Export Subsidies and Timing of Decision-Making*

—An Extension to the Sequential-Move Game

of Brander and Spencer (1985, JIE) Model—

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Abstract

This paper examines how the timing of decision-making affects the strategic trade policy. In this paper, I analyze the relationship between the different timing of the decision-making by exporting firms and their subsidizing governments and its impact on the export subsidy. The paper aims to extend the analysis by Brander and Spencer (1985), which is one of the seminar papers on the strategic trade policy, to the Stackelberg competition and the sequential-move decision on the subsidy choice by governments. I show some main results as follows: First, when the governments decide the export subsidies simultaneously in advance under the following Stackelberg quantity competition, the original leader firm produces as if it is the follower. Different from the Cournot model, under the Stackelberg model, the subsidy policy by the government that can subsidize the leader firm does not work effectively. Second, under the sequential-move game in which the government that can subsidize the leader firm decides its subsidy level at first, the profit of the leader firm is less than that of the follower in the Stackelberg model, although the first-mover advantage of the government is maintained. The result insists that the timing of decision-making affects the effect on the export subsidy policy significantly.

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1 Introduction

This paper examines how the timing of decision-making affects the strategic trade policy. I analyze the relationship between the different timing of the decision-making by exporting firms and their subsidizing governments and its impact on the export subsidy policy.

Although WTO reorganized from the GATT in 1995 and the FTAs conclude among many countries and tend to increase rapidly nowadays, the export subsidy policy have yet been practiced in many countries as a strategic means to induce more domestic surplus from exportation. In the WTO agreements (Agreement on Subsidies and Countervailing Measures), the export subsidies to the manufactured products are prohibited *per se*, but as for the agricultural products, the WTO members are in the very act of negotiating the reduction of the subsidy rate. The existence of the export subsidies is supported by the fact that the countervailing and the anti-dumping duties are justified under the WTO agreements in official, in order to allow the damaged government to countervail against the export subsidies. In actual fact, we can easily find a lot of cases about disputes between multinational firms on the export subsidies in the context of the international market competition.

From the theoretical point of view, a lot of existing literature has inquired various issues on export subsidies. In particular, since Brander and Spencer (1985), which is one of the seminar papers on the strategic trade policy, elucidated the strategic effect of subsidy policy, many articles has dealt with the exporting subsidies in the context of the strategic trade policy. Using the third country model, Brander and Spencer (1985) analyze the rent-shifting effect of the export subsidy and the strategic interaction between the export subsidies. They show that the export subsidy functions effectively to raise the domestic welfare, but it implies that the strategic subsidy choices by both governments in the exporting countries fall into the suboptimal excessive competition such as a prisoner's dilemma. Eaton and Grossman (1986), which is another pioneer work, analyze more generalized model than the Brander and Spencer. They extend the model of Brander and Spencer (1985) to allow the different conjectural variations from Cournot case, that is, the different competing environments. They show that the optimal trade policy is the exporting tax imposition to the domestic exporting firm under the Bertrand conjecture.

Although the above two representative papers and their successors deal with the general demand structure and illuminate the strategic aspects on the trade policy clearly, however, their papers have limited to the analyses of only the situation in which the choices of the strategic variables by the competitive firms are made simultaneously. For example, Brander and Spencer (1985) deal with only the Cournot quantity competition. Although Eaton and Grossman (1986) generalize the conjectural variations including Cournot, Bertrand, and consistent conjectures, these conjectural variations between firms are identical. The existing literature usually deals with only symmetric case between firms, that is, only the simultaneous-move game on output choice. I consider the Stackelberg leader–follower competition and deal with the asymmetric conjecture as a result. Extending the simultaneous-move game to the sequential one on output choice and also subsidy choice, I present a new point of view about the strategic subsidy policy that is influenced by the timing of decision-making.

In the actual international trade policy, we can imagine many situations in which the timing of decision-making about the trade policies by governments is different. For instance, it may take place that the governments of the developed countries first determine the subsidy levels in advance before the governments of the developing countries determine the subsidy levels and *vice versa*. Because of the different abilities between governments to implement and enforce the trade policy, there exists the time lag on the subsidy decisions by governments in usual. On the one hand, whether or not a country has the leading industry may affect the speed of policy determination positively. On the other hand, to facilitate the infant industry, the government may forestall the rival government and determine the subsidy level at first.

I introduce the difference on the timing of decision-making on output and subsidy level into the model. I examine how the different timing of the determination on strategic variables has the impact on the export subsidy policy under the imperfect competition environment.

The paper extends the analysis in the Cournot model by Brander and Spencer (1985) to the Stackelberg competition and the sequential-move game on the subsidy choice by governments. Although I limit the argument to the linear demand and linear cost model with any loss of generality, it is possible to make the comparative statics with regard to subsidy, output, profit and welfare levels in a clear way, in order to clarify the impact of the timing of the decision-making by exporting firms and their governments.

As a well-known result of Brander and Spencer (1985), the following result is obtained in Proposition 3 (p.89) in their paper:

> Proposition 3. The optimal export subsidy, s, moves the industry equilibrium to what would, in the absence of a subsidy, be the Stackelberg leader–follower position in output space with the domestic firm as leader.

Many articles have quoted this proposition. For a recent example, Maggi (1999, p.575) state as follows: The optimal unilateral subsidy is the one that shifts the domestic firm's reaction function in such a way that it intersects the foreign reaction function $R(q_i)$ at the Stackelberg point. However, there is little contribution that the original quantity competition is in the fashion of the Stackelberg competition in the context of the strategic trade policy. In this paper, I reexamine the optimal subsidy policy under the Stackelberg leader–follower competition.

The main interesting question is to investigate how the sequential-move between two exporting firms under the Stackelberg model and also the sequential decision-making between their subsidizing governments have the effects on the sizes of subsidy, firm's profit and national welfare. I pay attention to not only the simultaneous decision on subsidy by governments, which has usually analyzed the existing literature, but also the sequential decision.

As for the sequential-move game on strategic trade policy, there are several articles that I should refer. In the two-country model, Syropoulos (1994) shows that the governments may choose tariffs sequentially under perfect competition. Collie (1994) shows that the domestic government sets tariff at first and then the foreign government sets export subsidy under Cournot quantity competition. In the third-country model, Arvan (1991) concludes that demand uncertainty may cause the sequential-move of the policy choice by governments. Shivakumar (1993) introduces the export quota and shows that the restricted quantity competition and demand uncertainty bring the sequential decision of trade policy by governments. Although the existing literature analyzes the endogenous timing of policy-making by governments, in this paper in which the timing of policy-making is exogenous, I pay attention to examine the effects that the different timing on decision-making has on the effectiveness of trade policy.

Recently, Ohkawa, Okamura and Tawada (2002) endogenize the timing of government intervention under international oligopoly. Their paper is closely related with my paper in the sense that the sequential-move game by governments is analyzed in the third-country model. Different from my concern, however, they focus on the relationship between the number of firms and the endogenous timing of the policy decision by governments and they do not deal with

Stackelberg competition between firms. I introduce the sequential-move game by firms, that is, the Stackelberg quantity competition.[1]

Comparing between the simultaneous and sequential moves by firms and governments, I show some interesting results. Two main results are as follows: First, when the governments decide the export subsidies simultaneously in advance under the Stackelberg quantity competition, the original Stackelberg leader firm produces as if it is the follower. Different from the Cournot model, under the Stackelberg model, the subsidy policy by the government that can subsidize the leader firm is almost nullified. Second, under the sequential-move game in which the government that can subsidize the leader firm decides the subsidy level at first, the profit of the leader is less than that of the follower, although the first-mover advantage of the government is maintained and the leader produces more than the follower. The paper presents one of the theoretical foundations on the significance that the timing of policy decision has on the effectiveness of trade policy.

The remainder of the article is organized as follows. Section 2 describes the model. Section 3 derives subsidy, output, profit and domestic welfare in the equilibrium and analyze the relationship between the different structures. In Section 4, I summarize the calculating results about variables under each case and make the comparative statics with regard to the different structures on timing. I present some results on the different timing of decision-making. Section 5 is the concluding remarks.

[1] In the context of industrial organization, there are a lot of articles that argue the endogenous timing under duopolistic competition. For a representative paper, for example, see Hamilton and Slutsky (1990).

2 The model

Two identical firms, one from country i and one from country j, produce and sell in a third country. I consider the imperfect quantity competition model in the third country *à la* Brander and Spencer (1985). It is assumed that since both firms produce only for the third market, there is no consumption effects for the exporting countries.[2] They produce homogeneous goods. The firm in country i (j) are denoted by the index i (respectively j). Because of the identical firm, we can interpret both firms interchangeably.

Firm i (firm j) produces quantity q_i (resp. q_j). The total quantity is $Q \equiv q_i + q_j$. I limit the argument to the linear demand and linear cost for simplification of analysis. The inverse demand function is denoted by $P(Q) \equiv a - bQ$ and the constant marginal cost is denoted by c_i. It is assumed that $a > c_i$ and $b > 0$.

Government i that lies in country i can implement the per unit export subsidy, $s_i \geq 0$, as a means of the trade policy. It is defined that $e_i \equiv c_i - s_i$.[3]

The profit that firm i maximizes is denoted by $\pi^i(q_i, q_j; s_i, s_j) \equiv (P(Q) - c_i + s_i)q_i = (P(Q) - e_i)q_i$. The solution concept is the subgame perfect equilibrium.

The surplus of country i is denoted by $G^i(s_i, s_j)$, which consists of the profit from the exporting firm i minus the cost of the export subsidy: $G^i(s_i, s_j) \equiv \pi^i(q_i, q_j; s_i, s_j) - s_i q_i$. Government i maximizes this surplus.

The timing of the game is as follows:

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[2]This kind of assumption is usual in the context of the strategic trade policy as it makes the analysis simpler.

[3]It is shown that the sign of e_i is indeterminate in the following analysis. The government may compensate more than the marginal cost.

1st stage: Governments choose subsidy levels simultaneously or sequentially.

2nd stage: Firms choose output levels simultaneously or sequentially.

Subsidy policies can be committed by both governments and can be observed by both firms before the competition stage in advance.

In the next section, I derive the subsidy, the output, the profit and the welfare in the equilibrium by inducing backward. Both of the Cournot and the Stackelberg leader–follower duopolistic competition are analyzed.

3 The analysis

In this section, I examine the subsidy, the output, the profit and the domestic welfare in the equilibrium for all classified cases. As the first step, I solve the subgame at the second stage. At first, I examine the simultaneous output choice, that is, Cournot quantity competition at this subgame. Then I proceed to consider the sequential output choice, that is, Stackelberg duopoly.

3.1 The subgame at the second stage

3.1.1 Cournot competition

Given the subsidies (s_i, s_j), both firms maximizes their profits. The first-order condition for firm i to maximize the profit is as follows: $\pi^i_i = (a - b(q_i + q_j) - e_i) - bq_i = 0$.[4] The reaction function of firm i is $q_i = R_i(q_j) = \frac{a - bq_j - e_i}{2b}$. In order to obtain the output levels by the simultaneous choice

[4]The subscript i of the profit denotes the partial derivative by q_i, that is, $\pi^i_i \equiv \frac{\partial \pi^i}{\partial q_i}$. The second-order condition is satisfied because $\pi^i_{ii} = -2b < 0$.

under the Cournot duopolistic competition, I solve the intersection of the reaction functions as follows:

$$(q_i^C(s_i,s_j), q_j^C(s_i,s_j)) = (\frac{a-2e_i+e_j}{3b}, \frac{a-2e_j+e_i}{3b}).^5 \tag{1}$$

If there is no subsidy, the Cournot outcome is as follows: $(q_i^C(0,0), q_j^C(0,0)) = (\frac{a-2c_i+c_j}{3b}, \frac{a-2c_j+c_i}{3b})$.

The total quantity is $Q^C = \frac{2a-e_i-e_j}{3b}$, the price is $P(Q^C) = \frac{a+e_i+e_j}{3}$, and the profit margin is $P(Q^C) - e_i = \frac{a-2e_i+e_j}{3} = bq_i^C$. The profit under the Cournot competition is calculated as follows:

$$(\pi^{Ci}(s_i,s_j), \pi^{Cj}(s_i,s_j)) = (b(q_i^C)^2, b(q_j^C)^2) = (\frac{(a-2e_i+e_j)^2}{9b}, \frac{(a-2e_j+e_i)^2}{9b}). \tag{2}$$

3.1.2 Stackelberg competition

Under the Stackelberg competition as the sequential-move game, I consider that firm i is the Stackelberg leader and firm j is the follower without loss of generality. Anticipating the reaction of firm j to its own output choice q_i, that is, $q_j = R_j(q_i)$, firm i maximizes the profit function $\pi^i(q_i,q_j)$. That is, the following maximization problem is solved: $\max_{q_i} \pi^i(q_i, R_j(q_i))$. Note that $R_j'(q_i) = -\frac{1}{2}$.

The f.o.c. is $\pi_i^i + \pi_j^i R_j'(q_i) = ((a - b(q_i + R_j(q_i)) - e_i) - bq_i) - bq_i(-\frac{1}{2}) = 0$.[6] The Stackelberg output pairs are as follows:

$$(q_i^S(s_i,s_j), q_j^S(s_i,s_j)) = (\frac{a-2e_i+e_j}{2b}, \frac{a-3e_j+2e_i}{4b}).^7 \tag{3}$$

[5]For the output to be positive, it must be assumed that $a-2e_i+e_j > 0$. It is assumed that $a-2c_i+c_j > 0$ with no subsidy case.

[6]The s.o.c. is satisfied because $\pi_{ii}^i + \pi_{ij}^i R_j'(q_i) + (\pi_{ji}^i + \pi_{jj}^i R_j'(q_i))R_j'(q_i) + \pi_j^i R_j''(q_i) = -2b + b/2 + b/2 = -b < 0$.

[7]For the output to be positive, it must be assumed that $a-3e_j+2e_i > 0$. It is assumed that $a-3c_j+2c_i > 0$

If there is no subsidy, the Stackelberg outcome is as follows: $(q_i^S(0,0), q_j^S(0,0)) = (\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b})$.

As a well-known fact in the oligopoly theory, it is satisfied that $q_i^C(s_i, s_j) < q_i^S(s_i, s_j)$ and $q_j^C(s_i, s_j) > q_j^S(s_i, s_j)$ if the subsidy pairs (s_i, s_j) are identical.[8]

The total quantity is $Q^S = \frac{3a-2e_i-e_j}{4b}$ and the price is $P(Q^S) = \frac{a+2e_i+e_j}{4}$. It is satisfied that $Q^S > Q^C$ and $P(Q^C) > P(Q^S)$. The profit margin is $P(Q^S) - e_i = \frac{a-2e_i+e_j}{4} = \frac{b}{2}q_i^S$ and $P(Q^S) - e_j = \frac{a-3e_j+2e_i}{4} = bq_j^S$.

The profit under the Stackelberg competition is calculated as follows:

$$(\pi^{Si}(s_i, s_j), \pi^{Sj}(s_i, s_j)) = (\frac{b}{2}(q_i^S)^2, b(q_j^S)^2) = (\frac{(a-2e_i+e_j)^2}{8b}, \frac{(a-3e_j+2e_i)^2}{16b}). \quad (4)$$

It is satisfied that $\pi^{Ci}(s_i, s_j) < \pi^{Si}(s_i, s_j)$ and $\pi^{Cj}(s_i, s_j) > \pi^{Sj}(s_i, s_j)$ $\forall(s_i, s_j)$.

For the following analysis, I present the result of the comparative statics: $\frac{\partial q_i^C(s_i,s_j)}{\partial s_i} = \frac{2}{3b} > 0$, $\frac{\partial q_i^C(s_i,s_j)}{\partial s_j} = -\frac{1}{3b} < 0$. $\frac{\partial q_i^S(s_i,s_j)}{\partial s_i} = \frac{1}{b} > 0$, $\frac{\partial q_i^S(s_i,s_j)}{\partial s_j} = -\frac{1}{2b} < 0$, $\frac{\partial q_j^S(s_i,s_j)}{\partial s_j} = \frac{3}{4b} > 0$ and $\frac{\partial q_j^S(s_i,s_j)}{\partial s_i} = -\frac{1}{2b} < 0$.

3.2 The subsidy decision at the first stage

At the first stage, government i maximizes the surplus in country i as follows: $\max_{s_i \geq 0} G^i(s_i, s_j) \equiv \pi^i(q_i, q_j; s_i, s_j) - s_i q_i$. The f.o.c. for government i to welfare-maximize is as follows:

$$\frac{\partial G^i(s_i, s_j)}{\partial s_i} = \frac{\partial \pi^i(q_i, q_j; s_i, s_j)}{\partial s_i} - q_i - s_i \frac{\partial q_i}{\partial s_i} = 0, \quad (5)$$

if $s_i \geq 0$ (interior solution). If $\frac{\partial G^i(s_i,s_j)}{\partial s_i} < 0$, the solution is $s_i = 0$ (corner solution).[9] [10]

with no subsidy case.

[8] And also it is well-known that $q_i^S - q_i^C = \frac{a-2e_i+e_j}{6b} > q_j^C - q_j^S = \frac{a-2e_i+e_j}{12b}$ $\forall(s_i, s_j)$. That is, the total quantity expands under the Stackelberg competition.

[9] This is derived from the Kuhn-Tucker slackness condition.

[10] Under the following analysis, it is assumed that the s.o.c. is satisfied and the solution is interior, unique and stable, although I can confirm them by tedious calculation as the demand and cost are linear.

Finally, I classify the different timing of the decision-making among firms and governments into five cases. In Case A and Case B, I examine the unilateral and the bilateral intervention by government(s) under the Cournot competition. In Case C and Case D, I examine the unilateral and the bilateral intervention by government(s) under the Stackelberg competition. In Case E, the situation in which all players sequentially decide is analyzed. In the following subsection, I investigate all cases in turn. See also Figure 1. The superscripts, C and S stand for Cournot and Stackelberg equilibrium respectively for notational convenience.

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Figure 1 around here

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A. unilateral intervention under Cournot competition

First, I examine the unilateral intervention case in which only government i subsidizes under the Cournot competition.

As $s_j = 0$, that is, $e_j = c_j$, government i maximizes the following objective: $\max_{s_i \geq 0} G^{Ci}(s_i, 0) = \pi^i(q_i^C(s_i,0), q_j^C(s_i,0); s_i, 0) - s_i q_i^C(s_i,0)$. The f.o.c. for government i is as follows: $\frac{\partial G^{Ci}(s_i,0)}{\partial s_i} = \pi_i^i \frac{\partial q_i^C}{\partial s_i} + \pi_j^i \frac{\partial q_j^C}{\partial s_i} + \frac{\partial \pi^i}{\partial s_i} - q_i^C - s_i \frac{\partial q_i^C}{\partial s_i} = 0.$[11] It is calculated that $\pi_j^i = -bq_i$ and $\frac{\partial \pi^i}{\partial s_i} = q_i$ and it is satisfied that $\pi_i^i = 0$ by the f.o.c. of the Cournot equilibrium, Substituting them into the f.o.c., it is obtained that $(-bq_i)(-\frac{1}{3b}) + q_i - q_i - s_i \frac{2}{3b} = 0$, that is, $s_i = \frac{b}{2} q_i^C$ is derived. Arranging this equation, the optimal subsidy level, s_i^{uC}, is obtained as follows:

$$s_i^{uC} = \frac{a - 2c_i + c_j}{4}. \tag{6}$$

[11] The s.o.c. is satisfied because $\frac{\partial^2 G^{Ci}(s_i,s_j)}{\partial s_i^2} = \pi_{ii}^i \frac{\partial q_i^C}{\partial s_i} + \pi_{ji}^i \frac{\partial q_j^C}{\partial s_i} - \frac{\partial q_i^C}{\partial s_i} = (-2b)(\frac{2}{3b}) + (-b)(-\frac{1}{3b}) - \frac{2}{3b} = -(1 + \frac{2}{3b}) < 0$.

In this case, the Cournot output is as follows:

$$(q_i^C(s_i^{uC},0), q_j^C(s_i^{uC},0)) = (\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b})\ [= (q_i^S(0,0), q_j^S(0,0))]. \tag{7}$$

This result is summarized immediately in the following proposition.

Proposition 1. (*a corollary of Proposition 3 in Brander and Spencer* (1985))
Under the Cournot competition, the unilateral intervention by government i changes the market structure from the Cournot duopoly to the Stackelberg one in which firm i is the leader.

This proposition is just a corollary of Proposition 3 in Brander and Spencer (1985, p.89). The optimal subsidy has the profit-shifting effect and moves the Cournot competition to the Stackelberg leader–follower position. It is already well-known by a lot of articles in the context of strategic subsidy policy.[12] As a result of the unilateral subsidy, the profit of firm i that is subsidized by the government raises, that is, $\pi^{Ci}(s_i^{uC},0)(=\pi^{Si}(0,0)) > \pi^{Ci}(0,0)$ and $\pi^{Cj}(s_i^{uC},0)(=\pi^{Sj}(0,0)) < \pi^{Cj}(0,0)$. Also, it is obvious that the surplus in country i (j) expands (resp. contracts) by the subsidy of government i, that is, $G^{Ci}(s_i^{uC},0)(=\max_{s_i} G^{Ci}(s_i,0)) > G^{Ci}(0,0)$ and $G^{Cj}(s_i^{uC},0)(=\pi^{Cj}(s_i^{uC},0)) < G^{Ci}(0,0)(=\pi^{Cj}(0,0))$.[13]

B. bilateral intervention under Cournot competition

Next, I analyze the bilateral intervention case in which both governments subsidize under the Cournot competition. I examine the simultaneous decision and the sequential decision of subsidy in turn.

[12]It is immediately shown that $q_i^C(s_i^{uC},0) > q_i^C(0,0)$ and $q_j^C(s_i^{uC},0) < q_j^C(0,0)$.

[13]As $s_i^{uC} = \frac{b}{2}q_i^C$, the social surplus of country i is $G^{Ci}(s_i^{uC},0) = b(q_i^C)^2 - s_i^{uC}q_i^C = \frac{b}{2}(q_i^C)^2 = \frac{(a-2c_i+c_j)^2}{8b}(> G^{Ci}(0,0) = b(q_i^C)^2 = \frac{(a-2c_i+c_j)^2}{9b}$.

B-1. simultaneous decision of subsidy

Consider the simultaneous decision of subsidies (s_i, s_j) by both governments, under which has the similar timing of decision as the Cournot quantity competition. Given s_j, government i maximizes the surplus with regard to its own subsidy as follows: $\max_{s_i \geq 0} G^{Ci}(s_i, s_j) = \pi^i(q_i^C(s_i, s_j), q_j^C(s_i, s_j); s_i, s_j) - s_i q_i^C$.

The f.o.c. is as follows:

$$\frac{\partial G^{Ci}(s_i, s_j)}{\partial s_i} = \pi_i^i \frac{\partial q_i^C}{\partial s_i} + \pi_j^i \frac{\partial q_j^C}{\partial s_i} + \frac{\partial \pi^i}{\partial s_i} - q_i - s_i \frac{\partial q_i^C}{\partial s_i} = 0. \tag{8}$$

Like Case A, the f.o.c. is arranged as $s_i = \frac{b}{2} q_i^C = \frac{a-2e_i+e_j}{6}$. The reaction function is derived as $s_i = R_i(s_j) = \frac{-s_j+a-2c_i+c_j}{4}$.

In order to examine the simultaneous decision on the subsidy levels by both governments, I solve the intersection of the reaction functions of both governments. The subsidy level in the equilibrium is obtained as follows:

$$s_i^{bCC} = \frac{a - 3c_i + 2c_j}{5}. \tag{9}$$

Substituting this subsidy level, s_i^{bCC}, into the Cournot output, the Cournot output in the equilibrium is obtained under the simultaneous decision of subsidy.

$$(q_i^C(s_i^{bCC}, s_j^{bCC}), q_j^C(s_i^{bCC}, s_j^{bCC})) = (\frac{2(a - 3c_i + 2c_j)}{5b}, \frac{2(a - 3c_j + 2c_i)}{5b}). \tag{10}$$

First, comparing the subsidy levels under the unilateral and the bilateral cases, I am in the position to state this lemma.

Lemma 1. (*comparison of the subsidy level under Case A and B-1*)

The subsidy under the unilateral intervention is larger than under the bilateral intervention.

That is, $s_i^{uC} > s_i^{bCC}$.[14]

This lemma implies that under the bilateral intervention, there is the strategic interaction about the subsidy setting between governments and as a result, the impact on which the subsidy affects the output choice of the firm is smaller than under the unilateral intervention.

Then I proceed to compare the output levels under the unilateral and bilateral interventions.

Proposition 2. (*comparison of the output level under Case A and B-1*)

Under the Cournot competition,

(a) the output level of firm i (j) under the bilateral intervention is smaller (resp. larger) than under the unilateral intervention by government i. That is,

$q_i^C(s_i^{uC}, 0)(= q_i^S(0,0)) > q_i^C(s_i^{bCC}, s_j^{bCC})$, $q_j^C(s_i^{uC}, 0)(= q_j^S(0,0)) < q_j^C(s_i^{bCC}, s_j^{bCC})$.

(b) when each firm has almost identical marginal cost, the output level under the bilateral intervention is larger than under no intervention. That is, if $a - 8c_j + 7c_i > 0$,[15]

$q_i^C(0,0) < q_i^C(s_i^{bCC}, s_j^{bCC})$.

This proposition implies that by strategic substitutes on output competition, subsidizing by rival government results in the output reduction of the own firm. As the reaction functions of both firms shift outwards by subsidizing bilaterally, as a result, the output competition under the bilateral intervention becomes more severe than without intervention. This is an example of the prisoner's dilemma.

As for the profit of the firm, by $\pi^{Ci} = b(q_i^C)^2$, the relation about the profit size is immediately obtained from the relation about the above output size. By Proposition 2, it is obtained that

[14] $s_i^{uC} - s_i^{bCC} = \frac{a-2c_i+c_j}{4} - \frac{a-3c_i+2c_j}{5} = \frac{a-3c_j+2c_i}{20} > 0$.

[15] If the firm has the identical marginal cost, $c \equiv c_i = c_j$, this condition is satisfied, because $a - 8c_j + 7c_i = a - 3c_j + 2c_i + 5(c_i - c_j) > 0$.

$\pi^{Ci}(s_i^{uC},0) > \pi^{Ci}(s_i^{bCC},s_j^{bCC})$ and $\pi^{Cj}(s_i^{uC},0) < \pi^{Cj}(s_i^{bCC},s_j^{bCC})$. When each firm is almost identical, $\pi^{Ci}(0,0) < \pi^{Ci}(s_i^{bCC},s_j^{bCC})$. They imply that subsidizing by government i (j) makes the profit of firm i larger (resp. smaller) and the bilateral intervention makes the profits of both firms larger than without intervention.

Finally, I examine the effect of the subsidy on the social surplus. By direct calculation, it is obtained that $G^{Ci}(0,0) = \frac{(a-2c_i+c_j)^2}{9b}$, $G^{Ci}(s_i^{uC},0) = \frac{(a-2c_i+c_j)^2}{8b}$, $G^{Cj}(s_i^{uC},0) = \pi^{Cj}(s^{uC},0) = \frac{(a-3c_j+2c_i)^2}{16b}$ and $G^{Ci}(s_i^{bCC},s_j^{bCC}) = \frac{2(a-3c_i+2c_j)^2}{25b}$.[16] It is necessarily satisfied that $G^{Ci}(s_i^{uC},0) > G^{Ci}(0,0)$ and $G^{Cj}(s_i^{bCC},s_j^{bCC}) > G^{Cj}(s_i^{uC},0)$.[17] If the marginal cost is almost identical, it is satisfied that $G^{Ci}(0,0) > G^{Ci}(s_i^{bCC},s_j^{bCC})$.[18] That is, the bilateral intervention falls into the prisoner's dilemma for both governments. This is just a corollary of Proposition 5 in Brander and Spencer (1985, p.95). I restate this result in the following proposition. See also Table 1.

Proposition 3. (*comparison of the surplus under no intervention and Case B-1*)
Under the Cournot competition, when each firm has almost identical marginal cost, the surplus under the bilateral intervention is smaller than under no intervention. That is, the bilateral intervention falls into the prisoner's dilemma for both governments.

B-2. sequential decision of subsidy

Next, I consider the sequential decision of subsidy (s_i, s_j) as the sequential-move game between both governments. First, government i decides the subsidy level, s_i, and then government

[16] It is shown that $G^{Ci}(s_i^{uC},0) > G^{Ci}(s_i^{bCC},s_j^{bCC})$ if $9a+13c_j-22c_i > 0$. This condition is sufficiently satisfied when the marginal cost is identical, $c_i = c_j$, because $G^{Ci}(s_i^{uC},0) - G^{Ci}(s_i^{bCC},s_j^{bCC}) = \frac{(a-2c_i+c_j)^2}{8b} - \frac{2(a-3c_i+2c_j)^2}{25b} = \frac{(a-3c_j+2c_i)(9a+13c_j-22c_i)}{200b} > 0$, if $9a+13c_j-22c_i = 9(a-3c_j+2c_i)+40(c_j-c_i) > 0$.

[17] $G^{Cj}(s_i^{bCC},s_j^{bCC}) - G^{Cj}(s_i^{uC},0) = \frac{2(a-3c_j+2c_i)^2}{25b} - \frac{(a-3c_j+2c_i)^2}{16b} = \frac{7}{400}\frac{(a-3c_j+2c_i)^2}{b} > 0$.

[18] $G^{Ci}(s_i^{bCC},s_j^{bCC}) - G^{Ci}(0,0) = \frac{450(c_i-c_j)^2-(7a-11c_j+4c_i)^2}{7\times 225b} < 0$ if $c_i = c_j$.

		government j	
		no intervention	intervention
government i	no intervention	$G^{Ci}(0,0),\ G^{Cj}(0,0)$	$G^{Ci}(0,s_j^{uC}),\ G^{Cj}(0,s_j^{uC})$
	intervention	$G^{Ci}(s_i^{uC},0),\ G^{Cj}(s_i^{uC},0)$	$G^{Ci}(s_i^{bCC},s_j^{bCC}),\ G^{Cj}(s_i^{bCC},s_j^{bCC})$

If the cost is almost identical, $G^{Ci}(0,s_j^{uC}) < G^{Ci}(s_i^{bCC},s_j^{bCC}) < G^{Ci}(0,0) < G^{Ci}(s_i^{uC},0)$.

		government j	
		no intervention	intervention
government i	no intervention	$\frac{(a-2c_i+c_j)^2}{9b},\ \frac{(a-2c_j+c_i)^2}{9b}$	$\frac{(a-3c_i+2c_j)^2}{16b},\ \underline{\frac{(a-2c_j+c_i)^2}{8b}}$
	intervention	$\underline{\frac{(a-2c_i+c_j)^2}{8b}},\ \frac{(a-3c_j+2c_i)^2}{16b}$	$\underline{\frac{2(a-3c_i+2c_j)^2}{25b}},\ \underline{\frac{2(a-3c_j+2c_i)^2}{25b}}$

Table 1: the social surplus under no intervention, Case A and B-1

j decides s_j after observing s_i, as if government i (j) acts as the Stackelberg leader i (resp. follower j) with regard to the subsidy choice.

The follower government j decides the subsidy $s_j = R_j(s_i) = \frac{-s_i+a-2c_j+c_i}{4}$ given s_i. Note that $R_j'(s_i) = -\frac{1}{4} < 0$. The leader government induces this reaction and solves the following maximization problem: $\max_{s_i \geq 0} G^{Ci}(s_i, R_j(s_i)) = \pi^i(q_i^C(s_i, R_j(s_i)), q_j^C(s_i, R_j(s_i)); s_i, R_j(s_i)) - s_i q_i^C(s_i, R_j(s_i))$.

The f.o.c. is as follows:

$$\frac{\partial G^{Ci}(s_i,R_j(s_i))}{\partial s_i} = \frac{\partial \pi^i(q_i^C(s_i,R_j(s_i)), q_j^C(s_i,R_j(s_i)); s_i, R_j(s_i))}{\partial s_i} - q_i(s_i,R_j(s_i)) - s_i(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j}R_j'(s_i))$$
$$= \pi_i^i(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j}R_j'(s_i)) + \pi_j^i(\frac{\partial q_j}{\partial s_i} + \frac{\partial q_j}{\partial s_j}R_j'(s_i)) + (\frac{\partial \pi^i}{\partial s_i} + \frac{\partial \pi^j}{\partial s_j}R_j'(s_i)) - q_i - s_i(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j}R_j'(s_i)) = 0. \quad (11)$$

Like Case A, it is satisfied that $\pi_i^i = 0, \pi_j^i = -bq_i, \frac{\partial \pi^i}{\partial s_i} = q_i$ and $\frac{\partial \pi^i}{\partial s_j} = 0$. Arranging the f.o.c., it is derived that $\pi_j^i(\frac{\partial q_j}{\partial s_i} + \frac{\partial q_j}{\partial s_j}R_j'(s_i)) - s_i(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j}R_j'(s_i)) = 0$. That is, $s_i = \frac{2b}{3}q_i(s_i, R_j(s_i)) = \frac{2b}{3}\frac{a-2(c_i-s_i)+(c_j-R_j(s_i))}{3b}$. I obtain the subsidy level as follows:

$$s_i^{bSC} = \frac{a - 3c_i + 2c_j}{3},\ s_j^{bSC} = R_j(s_i^{bSC}) = \frac{a - 4c_j + 3c_i}{6}. \tag{12}$$

Substituting (s_i^{bSC}, s_j^{bSC}) into the Cournot output,

$$(q_i^C(s_i^{bSC}, s_j^{bSC}), q_j^C(s_i^{bSC}, s_j^{bSC})) = (\frac{a - 3c_i + 2c_j}{2b}, \frac{a - 4c_j + 3c_i}{3b}).^{19} \tag{13}$$

First, comparing the subsidy levels under the unilateral and the bilateral cases, I am in the position to state this lemma.

Lemma 2. (*comparison of the subsidy level under Case A and B-2*)

The subsidy under the unilateral intervention is smaller than that of the leader government under the bilateral intervention when each firm has almost identical marginal cost. The subsidy under the unilateral intervention is larger than that of the follower government under the bilateral intervention. That is, $s_i^{uC} < s_i^{bSC}$ *if* $a - 6c_i + 5c_j > 0$ *and* $s_j^{uC} > s_j^{bSC}$.[20]

Different from Case B-1, when the sequential decision of subsidy is made by governments under the bilateral intervention, the subsidy of the first-mover government is larger and that of the follower government is smaller than under the unilateral case.

Then I proceed to compare the output levels under the unilateral and the bilateral intervention.

Proposition 4. (*comparison of the output level under no intervention, Case A and B-2*)

Under the Cournot competition,

(a) whether the output level of firm i under the bilateral sequential intervention is smaller than

[19] For the output and the subsidy to be positive, it is assumed that $a - 4c_j + 3c_i > 0$ throughout the following analysis.

[20] $s_i^{uC} - s_i^{bSC} = -\frac{a-6c_i+5c_j}{12} < 0$ if $a - 6c_i + 5c_j = a - c_i - 5(c_i - c_j) > 0$. $s_j^{uC} - s_j^{bSC} = \frac{a-3c_i+2c_j}{12} > 0$.

under the unilateral intervention by government i depends on the relative sizes of the marginal costs between firms. That is, $q_i^C(s_i^{uC}, 0) \gtreqless q_i^C(s_i^{bSC}, s_j^{bSC}) \Leftrightarrow c_i \gtreqless c_j$.[21]

When each firm has almost identical marginal cost, the output level of firm j under the bilateral intervention is smaller than under the unilateral intervention by government i. That is, if $a - 7c_j + 6c_i > 0$, $q_j^C(s_i^{uC}, 0) < q_j^C(s_i^{bSC}, s_j^{bSC})$.[22]

When each firm has almost identical marginal cost, the output level of firm j under the bilateral intervention is smaller than under the unilateral intervention by government j. That is, if $a - 6c_i + 5c_j > 0$, $q_j^C(0, s_j^{uC}) > q_j^C(s_i^{bSC}, s_j^{bSC})$.[23]

(b) When each firm has almost identical marginal cost, the output level of firm i under the bilateral intervention is larger than under no intervention. That is, if $a - 5c_i + 4c_j > 0$, $q_i^C(0,0) < q_i^C(s_i^{bSC}, s_j^{bSC})$.[24]

Whether the output level of firm j under the bilateral intervention is smaller than under no intervention depends on the relative sizes of the marginal costs between firms. That is, $q_j^C(0,0) \gtreqless q_j^C(s_i^{bSC}, s_j^{bSC}) \Leftrightarrow c_i \lesseqgtr c_j$.[25]

As a corollary of this proposition, if the marginal cost is identical, that is, $c_i = c_j$, it is satisfied that $q_i^C(s_i^{uC}, 0) = q_i^C(s_i^{bSC}, s_j^{bSC})$ and $q_j^C(0,0) = q_j^C(s_i^{bSC}, s_j^{bSC})$. When the cost is identical, the output level of firm i under the bilateral intervention by the leader government i is equal to that under the unilateral intervention by government i. And also the output level of firm j under the bilateral intervention by the follower government j is equal to that under

[21] $q_i^C(s_i^{uC}, 0) - q_i^C(s_i^{bSC}, s_j^{bSC}) = \frac{c_i - c_j}{2b} \gtreqless 0 \Leftrightarrow c_i \gtreqless c_j$.

[22] $q_j^C(s_i^{bSC}, s_j^{bSC}) - q_j^C(s_i^{uC}, 0) = \frac{a - 7c_j + 6c_i}{12b} > 0$ if $a - 7c_j + 6c_i > 0$.

[23] $q_j^C(0, s_j^{uC}) - q_j^C(s_i^{bSC}, s_j^{bSC}) = \frac{a - 6c_i + 5c_j}{6b} > 0$ if $a - 6c_i + 5c_j > 0$.

[24] $q_i^C(s_i^{bSC}, s_j^{bSC}) - q_i^C(0,0) = \frac{a - 5c_i + 4c_j}{6b} > 0$ if $a - 5c_i + 4c_j > 0$.

[25] $q_j^C(0,0) - q_j^C(s_i^{bSC}, s_j^{bSC}) = -\frac{c_i - c_j}{b} \gtreqless 0 \Leftrightarrow c_i \lesseqgtr c_j$.

no intervention. The subsidy policy by the government has two effects on output. The one is to shift the reaction function outwards and have the home firm the advantage on output competition under strategic substitutes. The second is to adjust to competitive distortion by the cost difference. If the cost is identical, the second effect does not appear and the output is adjusted at the level of the Stackelberg leader by subsidizing.

As for the profit of the firm, by $\pi^{Ci} = b(q_i^C)^2$, the relation about the profit size is immediately obtained from the relation about the above output size. By Proposition 4, it is obtained that $\pi^{Ci}(s_i^{uC}, 0) \gtreqqless \pi^{Ci}(s_i^{bSC}, s_j^{bSC})$ if $c_i \gtreqqless c_j$, $\pi^{Cj}(s_i^{uC}, 0) < \pi^{Cj}(s_i^{bSC}, s_j^{bSC})$ if $a - 7c_j + 6c_i > 0$, and $\pi^{Cj}(0, s_j^{uC}) > \pi^{Cj}(s_i^{bSC}, s_j^{bSC})$ if $a - 6c_i + 5c_j > 0$. Moreover it is satisfied that $\pi^{Ci}(0,0) < \pi^{Ci}(s_i^{bSC}, s_j^{bSC})$ if $a - 5c_i + 4c_j > 0$, and $\pi^{Cj}(0,0) \gtreqqless \pi^{Cj}(s_i^{bSC}, s_j^{bSC})$ if $c_i \lesseqqgtr c_j$.

When each firm is almost identical, the firm that is subsidized by the leader government prefers the unilateral intervention to the bilateral one when the cost of its firm is higher than the rival one and vice versa. On the other hand, the firm that is subsidized by the follower government always prefers the bilateral intervention to the unilaterally intervention by the rival government, although this firm always prefers the unilaterally intervention by its home government to the bilateral intervention. Comparing no intervention with bilateral one, the leader government always prefers the bilateral intervention to no intervention, although the follower government prefers the bilateral intervention to no intervention if the marginal cost is lower and vice versa.

Finally, I examine the effect of the subsidy on the social surplus. By direct calculation, it is obtained that $G^{Ci}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-3c_i+2c_j)^2}{12b}$ and $G^{Cj}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-4c_j+3c_i)^2}{18b}$.[26] It is neces-

[26] Because $G^{Ci}(s_i^{uC}, 0) - G^{Ci}(s_i^{bCC}, s_j^{bCC}) = \frac{(a-c_j)^2 - 6(c_i - c_j)^2}{24b}$, it is shown that $G^{Ci}(s_i^{uC}, 0) > G^{Ci}(s_i^{bSC}, s_j^{bSC})$ if $a - c_j > \pm\sqrt{6}(c_i - c_j)$. It is satisfied when the marginal cost is almost identical.

sarily satisfied that $G^{Ci}(s_i^{uC}, 0) > G^{Ci}(0, 0)$ and $G^{Ci}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-3c_i+2c_j)^2}{12b} > G^{Ci}(0, s_j^{uC}) = \frac{(a-3c_i+2c_j)^2}{16b}$. If the marginal cost is almost identical, $G^{Cj}(s_i^{bSC}, s_j^{bSC}) < G^{Cj}(s_i^{uC}, 0)$.[27] If the marginal cost is identical, it is satisfied that $G^{Ci}(0, 0) > G^{Ci}(s_i^{bSC}, s_j^{bSC})$ and $G^{Cj}(0, 0) > G^{Cj}(s_i^{bSC}, s_j^{bSC})$.[28] Different from Case B-1, when the cost is almost identical, the leader government chooses to intervene and the follower government chooses not to intervene. The first-mover advantage of government i on the choice of subsidy can deter the rival follower government from exercising the subsidy. I state this result in the following proposition. See also Table 2.

Proposition 5. (*comparison of the surplus under no intervention and Case B-2*)
Under the Cournot competition, when each firm has almost identical marginal cost, the surplus under the bilateral intervention is smaller than under no intervention. In the equilibrium, the result is that only the leader government i intervenes and the follower government j does not intervene. The prisoner's dilemma of the bilateral intervention for both governments is avoided.

C. unilateral intervention under Stackelberg model

Now, I proceed to examine the unilateral intervention under the Stackelberg model.

C-1. unilateral intervention of government i

I examine the case in which government i whose firm i is the Stackelberg leader intervenes. As $s_j = 0$, that is, $e_j = c_j$, government i maximizes the following objectives: $\max_{s_i \geq 0} G^{Si}(s_i, 0) = \pi^i(q_i^S(s_i, 0), q_j^S(s_i, 0); s_i, 0) - s_i q_i^S$.

[27] $G^{Cj}(s_i^{bSC}, s_j^{bSC}) - G^{Cj}(s_i^{uC}, 0) = -\frac{(a-6c_i+5c_j)^2 - 72(c_i-c_j)^2}{144b} < 0$ if $(a - 6c_i + 5c_j)^2 - 72(c_i - c_j)^2 > 0$.

[28] $G^{Ci}(s_i^{bSC}, s_j^{bSC}) - G^{Ci}(0, 0) = -\frac{(a-2c_j+c_i)^2 - 12(c_i-c_j)^2}{36b} < 0$ if $(a-2c_j+c_i)^2 - 12(c_i-c_j)^2 > 0$. $G^{Cj}(s_i^{bSC}, s_j^{bSC}) - G^{Cj}(0, 0) = -\frac{(a-c_i)^2 - 8(c_i-c_j)^2}{18b} < 0$ if $(a - c_i)^2 - 8(c_i - c_j)^2 > 0$.

		government j	
		no intervention	intervention
government i	no intervention	$G^{Ci}(0,0),\ G^{Cj}(0,0)$	$G^{Ci}(0,s_j^{uC}),\ G^{Cj}(0,s_j^{uC})$
	intervention	$G^{Ci}(s_i^{uC},0),\ G^{Cj}(s_i^{uC},0)$	$G^{Ci}(s_i^{bSC},s_j^{bSC}),\ G^{Cj}(s_i^{bSC},s_j^{bSC})$

If the cost is almost identical, $G^{Ci}(0,0) < G^{Ci}(s_i^{uC},0)$, $G^{Ci}(0,s_j^{uC}) < G^{Ci}(s_i^{bSC},s_j^{bSC})$, $G^{Cj}(s_i^{bSC},s_j^{bSC}) < G^{Cj}(s_i^{uC},0)$. $G^{Ci}(0,0) > G^{Ci}(s_i^{bSC},s_j^{bSC})$ and $G^{Cj}(0,0) > G^{Cj}(s_i^{bSC},s_j^{bSC})$.

		government j	
		no intervention	intervention
government i	no intervention	$\frac{(a-2c_i+c_j)^2}{9b},\ \frac{(a-2c_j+c_i)^2}{9b}$	$\frac{(a-3c_i+2c_j)^2}{16b},\ \underline{\frac{(a-2c_j+c_i)^2}{8b}}$
	intervention	$\underline{\frac{(a-2c_i+c_j)^2}{8b}},\ \underline{\frac{(a-3c_j+2c_i)^2}{16b}}$	$\underline{\frac{(a-3c_i+2c_j)^2}{12b}},\ \frac{(a-4c_j+3c_i)^2}{18b}$

Table 2: the social surplus under no intervention, Case A and B-2

The f.o.c. is as follows: $\frac{\partial G^{Si}(s_i,0)}{\partial s_i} = \frac{d\pi^i(q_i^S(s_i,0),q_j^S(s_i,0);s_i,0)}{ds_i} - q_i - s_i\frac{\partial q_i}{\partial s_i} = \pi_i^i\frac{\partial q_i^S}{\partial s_i} + \pi_j^i\frac{\partial q_j^S}{\partial s_i} + \frac{\partial \pi^i}{\partial s_i} - q_i - s_i\frac{\partial q_i^S}{\partial s_i} \leq 0.$[29]

By the f.o.c. of Stackelberg leader, it is satisfied that $\pi_i^i = -\pi_j^i R_j'(q_i) = -\frac{b}{2}q_i$. $\pi_j^i = -bq_i$. $\frac{\partial \pi^i}{\partial s_i} = q_i$. Substituting them, the f.o.c. is as follows: $(-\frac{b}{2}q_i)(\frac{1}{b}) + (-bq_i)(-\frac{1}{2b}) + q_i - q_i - s_i\frac{1}{b} = -s_i\frac{1}{b} \leq 0$. That is, $s_i^{uS} = 0$. That is, the subsidy level is zero.

In this case, the Stackelberg output in the equilibrium is as follows:

$$(q_i^S(s_i^{uS},0), q_j^S(s_i^{uS},0))[= (q_i^S(0,0), q_j^S(0,0))] = (\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b}). \tag{14}$$

Also in Case C, a kind of corollary of Prop.3 in Brander and Spender (1985) is satisfied. The optimal subsidy moves to the Stackelberg leader-follower position. This result is summarized immediately in the following proposition.

[29]The s.o.c. is satisfied because $\frac{\partial^2 G^{Si}(s_i,s_j)}{\partial s_i^2} = \pi_{ii}^i\frac{\partial q_i^S}{\partial s_i} + \pi_{ji}^i\frac{\partial q_j^S}{\partial s_i} - \frac{\partial q_i^S}{\partial s_i} = (-2b)\frac{1}{b} + (-b)(-\frac{1}{2b}) - \frac{1}{b} = -(\frac{3}{2}+\frac{1}{b}) < 0.$

Proposition 6. (*a corollary of Proposition 3 in Brander and Spencer* (1985))

Under the Stackelberg competition in which firm i is the leader, the unilateral intervention by the government of the leader firm i is of no use. That is, there is no subsidy.

This proposition is just a corollary of Proposition 3 in Brander and Spencer (1985, p.89). The optimal subsidy has the profit-shifting effect and moves the Cournot competition to the Stackelberg leader–follower position. Under the Stackelberg competition, the government of the leader firm i has nothing to do. The profit of firm i that is not subsidized by the government is $\pi^{Si}(s_i^{uS}, 0) = \pi^{Si}(0,0) = \frac{b}{2}(q_i^S)^2$ and $\pi^{Sj}(s_i^{uS}, 0) = \pi^{Sj}(0,0) = b(q_j^S)^2$. The surplus is $G^{Si}(s_i^{uS}, 0) = G^{Si}(0,0) = \pi^{Si}(0,0)$ and $G^{Sj}(s_i^{uS}, 0) = \pi^{Sj}(0,0)$).

C-2. unilateral intervention of government j

I examine the case in which government j whose firm j is the Stackelberg follower intervenes. As $s_i = 0$, $e_i = c_i$. Government j maximizes the following objectives: $\max_{s_j \geq 0} G^{Sj}(0, s_j) = \pi^j(q_j^S(0, s_j), q_i^S(0, s_j); 0, s_j) - s_j q_j^S$.

The f.o.c. is as follows: $\frac{\partial G^{Sj}(0,s_j)}{\partial s_j} = \frac{d\pi^j(q_j^S(0,s_j), q_i^S(0,s_j); 0, s_j)}{ds_j} - q_j - s_j \frac{\partial q_j}{\partial s_j} = \pi_j^j \frac{\partial q_j^S}{\partial s_j} + \pi_i^j \frac{\partial q_i^S}{\partial s_j} + \frac{\partial \pi^j}{\partial s_j} - q_j - s_j \frac{\partial q_j^S}{\partial s_j} = 0.$[30]

By the f.o.c. of the Stackelberg follower, it is satisfied that $\pi_j^j = 0$. $\pi_i^j = -bq_j$. $\frac{\partial \pi^j}{\partial s_j} = q_j$. Substituting them, the f.o.c. is as follows: $(-bq_j)(-\frac{1}{2b}) + q_j - q_j - s_j \frac{3}{4b} = 0 \Leftrightarrow s_j = \frac{2b}{3} q_j = \frac{a - 3(c_j - s_j) + 2c_i}{6}$.

$$s_j^{uS} = \frac{a - 3c_j + 2c_i}{3}. \tag{15}$$

[30] The s.o.c. is satisfied because $\frac{\partial^2 G^{Sj}(s_i, s_j)}{\partial s_j^2} = \pi_{jj}^j \frac{\partial q_j^S}{\partial s_j} + \pi_{ij}^j \frac{\partial q_i^S}{\partial s_j} - \frac{\partial q_j^S}{\partial s_j} = (-2b)\frac{3}{4b} + (-b)(-\frac{1}{2b}) - \frac{3}{4b} = -(1 + \frac{3}{4b}) < 0$.

In this case, the Stackelberg output in the equilibrium is as follows:

$$(q_i^S(0, s_j^{uS}), q_j^S(0, s_j^{uS})) = (\frac{a - 4c_i + 3c_j}{3b}, \frac{a - 3c_j + 2c_i}{2b}). \tag{16}$$

This output level is equivalent to that in Case B-2 in essence. The following proposition is obtained.

Proposition 7. (*comparison of the output under Case B-2 and C-2*)
Under the Stackelberg competition in which firm i is the leader, the unilateral intervention by the government of the follower firm j yields the same result on output as the bilateral sequential intervention under Cournot competition in which government j first moves and then government i moves. That is, $(q_i^S(0, s_j^{uS}), q_j^S(0, s_j^{uS})) = (q_j^C(s_i^{bSC}, s_j^{bSC}), q_i^C(s_i^{bSC}, s_j^{bSC}))$.

This proposition implies that the subsidy of the government works as if it changes the competition mode from Stackelberg to Cournot. The optimal subsidy improves the Stackelberg-follower position to the Cournot one. Even if firm j is the Stackelberg follower under quantity competition, the optimal subsidy by government j makes the disadvantage of follower reduce to some extent.

Substituting (16) into $\pi^{Si}(s_i, s_j) = \frac{b}{2}(q_i^S)^2$ and $\pi^{Sj}(s_i, s_j) = b(q_j^S)^2$, the profit of the firm is immediately obtained: $\pi^{Si}(0, s_j^{uS}) = \frac{(a-4c_i+3c_j)^2}{18b}$ and $\pi^{Sj}(0, s_j^{uS}) = \frac{(a-3c_j+2c_i)^2}{4b}$. It is shown that $\pi^{Si}(s_i^{uS}, 0) = \frac{(a-2c_i+c_j)^2}{8b} > \pi^{Si}(0, s_j^{uS})$ and $\pi^{Sj}(s_i^{uS}, 0) = \frac{(a-3c_j+2c_i)^2}{16b} < \pi^{Sj}(0, s_j^{uS})$.[31]

Finally, the social surplus is derived. it is immediately obtained that $G^{Si}(0, s_j^{uS}) = \pi^{Si}(0, s_j^{uS}) = \frac{(a-4c_i+3c_j)^2}{18b} < G^{Si}(s_i^{uS}, 0) = \pi^{Si}(s_i^{uS}, 0) = \frac{(a-2c_i+c_j)^2}{8b}$. And it is obtained that $G^{Sj}(0, s_j^{uS}) =$

[31] Because $\pi^{Si}(0, s_j^{uS}) = (G^{Si}(0, s_j^{uS}) =)\frac{b}{2}(q_i^S(0, s_j^{uS}))^2$ and $\pi^{Si}(s_i^{uS}, 0) = (G^{Si}(s_i^{uS}, 0) =)\frac{b}{2}(q_i^S(s_i^{uS}, 0))^2$, it is satisfied that $\pi^{Si}(0, s_j^{uS}) < \pi^{Si}(s_i^{uS}, 0)$ if and only if $q_i^S(0, s_j^{uS}) < q_i^S(s_i^{uS}, 0)$. It is shown that $q_i^S(s_i^{uS}, 0) - q_i^S(0, s_j^{uS}) = \frac{a-3c_j+2c_i}{6b} > 0$.

$b(q_j^S)^2 - s_j^{uS} q_j^S = \frac{(a-3c_j+2c_i)^2}{12b} > G^{Sj}(s_i^{uS}, 0) = \pi^{Sj}(s_i^{uS}, 0) = \frac{(a-3c_j+2c_i)^2}{16b}$.

For simplification of analysis, it is assumed that $c \equiv c_i = c_j$ throughout the following analysis. Under this assumption, the following proposition is obtained:

Proposition 8. (*comparison of the profit and welfare under Case C-1 and C-2*)
Consider the Stackelberg competition in which firm i is the leader. Suppose that the cost is identical. When the government of the follower firm j unilaterally intervenes, the profit of the follower firm j (the leader firm i) is larger (resp. smaller) than that of the leader firm i (resp. the follower firm j) when the government of the leader firm i unilaterally intervenes. That is,

$\pi^{Si}(s_i^{uS}, 0) = \frac{(a-c)^2}{8b}$	$\pi^{Si}(0, s_j^{uS}) = \frac{(a-c)^2}{18b}$
$\pi^{Sj}(s_i^{uS}, 0) = \frac{(a-c)^2}{16b}$	$\pi^{Sj}(0, s_j^{uS}) = \frac{(a-c)^2}{4b}$

.

When government j unilaterally intervenes, the welfare of government j (government i) is smaller than that of government i (resp. government j) when government i unilaterally intervenes. That is,

$G^{Si}(s_i^{uS}, 0) = \frac{(a-c)^2}{8b}$	$G^{Si}(0, s_j^{uS}) = \frac{(a-c)^2}{18b}$
$G^{Sj}(s_i^{uS}, 0) = \frac{(a-c)^2}{16b}$	$G^{Sj}(0, s_j^{uS}) = \frac{(a-c)^2}{12b}$

.

Note that this proposition also holds when each firm has almost identical marginal cost. When the cost is almost identical, Although it looks at first glance that the leader firm may enjoy higher profit when the government of the leader firm can subsidize than that of the follower when its government can subsidize, the above proposition shows that this view is not correct. This implication is derived from the fact that government i does not subsidize at all because the advantage of the Stackelberg leader has already acquired by firm i in Case C-1. On the other hand, this intuition is correct when the welfare is considered. Even if the government makes the follower firm recover the first-mover advantage by intervention, it takes an extra cost to subsidize it.

D. bilateral intervention under Stackelberg model

I analyze the bilateral intervention under Stackelberg model. Both governments intervene.

D-1. simultaneous decision of subsidy

I consider the simultaneous decision of subsidy (s_i, s_j), like the simultaneous quantity choice in the Cournot model. Government i whose firm i is leader maximizes the following objective: Given s_j, $\max_{s_i \geq 0} G^{Si}(s_i, s_j) = \pi^i(q_i^S(s_i, s_j), q_j^S(s_i, s_j); s_i, s_j) - s_i q_i^S$.

The f.o.c. is as follows: $\frac{\partial G^{Si}(s_i,s_j)}{\partial s_i} = \frac{d\pi^i(q_i^S(s_i,s_j),q_j^S(s_i,s_j);s_i,s_j)}{ds_i} - q_i - s_i\frac{\partial q_i}{\partial s_i} = \pi_i^i\frac{\partial q_i^S}{\partial s_i} + \pi_j^i\frac{\partial q_j^S}{\partial s_i} + \frac{\partial \pi^i}{\partial s_i} - q_i - s_i\frac{\partial q_i^S}{\partial s_i} = 0$.

By the f.o.c. of the Stackelberg leader, it is satisfied that $\pi_i^i = -\pi_j^i R_j'(q_i) = -\frac{b}{2}q_i$. $\pi_j^i = -bq_i$. $\frac{\partial \pi^i}{\partial s_i} = q_i$. Substituting them, the f.o.c. is as follows: $(-\frac{b}{2}q_i)\frac{1}{b} + (-bq_i)(-\frac{1}{2b}) + q_i - q_i - s_i\frac{1}{b} = -s_i\frac{1}{b} \leq 0 \Rightarrow s_i^{bCS} = 0$. The reaction function is $s_i^{bCS} = R_i(s_j^{bCS}) = 0$.

Government j whose firm j is the follower maximizes the following objective: Given s_i, $\max_{s_j \geq 0} G^{Sj}(s_i, s_j) = \pi^j(q_j^S(s_i, s_j), q_i^S(s_i, s_j); s_i, s_j) - s_j q_j^S$.

The f.o.c. is as follows: $\frac{\partial G^{Sj}(s_i,s_j)}{\partial s_j} = \frac{d\pi^j(q_j^S(s_i,s_j),q_i^S(s_i,s_j);s_i,s_j)}{ds_j} - q_j - s_j\frac{\partial q_j}{\partial s_j} = \pi_j^j\frac{\partial q_j^S}{\partial s_j} + \pi_i^j\frac{\partial q_i^S}{\partial s_j} + \frac{\partial \pi^j}{\partial s_j} - q_j - s_j\frac{\partial q_j^S}{\partial s_j} = 0$.

By the f.o.c. of the Stackelberg follower, it is satisfied that $\pi_j^j = 0$. $\pi_i^j = -bq_j$. $\frac{\partial \pi^j}{\partial s_j} = q_j$. Substituting them, the f.o.c. is as follows: $(-bq_j)(-\frac{1}{2b}) + q_j - q_j - s_j\frac{3}{4b} = 0 \Leftrightarrow s_j = \frac{2b}{3}q_j = \frac{a-3(c_j-s_j)+2(c_i-s_i)}{6}$. The reaction function is $s_j^{bCS} = R_j(s_i^{bCS}) = \frac{-2s_i+a-3c_j+2c_i}{3}$.

In order to solve the simultaneous decision of the subsidy levels by both governments, I solve the intersection of the reaction function.

$$s_i^{bCS} = 0,\ s_j^{bCS} = \frac{a - 3c_j + 2c_i}{3}. \tag{17}$$

Note that $s_i^{bCS} = s_i^{uS} = 0$ and $s_j^{bCS} = s_j^{uS} = \frac{a-3c_j+2c_i}{3}$.

Substituting s_i into the Stackelberg output, the optimal Stackelberg output level is obtained as follows:

$$(q_i^S(s_i^{bCS}, s_j^{bCS}), q_j^S(s_i^{bCS}, s_j^{bCS})) = (\frac{a - 4c_i + 3c_j}{3b}, \frac{a - 3c_j + 2c_i}{2b}). \tag{18}$$

It is satisfied that $q_i^S(s_i^{bCS}, s_j^{bCS}) = q_i^S(0, s_j^{uS})$ and $q_j^S(s_i^{bCS}, s_j^{bCS}) = q_j^S(0, s_j^{uS})$.

In this case, as a result of the simultaneous decision of subsidy levels, the subsidy policy of the government to the leader firms does not work. Different from the Cournot model, under the Stackelberg model, the subsidy policy of the government of the leader firm is nullified. In Case D-1, the Stackelberg leader and the follower behave as if they do in Case C-2.

Proposition 9. (*equivalence between Case D-1 and C-2*)

Consider the Stackelberg competition in which firm i is the leader. The result in the equilibrium under Case D-1 is the same as that under Case C-2. The government whose firm is the leader does not subsidize its firm.

Also the profit and the welfare are the same as that under Case C-2. That is, the profit is $\pi^{Si}(s_i^{bCS}, s_j^{bCS})(= \pi^{Si}(0, s_j^{uS}) = \frac{(a-4c_i+3c_j)^2}{18b}$ and $\pi^{Sj}(s_i^{bCS}, s_j^{bCS})(= \pi^{Sj}(0, s_j^{uS})) = \frac{(a-3c_j+2c_i)^2}{4b}$. The social surplus is $G^{Si}(s_i^{bCS}, s_j^{bCS}) = \pi^{Si}(s_i^{bCS}, s_j^{bCS}) = \frac{(a-4c_i+3c_j)^2}{18b}$ and $G^{Sj}(s_i^{bCS}, s_j^{bCS}) = G^{Sj}(0, s_j^{uS}) = \frac{(a-3c_j+2c_i)^2}{12b}$.

D-2. sequential decision of subsidy ($s_i \to s_j$)

Then, I examine the sequential decision of subsidy ($s_i \to s_j$). In this case, the government i of the Stackelberg leader i moves first and then the government j of the follower j decides the subsidy level, s_j after observing s_i. Looking at s_i, the follower government j decides the subsidy $s_j = R_j(s_i)$.

Government j whose firm j is the follower maximizes the following objective: Given s_i, $\max_{s_j \geq 0} G^{Sj}(s_i, s_j) = \pi^j(q_j^S(s_i, s_j), q_i^S(s_i, s_j); s_i, s_j) - s_j q_j^S$.

The f.o.c. is as follows: $\frac{\partial G^{Sj}(s_i,s_j)}{\partial s_j} = \frac{d\pi^j(q_j^S(s_i,s_j),q_i^S(s_i,s_j);s_i,s_j)}{ds_j} - q_j - s_j \frac{\partial q_j}{\partial s_j} = \pi_j^j \frac{\partial q_j^S}{\partial s_j} + \pi_i^j \frac{\partial q_i^S}{\partial s_j} + \frac{\partial \pi^j}{\partial s_j} - q_j - s_j \frac{\partial q_j^S}{\partial s_j} = 0$.

By the f.o.c. of the Stackelberg follower, it is satisfied that $\pi_j^j = 0$. $\pi_i^j = -bq_j$. $\frac{\partial \pi^j}{\partial s_j} = q_j$. Substituting them, the f.o.c. is as follows: $(-bq_j)(-\frac{1}{2b}) + q_j - q_j - s_j \frac{3}{4b} = 0 \Leftrightarrow s_j = \frac{2b}{3} q_j = \frac{a-3(c_j-s_j)+2(c_i-s_i)}{6}$. The reaction function is $s_j^{bSiS} = R_j(s_i^{bSiS}) = \frac{-2s_i+a-3c_j+2c_i}{3}$. $R_j'(s_i^{bSiS}) = -\frac{2}{3}$. This maximization problem is the same procedure as the follower government in Case D-1.

The leader government induces this reaction and solves the following maximization problem: $\max_{s_i \geq 0} G^{Si}(s_i, R_j(s_i)) = \pi^i(q_i^S(s_i, R_j(s_i)), q_j^S(s_i, R_j(s_i)); s_i, R_j(s_i)) - s_i q_i^S(s_i, R_j(s_i))$.

The f.o.c. is as follows: $\frac{dG^{Si}(s_i,R_j(s_i))}{ds_i} = \frac{d\pi^i(q_i^S(s_i,R_j(s_i)),q_j^S(s_i,R_j(s_i));s_i,R_j(s_i))}{ds_i} - q_i^S(s_i, R_j(s_i)) - s_i(\frac{\partial q_i^S}{\partial s_i} + \frac{\partial q_i^S}{\partial s_j} R_j'(s_i)) = \pi_i^i(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j} R_j'(s_i)) + \pi_j^i(\frac{\partial q_j}{\partial s_i} + \frac{\partial q_j}{\partial s_j} R_j'(s_i)) + (\frac{\partial \pi^i}{\partial s_i} + \frac{\partial \pi^i}{\partial s_j} R_j'(s_i)) - q_i - s_i(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j} R_j'(s_i)) = 0$.

Like Case C, it is satisfied that $\pi_i^i = -\pi_j^i R_j'(q_i) = -\frac{b}{2} q_i, \pi_j^i = -bq_i, \frac{\partial \pi^i}{\partial s_i} = q_i, \frac{\partial \pi^i}{\partial s_j} = 0$. Substituting them, the f.o.c. is as follows: $-\frac{b}{2} q_i(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j} R_j'(s_i)) - bq_i(\frac{\partial q_j}{\partial s_i} + \frac{\partial q_j}{\partial s_j} R_j'(s_i)) - s_i(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j} R_j'(s_i)) = 0. \Leftrightarrow s_i = \frac{b}{4} q_i = \frac{a-2(c_i-s_i)+(c_j-R_j(s_i))}{8}$.

Under the sequential decision, the optimal subsidy level is as follows:

$$s_i^{bSiS} = \frac{a - 4c_i + 3c_j}{8}, \ s_j^{bSiS} = R_j(s_i^{bSiS}) = \frac{a - 5c_j + 4c_i}{4}. \tag{19}$$

Note that $s_j^{bSiS} > s_i^{bSiS}$ if the cost is almost identical.[32] When the cost is almost identical, the subsidy to the follower is larger than that to the leader.

Substituting s_i into the Stackelberg output, the optimal Stackelberg output level is obtained as follows:

$$(q_i^S(s_i^{bSiS}, s_j^{bSiS}),\ q_j^S(s_i^{bSiS}, s_j^{bSiS})) = (\frac{a-4c_i+3c_j}{2b}, \frac{3(a-5c_j+4c_i)}{8b}). \tag{20}$$

Note that $q_i^S(s_i^{bSiS}, s_j^{bSiS}) > q_j^S(s_i^{bSiS}, s_j^{bSiS})$ if the cost is almost identical.[33] In particular, when cost is identical ($c \equiv c_i$), $q_i^S(s_i^{bSiS}, s_j^{bSiS}) = \frac{a-c}{2b} > q_j^S(s_i^{bSiS}, s_j^{bSiS}) = \frac{3(a-c)}{8b}$. When the cost is almost identical, the output of the leader is larger than that of the follower.

As for the profit, because it is satisfied that $(\pi^{Si}, \pi^{Sj}) = (\frac{b}{2}(q_i^S)^2, b(q_j^S)^2)$, the profit is calculated as follows: $\pi^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-4c_i+3c_j)^2}{8b}$ and $\pi^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{9(a-5c_j+4c_i)^2}{64b}$. When the cost is identical, it is worth noting that $\pi^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-c)^2}{8b} < \pi^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{9(a-c)^2}{64b}$. In other words, the profit of the follower is larger than that of the leader under the identical cost.

By $G^{Si} = \pi^{Si} - s_i q_i$, the welfare is calculated as follows: $G^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-4c_i+3c_j)^2}{16b}$ and $G^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{3(a-5c_j+4c_i)^2}{64b}$. When the cost is identical, it is satisfied that $G^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-c)^2}{16b} > G^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{3(a-c)^2}{64b}$. With regard to the social welfare, the welfare of the leader government is larger than that of the follower under the identical cost. The above result is summarized as the following proposition.

Proposition 10. (*the profit and the welfare under Case D-2*)

Consider the Stackelberg competition in which firm i is the leader. In the symmetric equilibrium

[32] It is satisfied that $s_j^{bSiS} > s_i^{bSiS}$ if $a - 13c_j + 12c_i = a - 3c_j + 2c_i + 10(c_i - c_j) > 0$, because $s_j^{bSiS} - s_i^{bSiS} = \frac{a-13c_j+12c_i}{8}$.

[33] $q_i^S(s_i^{bSiS}, s_j^{bSiS}) - q_j^S(s_i^{bSiS}, s_j^{bSiS}) = \frac{a-c_i-27(c_i-c_j)}{8b} > 0$ if $a - 28c_i + 27c_j > 0$.

under Case D-2, the profit of the leader is less than that of the follower. The welfare of the leader government i is larger than that of the follower government j.

Although the leader produces more than the follower, the government of the leader subsidizes less than that of the follower. As a result, it is shown that the profit of the leader is less than that of the follower and the welfare of the first-mover government is larger than that of second-mover government. The result so that the profit of the leader is smaller is a new viewpoint that is obtained from considering the subsidy policy.

Further by comparing the surplus under no intervention with that under Case D-2, it is obtained that $G^{Si}(0,0) = \frac{(a-2c_i+c_j)^2}{8b} > G^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-4c_i+3c_j)^2}{16b}$ and $G^{Sj}(0,0) = \frac{(a-3c_j+2c_i)^2}{16b} > G^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{3(a-5c_j+4c_i)^2}{64b}$ if the cost is almost identical.[34] Thus I am in the position to state the following proposition. Under the bilateral intervention, both governments obtain fewer surpluses than under no intervention.

Proposition 11. (*comparison of the surplus under no intervention and Case D-2*)
Under the Stackelberg competition in which firm i is the leader, when the marginal cost is almost identical, the surplus under the bilateral intervention in Case D-2 is smaller than under no intervention. That is, the bilateral intervention falls into the prisoner's dilemma for both governments.

D-3. sequential decision of subsidy ($s_j \to s_i$)

Finally, I examine the sequential decision of subsidy ($s_j \to s_i$). In this case, the government j of the Stackelberg follower firm j moves first and then the government i of the leader i decides the subsidy level, s_i after observing s_j. This case is the adverse case of Case D-2 with regard

[34] $G^{Si}(0,0) - G^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-c_j)^2-8(c_i-c_j)^2}{16b}$ and $G^{Sj}(0,0) - G^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-4c_i+3c_j)^2-48(c_i-c_j)^2}{64b}$.

to the timing of the decision-making by governments. Looking at s_j, the follower government i decides the subsidy $s_i = R_i(s_j)$.

The follower government i whose firm i is the leader maximizes the following objective: Given s_j, $\max_{s_i \geq 0} G^{Si}(s_i, s_j) = \pi^i(q_i^S(s_i, s_j), q_j^S(s_i, s_j); s_i, s_j) - s_i q_i^S$.

The f.o.c. is as follows: $\frac{\partial G^{Si}(s_i,s_j)}{\partial s_i} = \frac{d\pi^i(q_i^S(s_i,s_j),q_j^S(s_i,s_j);s_i,s_j)}{ds_i} - q_i - s_i \frac{\partial q_i}{\partial s_i} = \pi_i^i \frac{\partial q_i^S}{\partial s_i} + \pi_j^i \frac{\partial q_j^S}{\partial s_i} + \frac{\partial \pi^i}{\partial s_i} - q_i - s_i \frac{\partial q_i^S}{\partial s_i} = 0$.

By the f.o.c. of Stackelberg leader, it is satisfied that $\pi_i^i = -\pi_j^i R_j'(q_i) = -\frac{b}{2} q_i$. $\pi_j^i = -bq_i$. $\frac{\partial \pi^i}{\partial s_i} = q_i$. Substituting them, the f.o.c. is as follows: $(-\frac{b}{2} q_i)\frac{1}{b} + (-bq_i)(-\frac{1}{2b}) + q_i - q_i - s_i \frac{1}{b} = -\frac{s_i}{b} \leq 0 \Rightarrow s_i = 0$. The reaction function is $s_i^{bSjS} = R_i(s_j^{bSjS}) = 0$. This is the same procedure as the leader government i in D-1 Case.

The leader government j induces this and solves the following maximization problem with substituting $s_i^{bSjS} = R_i(s_j^{bSjS}) = 0$: $\max_{s_j \geq 0} G^{Sj}(0, s_j) = \pi^j(q_j^S(0, s_j), q_i^S(0, s_j); 0, s_j) - s_j q_j^S(0, s_j)$.

The f.o.c. is as follows: $\frac{dG^{Sj}(0,s_j)}{ds_j} = \frac{d\pi^j(q_j^S(0,s_j),q_i^S(0,s_j);0,s_j)}{ds_j} - q_j^S(0, s_j) - s_j \frac{\partial q_j^S}{\partial s_j} = \pi_j^j \frac{\partial q_j}{\partial s_j} + \pi_i^j \frac{\partial q_i}{\partial s_j} + \frac{\partial \pi^j}{\partial s_j} - q_j - s_j \frac{\partial q_j}{\partial s_j} = 0$.

Like Case C-2, it is satisfied that $\pi_j^j = 0, \pi_i^j = -bq_j, \frac{\partial \pi^j}{\partial s_j} = q_j$. Substituting them, the f.o.c. is as follows: $-bq_j \frac{\partial q_i}{\partial s_j} - s_j \frac{\partial q_j}{\partial s_j} = 0$. $\Leftrightarrow s_j = \frac{2b}{3} q_j = \frac{a-3(c_j-s_j)+2c_i}{6} \Leftrightarrow s_j = \frac{a-3c_j+2c_i}{3}$.

Under the sequential decision, the optimal subsidy level is as follows:

$$s_i^{bSjS} = 0, \ s_j^{bSjS} = \frac{a - 3c_j + 2c_i}{3}. \tag{21}$$

Substituting s_i into the Stackelberg output, the optimal Stackelberg output level is obtained that

$$(q_i^S(s_i^{bSjS}, s_j^{bSjS}), \ q_j^S(s_i^{bSjS}, s_j^{bSjS})) = (\frac{a - 4c_i + 3c_j}{3b}, \frac{a - 3c_j + 2c_i}{2b}). \tag{22}$$

The equilibrium in this case is the absolutely same as Case D-1 (and also C-2 case).

Proposition 12. (*equivalence between Case D-3 and D-1* (*and also C-2*))

Consider the Stackelberg competition in which firm i is the leader. The result in the equilibrium under Case D-3 is the same as that under Case D-1 (and also Case C-2). The government whose firm is the leader does not subsidize its firm.

In this case, the follower subsidy setter in the country where there is the Stackelberg leader firm has nothing to do by the same reason as Case D-1 and C-2. Also the profit and the welfare are the same as that under Case D-1 (C-2). That is, the profit is $\pi^{Si}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-4c_i+3c_j)^2}{18b}$ and $\pi^{Sj}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-3c_j+2c_i)^2}{4b}$. The social surplus is $G^{Si}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-4c_i+3c_j)^2}{18b}$ and $G^{Sj}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-3c_j+2c_i)^2}{12b}$.

I compare the profit and the welfare in Case D-2 with that in Case D-3. When the cost is identical, the following inequality is satisfied: $\pi^{Si}(s_i^{bSjS}, s_j^{bSjS}) < \pi^{Si}(s_i^{bSiS}, s_j^{bSiS}) < \pi^{Sj}(s_i^{bSiS}, s_j^{bSiS}) < \pi^{Sj}(s_i^{bSjS}, s_j^{bSjS})$. [35] And it is satisfied that $G^{Si}(s_i^{bSjS}, s_j^{bSjS}) < G^{Sj}(s_i^{bSiS}, s_j^{bSiS}) < G^{Si}(s_i^{bSiS}, s_j^{bSiS}) < G^{Sj}(s_i^{bSjS}, s_j^{bSjS})$. [36] For the government, the first-mover advantage still exists.

E. the wholly sequential decision

Finally, I examine the wholly sequential decision ($s_i \to q_i \to s_j \to q_j$). I consider the bilateral intervention of sequential decision-making: $s_i \to q_i \to s_j \to q_j$.[37] I solve the equilibrium by backward induction.

In the subgame at fourth stage, the f.o.c. for profit maximization of firm j given (s_i, q_i, s_j) is

[35] By calculating directly, it is obtained that $\frac{(a-c)^2}{18b} < \frac{(a-c)^2}{8b} < \frac{9(a-c)^2}{64b} < \frac{(a-c)^2}{4b}$ in turn.

[36] By calculating directly, it is obtained that $\frac{(a-c)^2}{18b} < \frac{3(a-c)^2}{64b} < \frac{(a-c)^2}{16b} < \frac{(a-c)^2}{12b}$ in turn.

[37] Note that if $s_i = 0$, the unilateral intervention of sequential decision-making is analyzed: $q_i \to s_j \to q_j$. If $s_j = 0$, This is C-1 case.

as follows: $\pi_j^j = (a - b(q_i + q_j) - e_j) - bq_j = 0$. The reaction function is $q_j = R_j^s(q_i, s_j) = \frac{a - bq_i - e_j}{2b}$. Note that this reaction function does not depend on s_i directly.

In the subgame at third stage, the subsidy decision by government j is determined by maximizing $G^j(s_i, q_i, s_j, q_j) \equiv \pi^j(q_j, q_i; s_i, s_j) - s_j q_j$. The government j maximizes the following objective: Given (s_i, q_i), $\max_{s_j \geq 0} G^j(s_i, q_i, s_j, q_j), s.t.\ q_j = R_j^s(q_i, s_j) = \frac{a - bq_i - e_j}{2b}$. That is, $\max_{s_j \geq 0} G^j(s_i, q_i, s_j, R_j^s(q_i, s_j))$. $\frac{\partial R_j^s(q_i, s_j)}{\partial s_j} = \frac{1}{2b}$.

The f.o.c. for domestic welfare maximization of government j is as follows: $\frac{dG^j(s_i, q_i, s_j, R_j^s(q_i, s_j))}{ds_j} = \frac{d\pi^j(R_j^s(q_i, s_j), q_i; s_i, s_j)}{ds_j} - q_j - s_j \frac{\partial q_j}{\partial s_j} = 0$ if the solution is interior $(s_j \geq 0)$. If $\frac{dG^j(s_i, q_i, s_j, R_j^s(q_i, s_j))}{ds_j} < 0$, the solution is corner, $s_j = 0$.

The f.o.c. is as follows: $\frac{\partial G^j(s_i, q_i, s_j, R_j^s(q_i, s_j))}{\partial s_j} = \frac{d\pi^j(R_j^s(q_i, s_j), q_i; s_i, s_j)}{ds_j} - q_j - s_j \frac{\partial q_j}{\partial s_j} = \pi_j^j \frac{\partial R_j^s(q_i, s_j)}{\partial s_j} + \frac{\partial \pi^j}{\partial s_j} - q_j - s_j \frac{\partial R_j^s(q_i, s_j)}{\partial s_j} = 0$. By the f.o.c. of firm j, it is satisfied that $\pi_j^j = 0$. $\frac{\partial \pi^j}{\partial s_j} = q_j$. Substituting them into the f.o.c., the equation is arranged as follows: $0 \times \frac{1}{2b} + q_j - q_j - s_j \frac{1}{2b} = -s_j \frac{1}{2b} \leq 0 \Rightarrow s_j^{bs} = 0$. As a result, the subsidy level is zero regardless of any q_i. The reaction function is $s_j^{bs} = R_j^s(q_i) = 0$. $q_j = R_j^s(q_i, 0) = \frac{a - bq_i - c_j}{2b}$.

In the subgame at second stage, the output choice by firm i is solved as follows: Given s_i, inducing $s_j^{bs} = 0$ and $q_j = R_j^s(q_i, 0) = \frac{a - bq_i - c_j}{2b}$, the firm i maximizes the profit function $\pi^i(q_i, R_j^s(q_i, 0); s_i, 0)$. That is, the maximizing problem for the firm i is $\max_{q_i} \pi^i(q_i, R_j^s(q_i, 0); s_i, 0)$. $R_j^{s\prime}(q_i) = -\frac{1}{2}$.

The f.o.c. is $\pi_i^i + \pi_j^i R_j^{s\prime}(q_i) = ((a - b(q_i + R_j^s(q_i, 0)) - e_i) - bq_i) - bq_i(-\frac{1}{2}) = 0$. It is the same as the Stackelberg equilibrium without any subsidy: $q_i = R_i^s(s_i) = \frac{a - 2e_i + c_j}{2b} = q_i^S(s_i, 0)$, $q_j = R_j(q_i, 0) = \frac{a - 3c_j + 2e_i}{4b} = q_j^S(s_i, 0)$.

The f.o.c. is as follows: $\pi_i^i + \pi_j^i R_j^{s\prime}(q_i) = ((a - b(q_i + R_j^s(q_i, 0)) - c_i) - bq_i) - bq_i(-\frac{1}{2}) = 0$. It is the same as the Stackelberg equilibrium without any subsidy:

In the subgame at first stage, the subsidy decision by government i is determined by the same procedure as C-1 case. That is, there is no subsidy, $s_i = 0$.

$$(s_i^{bs}, s_j^{bs}) = (0, 0). \tag{23}$$

$$(q_i^{bs},\ q_j^{bs}) = (\frac{a - 2c_i + c_j}{2b},\ \frac{a - 3c_j + 2c_i}{4b}). \tag{24}$$

It is satisfied that $(q_i^{bs}, q_j^{bs}) = (q_i^S(0,0), q_j^S(0,0))$. The profit and the welfare is as follows: $\pi^{Si}(0,0) = G^{Si}(0,0) = \frac{(a-2c_i+c_j)^2}{8b}$ and $\pi^{Sj}(0,0) = G^{Sj}(0,0) = \frac{(a-3c_j+2c_i)^2}{16b}$.

4 The result

In this section, I compare the different structures with regard to the timing of the decision-making on the subsidy by governments and the output by the firms. In particular, I compare the simultaneous decision with the sequential one in the equilibrium which is derived in the above section. Before proceeding to the analysis, it is convenient to digest the equilibrium outcome under the different timing of the decision-making in the table. See Table 3. In order to figure out the output level in the equilibrium graphically, see also Figure 2 and 3.[38]

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Table 3 around here

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Figure 2 and 3 around here

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[38]For simplification, I illustrate only the case in which the cost is identical.

By Table 3, I can examine how the different structures about the timing of the decision-making by firms and governments affects the efficiency of the subsidy policy. In the following comparison, I limit the argument on the situation in which the cost is identical for simplification. Although this paper does not present exhaustive comparison in the comprehensive way, it shows the several noticeable results as the following propositions.[39]

Proposition 13. (*comparison of the profit between Cournot and Stackelberg competition under the unilateral intervention*)

Consider the unilateral intervention of the government. When the firm faces the Cournot competition, the profit is larger than when it competes as the Stackelberg leader and it is equal to that when it competes as the Stackelberg follower. That is,

$\pi^{Ci}(s_i^{uC}, 0) = \pi^{Sj}(0, s_j^{uS}) = \frac{(a-c)^2}{4b} > \pi^{Si}(s_i^{uS}, 0) = \frac{(a-c)^2}{8b}$.

This proposition implies that although it looks at first glance that the Stackelberg leader has more advantage than under Cournot competition, this impression is not correct. If the unilateral intervention of the government (and no intervention of the rival government) is necessarily guaranteed, the firm prefers to compete in the Cournot way rather than become the Stackelberg leader. The reason is that the government has nothing to do under the Stackelberg competition, but as the government under the Cournot competition subsidizes the firm, the acquisition of this subsidy raises the firm's profit. Whereas, the proposition also implies that the Stackelberg

[39] Although I do not compare the welfare in the third country, it is worth noting that the third country's welfare is reduced by the size of total quantity, Q, because this welfare consists of only the consumer's surplus, $CS(Q) = \frac{b}{2}Q^2$. Comparing the price level in Table 3, we can compare the third country's welfare level immediately. Likewise, the sum of the welfare of two exporting countries and the world welfare are immediately derived by tedious calculation, although I omit these comparisons.

follower recovers the competitive position from the Stackelberg follower to the Cournot and it can compete on equal terms with the rival firm by subsidization of the government.

Next I compare the profits between two competitive forms under the bilateral intervention. I compare Case B with Case D. At first, I compare Case B-1 with Case D-1.

Proposition 14. (*comparison of the profit between Cournot and Stackelberg competition under the bilateral simultaneous intervention*)
Consider the bilateral simultaneous intervention of the governments. When the firm faces the Cournot competition, the profit is larger than when it competes as the Stackelberg leader. On the other hand, the profit under the Cournot competition is smaller than when it competes as the Stackelberg follower. That is,
$\pi^{Si}(s_i^{bCS}, s_j^{bCS}) = \frac{(a-c)^2}{18b} < \pi^{Ci}(s_i^{bCC}, s_j^{bCC}) = \frac{4(a-c)^2}{25b} < \pi^{Sj}(s_i^{bCS}, s_j^{bCS}) = \frac{(a-c)^2}{4b}$.

This proposition is the extensive version of **Proposition 13** to the bilateral intervention. Although it looks at first glance that the Stackelberg leader has more advantage than under Cournot competition, the firm prefers to compete in the Cournot way rather than become the Stackelberg leader and moreover prefers to become the follower. The reason is similar to that of **Proposition 13**. Under the bilateral simultaneous intervention, the government that has the Stackelberg leader has nothing to do, that is, it does not have any influence to change the market structure. As both governments under the Cournot competition subsidize the firm, their subsidies influence market structure and raise the firm's profits. Whereas, the Stackelberg follower supports by its government fully and improves the competitive position vastly. It can compete on more advantageous terms than the rival firm in Case D-1.

This proposition suggests the following important assertion on the trade policy: When governments can intervene in its domestic firm with a certain policy instrument in advance, the difference of the competitive mode between firms such as Cournot or Stackelberg competition does not necessarily influence the firm's advantage on the competition. In other words, even if a firm is the Stackelberg follower in the third-market, the subsidization by the government can get rid of the competitive disadvantage entirely.

Finally, I compare Case B-2 with Case D-2 and D-3.

Proposition 15. (*comparison of the profit between Cournot and Stackelberg competition under the bilateral sequential intervention*)
Consider the bilateral sequential intervention of the governments. Suppose that government i first moves.
(i) When firm i faces the Cournot competition, the profit is larger than when it competes as the Stackelberg leader. On the other hand, the profit of firm j under the Cournot competition is smaller than when it competes as the Stackelberg follower. That is, $\pi^{Ci}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-c)^2}{4b} > \pi^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-c)^2}{8b}$ *and* $\pi^{Cj}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-c)^2}{9b} < \pi^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{9(a-c)^2}{64b}$.
(ii) When firm i faces the Cournot competition, the profit is equivalent when it competes as the Stackelberg follower. And also, the profit of firm j under the Cournot competition is larger than when it competes as the Stackelberg leader. That is, $\pi^{Ci}(s_i^{bSC}, s_j^{bSC}) = \pi^{Sj}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-c)^2}{4b}$ *and* $\pi^{Cj}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-c)^2}{9b} > \pi^{Si}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-c)^2}{18b}$.[40]

Part (i) in this proposition states as follows: Under the bilateral sequential intervention, like the simultaneous intervention, the firm prefers to compete in the Cournot way rather than

[40]Note that the index of i interchanges that of j in Case D-3, because government i first moves and firm i is the Stackelberg follower.

become the Stackelberg leader. On the other hand, the firm prefers to become the follower rather than compete in the Cournot way. At first glance, it seems that the change of decision structure from Case B-2 to D-2 gives the firm that becomes the Stackelberg leader more advantage about output choice. However, this proposition implies that this shift of decision structure does not bring any advantage and in the contrary, the profit of the leader firm decreases. On the other hand, the firm that becomes the Stackelberg follower acquires more profit than under the previous Cournot competition.

The basic logic of the proposition is similar to that of **Proposition 14**, although this logic is a little bit complicated. Under the bilateral sequential intervention, the Stackelberg leader needs less subsidy than if this firm is engaged in Cournot competition, that is, $s_i^{bSC} = \frac{a-c}{3} > s_i^{bSiS} = \frac{a-c}{8}$, because the firm has already enjoyed the first-mover advantage. Whereas, the Stackelberg follower is supported by its government with great care and improves the competitive position, that is, $s_j^{bSC} = \frac{a-c}{6} < s_j^{bSiS} = \frac{a-c}{4}$. As a result of the asymmetric subsidy policy, it occurs that when both governments subsidize the firm under the Cournot competition, the profit of the leader (the follower) is less (resp. more) than when they do under the Stackelberg competition.

It implies that the difference on the timing of the decision-making by governments, in particular, which simultaneous or sequential are the decision-move of governments, affects the size of the firm' s profit and the efficiency of the trade policy significantly.

Part (ii) in this proposition states as follows: Under the bilateral sequential intervention, when the competition is in the Cournot way, the profit of the firm is equivalent to that when this firm competes as the Stackelberg follower. Moreover, the firm prefers to compete in the Cournot way rather than become the Stackelberg leader in Case D-3. It implies that when government i first moves, even if the competition form shifts from Cournot to Stackelberg and firm i becomes the follower, its profit does not change. The profit of firm j that becomes the leader in place of firm i becomes less than under the previous Cournot competition. Although it seems at first glance that the change of decision structure from Case B-2 to D-3 gives the firm that becomes the Stackelberg follower the disadvantage on output choice, this shift of decision structure does not change the profit and only the profit of the firm that becomes the leader decreases.

It implies that it is desirable for both firms to shift the competitive mode from the Stackelberg competition under which the government of the Stackelberg follower first decides the subsidy, to the Cournot one under bilateral sequential intervention. This shift of the competitive mode avoids the excessive subsidy allocation by governments and saves the subsidy that does not have quite effect on the advantage of the domestic firm in the market.

These propositions insists that the difference in timing of policy execute (and announcement) by the government affects the profit of the firm and the resulting welfare. And also they insist that the government should exercise the trade policy, taking what is the competitive mode between firms into consideration.

5 The concluding remarks

This paper analyzes the relationship between the different timing of the decision-making by exporting firms and their subsiding governments and its impacts on the export subsidy. Two main results are as follows: First, when the governments decide the export subsidies simultaneously in advance under the Stackelberg quantity competition, the original Stackelberg leader firm in the output competition produces as if it is the follower. Different from the Cournot model, under the

Stackelberg model, the subsidy policy by the government that can subsidize the leader firm is nullified. Second, under the sequential-move game in which the government that can subsidize the leader firm decides the subsidy level at first, the profit of the leader is less than that of the follower, although the first-mover advantage is maintained. I conclude that the timing of decision-making affects the effectiveness on the export subsidy policy more significantly.

Although the paper mainly focuses on the theoretical aspect, the results in this paper are applicable to make some proper suggestions to the actual strategic trade policies. For example, in the realistic context of the international exporting competition, suppose that there is the leader firm that is the predecessor and lies in the dominant position in the exporting market. When the successor entries and the Stackelberg competition is made, the predecessor government of the leader firm may anticipate the successor's government in deciding the trade policy. In this situation, how does the government implement the strategic subsidy policy? **Proposition 15** in Section 4 tells us that the predecessor government dares to make its firm acquire less profit than the rival firm with the less subsidy than the successor government and should save the subsidy in order to attain more welfare. Moreover, by **Proposition 15**, it is indicated that even if the government can choose the timing of subsidy policy, it should defend the position as the first-mover policy maker and maintain the first-mover advantage. On the other hand, **Proposition 15** suggests also that in order to bring more profit to the successor firm, its government should announce the subsidy policy faster than the rival government of the leader and take the first-mover advantage if possible. This may present one of the reasons that the trade war is triggered.

Further extension in this paper can be considered. The above analysis is the linear demand and linear cost model. First, the more general model in which the demand and cost functions have the general forms can be analyzed. I guess that the basic logic is the same as the linear case. I think that I can generalize the above analysis directly. Moreover I limit the argument to the homogeneous goods. Second, product differentiation should be analyzed although the basic results that I argued in the above analysis have remained unchanged.

Thirdly, the extension to strategic complements should be considered as an extension of Eaton and Grossman (1986). They argue the generalization of the analysis by Brander and Spencer (1935). However, they deal with only the symmetric conjectural variations like Cournot, Bertrand, and consistent conjectures. That is, they deal with only the simultaneous-move game on output choice. I deal with the asymmetric conjecture such as the Stackelberg leader-follower competition. Using the similar method as theirs, the generalized analysis can be applied. This is another issue because under price competition (or strategic complements in general), the optimal trade policy is to adopt the exporting tariff. This result may be extended to strategic complements and other conjectural variables, although the notice should be given that most of the conjectural variables do not have any economic justification.

Finally, the endogenous timing of the trade policy by policy maker is another important topic. As for the endogenous timing, Ohkawa, Okamura and Tawada (2002) tackle this issue under the Cournot oligopoly. I think that the result in this paper is applicable to deal with this problem.

As a possibility of further extensions of my analysis that investigates the strategic subsidy policy in the sequential-move game, other arguments on the strategic trade policy can be analyzed, with considering the timing of decision-making. For example, sequential timing on other variables such as export tariff, quota, and investment choice of FDI can be investigated. Also the comparative statics of the parameter may be able to be considered, such as the externality and spillover, for example, the environmental diseconomy. Moreover we may be able to deal with the informational asymmetry under this sequential move game.

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	subsidy	output	price
1. no-subsidy Cournot	nothing	$(\frac{a-2c_i+c_j}{3b}, \frac{a-2c_j+c_i}{3b})$	$\frac{a+c_i+c_j}{3}$
2. no-subsidy Stackelberg	nothing	$(\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b})$	$\frac{a+2c_i+c_j}{4}$
A. unilateral Cournot	$(\frac{a-2c_i+c_j}{4}, 0)$	$(\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b})$	$\frac{a+2c_i+c_j}{4}$
B. bilateral Cournot			
B-1. simultaneous	$(\frac{a-3c_i+2c_j}{5}, \frac{a-3c_j+2c_i}{5})$	$(\frac{2(a-3c_i+2c_j)}{5b}, \frac{2(a-3c_j+2c_i)}{5b})$	$\frac{a+2c_i+2c_j}{5}$
B-2. sequential	$(\frac{a-3c_i+2c_j}{3}, \frac{a-4c_j+3c_i}{6})$	$(\frac{a-3c_i+2c_j}{2b}, \frac{a-4c_j+3c_i}{3b})$	$\frac{a+3c_i+2c_j}{6}$
C. unilateral Stackelberg			
C-1. government i	$(0, 0)$	$(\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b})$	$\frac{a+2c_i+c_j}{4}$
C-2. government j	$(0, \frac{a-3c_j+2c_i}{3})$	$(\frac{a-4c_i+3c_j}{3b}, \frac{a-3c_j+2c_i}{2b})$	$\frac{a+2c_i+3c_j}{6}$
D. bilateral Stackelberg			
D-1. simultaneous	$(0, \frac{a-3c_j+2c_i}{3})$	$(\frac{a-4c_i+3c_j}{3b}, \frac{a-3c_j+2c_i}{2b})$	$\frac{a+2c_i+3c_j}{6}$
D-2. sequential gov. i	$(\frac{a-4c_i+3c_j}{8}, \frac{a-5c_j+4c_i}{4})$	$(\frac{a-4c_i+3c_j}{2b}, \frac{3(a-5c_j+4c_i)}{8b})$	$\frac{a+4c_i+3c_j}{8}$
D-3. sequential gov. j	$(0, \frac{a-3c_j+2c_i}{3})$	$(\frac{a-4c_i+3c_j}{3b}, \frac{a-3c_j+2c_i}{2b})$	$\frac{a+2c_i+3c_j}{6}$
E. wholly sequential	$(0, 0)$	$(\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b})$	$\frac{a+2c_i+c_j}{4}$

	profit	welfare
1. no-subsidy Cournot	$(\frac{(a-2c_i+c_j)^2}{9b}, \frac{(a-2c_j+c_i)^2}{9b})$	$(\frac{(a-2c_i+c_j)^2}{9b}, \frac{(a-2c_j+c_i)^2}{9b})$
2. no-subsidy Stackelberg	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$
A. unilateral Cournot	$(\frac{(a-2c_i+c_j)^2}{4b}, \frac{(a-3c_j+2c_i)^2}{16b})$	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$
B. bilateral Cournot		
B-1. simultaneous	$(\frac{4(a-3c_i+2c_j)^2}{25b}, \frac{4(a-3c_j+2c_i)^2}{25b})$	$(\frac{2(a-3c_i+2c_j)^2}{25b}, \frac{2(a-3c_j+2c_i)^2}{25b})$
B-2. sequential	$(\frac{(a-3c_i+2c_j)^2}{4b}, \frac{(a-4c_j+3c_i)^2}{9b})$	$(\frac{(a-3c_i+2c_j)^2}{12b}, \frac{(a-4c_j+3c_i)^2}{18b})$
C. unilateral Stackelberg		
C-1. government i	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$
C-2. government j	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{4b})$	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{12b})$
D. bilateral Stackelberg		
D-1. simultaneous	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{4b})$	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{12b})$
D-2. sequential gov. i	$(\frac{(a-4c_i+3c_j)^2}{8b}, \frac{9(a-5c_j+4c_i)^2}{64b})$	$(\frac{(a-4c_i+3c_j)^2}{16b}, \frac{3(a-5c_j+4c_i)^2}{64b})$
D-3. sequential gov. j	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{4b})$	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{12b})$
E. wholly sequential	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$

Table 3: the equilibrium result

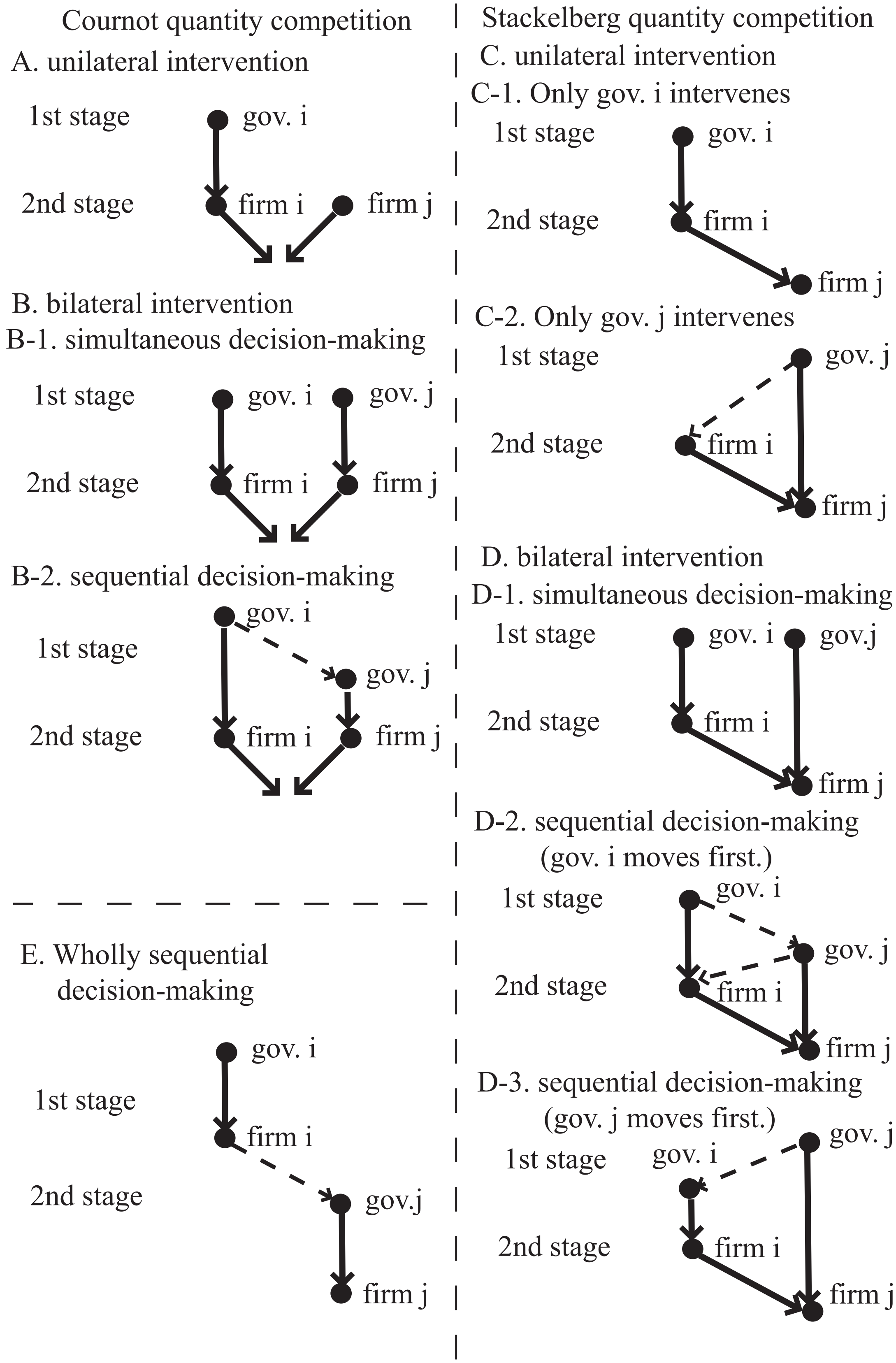


Figure 1: decision node

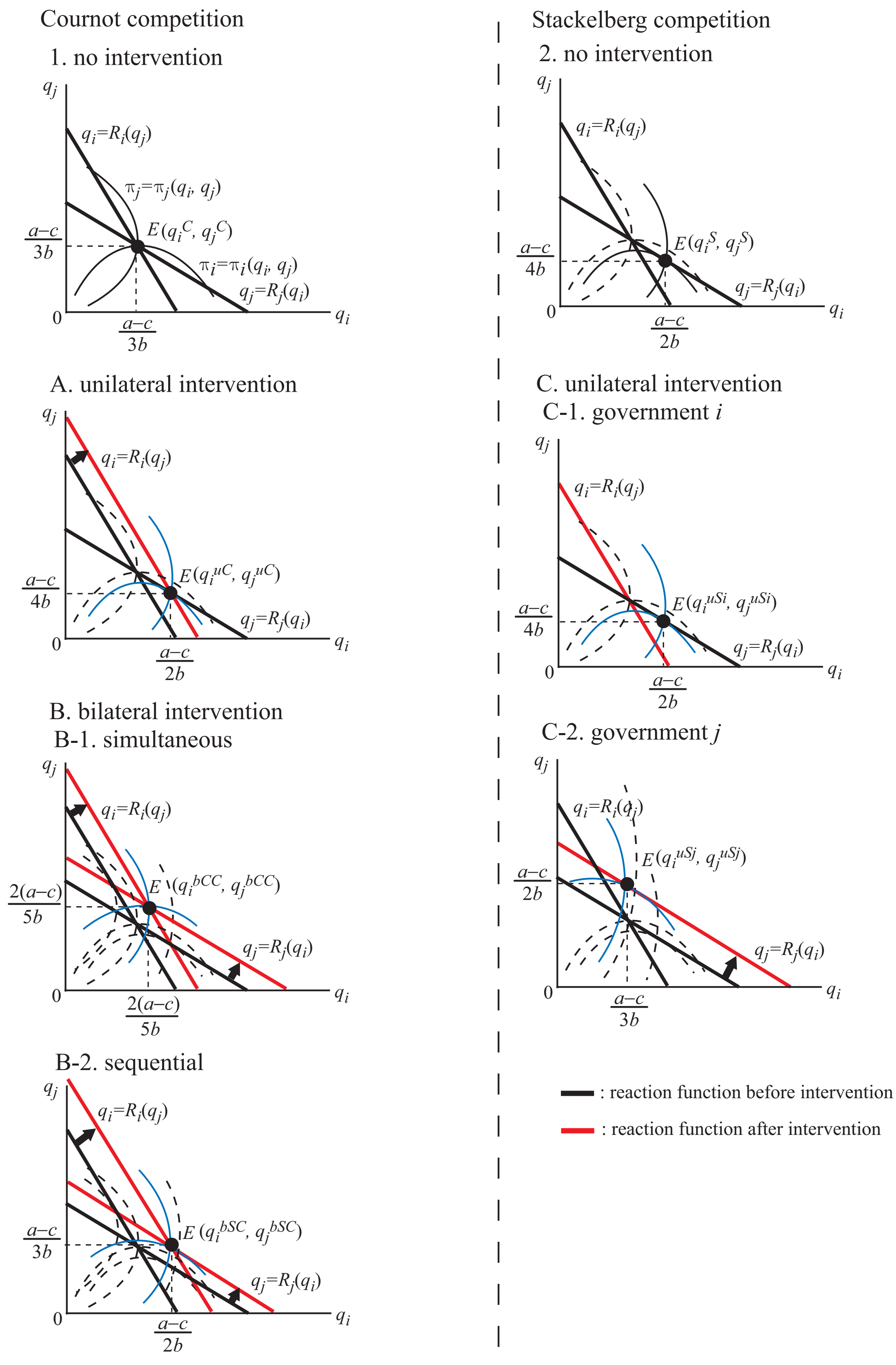
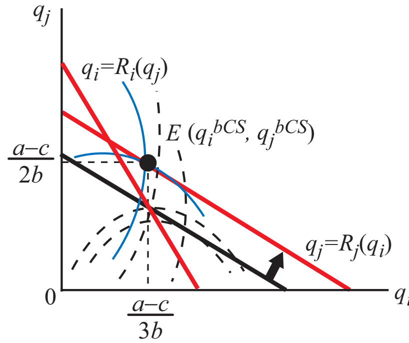


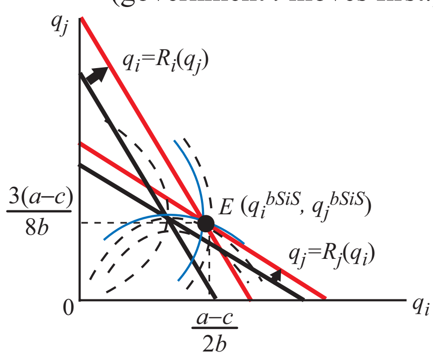
Figure 2: reaction functions and equilibrium output levels

Stackelberg competition (continued)

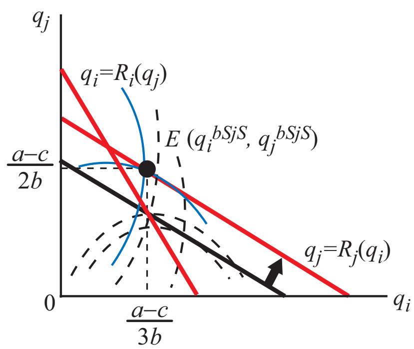
D-1. simultaneous decision-making



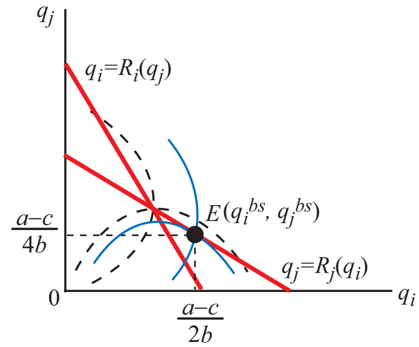
D-2. sequential decision-making
(government i moves first.)



D-3. sequential decision-making
(government j moves first.)



E. Wholly sequential decision-making



: reaction function before intervention

: reaction function after intervention

Figure 3: reaction functions and equilibrium output levels (continued)