can be described as

$$\dot{L}_1 = g(w_1 - w_2), \quad dg(z)/dz > 0, \quad g(0) = 0, \tag{10}$$

$$\dot{L}_1* = g*(w_1* - w_2*), \quad dg*(z)/dz > 0, \quad g*(0) = 0, \tag{11}$$

where dot denotes a time derivative.

3. Autarky equilibrium

Before turning to the determination of free trade equilibrium, we will investigate the equilibrium allocation of the home country in autarky. Exactly the same argument applies to the foreign country.

In the autarky equilibrium, the home country's spending on the homogenous good, $(1-\gamma)I$, equals the amount of production in its competitive sector, $w_2(L-L_1)$, where L is the home country's labor endowment. Thus the market-clearing condition under autarky is given by

$$(1-\gamma)[w_1 L_1 + w_2(L-L_1)] = w_2(L-L_1),$$

which can be rewritten as

$$w_1 / w_2 = \gamma(L-L_1)/(1-\gamma)L_1. \tag{12}$$

This is the cross-sector wage ratio in the short-run equilibrium. This schedule is drawn in Figure 1 as the downward sloping curve AA'; that is, the relative wage rate of the

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[2] This adjustment process was presented in Tawada (1989), which explored the influence of external economies of scale on trade patterns and gains from trade.

monopolistically competitive sector is decreasing in the employment level in the sector. Therefore, under the adjustment process defined in (10), the autarky equilibrium is unique and globally stable. In the figure, L_{1A} represents the level of L_1 in the autarky equilibrium. By setting $w_1 / w_2 = 1$ in (12) and solving for L_1, this employment level is obtained as

$$L_{1A} = \gamma L . \tag{13}$$

[Figure 1]

This section is closed by giving the real wage rate in the autarky equilibrium. Under Cobb-Douglas preferences given in (1), the real wage rate (hereafter ω) is defined as

$$\omega = w / G^{\gamma} P^{1-\gamma} , \tag{14}$$

where w is the wage rate in the long-run equilibrium and P is the price of the homogeneous good. Note that in the autarky equilibrium, the price index (2) can be rewritten as

$$G = w(\gamma L)^{[\alpha(\sigma-1)+1]/(1-\beta)(1-\sigma)},$$

where (5), (8) and (13) has been used. In the autarky equilibrium, the output of the homogenous good is positive, so $P = w$ holds. Hence, the real wage rate in the autarky equilibrium, ω_A, is given by

$$\omega_A = (\gamma L)^{\gamma[\alpha(\sigma-1)+1]/(1-\beta)(\sigma-1)} \tag{15}$$

4. Free trade equilibria and specialization patterns

Suppose that the two countries open their goods markets. Then, if both countries continue to produce the differentiated product, the output level at which each firm makes zero profit is equal to the firm’s total sales in both countries. By (3), (4) and (9), the equilibrium conditions are given by

$$\gamma p(i)^{-\sigma} G^{\sigma-1}(I+I^*) = L_1^{(\alpha-\beta)/(1-\beta)}, \tag{16}$$

$$\gamma p(i^*)^{-\sigma} G^{\sigma-1}(I+I^*) = L_1{}^{*(\alpha-\beta)/(1-\beta)}, \tag{17}$$

Dividing (16) by (17) and the use of (5) and (8) yield

$$w_1{}^*/w_1 = (L_1{}^*/L_1)^{\theta}, \tag{18}$$

where $\theta \equiv [\alpha(\sigma-1)+\beta]/\sigma(1-\beta)$. Hence, if the differentiated product is produced in both countries under free trade, the cross-country wage ratio in the monopolistically competitive sector, $w_1{}^*/w_1$, is determined by the cross-country labor ratio in this sector, $L_1{}^*/L_1$.

Now, let us turn to the homogeneous good market. Under free trade, the market clearing condition for the homogenous good is expressed as

$$\begin{aligned}(1-\gamma)[w_1L_1 + w_1{}^*L_1{}^* + w_2(L-L_1) + w_2{}^*(L^*-L_1{}^*)]\\ = w_2(L-L_1) + w_2{}^*(L-L_1{}^*).\end{aligned} \tag{19}$$

If the output of the homogeneous good is positive in both countries, w_2 and $w_2{}^*$ are equalized. Thus (19) can be rewritten as

$$\frac{w_1}{w_2} = \frac{\gamma(L+L^*-L_1-L_1{}^*)}{(1-\gamma)[L_1+(w_1{}^*/w_1)L_1{}^*]}. \tag{20}$$

Using (18) in (20) and slightly manipulating, the home country's cross-sector wage ratio, w_1/w_2, is given by

$$\frac{w_1}{w_2} = \frac{\gamma(L+L^*-L_1-L_1{}^*)L_1^{\theta}}{(1-\gamma)(L_1^{1+\theta}+L_1{}^{*1+\theta})}, \tag{21}$$

By a similar calculation, the foreign country's cross-sector wage ratio, $w_1{}^*/w_2{}^*$, is obtained as

$$\frac{w_1*}{w_2*} = \frac{\gamma(L + L* - L_1 - L_1*)L_1*^{\theta}}{(1-\gamma)(L_1^{1+\theta} + L_1*^{1+\theta})}. \tag{22}$$

From (21) and (22), it is obvious that each country's wage ratio is decreasing in the other country's employment level in the monopolistically competitive sector.

We are ready to determine the patterns of specialization in the long-run equilibrium. Note that the dynamic adjustment process under free trade is described by the simultaneous differential equations defined in (10) and (11). Then the phase diagram can be depicted as Figure 2. In the figure, the locus OAB illustrates the collection of L_1 and L_1* for which the cross-sector wage ratio in the home country equals unity, or equivalently, $w_1 = w_2$. Following Ethier (1979, 1982), we call this locus the home country's *allocation curve*. By the declining property of w_1 / w_2 with respect to L_1*, we have $\dot{L}_1 > 0$ ($\dot{L}_1 < 0$) in the lower (upper) region of the home country's allocation curve. Thus L_1 increases (decreases) over time in the lower (upper) region of OAB. The locus OAB' illustrates the foreign country's allocation curve, which represents the collection of L_1 and L_1* such that $w_1* = w_2*$. By the same token, L_1* increases (decreases) over time in the left-hand (right-hand) side region of OAB'.

[Figure 2]

To determine specialization patterns, it is necessary to know where the autarky equilibrium is located in Figure 2. The line WW' shows the locus of pairs (L_1, L_1*) for which $L_1 + L_1* = \bar{L}$, where $\bar{L}$ is constant. Suppose that the level of each country's labor endowment is as indicated by point F. Then, from (13) it is easy to verify that the autarky equilibrium is given by the intersection of BB' and OF. In view of the dynamic behaviors of L_1 and L_1* shown by arrows, we find that the long-run equilibrium is attained at point G.[3] Evidently, at this point the home country specializes in the homogeneous good, while the foreign country specializes in the differentiated product. Note that any change in $L*/L$ does not shift both countries' allocation curves as long as $L+L*$ is held constant at $\bar{L}$ (see (21) and (22)). Hence, for all levels of $L*/L$, the specialization patterns can be identified.

Proposition 1. *Suppose that $L* > L$ and that the world moves from autarky to free trade. Then, the following statements apply to the free trade equilibrium.*

(i) The home country specializes in the homogeneous good and the foreign country diversifies if and only if the following condition is satisfied:

$$L*/L > \gamma/(1-\gamma)\,. \tag{23}$$

(ii) The home country specializes in the homogeneous good and the foreign country specializes in the differentiated product if and only if the following condition is satisfied:

$$\gamma/(1-\gamma) \geq L*/L > x(\gamma)\,, \tag{24}$$

where $x(\gamma)$ is the slope of OE in Figure 2 and is given by

$$x(\gamma) = \frac{\theta^{\theta/(1+\theta)}}{1+\theta} \cdot \frac{\gamma}{(1-\gamma)^{\sigma(1-\beta)/[\alpha(\sigma-1)+\beta+\sigma(1-\beta)]}}\,.$$

(iii) The home country diversifies and the foreign country specializes in the differentiated product if and only if the following condition is satisfied:

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[3] The labor employment levels in both countries can not increase beyond their resource constraints. Thus the allocation curve outside the rectangle shaped by the origin and the endowment point on WW' is out of consideration.

$$x(\gamma) \geq L^*/L. \tag{25}$$

Proof. The proof highly resorts to Figure 2. Consider first the case where the labor endowment pair (L, L^*) is located on W'H in Figure 2. Then, L^* exceeds $\gamma\bar{L}$, namely the level of $L_1{}^*$ implied by point B'. Since $L+L^*$ is equal to $\bar{L}$, condition $L^* > \gamma\bar{L}$ can be rewritten as (23). From arrows in the figure, we find that the long-run equilibrium is attained at point B'. Thus, if condition (23) is satisfied, the foreign country produces both types of goods, whilst the home country produces the homogeneous good only.

Consider next the case where the labor endowment pair (L, L^*) is located on HE, like point F. In the figure, HE represents the segment of WW' for which $L_1{}^*/L_1 > x(\gamma)$ and $L_1{}^* \leq \gamma\bar{L}$. Substituting $L_1{}^* = L^*$ and $\bar{L} = L + L^*$ into the latter condition and rearranging the terms, we have $\gamma/(1-\gamma) \geq L^*/L$. Hence, if the labor endowment pair is located on HE, condition (24) is satisfied (for derivation of a lower bound $x(\gamma)$, see Appendix A). As mentioned earlier, in this case the home country specializes in the homogeneous good, while the foreign country specializes in the differentiated product.

Finally, consider the case where the labor endowment pair (L, L^*) is located on EC, or equivalently, condition (25) is satisfied. Then, the long-run equilibrium is attained on the segment JA of the home country's allocation curve, with $L_1{}^* = L^*$. Thus the foreign country specializes in the differentiated product, whereas the home country diversifies.

Hence, the proposition is proved. **QED.**

With external economies of scale, the differentiated-product industry tends to agglomerate into the larger country under free trade. This tendency reduces the likelihood of intra-industry trade emerging in the free trade equilibrium. In fact, Proposition 1 shows that, only if (25) is satisfied, the output of the differentiated product is positive in both countries; that is, the emergence of intra-industry trade requires that the taste for the differentiated product is strong[4] and that both countries are close in their sizes. In any case, it can be said that the larger country becomes a net exporter of differentiated products in the free trade equilibrium.

5. The normative implications of free trade and agglomeration

This section investigates the welfare implications of free trade and the trade-induced industrial agglomeration. The following argument is developed on the assumption that the foreign country is larger than the home country, i.e., $L^* > L$.

5.1. The welfare effects of free trade

Preliminary to the subsequent analysis, we first derive both countries' real wage rates under free trade. Note that the price index (2) can be rewritten as

$$G = \left\{ w^{1-\sigma} L_1^{[\alpha(\sigma-1)+1]/(1-\beta)} + w^{*1-\sigma} L_1^{*[\alpha(\sigma-1)+1]/(1-\beta)} \right\}^{1/(1-\sigma)}, \tag{26}$$

from (5) and (8). By proposition 1, the home (smaller) country continues to produce the homogeneous good after the opening of trade, so we have $P = w$. Inserting (26) and $P = w$ into (14) and slightly rearranging the terms, we get the home country's real wage rate under free trade:

[4] Note that $x(\gamma)$ is increasing in γ.

$$\omega = \left\{ L_1^{[\alpha(\sigma-1)+1]/(1-\beta)} + \left(\frac{w}{w^*} \right)^{\sigma-1} L_1^{*[\alpha(\sigma-1)+1]/(1-\beta)} \right\}^{\gamma/(\sigma-1)} . \tag{27}$$

Similarly, by using (26) and $P = w$ in a similar equation to (14) with the corresponding foreign variables, the foreign country's real wage rate can be obtained as

$$\omega^* = \left(\frac{w^*}{w} \right)^{1-\gamma} \left\{ \left(\frac{w^*}{w} \right)^{\sigma-1} L_1^{[\alpha(\sigma-1)+1]/(1-\beta)} + L_1^{*[\alpha(\sigma-1)+1]/(1-\beta)} \right\}^{\gamma/(\sigma-1)} . \tag{28}$$

We are now ready to explore the welfare effects of free trade. Consider first the case where both countries produce the homogeneous good at the free trade equilibrium, or equivalently, condition (23) is satisfied. Then, the long-run equilibrium is attained at point B', where $L_1 = 0$ and $L_1^* = \gamma \bar{L} = \gamma(L + L^*)$, and the wage rates in both countries are equalized. Hence, both countries' real wage rates in the free trade equilibrium, ω_T and ω_T^*, are given by

$$\omega_T = \omega_T^* = \left[\gamma(L + L^*) \right]^{\gamma[\alpha(\sigma-1)+1]/(1-\beta)(\sigma-1)}, \tag{29}$$

where use has been made of (27) and (28). Since the level of ω in the autarky equilibrium is given by (15), the home country's real wage rate is higher in the free trade equilibrium than in the autarky equilibrium. The same is true of the foreign country. Therefore, free trade is beneficial to both countries if they continue to produce the homogeneous good after the opening of trade.

Next, consider the case where condition (24) is satisfied, so that both countries completely specialize at the free trade equilibrium. Then, by setting $L_1 = 0$ and $L_1^* = L^*$ in (19), the cross-country wage ratio is obtained as

$$w^*/w = w_1^*/w_2 = \gamma L/(1-\gamma)L^* . \tag{30}$$

By using (30) in (27) and (28), both countries' real wage rates are given by

$$\omega_T = [(1-\gamma)L^*/\gamma L]^{\gamma} L^{*\gamma[\alpha(\sigma-1)+1]/(1-\beta)(\sigma-1)}, \tag{31}$$

$$\omega_T^* = [\gamma L/(1-\gamma)L^*]^{1-\gamma} L^{*\gamma[\alpha(\sigma-1)+1]/(1-\beta)(\sigma-1)}. \tag{32}$$

Since $\gamma/(1-\gamma) \geq L^*/L$ is satisfied in this case (see part (ii) in Proposition 1), the cross-country wage ratio (30) is greater than unity. Keeping this in mind and comparing (32) with the foreign country's real wage rate in the autarky equilibrium, say, $\omega_A^* = (\gamma L^*)^{[\alpha(\sigma-1)+1]/(1-\beta)(\sigma-1)}$, we find that the foreign country becomes better off relative to the autarky equilibrium. On the other hand, the home country does not necessarily become better off from free trade. Indeed, making a comparison between (15) and (31) reveals that the home country gains if and only if

$$L^*/L > \gamma/(1-\gamma)^{(\sigma-1)(1-\beta)/[(\sigma-1)(\alpha+1-\beta)+1]}. \tag{33}$$

Note that (23) implies (33). In addition, as we have already seen, the home country becomes better off from free trade if (23) is satisfied. By pulling these facts together, it is shown that (33) is the necessary and sufficient condition for the home country to gain from free trade when it completely specializes, namely $L^*/L > x(\gamma)$.

Finally, consider the case where both countries produce the differentiated product at the free trade equilibrium, that is, condition (25) is satisfied. Then, the cross-country wage ratio, w^*/w, is expressed as (18). Hence, letting L_{1T} denote the level of L_1 in the free trade equilibrium, both countries' real wage rates are given by

$$\omega_T = \left\{ L_{1T}^{[\alpha(\sigma-1)+1]/(1-\beta)} + \left(\frac{L_{1T}}{L^*}\right)^{\theta(\sigma-1)} L^{*[\alpha(\sigma-1)+1]/(1-\beta)} \right\}^{\gamma/(\sigma-1)}, \tag{34}$$

$$\omega_T^* = \left(\frac{L^*}{L_{1T}}\right)^{\theta(1-\gamma)} \left\{\left(\frac{L^*}{L_{1T}}\right)^{\theta(\sigma-1)} L_{1T}^{[\alpha(\sigma-1)+1]/(1-\beta)} + L^{*[\alpha(\sigma-1)+1]/(1-\beta)}\right\}^{\gamma/(\sigma-1)}. \qquad (35)$$

Noticing that the foreign country's real wage rate in the autarky equilibrium is given by a similar equation to (15) and taking a careful look at (35), it can be seen that free trade confers a welfare gain on the foreign country. As for the home country, it gains from trade if and only if

$$L_{1T}^{[\alpha(\sigma-1)+1]/(1-\beta)} + \left(\frac{L_{1T}}{L^*}\right)^{\theta(\sigma-1)} L^{*[\alpha(\sigma-1)+1]/(1-\beta)} > (\gamma L)^{[\alpha(\sigma-1)+1]/(1-\beta)}, \qquad (36)$$

which is obtained by comparing (15) with (34).

To summarize, we have the following proposition.

Proposition 2. *Suppose that* $L^* > L$ *and that the world moves from autarky to free trade. Then, the foreign country is always better off, relative to autarky, at the free trade equilibrium. The welfare effects of free trade on the home country are as follows.*

(i) The case with complete specialization by the home country, i.e., $L^*/L > x(\gamma)$ *: free trade is welfare-improving if and only if condition (33) is satisfied.*

(ii) The case with diversification by the home country, i.e., $L^*/L \le x(\gamma)$ *: free trade is welfare-improving if and only if condition (36) is satisfied.*

The assertion of Proposition 2 is that the larger country becomes better off by a movement from autarky to free trade, whereas the smaller country may become worse off. Condition (33) indicates that, if the smaller country completely specializes under free trade, the likelihood of its being worse off relative to autarky is strengthened as both countries is more analogous in their sizes. This is because a relatively larger size of the smaller country leads to its relatively larger demand for the differentiated product, which, in turn, causes an unfavorable change in its terms of trade. This observation is consistent with Ethier's (1982) proposition concerning the possibility of losses from trade (p. 1261).

In the case where the home country diversifies, i.e., (25) is satisfied, the outcome is distinct from that of Ethier (1982), which assumed no differentiated product and demonstrated that the smaller country loses from trade when it diversifies under free trade. In the present monopolistically competitive framework, free trade makes it possible for the smaller country to gain from the availability of a greater variety of products supplied by the larger country. Actually, if the taste for the product diversity is so strong that σ is nearly equal to unity, the following condition holds:[5]

$$\left(\lim_{\sigma\to 1} L_{1T}\right)^{1/(1-\beta)} + L^{*1/(1-\beta)} > (\gamma L)^{1/(1-\beta)},$$

which is obtained by letting σ approach unity in (36). Hence, in our model there is a possibility that the home (smaller) country gains from trade while diversifying under free trade.[6]

5.2. The welfare effects of industrial agglomeration

This subsection examines the normative implications of the trade-induced industrial

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[5] Note that the small value of σ indicates the small degree of substitutability between every pair of varieties and thus implies the strong taste for the product diversity.

[6] Kikuchi (2002) also examines the implications of a declining fixed cost for gains from trade. However, the study confines its attention to the equilibrium with complete specialization by the smaller country.

agglomeration. First, let us clarify the welfare effect of two-way trade in differentiated products. It is instructive to consider a *hypothetical* situation where trade in the homogeneous good is prohibited while the differentiated products are traded. Then, the equilibrium condition for the homogeneous good is the same as in autarky, so each country's cross-sector wage ratio is expressed as (12). Hence, the employment level in the monopolistically competitive sector will eventually reach the level in the autarky equilibrium. In the long-run equilibrium, each country gains from the product diversity in the other country via intra-industry trade and thus becomes better off relative to the autarky equilibrium. Letting ω_I and ω_I* denote the real wage rates in this intra-industry trade equilibrium, we have

$$\omega_I = \left\{(\gamma L)^{[\alpha(\sigma-1)+1]/(1-\beta)} + (L/L*)^{\theta(\sigma-1)}(\gamma L*)^{[\alpha(\sigma-1)+1]/(1-\beta)}\right\}^{\gamma/(\sigma-1)}, \tag{37}$$

$$\omega_I* = \left\{(L*/L)^{\theta(\sigma-1)}(\gamma L)^{[\alpha(\sigma-1)+1]/(1-\beta)} + (\gamma L*)^{[\alpha(\sigma-1)+1]/(1-\beta)}\right\}^{\gamma/(\sigma-1)}, \tag{38}$$

which are obtained by substituting (18) into (27) and (28) and evaluating them at the levels of L_1 and L_1* in the autarky equilibrium.

According to Proposition 1, the differentiated-product industry shows the tendency toward agglomeration once both countries open their homogeneous good markets. This industrial agglomeration increases the diversity of products supplied by the foreign (larger) country, which has a welfare-improving effect on both countries. On the other hand, the corresponding trade-induced reduction in the product diversity in the home (smaller) country has a welfare-decreasing effect on both countries. However, if the two countries produce the homogeneous good under free trade, i.e., condition (23) holds, we can show that the welfare-increasing effect prevails over the welfare-decreasing effect. Comparing the real wage rates in the free trade equilibrium with those in the intra-industry trade equilibrium yields

$$\left(\frac{\omega_T}{\omega_I}\right)^{(\sigma-1)/\gamma} = \frac{(L+L^*)^{[\alpha(\sigma-1)+1]/(1-\beta)}}{L^{[\alpha(\sigma-1)+1]/(1-\beta)} + (L/L^*)^{\theta(\sigma-1)} L^{*[\alpha(\sigma-1)+1]/(1-\beta)}}, \tag{39}$$

$$\left(\frac{\omega_T^*}{\omega_I^*}\right)^{(\sigma-1)/\gamma} = \frac{(L+L^*)^{[\alpha(\sigma-1)+1]/(1-\beta)}}{L^{[\alpha(\sigma-1)+1]/(1-\beta)}(L^*/L)^{\theta(\sigma-1)} + L^{*[\alpha(\sigma-1)+1]/(1-\beta)}}, \tag{40}$$

where use has been made of (29), (37) and (38). Algebraic manipulation reveals that the right-hand sides of both equations are greater than unity (for the details, see Appendix B). Therefore, in this case the home and foreign countries gain from the trade-induced industrial agglomeration.

On the other hand, the free trade equilibrium that involves complete specialization by both countries is not necessarily superior to the intra-industry trade equilibrium from the normative point of view. If both countries completely specialize under free trade, the home country gains from agglomeration if and only if

$$\frac{L^*/L}{\left[1+(L^*/L)^{1+\theta}\right]^{(1-\beta)/[(\sigma-1)(\alpha+1-\beta)+1]}} > \gamma/(1-\gamma)^{(\sigma-1)(1-\beta)/[(\sigma-1)(\alpha+1-\beta)+1]}, \tag{41}$$

which is obtained by comparing (31) and (37). Because the left-hand side of (41) is increasing in L^*/L, the likelihood of the smaller country losing from agglomeration is strengthened as L^*/L decreases. The intuition behind it is the same as (33), but the possibility that the smaller country loses is greater than when it moves from autarky to free trade.[7] This conclusion is immediately derived from the fact that welfare is higher under intra-industry trade than under autarky. As for the foreign country, the necessary and sufficient condition for its gains from agglomeration is obtained as

[7] Note that (41) is possibly violated when (33) is satisfied.

$$\left[\frac{\gamma L}{(1-\gamma)L^*}\right]^{1-\gamma} > \left\{\gamma^{[\alpha(\sigma-1)+1]/(1-\beta)}\left[\left(\frac{L}{L^*}\right)^{[\sigma(\alpha+1-\beta)+\beta-\alpha]/\sigma(1-\beta)}+1\right]\right\}^{\gamma/(\sigma-1)}, \qquad (42)$$

from (32) and (38). The violation of this condition implies that the lager country becomes worse off due to the welfare-decreasing effect associated the trade-induced agglomeration, say, the reduction in the diversity of products supplied by the smaller country.[8]

Finally, let us derive the conditions for both countries' gains from agglomeration in the case where both countries produce the differentiated product under free trade. From (34) and (37), it is shown that the home country becomes better off from agglomeration if and only if

$$L_{1T}{}^{[\alpha(\sigma-1)+1]/(1-\beta)} + \left(\frac{L_{1T}}{L^*}\right)^{\theta(\sigma-1)} L^{*[\alpha(\sigma-1)+1]/(1-\beta)}$$

$$> (\gamma L)^{[\alpha(\sigma-1)+1]/(1-\beta)} + \left(\frac{L}{L^*}\right)^{\theta(\sigma-1)} (\gamma L^*)^{[\alpha(\sigma-1)+1]/(1-\beta)}, \qquad (43)$$

where L_{1T} denote the level of L_1 in the free trade equilibrium. On the other hand, the foreign country becomes better off from agglomeration if and only if

$$\left(\frac{L^*}{L_{1T}}\right)^{(1-\gamma)(\sigma-1)/\gamma}\left[\left(\frac{L^*}{L_{1T}}\right)^{\theta(\sigma-1)} L_{1T}{}^{[\alpha(\sigma-1)+1]/(1-\beta)} + L^{*[\alpha(\sigma-1)+1]/(1-\beta)}\right]$$

$$> \left(\frac{L^*}{L}\right)^{\theta(\sigma-1)} (\gamma L)^{[\alpha(\sigma-1)+1]/(1-\beta)} + (\gamma L^*)^{[\alpha(\sigma-1)+1]/(1-\beta)}, \qquad (44)$$

which is derived from (35) and (38). In this case, there will be various situations in

[8] Suga (2005) showed the possibility of the larger country losing from the trade-induced agglomeration, assuming international economies of scale and no differentiated product.

which (43) or (44) is violated.

To summarize, we have the following proposition.

Proposition 3. *Suppose that* $L^* > L$ *and that the world enters into free trade from a situation in which only the differentiated product is traded. Then, the following statements apply to the welfare effects of the trade-induced agglomeration.*

(i) The case with diversification by the foreign country, i.e., condition (23): both countries gain from agglomeration.

(ii) The case with complete specialization by both countries, i.e., condition (24): the home country gains from agglomeration if and only if condition (41) is satisfied, while the foreign country gains from agglomeration if and only if condition (42) is satisfied.

(iii) The case with diversification by the home country, i.e., condition (25): the home country gains from agglomeration if and only if condition (43) is satisfied, while the foreign country gains from agglomeration if and only if condition (44) is satisfied.

Consequently, the trade-induced agglomeration is not always beneficial to both countries. In particular, when the two countries completely specialize under free trade, or equivalently, condition (24) is satisfied, we can specify the situation in which both countries simultaneously become worse off from agglomeration. Note first that the right-hand side of (42) exceeds unity if and only if

$$\frac{\gamma^{[\alpha(\sigma-1)+1]/(1-\beta)}}{1-\gamma^{[\alpha(\sigma-1)+1]/(1-\beta)}} > \left(\frac{L^*}{L}\right)^{[\sigma(\alpha+1-\beta)+\beta-\alpha]/\sigma(1-\beta)}. \tag{45}$$

Therefore, if the relative size of the smaller country is so large that (45) holds and σ is sufficiently small, (42) is violated. Then, by taking account of (41), which becomes more stringent as L^*/L decreases, we reach the conclusion that both countries lose from agglomeration if they are analogous in their sizes and the taste for product diversity is sufficiently strong.

6. Concluding remarks

A monopolistic competition model of international trade with external economies of scale has been developed to investigate the positive and normative implications of free trade and industrial agglomeration. The analysis has shown that the two main findings in Ethier's (1982) perfect-competition and homogeneous-good model have turned out to be valid in our model. One is that the industry subject to external economies of scale tends to agglomerate into the larger country under free trade. In the present monopolistically competitive framework, this implies that the emergence of intra-industry trade is very unlikely. Another is that the larger country becomes better off by moving from autarky to free trade, whereas the smaller country may become worse off.

The key feature distinguishing our model from Ethier's (1982) is that each country is able to gain from the other country's product diversity via trade in differentiated products. Therefore, the smaller country possibly becomes better off by a movement from autarky to free trade even if it diversifies under free trade. This possibility is not observed in the homogeneous-good case, where the diversifying smaller country loses from its reduced productivity in the increasing-returns industry. The most remarkable one among our findings is the welfare implications of the industrial agglomeration that is caused by inter-industry trade. The results indicate that such industrial agglomeration may be less beneficial to both countries, compared with a hypothetical situation with trade in differentiated products only. In particular, when the taste for product diversity is sufficiently strong, the larger country as well as the smaller country may lose from agglomeration.

In this paper, the analysis has been conducted without specifying the source of technological externalities. However, we believe that the virtue of our model lies in its tractability and that our geometric approach, which is based on Ethier's (1979, 1982) allocation curve technique, will have many directions of application such as trade and industrial policy. It is hoped that the present analysis will provide a useful and alternative framework for considering the policy implications under imperfect competition and industrial agglomeration.

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Appendix A. Derivation of $x(\gamma)$

Letting λ denote the level of L_1* implied by point E in Figure 2, the slope of OE, namely $x(\gamma)$, can be expressed as

$$x(\gamma) = \lambda / (\bar{L} - \lambda) . \tag{A. 1}$$

In order to derive $x(\gamma)$, therefore, it is necessary to clarify the value of λ. First of all, totally differentiate (21) with respect to L_1 and L_1*. Then, we have

$$d(w_1 / w_2) = \chi dL_1 + \chi * dL_1 * , \tag{A. 2}$$

where χ and $\chi*$ represent the partial derivatives of (21) in regard to L_1 and L_1*, respectively. Particularly, χ is given by

$$\chi = \frac{w_1 / w_2}{L_1}\left[\theta - \frac{L_1}{\bar{L} - L_1 - L_1 *} - \frac{(1+\theta)L_1^{1+\theta}}{L_1^{1+\theta} + L_1 *^{1+\theta}}\right].$$

Setting $d(w_1 / w_2) = 0$ in (A. 2) and solving for $dL_1 * / dL_1$, we obtain the slope of the home allocation curve:

$$\left.\frac{dL_1 *}{dL_1}\right|_{w_1 / w_2 = 1} = -\frac{w_1 / w_2}{L_1 \chi *}\left[\theta - \frac{L_1}{\bar{L} - L_1 - L_1 *} - \frac{(1+\theta)L_1^{1+\theta}}{L_1^{1+\theta} + L_1 *^{1+\theta}}\right].$$

Now it is useful to rewrite this equation as

$$\left.\frac{dL_1 *}{dL_1}\right|_{w_1 / w_2 = 1} = -\frac{w_1 / w_2}{L_1 \chi *} \cdot \frac{\theta L_1^{1+\theta}}{L_1^{1+\theta} + L_1 *^{1+\theta}}\left[\left(\frac{L_1 *}{L_1}\right)^{1+\theta} - \frac{1}{\theta(1-\gamma)}\right],$$

where use has been made of the fact that (21) holds with $w_1 / w_2 = 1$ on the home allocation curve. At point J, where the home allocation curve is maximized, it follows that $(dL_1 * / dL_1)|_{w_1 / w_2 = 1} = 0$, or equivalently,

$$L_1 = [\theta(1-\gamma)]^{1/(1+\theta)} L_1 * . \tag{A. 3}$$

Inserting (A. 3) into (21) while setting $w_1 / w_2 = 1$ and a little manipulation yield

$$\lambda = \frac{\gamma \bar{L}}{\gamma + [\theta(1-\gamma)]^{1/(1+\theta)}(1+1/\theta)}. \tag{A. 4}$$

By using (A. 4) in (A. 1), we obtain $x(\gamma)$.

Appendix B. Proof of gains from agglomeration

This appendix proves that both countries gain from agglomeration if the foreign country diversifies under free trade, i.e., condition (23) holds. Let us first demonstrate that in the free trade equilibrium, the home country becomes better off, relative to the hypothetical equilibrium that involves intra-industry trade only. By (39) and the assumption that $L^* > L$, it follows that

$$\left(\frac{\omega_T}{\omega_I}\right)^{(\sigma-1)/\gamma} > \frac{(L+L^*)^{[\alpha(\sigma-1)+1]/(1-\beta)}}{L^{[\alpha(\sigma-1)+1]/(1-\beta)} + L^{*[\alpha(\sigma-1)+1]/(1-\beta)}}. \quad \text{(B. 1)}$$

Because of $[\alpha(\sigma-1)+1]/(1-\beta) > 1$, the numerator in the right-hand side of (B. 1) is greater than the denominator, which implies $\omega_T > \omega_I$. Hence, the home country gains from agglomeration under condition (23). Now, let us turn to the foreign country. Note first that (40) can be rewritten as

$$\left(\frac{\omega_T^*}{\omega_I^*}\right)^{(\sigma-1)/\gamma} = \frac{(L+L^*)^{1+\theta}}{L^{1+\theta} + L^{*1+\theta}}\left(\frac{L+L^*}{L^*}\right)^{\theta(\sigma-1)}. \quad \text{(B. 2)}$$

Obviously, the second term in the right-hand side, $[(L+L^*)/L^*]^{\theta(\sigma-1)}$, exceeds unity. In addition, by applying a similar argument to the case of (B. 1) we can verify that the numerator in the first term of the right-hand side is greater than the denominator. Combining these findings, we have $\omega_T^* > \omega_I^*$. Consequently, the foreign country gains from agglomeration if condition (23) is satisfied.

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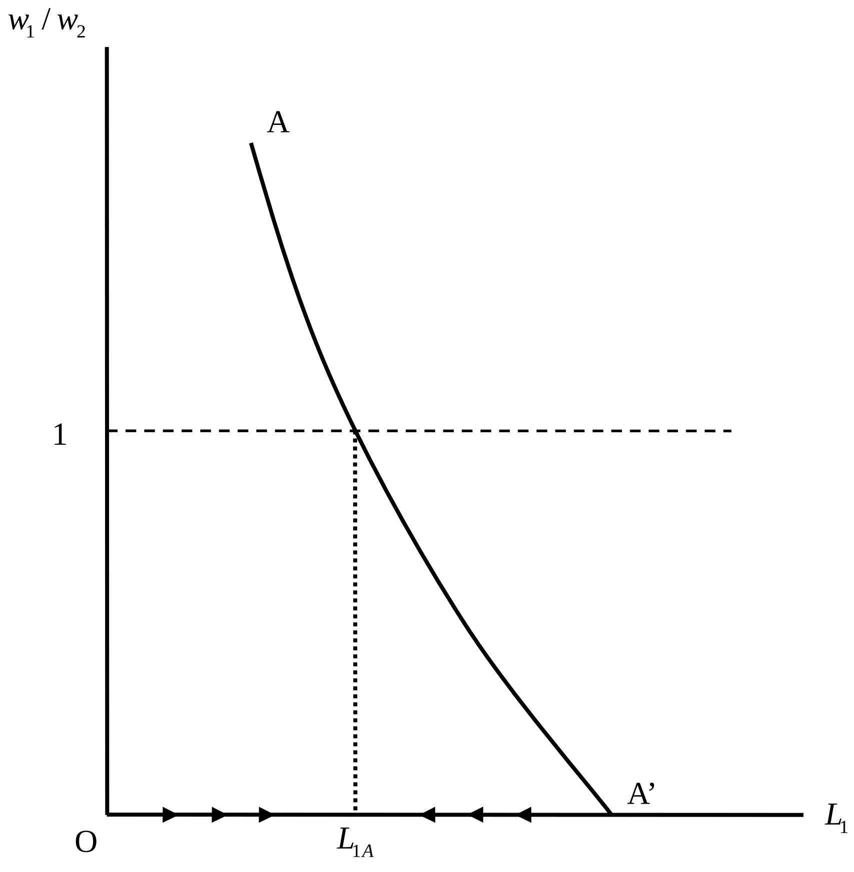
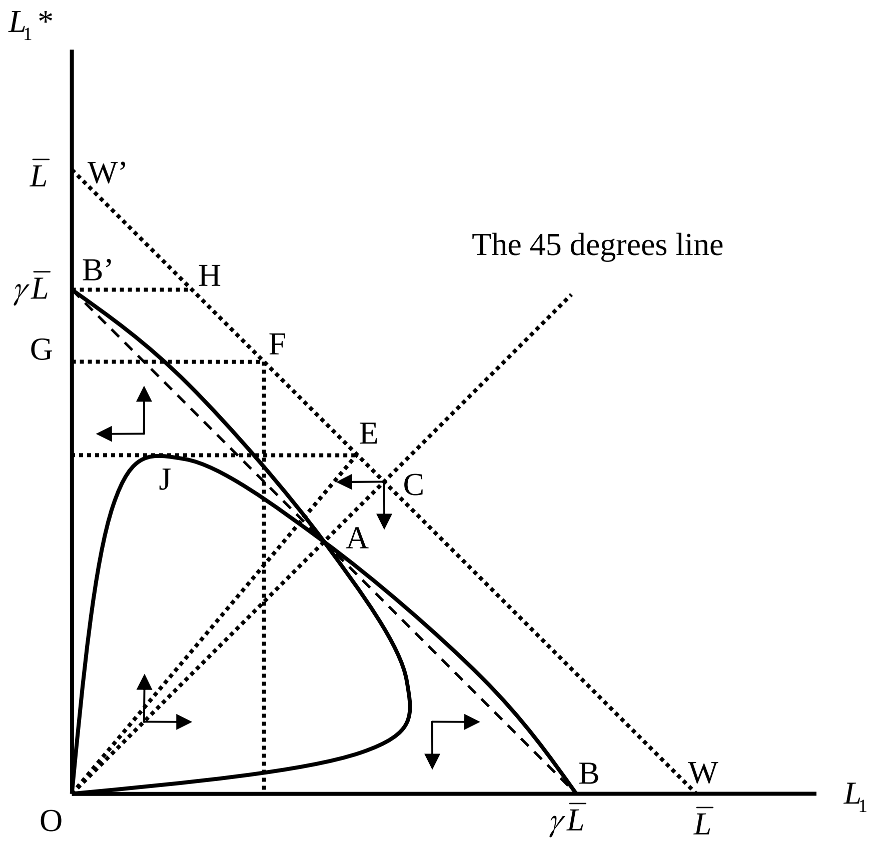


Figure 1



$L_1{}^*$
$\bar{L}$
W'
The 45 degrees line
B'
H
$\gamma\bar{L}$
G
F
E
C
J
A
B
W
L_1
O
$\gamma\bar{L}$
$\bar{L}$

Figure 2