The Forward Premium Puzzle and Intervention Risk

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ABSTRACT

This paper presents a new interpretation regarding the forward premium puzzle (FPP). We postulate that the negative correlation between the expected rate of depreciation and interest differential is due to an expected intervention risk. We define a target zone by using a Multiple-Regime STAR. It is confirmed that deviations from UIP follow a random-walk within a target zone, but display mean-reversion once the deviations exceed the boundaries. We also confirm that the adjustment speed in the case of appreciation is faster than that in the case of depreciation. This indicates that the expected intervention risk provides an alternative explanation for FPP.

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1. Introduction

This article focuses on one central question of whether central intervention risk can account for the well-known empirical failure of the forward premium as an unbiased predictor of future exchange rate movements. This failure has been one of the unresolved issues in the forefront of modern international finance, known as the forward premium puzzle (FPP hereafter).\(^1\) To be more specific, it refers to the empirical finding that the forward premium is not only a biased predictor, but also generally predicts the future spot exchange rate in the wrong direction.

The puzzle has been challenged with two main explanations: the time varying risk premium and expectations errors. However, the puzzle still remains unresolved and thus serious, in the sense that none of the subsequent empirical studies, using elaborated estimation techniques and different data sets, can account for the puzzle (Froot and Thaler, 1990; Wu and Zhang, 1996; Sarno and Taylor, 2002).

This empirical failure has led to two alternatives, but not exclusive, approaches to the puzzle. One is an open economy macroeconomics approach to construct a plausible model that gives rise to the two characteristics of the FPP. One characteristic is that the risk premium is more volatile than the expected depreciation, and the other characteristic is that their covariation is negative (Fama, 1984; Engel, 1996; Obstfeld and Rogoff, 1996). Specifically, a consumption and/or money based general equilibrium model, or an intertemporal CAPM have been employed to examine the characteristics of the risk premium. However, most of the empirical studies either rejected the consumption Euler equations, or gave rise to implausibly large coefficient values of the relative risk aversion.\(^2\)\(^3\)

The second alternative, a slightly more straightforward model-based explanation of the FPP has also been offered by deriving the UIP condition from a rational expectations macro model that

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\(^1\) For an earlier survey on the FPP, see Hodrick (1987). For a recent comprehensive survey, see Engel (1996). For various unresolved puzzles in international finance, see Lewis (1995). For a brief exposition that the FPP contains a wide range of serious problems in international finance, see Obstfeld and Rogoff (1996, chapter 8).

\(^2\) In this sense the FPP resembles, but is slightly more complicated than, the familiar equity premium puzzle by Mehra and Prescott (1985) (see Engel, 1996).

\(^3\) See a comprehensive survey in Engel (1996, sections 3.1 and 3.2). Most of the studies that analyzed the FPP within a general equilibrium model employed a two-country model by Lucas (1982) (see, e.g. Hodrick and Srivastava, 1984; Bekaert, 1996).
embodies monetary policy endogeneity (McCallum, 1994; Meledith and Ma, 2002). But none of these successfully accounts for the FPP in the short-run.

Another strand of tests has elaborated on estimations for the FPP by using new techniques and data sets. Several interesting attempts have been made along this line. Recent studies include, for example, consideration of structural changes (Wu, 1997), examination of the term structure of the risk premia (Clarida and Taylor, 1997) and interest rates (Bansal, 1997), non-parametric tests (Wu and Zhang, 1997), tests by the fully modified OLS (Goodhart, McMahon, and Ngama, 1997), tests by co-integration (Luintel and Paudyal, 1998), uses of the EMS data set (Flood and Rose, 1996) or a high-frequency, cross-country data set (Flood and Rose, 2002), and the random time effects panel data (Huisman, Koedijk, Kool and Nissen, 1998) etc. Unfortunately, none of these could successfully account for the FPP, although some improvements have been made. A general conclusion is that the FPP continues to remain unresolved.

Baillie and Bollerslev (2000) suggest that, by examining the monthly Deutsche mark (against the U.S. dollar) exchange rate for 1974-1991, the FPP is attributable to the slow speed of adjustment in the foreign exchange markets and thus is a short-run phenomenon that will disappear in the longer time horizon. McDonald and Taylor (2001) conclude, by examining the monthly exchange rates of the EMS currencies against the Deutsche mark and the U.S. dollar for 1978-1994 by a vector error-correction model, that the FPP is due to the forward exchange rates that are usually not weakly exogenous. Bansal and Dahlquist (2000, B-D hereafter), using monthly observations of 28 economies for the period 1976-1998, found that the FPP is confined only to high income economies, and that the forward premium responds asymmetrically to the interest rate differentials.

In view of these unsuccessful developments, further research is necessary in four general directions as mentioned by Engel (1996) to explain why the FPP occurs. They are extension of the Peso problem, expectations using survey data, inefficiency in the international financial markets arising from various frictions and the risk premium analysis. This paper takes up the last direction; we focus on the risk premium which is defined slightly differently than in the literature, but similar to the one defined by Garber and Svensson (1995, ch36).

When the central bank announces a target zone and credibly intervenes in the market when the exchange rate approaches the upper and lower edges, or when investors expect a credible intervention, it was proved by Krugman (1992) that exchange rate movement is approximated by a S-shaped curve.
within a target zone. That is, exchange rates are expected to follow non-linear processes and independent of fundamentals.

We introduce a risk premium which is defined slightly differently in the next section and empirically examine the effects on the FPP using a data set of 15 high-income countries where the puzzle was confirmed by Bansal and Dahlquist (2000, hereafter B-D). The purpose of this paper is fourfold.

First, we empirically analyze the asymmetry reflected in different speeds of adjustment of risk premium, which is introduced by the expected intervention. While asymmetry is also confirmed by B-D (2000) and Wu and Zhang (1996) in their studies of the FPP, the source of the asymmetry remains unsolved. In contrast with this, our analysis clarifies that asymmetry is due to different speeds of adjustment of risk premium.

Second, we also empirically address non-linearity introduced by a S-shaped curve within a target zone. If the exchange rate follows an S-shape in the target zone, the FPP is an inevitable consequence, as remarked by Garber and Svensson (1995, ch36).

Third, since the above two purposes are attained by using a Multiple-regime Smooth Transition Autoregressive (MRSTAR) model, whose prominent characteristics are asymmetric dynamics, regime-switching and non-linearity, among others, we are in a position to examine exchange rate behavior under a target zone, which reflects both a publicly announced zone and a privately conceived one.

Last, our fourth purpose is, utilizing the results of the above three purposes, to show empirically that a possible cause of the FPP is the risk premium introduced by the expected intervention.

The rest of this paper is organized as follows: The next section briefly discusses the FPP and the risk premium resulting from central bank intervention. Section 3 is devoted to an empirical examination of the FPP. Our strategy for empirical analysis is to first define a target zone by using a Multiple-Regime STAR model and then to examine behaviors of risk premium for each zone. It is shown that our results successfully explain the FPP in almost all of the 15 economies in which the FPP was detected by B-D (2000). Section 4 concludes the paper.

2 The Forward Premium Puzzle
2.1 Background

This section briefly summarizes the forward premium puzzle, or the FPP. If the exchange market uses all information available up to the present, the forward exchange rate at time $t+1$, $F_t$ (in log form) contracted at $t$, is an unbiased predictor of the future spot exchange rate $S_{t+1}$ (in log form). This so-called unbiasedness hypothesis is expressed in the following equation:

$$E [S_{t+1} | I_t] = F_t$$

where $E$ represents the mathematical expectations and $I_t$ the information set available at time $t$. Subtracting the spot rate $S_t$ (in log form) from both sides of equation (1) and transforming it into a regression equation yields:

$$S_{t+1} - S_t = \alpha + \beta (F_t - S_t) + u_{t+1}$$

which has been empirically examined in the literature. $u_{t+1}$ denotes disturbance, which is a white noise error term with a zero mean and finite variance and uncorrelated with information available at time $t$ according to the efficient market hypothesis. $F_t - S_t$ is called the forward premium. Equation (2) tells us whether the forward premium has the predictive power of future changes in the spot rate. The unbiasedness hypothesis maintains that the coefficients $\alpha$ and $\beta$ should be equal to zero and one, respectively.

We assume, based on accumulated evidence (e.g., Frankel and MacArthur, 1988), that the covered interest parity condition (CIP hereafter) holds and this implies that the forward premium is equivalent to the interest rate differential. Thus we formally write equation (2) as follows:

$$S_{t+1} - S_t = \alpha + \beta (i_t - i_t^*) + u_{t+1}$$

where $i_t$ and $i_t^*$ are the domestic and the foreign interest rates observed at $t$. Equation (3) is the so-called uncovered interest parity condition (UIP hereafter) in a regression form. The null hypothesis of UIP embodies the joint hypotheses of rational expectations, perfect capital movements and perfect substitution among assets. The hypothesis still maintains that $\alpha = 0$ and $\beta = 1$. However, many preceding studies show that these joint hypotheses are strongly rejected. To be more specific, the estimate of the slope coefficient $\beta$ is not only significantly different from the theoretically

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Equation (1) is based on the Efficient Market Hypothesis that applies a rational expectation hypothesis to the price formation of foreign exchange markets.
prescribed value of unity, but also negative.\textsuperscript{5} This negative empirical correlation between the interest rate differential (or the forward premium) and the expected rate of depreciation is puzzling and thus, has been called the forward premium puzzle (FPP).

Fama (1984) proposes that the rational expectations risk premium on foreign exchange markets exists. He defines risk premium $rp_t$ as the bias in the (log) forward premium

$$F_{t+1} - E[S_{t+1} \mid I_t] = rp_t$$

which is identically expressed as:

$$F_{t+1} - S_t = E[S_{t+1} \mid I_t] - S_t + rp_t$$

Under rational expectations, the difference between the expected and the realized exchange rate must be uncorrelated with forward premium on date $t$, which implies

$$E[(F_{t+1} - S_t)(S_{t+1} - E[S_{t+1} \mid I_t]) \mid I_t] = 0$$

Assuming a linear specification for estimation of equation (5),

$$E[S_{t+1} \mid I_t] - S_t = \alpha + \beta_{OLS} (F_{t+1} - S_t) + u_t$$

We can describe the OLS estimator as

$$p \lim(\beta_{OLS}) = \frac{\text{cov}(F_{t+1} - S_t, E[S_{t+1} \mid I_t] - S_t)}{\text{Var}(F_{t+1} - S_t)}$$

Substituting equation (5) into the numerator of equation yield

$$\text{Cov}(F_{t+1} - S_t, E[S_{t+1} \mid I_t] - S_t) = \text{Var}(E[S_{t+1} \mid I_t] - S_t) + \text{Cov}(E[S_{t+1} \mid I_t] - S_t, rp_t)$$

Since the variances are nonnegative, $\beta_{OLS} < 0$ only if

$$\text{Cov}(E[S_{t+1} \mid I_t] - S_t, rp_t) < 0$$

\textsuperscript{5} Froot and Thaler (1999) show that the average value of $\beta$ across 75 previously published papers is -0.88.
And the implication of \( \frac{\text{OLS}}{\text{OLS}} \) implies:

\[
Var(rp_t) > Var(E[S_{t+1} | I_t] - S_t) \quad \Box
\]

In short, Fama (1984) suggests that when the FPP is observed, the correlation between the risk premium and the expected change in the spot rate must be negative and the risk premium must be more variable than expected future exchange changes.

Although many preceding studies consider that the FPP is due to the existence of the risk premium inspired by Garber and Svensson (1995, ch36). To be more specific in the following section, this paper introduces a similar, but slightly differently defined, risk premium perceived by investors when central bank intervention is expected.

### 2.2 Intervention and Risk Premium

Most of the central banks intervene in a foreign exchange market in a variety of ways. Frequency and Volume of intervention depend on the exchange rate systems. One of the most familiar ways of intervention is the one adopted in a target zone system, e.g. in the European Union. The multilateral exchange-rate grid of the EMS included an official bilateral zone of \( \pm 2.25\% \) (or \( \pm 6\% \) and so on). The central banks intervened in foreign exchange markets when the exchange rate exceeds the limits of the zone, but they allowed the exchange rate to be free-floating within the zone. Other examples include a system in which the exchange rate is managed so as to fix it to the currency or currencies of the main trading partners, or the dirty float exchange rate system which monetary authorities intervene frequently, despite a free-floating system. Under these systems, foreign exchange rates do not only reflect the fundamentals through the demand and supply of the market, but are also determined by intervention, policy stance, foreign environment, etc.

Thus, investors expect intervention when the central bank is in active controlling of the money supply, or for changing exchange policy, even if there is no announcement. As a result, investors come to bear a zone for exchange rate in their mind. This can be thought of as an implicit target zone, even under free-float systems. In the following we examine the effects of intervention on the risk premium both within a clearly announced target zone and the implicit target zone.
Krugman (1992) proved that the exchange rate is determined by the fundamentals that are subject to a random walk process within a target zone, but that the movement is approximated by a S-shaped curve near the edges of the target zone. If the exchange rate actually moves along this line of thought, then it is expected to follow a non-linear process. Applying this idea to UIP, it provides a specific forecast as explained below (See, Garber and Svensson (1995, section 3.3.2). When the exchange rate approaches, for example, the upper edge of the zone, it either remains at the edge or drifts back towards the interior of the zone. That is, the expected rate of depreciation within the zone is negatively-correlated with the exchange rate. At the same time, the exchange rate level is expected to be negatively-correlated with the interest rate differential under UIP. However, this prediction has predominately been rejected by empirical studies. Although these two negative correlations imply a positive relationship between the exchange rate of depreciation and interest rate differentials, this prediction has predominantly been rejected by preceding empirical studies. This means that UIP under a target zone is rejected and Garber and Svensson (1995) attribute the rejection to the two assumptions imposed by Krugman (1992), which will be discussed below.

The first assumption is that the central bank intervenes at the margin of the zone and the second is that there is perfect credibility to intervention. These assumptions seem to be unrealistic, given the actual behavior of central banks. Garber and Svensson (1995, chapter 36) thus relax the second assumption, and suggest that UIP failure is observed if a time-varying risk premium occurs by intervention. They divide the exchange rate in a target zone into two components as follows:

\[ S_t = X_t + C_t \]  

where \( C_t \) is the (log) level of the central parity and \( X_t \) is the deviation of the (log) exchange rate from the central parity. Updating and taking expectations of equation (7) and subtracting (7) yields

\[
E[S_{t+1} \mid I_t] - S_t = (E[X_{t+1} \mid I_t] - X_t) + (E[C_{t+1} \mid I_t] - C_t).
\]

The first term on the right-hand side means the expected rate of depreciation within a zone and the second term the expected rate of devaluation. The latter reflects a premium which is called for the time-varying realignment risk, according to Garber and Svensson (1995, chapter 36). Substituting (7)

\^[6\] Using data of EMS, Vilasuso and Cunningham (1996) show that the exchange rates follow a non-linear process. Another example is the study of Hsu and Krugler (1997) by using EGARCH.
into (1), subtracting $C_t$ from the both sides and comparing with the definition of the risk premium in Fama (1984) implies

$$F_{t+1} - (E[X_{t+1} | I_t] + C_t) = E[C_{t+1} | I_t] - C_t.$$ 

The second term on the left-hand side is the expected exchange rate of the central parity at $t+1$ and we define the right-hand side as the risk premium, $\kappa_t$.

$$\kappa_t = (E[C_{t+1} | I_t] - C_t). \quad (8)$$

Equation (8) means that the time-varying risk premium is the depreciation rate resulting from the expected realignment$^7$.

Realignment is assumed that the central parity at time $t$ is set to the actually observed exchange rate at the time $t-1$, i.e., $C_t = S_{t-1}$. Substituting this into equation (7) yields,

$$C_{t+1} - C_t = X_t.$$ 

The expected value of the left-hand side is, recalling equation (8), is a risk discount.

We assume that a change in deviation form the central parity $C_t$, $X_{t+1} - X_t$, is decomposed into two components; one is an expected change approximated by interest rate differential ($i_t - i^*_t$) and the other an unexpected one (expressed by $\omega_t$). In other words,

$$\omega_t = X_{t+1} - X_t - (i_t - i^*_t).$$

Using this equation with equation (7)' and (8),

$$E[S_{t+1} | I_t] - S_t = i_t - i^*_t + (\omega_t + \kappa_t). \quad (8')$$

$^7$ We interpret that realignment means a change in $C_t$ by the Central Bank.
Thus, the expected change in the exchange rate consists of the fundamental component \((i_t - i_t^*)\) and the composite risk component composing of the realignment risk premium, \(\theta_t\), and the intra-marginal risk, \(\delta_t\). We propose this intervention risk premium as the sum of the two risk components.

\[
rp_t = k_t + \omega_t \tag{8''}
\]

Obstfeld and Rogoff (1996, chapter 8) point out that central bank intervention explains a part of the large fluctuation of risk premium through a portfolio channel and influences the risk premium when investors interpret the intervention as the signal of future changes in macroeconomic policy.\(^8\)

Garber and Svensson (1995, chapter 36) suggest that FPP is observed when UIP depends on the fluctuation of the time-varying risk premium. However, while their risk premium refers to the realignment risk \(\theta_t\), only, our generalized UIP suggests that the intra-marginal risk \(\delta_t\), should not be overlooked for FPP.

The estimated equation for (8’) is linearly specified as follows.

\[
S_{t+1} - S_t = \alpha + \beta(i_t - i_t^*) + u_{t+1} \tag{9}
\]

In equation (9) both the realignment risk \(\theta_t\) and the intra-marginal risk \(\delta_t\) are assumed convoluted in the disturbance term \(u_{t,t}\). We impose the restriction \(\theta_t = 1\) on equation (9) according to the suggestion of Garber and Svensson (1995, chapter 36) and divide the region of exchange rate into three regimes\(^9\) in section 3. Comparison and examination of the time-series properties of the risk premium shed some light on possible causes of FPP.

2.3 Analysis of FPP by OLS

We extend the empirical analysis of FPP with the data of the same 15 high-income countries where B-D (2000) confirmed FPP: Switzerland, Singapore, Japan, Belgium, Austria, Denmark, Canada, France, Germany, the Netherlands, Italy, the U.K., Australia, Sweden, and Spain\(^10\). We use monthly

\(^8\) It is still controversial how intervention theoretically influences a risk premium. Baillie and Osterberg (1997) empirically show that intervention affects a risk premium in forward exchange markets.

\(^9\) In the time-series literature, ‘regime’ means that time-series behavior lies in a state and ‘regime switch’ is defined as a transition from one state to the another. We follow these definitions.

\(^10\) B-D (2000) include Hong Kong as a high-income country in addition to the above countries. But we exclude Hong Kong because the interest rates for Hong Kong are not available in IFS-IMF.
data of the spot exchange rates and the interest rates from the IFS CD-ROM (the 2001 edition) compiled by the International Monetary Fund (IMF).

In order to make our results comparable with B-D (2000), the sample period for the 15 countries is from January 1976 to December 1998. The maximum number of observations is 275. The interest rate used for 14 countries excluding France is the end-of-period values of the money market rate. Due to missing observations in the interest rate data for France, we use the end-of-period values of the Treasury bill rate from the IFS-IMF. The IFS exchange rate data consists of end-of-period values of the market rate, defined by the home currency price per unit of the US dollar.

We use the above data and estimate equation (9) by ordinary least squares with the US interest rate as \( i^* \). 11 The estimated results are reported in Table 1. The average value of \( \bar{A} \) for 15 countries is -0.623 and we confirm that the FPP is observed for 13 out of the total 15 countries, which is quite similar to the conclusions reached in B-D (2000). The exceptions are Italy and Sweden, for which the estimated \( \bar{A} \) is positive (and significant for Sweden).

In the next section, using the data for which FPP is suspected, we examine the properties of the risk premium resulting from central bank intervention and realignment.

3. MRSTAR Model and Risk Premium

3.1 MRSTAR Model and Target Zone System

So far, we have demonstrated that intervention under a managed float system gives rise to the two conceptually different risks, \( \bar{A}_1 \) and \( \bar{A}_2 \), in both an announced and implicit target zones. That is,

(1) Central bank Intervention influences the risk premium in the exchange market, and

(2) There is an announced target zone and an implicit zone with a possibility of intervention in exchange rates.

11 In the IFS data the money market rate is regarded as the short-term interbank rate. It is unclear whether it is the rate of interest that has the same maturity for all sample countries and periods. Despite these possible shortcomings, we use the IFS data because of the high reliability and accuracy for using the same source for data.
In addition, several preceding studies suggest that exchange rates have an asymmetric property depending on signs of risk premium. In Huisman, Koedijk, Kool and Nissen (1998), it was observed that the relationship between the forward premium and the future change in the spot exchange rate, as suggested by UIP, tends to be particularly strong in periods with large forward premiums. Wu and Zhang (1996), using the Deutschmark and the Japanese yen exchange rates against the US dollar for the period of March 1973 to May 1993, show that UIP holds in periods when the forward US dollar is quoted at a premium, but fails when it is quoted at a discount. The asymmetrical results observed by Wu and Zhang are reconfirmed by the empirical results of Bansal (1997) and Bansal and Dalhquist (2000). Kitamura, Sato and Akiba (2002a, 2003a, b) point out that transactions costs in financial markets explain a part of the asymmetry for UIP. In view of these empirical finding of asymmetry observed in the foreign exchange rate, it is essential to consider:

(3) Buying currencies and selling them have different impacts on the market, and adjustment speed is different between depreciation and appreciation.

We focus on the above characteristics and model them.

For an empirical study of the target zone, Flood and Rose (1996) used the pooled daily data of the EMS, for the period of 1981 to October 1994, and found a more significant departure from UIP in floating rate date than in fixed rate data. Besides, Osterberg (1997), using the daily exchange rate of the Deutschmark and the Japanese yen against the US dollar for the period of August 6, 1985 through September 6, 1991, demonstrated that the coefficient of $\delta$ in equation (3) not only significantly different from unity, but is also significantly negative for intervention periods. Although these studies are valuable in that they classify exchange rate regimes in a target zone, they need to clarify the correspondence between their choice of intervention periods and actual shifts from floating- to fixed-rate regimes. However, correspondence in short-run data is difficult, because the central bank actually intervenes in the interior of a zone, or the credibility for a zone is imperfect. Furthermore, implicit target zones seem to be prevalent in many countries and they are impossible to observe by the definition of the implicit target zone.
We use the Smooth Transition Auto Regressive (STAR) family, which is a non-linear and regime-switching model, to account for such a difficulty in classifying the exchange rate regimes in a target zone. We define three regimes by fitting this model to the residuals of equation (9) and call the middle regime the target zone.

Michael, Nobay and Peel (1997, hereafter MNP) use an exponential STAR (ESTAR) model, and analyze that the deviation from Purchasing Power Parity (PPP) is affected by transactions costs. However, since an exponential function exhibits symmetry by definition, we encounter a difficulty in applying an ESTAR model for analyzing residuals from an asymmetric nature as remarked above. Hence, we use the Multiple-regime STAR (MRSTAR) model which extends the STAR model to take the asymmetry of UIP residuals into account.

The Disturbance \( u \) in equation (9) is assumed to be described by the following MRSTAR model:

\[
\Delta u_t = \lambda_0 u_{t-1} + \sum_{j=1}^{p-1} \phi_{0j} \Delta u_{t-j} + (\lambda_2 u_{t-1} + \sum_{j=1}^{p-1} \phi_{2j} \Delta u_{t-j}) F_1(u_{t-d}; \gamma_1, c_1) + (\lambda_2 u_{t-1} + \sum_{j=1}^{p-1} \phi_{2j} \Delta u_{t-j}) F_2(u_{t-d}; \gamma_2, c_2)
\]

(10)

The first transition function \( F_1(u_{t-d}; \gamma_1, c_1) \) is assumed as the following ESTAR model.

\[
F_1(u_{t-d}; \gamma_1, c_1) = 1 - \exp\{-\gamma_1 (u_{t-d} - c_1)^2\}, \quad 0 \leq F_1 \leq 1
\]

(11)

The second transition function \( F_2(u_{t-d}; \gamma_2, c_2) \) is assumed as the following Logistic STAR (LSTAR) model.

\[
F_2(u_{t-d}; \gamma_2, c_2) = \left[1 + \exp\{-\gamma_2 (u_{t-d} - c_2)\}\right]^{-1}, \quad 0 \leq F_2 \leq 1
\]

(12)

In equations (11) and (12), \( d \) is a delay parameter. \( \gamma_1 \) measures the speed of transition, and \( c_i \) indicates the half-way point \((i=1,2)\).

Equation (10) is used as a non-linear cointegration test, where the speed of adjustment varies with the extent of the deviation from UIP. This is in contrast with linear cointegration with constant speed. It is postulated that the further the deviations depart for UIP the faster the speed of mean-reversion because of intervention. Substituting equations (11) and (12) into (10) and then estimating it allows us to classify the region of residuals into three sub-regions. Since states in each sub-region are statistically different, we denote the middle sub-region as the middle regime (corresponding to \( u_{t-d} = c_1 = c_2 \)) and two outer sub-region as upper regime (corresponding to \( u_{t-d} = + \square \)) and lower regime (corresponding to \( u_{t-d} = - \square \)). We define the middle regime as a “Target Zone.” It should be
emphasized that the adjustment process in two outer regimes is different and asymmetrical\textsuperscript{12}. The rest of this section briefly presents a theoretical base for asymmetry and difference\textsuperscript{13} in adjustment speed.

Substituting $u_{t,d} = c_1 = c_2$ into equations (11) and (12), we obtain from equation (10):

$$\Delta u_t = \left( \lambda_0 + \frac{\lambda_2}{2} \right) u_{t-1}.$$  \hspace{1cm} (13)

The coefficient $(\lambda_0 + \lambda_2/2)$ in equation (13) shows the stability and the speed of adjustment of $u_{t,d}$ in the target zone.

The deviation lying on or above (below) the upper (lower) edge of the target zone implies that the exchange rate exhibits excessive depreciation (appreciation). For excessive depreciation, substituting $u_{t,d} = + \square$ yields

$$\Delta u_t = (\lambda_0 + \lambda_1 + \lambda_2) u_{t-1}.$$  \hspace{1cm} (14)

Similarly, for excessive appreciation, substituting $u_{t,d} = - \square$ yields

$$\Delta u_t = (\lambda_0 + \lambda_1) u_{t-1}.$$  \hspace{1cm} (15)

Hence, it is clear that $\lambda_0$, $\lambda_1$ and $\lambda_2$ are important parameters, in the sense that we can show that the disturbances in two regimes outside the target zone have different speeds of adjustment.

3.2 Estimation of MRSTAR model.

For estimation of the MRSTAR model, we first need to specify a linear autoregressive model AR ($p$), where $p$ denotes the order of lag. The order is selected as $p=1$ for all countries\textsuperscript{14}. Next, for each

\textsuperscript{12} See, appendix for details.
\textsuperscript{13} In equations (13), (14) and (15) the summation terms including $\square ij$ ($i=0,1,2$, $j=1,2,\ldots,p-1$) are neglected.
linear AR \((p)\) model for all fifteen countries, we test linearity \((F\) test\) for the delay parameter \(d=1,2\) by using the following auxiliary equation\(^{15}\):

\[
u_t = \beta_0 + \sum_{j=1}^{p} (\beta_1 u_{t-j} + \beta_2 u_{t-j} u_{t-d} + \beta_3 u_{t-j} u_{t-d}^2 + \beta_4 u_{t-j} u_{t-d}^3) + \varepsilon_t'.
\] (16)

\(\varepsilon_t' \sim n.i.d(0, \sigma^2)\)

The hypothesis to be tested is

\(H_0: \beta_2 = \beta_3 = \beta_4 = 0.\)

As reported in Table 2, it is concluded that the linearity in the residuals of equation (9) are strongly rejected for \(d=1,2\) and a nonlinear stochastic process is accepted in all countries. We select the larger \(F\) statistics for a delay parameter \(d\) of \(d=1,2\).

Finally, we use Nonlinear Least Squares to estimate equation (10)\(^{16}\). Terasvirta (1994) points out that the estimation of parameters may cause some problems (e.g. slow convergence, overestimation and so on)\(^{17}\). We therefore follow his recommendation in scaling the argument of the transition function \(F(u_{t-d})\) by dividing it by the standard deviation of \(u_t\), in the case of the LSTAR model and by variance for the ESTAR model, respectively. We set \(\hat{\phi}_i\), at 1 \((i=1,2)\) and use the estimates in equation (17)(see, footnote16) for other parameters as the initial value. The results are reported in Table 3.

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\(^{14}\) If the order is determined by the AIC for a finite sample, it is known that it has a positive probability of overfitting, but it is not consistent for a large sample. In contrast, it is known that the SBIC is consistent. Since the sample size of our data is relatively large, we adopt the SBIC. We also check the validity of the selected order of lag with the Ljung-Box \((Q)\) statistics.

\(^{15}\) See, Terasvirta and Anderson (1992).

\(^{16}\) After the linearity test, we carry out the test for no remaining Nonlinearity of Eitrheim and Terasvirta (1996). We estimate equation (10) with \(F_1=\text{ESTAR}\) and \(F_2=0\). After this estimation, we assume that \(F_2\) is LSTAR and estimate the following auxiliary equation which includes its third-order Taylor series approximation term.

\[
u_t = g_{00} + \sum_{j=1}^{q_0} g_{0j} u_{t-j} + \sum_{j=1}^{q_1} \hat{\phi}_j u_{t-j} F_1(u_{t-d}; \hat{\gamma}_1, \hat{\xi}_1) + \sum_{j=1}^{q_2} (g_1 u_{t-j} u_{t-d} + g_2 u_{t-j} u_{t-d}^2 + g_3 u_{t-j} u_{t-d}^3) + \varepsilon_t'.
\] (17)

\(\varepsilon_t' \sim n.i.d(0, \sigma^2)\)

If the null hypothesis \(H_0: g_{1j} = g_{2j} = g_{3j} = 0\) is rejected, there exists an additive LSTAR component in equation (10). The results (Table 2) suggest that there exists the additive LSTAR component for the 13 countries other than Denmark and Netherlands. We use MRSTAR for all countries, although the residual of UIP in Denmark and Netherlands is described by ESTAR. Since both MRSTAR and ESTAR have three regimes, we regard ESTAR as being contained in MRSTAR.
Terasvirta (1994) suggests that the estimate of the parameter $c$ be within the observed range of $u$. Our estimated $c$ satisfies this criterion for all countries. The standard deviations for the estimates other than Japan are relatively small and almost all of the fitted parameters, other than $\alpha_2$, are rejected at 5% level of significance by the $t$ test.

We examine the validity of restriction $c_1 = c_2$ by Wald test. If the restriction is not rejected, we conclude that there exist three regimes for exchange rates. The results reported in Table 4 indicate that the restriction is rejected at 5% level of significance for France and at 1% level for Spain. Therefore, this implies that our model is appropriate for modeling the target zone in 13 countries: Switzerland, Singapore, Japan, Belgium, Austria, Denmark, Canada, Germany, Netherlands, Italy, the UK, Australia and Sweden. In the next subsection 3.3, we analyze the behavior of risk premium defined by equation (8''), using the data of these counties.

### 3.3. Properties of Risk Premium

Up to now we have treated the risk premium $\rho$, as an exogenous variable. Krugman (1992) assumes that the fundamentals within target zones follow a random walk process. Meese and Rogoff (1983) empirically show that movement of free-float exchange rates is closely approximated by a random walk process for a time horizon of up to 12 months. Thus, it seems plausible that the risk premium is also closely approximated by a random walk process, if investors expect that the central banks do not intervene within the target zone. We test the following null-hypothesis for a random walk for the residuals of exchange rates concerning the inside of target zone (regime (a) in Figure 1) derived from equation (13):

$$H_0^a: \lambda_0 + \frac{\hat{\lambda}_2}{2} = 0.$$ 

According for Table 4, $H_0^a$ is rejected at 5% level of significance for Switzerland and the UK and at 10% level for Australia. This suggests that the risk premium is consistent with a random walk process for the rest of 10 countries.

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17 When we estimate STAR, it is well known that it is difficult to obtain a precise estimate of $\alpha$. However, an apparently insignificant estimate of $\alpha$ should not be interpreted as insignificance of the regime switching (see, Terasvirta and Anderson (1992)).

18 See appendix for the necessity of the restriction $c_1 = c_2$.  

17

18
Following our previous argument of mean reversion through intervention, for risk premium outside of the target zone, we presuppose that the disturbance is stable and converging toward the target zone. For \( u \) to be stable, \( \bar{\sigma}_0 + \bar{\sigma}_1 + \bar{\sigma}_2 \) and \( \bar{\sigma}_0 + \bar{\sigma}_2 \) must lie in between \(-2\) and \(0\). In fact, looking at Table 3 confirms that the average of \( \bar{\sigma}_0 + \bar{\sigma}_1 + \bar{\sigma}_2 = -1 \) for Regime (b) (in Figure 1) and that the average of \( \bar{\sigma}_0 + \bar{\sigma}_2 = -1.64 \) for regime (c) (in Figure 1). These results mean that deviations outside of zones display mean reversion and in particular, \( u_t \) in regime (b) converges into the target zone relatively rapidly, while \( u_t \) in regime (c) first overshoots because of the very fast speed of adjustment and then gradually converges to the target zone. We conduct a complementary test of whether the process of \( u_t \) is white noise. The hypotheses to be tested are formally stated as,

\[
H_0^b: \lambda_0 + \lambda_1 + \lambda_2 = -1, \quad \text{and} \quad H_0^c: \lambda_0 + \lambda_1 = -1.
\]

\( H_0^b \) implies \( u_t \) lies in regime (b) and \( H_0^c \) in regime (c). Table 4 shows \( H_0^b \) is rejected in Australia, and \( H_0^c \) is rejected in Austria, Denmark and Netherlands. Consequently, this implies that the risk premium inside of the target zone follows a random walk process for the 10 countries other than Switzerland, Australia and the UK, but it is stable outside of the zone for all countries. Recognizing that UIP residuals reflect a cointegration relationship between expected rate of depreciation and interest rate differentials, this in turn suggests that UIP holds in the outer zones (regimes (b) and (c)), but it does not hold in the interior of the zone (regime (a)). This conclusion is consistent with the findings of Flood and Rose (1996) and Osterberg (1997).

Furthermore, we also find the asymmetry in UIP, in the sense that the adjustment speed in regime (c), which is interpreted as the speed in the case of appreciation (dollar depreciation), is faster than that in regime (b) for the 10 countries and thus the disturbances seem to overshoot because the speed in absolute value is greater than 1. The overshooting of UIP residuals in regime (c) means that investors expect appreciation of dollar, which drives the overshooting of the exchange rate UIP predicts. These empirical results are consistent with those found by Wu and Zhang (1996) and B-D (2000), in the sense that when the dollar is expected to appreciate, FPP is observed. In view of these empirical developments, it should be emphasized that our contribution of the present study rests on the finding that when the central bank is expected to sell (buy) dollar, investors sell (buy) the rational
ly expected amount of dollar. According to our empirical examination, it is interpreted that the buying pressure seems to be the stronger than the selling pressure in the foreign exchange market, which results in the difference in the adjustment speed.

4. Conclusions

This paper examined the forward premium puzzle, which is one of the most important unresolved puzzles in the modern theory of international finance, by focusing on exchange rate intervention. Following the definition by Fama (1984), we assume the time-varying risk premium as the general risk premium containing the realignment risk and the intra-marginal risk. We employ an MRSTAR model to define the target zone (including the implicit one), because we consider a non-linear and asymmetric process for deviation from UIP.

The first contribution of this paper is to provide evidence that the deviations from UIP follow a nonlinear process for all countries where the FPP has been observed in a recent study by B-D (2000). Second, we hypothesize that the risk premium resulting from the expected intervention and/or realignment has an adjustment speed dependent on the extent of the deviation. Then, we fitted the risk premium to the MRSTAR model to test the hypothesis and found that the risk premium within the target zone follows a random-walk process for 13 out of the 15 countries, but the risk premium outside of the target zone follows a mean-reverting process for all countries. The third contribution is to confirm that the adjustment speed in the case of appreciation is faster than that in the case of depreciation. The risk premium in the case of appreciation gradually converges into UIP with overshooting, while the risk premium in the case of depreciation converges into UIP after one term.

In this paper, we do not directly estimate \( \delta \) in equation (3), but examine the extent to which the FPP is accounted for by the risk premium resulting from the intervention expectation. We found that the behavior of risk premium is different among three regimes. As discussed in the last section, the risk premium inside of regime (a) (the target zone) follows a random walk process, meaning that the FPP is observed because of no-cointegration. In contrast, the risk premium in regime (b) and (c) follows a stable process, indicating that the FPP is not observed. Besides, our finding that a

\[ H_0^{\epsilon} \]

is also rejected for Spain. However, Spain is disregarded because is does not pass the Wald test for \( c_1 = c_2 \).
non-linear process in risk premium implies that the restriction $\mathfrak{a} = 1$ on equation (3) and estimates a linear regression model is unjustifiable.

Obstfeld and Rogoff (1996, chapter 8) suggest that target zones allow interest rate differential to be more independent from exchange rates than in a rigid peg. According to our present investigation, it is suggested that the FPP is accounted for by the risk premium, because investors also consider the risk premium, as well as interest rate differentials. As we mentioned in the previous section, investors’ behavior would be based on the implicit target zones they set up themselves, even if central banks make no announcement in terms of the zones. Our findings that, in some cases, the intervention expectation may have an adverse effect on UIP through a change in the risk premium.

Appendix The Extension of MRSTAR

A typical STAR model is classified into the following two classes;

\[
F(u_{t-d}; \gamma, c) = 1 - \exp\{ -\gamma(s_t - c)^2 \} \quad (A1)
\]

\[
F(u_{t-d}; \gamma, c) = [1 + \exp\{ -\gamma(s_t - c) \}]^{-1}. \quad (A2)
\]

(A1) is called Exponential STAR (ESTAR) and (A2) Logisitc STAR (LSATR). An ESTAR model has an inner regime corresponding to $u_{t-d} = c$, and has two outer regimes corresponding to $u_{t-d} = \square \, \square$, which are restricted to symmetric adjustment dynamics. On the other hand, an LSTAR model has only two asymmetric regimes.

Duk and Franses (1999) propose an alternative model to allow for multiple asymmetric dynamics and call it a Multiple-Regime STAR (MRSTAR) model;

\[
y_t = \kappa_0 + \sum_{j=1}^{p} \phi_{0,j} y_{t-j} + \left( \kappa_1 + \sum_{j=1}^{p} \phi_{1,j} y_{t-j} \right) F_1(s_t; \gamma_1, c_1) + \left( \kappa_2 + \sum_{j=1}^{p} \phi_{2,j} y_{t-j} \right) F_2(s_t; \gamma_2, c_2) + \varepsilon_t. \quad (A3)
\]

Where $F_1$ and $F_2$ are the equation (A2). $s_t$ is a transition variable. For example, if $c_1 < c_2$, then $F_1$ changes from zero to one prior to $F_2$ in process that $s_t \square \, \square \, \square$. Equation (A3) can describe three asymmetric regimes (see Graph 1), to be explained in detail as follows.

When $s_t \square - \square$, $F_1$ and $F_2$ are equal to zero, and equation (A3) becomes the following AR (p):
\[
y_t = \kappa_0 + \sum_{j=1}^{p} \phi_{0j} y_{t-j} + \epsilon_t.
\]

This is illustrated as Regime (d) in Graph 1. In the same way, when \( \phi_1 \) and \( \phi_2 \) are quite large\(^{20}\), \( F_1 \) and \( F_2 \) approach to one and zero, respectively in process that \( s \) increases. At the limit, we obtain
\[
y_t = \kappa_0 + \kappa_1 + \left( \sum_{j=1}^{p} \phi_{0j} + \sum_{j=1}^{p} \phi_{1j} \right) y_{t-j} + \epsilon_t.
\]

This is illustrated as Regime (e) in Graph 1. If \( s \) approach to \( + \infty \) in the limit, \( F_2 \) is also close to one, and we have,
\[
y_t = \kappa_0 + \kappa_1 + \kappa_2 + \left( \sum_{j=1}^{p} \phi_{0j} + \sum_{j=1}^{p} \phi_{1j} + \sum_{j=1}^{p} \phi_{2j} \right) y_{t-j} + \epsilon_t.
\]

This is illustrated as Regime (f) in Graph 1. We should note that AR (p) processes depend on which of \( c_1 \) and \( c_2 \) is larger. They are obtained by estimation, but even if they are significant, there is no guarantee to which of them is greater. Hence, we extend MRSTAR and impose restrictions, \( F_1 = \text{ESTAR}, F_2 = \text{LSTAR}, \) and \( c=c_1=c_2. \)

When \( s \) \( - \infty \), \( F_1 \) and \( F_2 \) is equal to one and zero, respectively. Therefore, equation (A3) reduces to
\[
y_t = \kappa_0 + \kappa_1 + \left( \sum_{j=1}^{p} \phi_{0j} \right) y_{t-j} + \epsilon_t.
\]

This is illustrated as Regime (a) in Graph 2. In the same way, when \( s = c, F_1 = 0 \) and \( F_2 = 1/2. \) This is illustrated as Regime (b) in Graph 2\(^{21}\).
\[
y_t = \frac{1}{2} \kappa_0 + \kappa_1 + \left( \sum_{j=1}^{p} \phi_{0j} + \frac{1}{2} \sum_{j=1}^{p} \phi_{1j} \right) y_{t-j} + \epsilon_t.
\]

When \( s = + \infty, F_1=F_2=1. \) Regime (c) in Graph 2 is expressed as follows.
\[
y_t = \kappa_0 + \kappa_1 + \kappa_2 + \left( \sum_{j=1}^{p} \phi_{0j} + \sum_{j=1}^{p} \phi_{1j} + \sum_{j=1}^{p} \phi_{2j} \right) y_{t-j} + \epsilon_t
\]

\(^{20}\) Unless \( \phi_1 \) and \( \phi_2 \) are large, it is difficult to define AR model. This prevents us from investigating the asymptotic property of risk premium.

\(^{21}\) Unless we impose the restriction \( c = c_1 = c_2 \) on equation (A3), it is difficult to define AR model as well.
Obviously, the extended MRSTAR imposing both ESTAR and LSTAR, also reflects three asymmetric regimes. In processes where \( s \) increases, for example, if \( \alpha_1 < \alpha_2 \), F2 can change from zero to one faster than F1. This means there are four regimes in exchange rates. However, we exclude this situation and assume that F1 and F2 become one at the same time. Because the estimates of \( \alpha_1 \) and \( \alpha_2 \) are mostly insignificant and very large; when \( \alpha \) is large, the switching of the transition function is almost instantaneous at \( c \) and F1 and F2 must change at the same time (see, Terasvirta and Anderson(1992)).