

Effective and Nominal Rates of Protection

– Role of Market Structure in a Specific-Factor Model– *

Kazuharu Kiyono[†] Fang Wei[‡]

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Abstract

This paper explores the relationship between effective rate of protection and factor substitution in different market structures when the protected industry employs specific labor. We will show that there is a simple criterion, independent of the market structure, to judge when the effective rate of protection exceeds the nominal one. When the market is in international oligopoly, tariff escalation policy yields (i) the market power effect and (ii) the factor substitution effect over effective protection of the industry.

1 Introduction

The effective protection concept is widely used in the evaluation of protected industry regimes. It is initially developed by Corden (1966) who defined it as the percentage increase in value added per unit. Given the assumption of nonsubstitutability between domestic primary factors and imported inputs, Corden showed that the effective rate of protection shed light on the resource allocation towards activities enjoying relatively high effective protective rates.

Corden's assumption of no substitution only exists in some particular industries with no factor movement. When substitution between domestic primary factors and imported inputs is allowed, the effective tariff structure will be changed. Pioneering work has been done by many economists using different technology to evaluate it. Corden (1971) assumed the quantity of factors used only depends on their price ratio to the final good. Under factor substitution, the tariff will pull

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[†]School of Political Science & Economics, Waseda University. E-mail: kazr@waseda.jp

[‡]Graduate School of Economics, Waseda University. E-mail address: fwei@suou.waseda.jp

resources toward activities enjoying relatively high effective protection rate. Corden's result is challenged by Ramaswami and Srinivasan (1971) who provided a counter-example. They showed that if substitution effects are biased, the industry which receives effective protection may, as a consequence, lose resources to the other industry. Jones (1971) developed into a general equilibrium model and explored the relationship between the effective protection rate and relative outputs. Jones reinforced the viewpoint of Ramaswami and Srinivasan that the effective protection rate may diminish the output of that industry even if substitution is unbiased.

Factor substitution is an important extension in the effective protection concept. Most of their work assume simply one intermediate good. Recently, more papers on the effective protection take into account the extension of multiple intermediate goods, tradables as well as non-tradables. Ohyama and Suzuki (1980) examined the response of net and gross outputs of tradable goods to changes in tariff structure in a comprehensive general equilibrium with interindustry flows. Greenaway, Reed, and Hassan (1994) allowed for the existence of by-product by treating them as non-tradable outputs and modified Corden's measure of protection in the traded and non-traded by-product cases. Londero (2001) analyzed the effective protective rates in the joint-production when tradables are jointly produced with non-tradables. Also, Some Related issues treat the effective protection in the political economy. As Grossman and Helpman (1994), Anderson (1998) showed, Effective protection implies welfare loss sacrificed to the interest groups.

The challenge to Corden (1966) and Corden (1971) critically hinges on the assumption that all the domestic factors employed in the protected industry are completely mobile over the economy. In fact, the assumption of mobile factors make the concept of effective protection vague, and thus triggers the works to invent the more suitable criterion of protection such as Ethier (1977) with only limited results.

This paper instead return to the original idea of Corden. That is, we explicitly consider presence of a certain factor specific to the protected industry and assume that factor substitution is limited or more specifically none. This highlights the effect of the so-called tariff escalation policy from the short-run point of view. By focusing on such a short-run effect within a framework of industry-specific factor, which we call "labor", we are rather interested in the market structure problem. Does the notion of effective protection as well as tariff escalation keeps making sense? This is what we explore through the present paper.

The rest of the paper is concluded as follows. In section 2, we build up a basic model of effective protection for a competitive industry with specific labor. We will show a very simple criterion to judge when the effective rate of protection exceeds the nominal one. Something surprising is that this simple criterion makes sense regardless of the market structure. After discussing the role of factor

substitution and industry-specific labor for effective protection for the competitive industry in section 3, we extend the model to international duopoly in section 4. The effects of tariff escalation are decomposed to (i) the market power effect and (ii) the factor substitution effect. Section 5 concludes the paper.

2 Model

Consider a small open economy producing a certain final good using the domestic labor and the imported intermediate good subject to constant returns to scale under perfect competition.¹ The production technology is expressed by the following unit cost function

$$c = c(w_M, w_L), \quad (1)$$

where w_M denotes the domestic price of the intermediate good and w_L the wage rate of the specific labor. By virtue of Shephard's lemma, $c_i(w_M, w_L) \stackrel{\text{def}}{=} \frac{\partial c(w_L, w_M)}{\partial w_i}$ ($i = L, M$) represents the input coefficient of the factor in question, so that the domestic labor market equilibrium is given by

$$\bar{L} = c_L(w_L, w_M)x, \quad (2)$$

where \bar{L} is the labor supply, and x the output of the final good.

Let p denote the domestic price of the final good and v the value-added per unit of final output. Then the per-unit value-added v is expressed by

$$v = p - w_M c_M(w_M, w_L).$$

When the government imposes taxes over the final and intermediate goods, there will be wedges between the domestic and foreign prices. Let t_F (t_M) denote the ad valorem tariff rate on the imported final (or intermediate) good, p^* (w_M^*) the world price of the final (or intermediate) good, and superscript t (or 0) associated with domestic prices the tariff-ridden (or free-trade) equilibrium values. Then the following relations hold in free trade equilibrium as well as tariff-ridden one.

$$p^t = (1 + t_F)p^*, \quad p^0 = p^*, \quad w_M^t = (1 + t_M)w_M^*, \quad w_M^0 = w_M^*. \quad (3)$$

We also often let $\tau_F \stackrel{\text{def}}{=} 1 + t_F$ express the gross tariff rate on the imported final good and $\tau_M \stackrel{\text{def}}{=} 1 + t_M$ the counterpart for the imported intermediate good. Using these notations, we may express the two notions of the rates of protection, i.e.,

¹For simplicity of exposition, we assume that the production requires no other factors. We will often make some remarks on how the results change when we relax this assumption.

the nominal rate of protection for the final good industry denoted by R_n and the associated effective one denoted by R_e as follows.

$$R_e \stackrel{\text{def}}{=} \frac{v^t - v^0}{v^0} = \frac{(p^t - w_M^t c_M(w_L^t, w_M^t)) - (p^0 - w_M^0 c_M(w_L^0, w_M^0))}{p^0 - w_M^0 c_M(w_L^0, w_M^0)}, \quad (4)$$

$$R_n \stackrel{\text{def}}{=} \frac{p^t - p^0}{p^0}. \quad (5)$$

To explore the relation between the two rate of protection, we define the following two share variables. The first is the factor cost share expressed by $\theta_i (i = M, L)$, where θ_M stands for the cost share of the intermediate good and θ_L the cost share of the domestic labor. By applying Shephard's lemma to the unit cost function (1), we find that each factor cost share is given by

$$\theta_i(w_M, w_L) = \frac{w_i c_i(w_M, w_L)}{c(w_M, w_L)} \quad (i = M, L),$$

where $\theta_M + \theta_L = 1$ must hold.

The second is the revenue share of the intermediate good, ϕ , which is given by

$$\phi(p, w_M, w_L) = \frac{w_M c_M(w_M, w_L)}{p} = \frac{c(w_M, w_L)}{p} \theta_M(w_M, w_L). \quad (6)$$

This represents the ratio of the intermediate good costs relative to the total revenue. This should be distinguished from the cost share of the intermediate good defined by θ_M . The two coincide only when the final-good producers are price-takers both in the final- and intermediate-good markets.

Using this revenue share of the intermediate good, the difference between the effective and nominal rates of protection is expressed by

$$\begin{aligned} R_e - R_n &= \frac{p^t - w_M^t c_M^t}{p^0 - w_M^0 c_M^0} - \frac{p^t}{p^0} \\ &= \frac{p^t}{p^0 - w_M^0 c_M^0} (\phi^0 - \phi^t) \\ &= (1 + t_F) \frac{\phi^0 - \phi^t}{1 - \phi^0} \end{aligned} \quad (7)$$

where $c_M^k \stackrel{\text{def}}{=} c_M(w_L^k, w_M^k) (k = 0, t)$, and ϕ^0 (or ϕ^t) represents the revenue share of the intermediate good under the free-trade (or tariff-ridden) equilibrium. The above equation gives us the fundamental proposition concerning the relation between the effective rate of protection and the nominal one summarized as follows.

Proposition 1 *The effective rate of protection exceeds the nominal one if and only if the tariff policy lowers the revenue share of the intermediate good compared with the free-trade equilibrium, i.e., $d\phi < 0$.*

Note the above proposition is independent on the market structure of final good and intermediate good. In the following sections, we explore the factors behind the change of intermediate-good-cost-price ratio $\phi(p, w_M, w_L)$ under different market structures and technology assumptions.

3 Competitive Market

First consider the case when the domestic firm is a price-taker both in the final good and intermediate good sector. The final good price is equal to the unit cost in equilibrium, i.e.,

$$p = c(w_L, w_M). \quad (8)$$

The revenue share of the intermediate good (6) is equal to the factor cost share of the intermediate good, i.e.,

$$\phi_c(\tau_F, \tau_M, w_L) = \frac{w_M c_M}{c(w_M, w_L)} = \theta_M(w_L, w_M).$$

So the change of ϕ_c is equivalent to the change of θ_M . It is easy to establish the following corollary from Proposition 1

Corollary 1 *When the domestic final-good producers are price-takers both in the final good and the intermediate good markets, the effective rate of protection exceeds the nominal one if and only if the tariff policy lowers the intermediate-good-cost share when both the final good and intermediate good industry are in perfect competition, i.e. $d\theta_M < 0$.*

The factor cost share of the intermediate good depends on the relative factor price w_M/w_L and the degree of factor substitution, expressed by the elasticity of factor substitution σ_{LM} . To make the succeeding results clear, we assume

Assumption 1 *The elasticity of factor substitution between the domestic labor and the intermediate good σ_{LM} satisfies either of the following:*

(i) $\sigma_{LM} > 1$, (ii) $\sigma_{LM} < 1$, (iii) $\sigma_{LM} = 1$
regardless of the factor prices.

For example, when $\sigma_{LM} > 1$ the factor cost share of the intermediate good increases if and only if the relative factor price of the domestic labor w_L/w_M gets

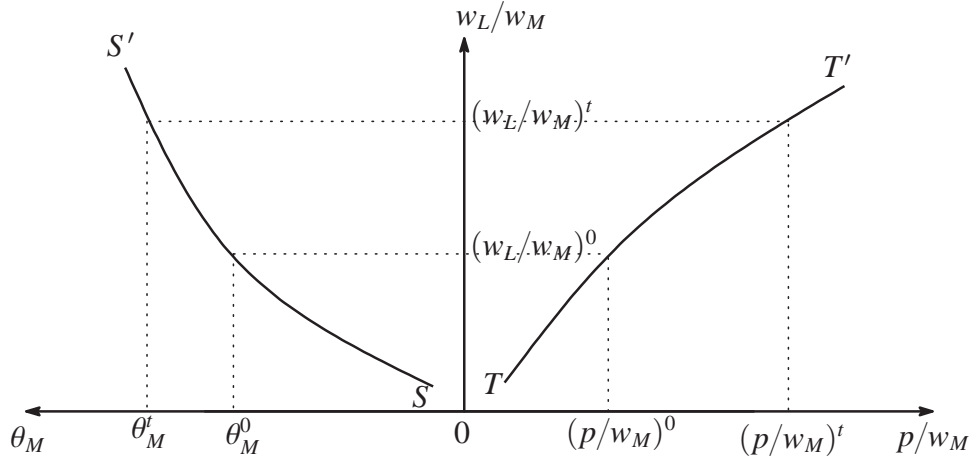


Figure 1: Tariffs and Labor-Cost Share when $\sigma_{LM} > 1$

higher. The relation between the factor cost share of the intermediate good and the relative factor price is shown by the upward sloping curve SS' in the left panel of Figure 1.

Now the question is when the effective rate of protection exceeds the nominal one. There are two clues to this question. First, how does the tariff policy affects the domestic price of the final good relative to the intermediate good? Second, how does the change in the domestic price of the final good relative to the intermediate good affect the relative factor price between the domestic labor and the intermediate good? We consider these two questions for $\sigma_{LM} > 1$ as our benchmark.

The answer to the first question is easy from (3). That is, p/w_M becomes higher than at free trade if and only if the tariff rate of the final good t_F exceeds the counterpart for the intermediate good t_M .

The answer to the second question requires us to rewrite the unit cost equation (1) as below

$$\frac{P}{w_M} = c \left(\frac{w_L}{w_M}, 1 \right), \quad (9)$$

where use was made of the linear homogeneity of the unit cost function. Thus the relative factor price w_L/w_M increases if and only if the relative price p/w_M becomes higher. This relation is expressed by the upward-sloping curve TT' in the right panel of Figure 1.

Therefore by virtue of Corollary 1, in our benchmark case, the higher tariff rate of the final good compared with that of the intermediate good leads to the

effective rate of protection higher than the nominal one. The results including the cases of σ_{LM} taking other values are summarized as below.²

Proposition 2 *Suppose that the domestic final-good producers are price-takers both in the final good and the intermediate good markets. Then given a constant elasticity of substitution between the domestic labor and the imported intermediate good σ_{LM} , the higher tariff on the final good than on the intermediate good leads to $R_e > R_n$ if and only if $\sigma_{LM} < 1$.*

Since the economically meaningful notion of protection requires an increase in the output after tariffs, let us explore the change in the final-good outputs resulting from tariffs. In view of (2), the change in the output is expressed by

$$\begin{aligned} d \ln x &= -\frac{dc_L(w_L, w_M)}{c_L} = -\left(\frac{w_L c_{LL}}{c_L} d \ln w_L + \frac{w_M c_{LM}}{c_L} d \ln w_M \right), \\ &= -\frac{w_L c_{LL}}{c_L} d \ln(w_L/w_M), \end{aligned}$$

where use was made of the zero-homogeneity of each factor input coefficient. i.e., $w_L c_{LL} + w_M c_{LM} = 0$.

By the virtue of cost function (9) and $c_{LL} < 0$, the above equation shows the higher tariff on the final good than on the intermediate good leads to the greater output of final good, i.e., $dx > 0$. Note that this change in the output does not depend on the value of the elasticity of factor substitution. Thus the change of the output also gives rise to higher rate of the effective protection summarized in the following corollary.

Corollary 2 *Given the tariff rate on the final good higher than that on the imported good, i.e., $t_F > t_M$, the final good industry always expands the output compared with free trade, regardless of the value of the elasticity of factor substitution.*

In view of the above result coupled with Proposition 2, Corden's notion of effective protection makes sense only when the elasticity of factor substitution is less than unity. It also hinges on the implicit assumption that the domestic labor is specific to the final good industry, for when the industry in question is very small from the view point of the national economy as a whole the domestic wage cannot change but stay at a constant.

The two assumptions of small elasticity of factor substitution and industry-specific factor imply that Corden's notion of effective protection may lose its sense

²The result can be extended easily into the case of two or more imported intermediate goods. See Section A in Appendix.

in the long-run where factors are sufficiently substitutable and mobile among the sectors. However insofar as we confine ourselves to the short-run analysis, it makes its sense at least in a competitive framework. A next step is to examine whether his effective protection keeps its economic sense in other alternative market structures. We will tackle this question within an international duopoly where the domestic final good producer competes with the foreign one.

4 International Duopoly

Let us extend the model in the previous section to international duopoly in the final good sector. There are two firms, a domestic firm and a foreign one, producing a homogeneous product and competing à la Cournot in the domestic market. Let x denote the output produced by the domestic firm, c its unit cost of production. The foreign counterparts are denoted with $*$. Let p denote the final good price in the domestic market, X the total consumption, and $p = P(X)$ the inverse demand function. We also assume the intermediate good market is in perfect competition with its world price given by a constant w_M^* . We also let τ_F and τ_M denote the gross ad valorem tariff on the final good and on the intermediate good, i.e., $\tau_F \stackrel{\text{def}}{=} 1 + t_F$ and $\tau_M \stackrel{\text{def}}{=} 1 + t_M$.

4.1 Model Structure

Given the tariff policy of the home country's government (τ_F, τ_M) as well as the world intermediate good price w_M^* and the initial constant marginal costs of the foreign firm c^* , each firm's profit function is expressed by

$$\begin{aligned} \text{Home firm's profit} \quad \tilde{\pi}(x, x^*, c) &= \{P(x + x^*) - c\}x & (10) \\ \text{Foreign firm's profit} \quad \tilde{\pi}^*(x, x^*, \bar{c}^*) &= \left\{ \frac{P(x + x^*)}{\tau_F} - c^* \right\} x^* \end{aligned}$$

The foreign firm's profit function can be rewritten as

$$\tilde{\pi}^*(x, x^*, \bar{c}^*) = \frac{1}{\tau_F} \{P(x + x^*) - \bar{c}^*\} x^*, \quad (11)$$

where $\bar{c}^* = \tau_F c^*$ represents the tariff-inclusive effective unit cost of the foreign firm.

Our model of international duopoly is captured by Figure 2. First, the government of the home country decides on the tariff rates on the final good τ_F and on the imported intermediate good τ_M . Given the domestic wage rate w_L , this tariff

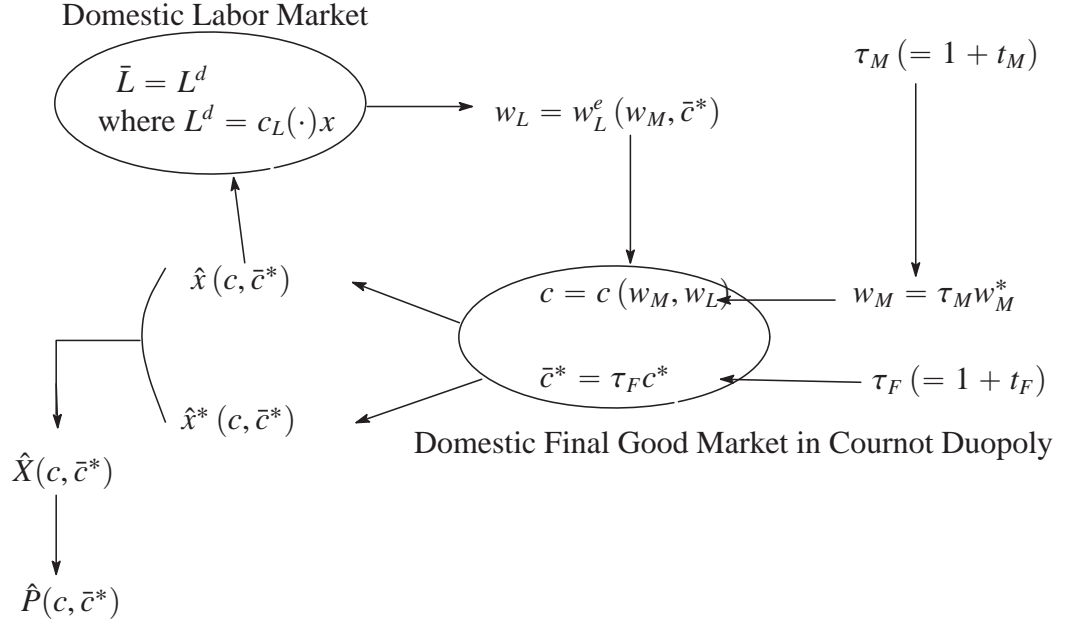


Figure 2: Effective and Nominal Rates of Protection in International Duopoly

package (τ_F, τ_M) determines the domestic price of the intermediate good

$$w_M = \tau_M w_M^*, \quad (12)$$

and the effective unit cost of each firm,

$$c = c(w_L, w_M) \quad (13)$$

for the home firm and $\bar{c}^* = \tau_F c^*$ for the foreign firm. Both firms simultaneously decide on the output to sell in the domestic market, which yields the equilibrium output of each firm as a function of the unit cost profile, $\hat{x}(c, \bar{c}^*)$ for the home firm and $\hat{x}^*(c, \bar{c}^*)$ for the foreign firm. The total output as well as the market price also depends on the effective unit cost profile, and we let $\hat{X}(c, \bar{c}^*)$ denote the equilibrium total output and $\hat{P}(c, \bar{c}^*)$ the associated market price. The domestic wage initially treated as a parameter is in fact determined by the market clearing condition for the domestic labor market where the labor supply is fixed at \bar{L} and its demand is given by

$$L^d = c_L(w_L, w_M) \hat{x}(c, \bar{c}^*), \quad (14)$$

as a result of the interaction with the domestic final good market. Coupled with (12), (13), $\bar{c}^* = \tau_F c^*$, and (14), the domestic labor market equilibrium condition determines the equilibrium domestic wage rate as a function of the tariff package (τ_F, τ_M) , the relation of which we express by $w_L^e(\tau_F, \tau_M)$. Its substitution into

(13) and the equilibrium market price function, the unit cost of the home firm becomes a function of the tariff package and so does the equilibrium market price. We express these with $c^e(\tau_F, \tau_M)$ and $P^e(\tau_F, \tau_M)$. Thus the revenue share of the intermediate good becomes a function of the tariff package. We represent this relation by $\phi^e(\tau_F, \tau_M)$. What is of our interest is the change in this revenue share of the intermediate good $\phi^e(\tau_F, \tau_M)$ as a result of the tariff package introduction. Our strategy consists of the following steps.

- (Step 1) Effects of the changes in the unit costs on the equilibrium outputs and market price.
- (Step 2) Effect of the changes in the unit costs on the domestic labor demand.
- (Step 3) Effect of the change in the tariff package on the equilibrium domestic wage.
- (Step 4) Full effects of the change in the tariff package on the revenue share of the intermediate good.

As a preliminary analysis, we confirm the equilibrium conditions for the present model. Without loss of generality, we assume

Assumption 2 *The output of each firm is strictly positive at all relevant equilibria.*

Assumption 3 *Each firm's profit function is twice-continuously differentiable and $\pi_{xx} < 0$, $\pi_{x^*x^*}^* < 0$ at all relevant equilibria.*

In view of Assumptions 2 and 3, the following first order conditions for maximizing (10)(11) with respect to the own output are enough to delineate the duopoly equilibrium.

$$\begin{aligned} 0 = \pi_x &= p - c + xP'(X), \\ 0 = \pi_{x^*} &= p - \bar{c}^* + x^*P'(X), \end{aligned} \tag{15}$$

or alternatively

$$\begin{aligned} p \left(1 - \frac{s}{\varepsilon}\right) &= c, \\ p \left(1 - \frac{s^*}{\varepsilon}\right) &= \bar{c}^* \end{aligned}$$

where s (or s^*) denotes the market share of the home firm (or the foreign firm) and ε denotes the price-elasticity of the demand, i.e., $\varepsilon = -\frac{X'(P)P}{X}$. Assumption 2 implies that $\varepsilon > \max\{s, s^*\}$ always holds at all relevant equilibria.

The second-order condition for profit maximization further characterizes the structure of our model. Let E denote the elasticity of the slope of the demand curve by $E = -\frac{XP''(X)}{P'(X)}$. Since the second partial derivatives of each firm's profit function are given by

$$\begin{aligned} 0 > \pi_{xx} &= 2P'(X) + xP'' = P'(X)(2 - sE), \\ 0 > \pi_{x^*x^*} &= 2P'(X) + x^*P''(X) = P'(X)(2 - s^*E), \end{aligned}$$

Assumption 3 implies $E < \min\{\frac{2}{s}, \frac{2}{s^*}\}$.

Application of the implicit function theorem to the first-order conditions for profit maximization enables us to define each firm's reaction function as follows.

$$x = r(x^*, c), \quad x^* = r^*(x, \bar{c}^*).$$

The above reaction functions have the following properties.

$$\begin{aligned} r_{x^*} &\stackrel{\text{def}}{=} \frac{\partial r}{\partial x^*} = -\frac{1 - sE}{2 - sE} & r_x^* &\stackrel{\text{def}}{=} \frac{\partial r^*}{\partial x} = -\frac{1 - s^*E}{2 - s^*E} \\ r_c &\stackrel{\text{def}}{=} \frac{\partial r}{\partial c} = \frac{1}{p'(2 - sE)} < 0 & r_{\bar{c}^*}^* &\stackrel{\text{def}}{=} \frac{\partial r^*}{\partial \bar{c}^*} = \frac{1}{p'(2 - s^*E)} < 0 \end{aligned}$$

Note when each firm's output is mutually a strategic substitute, the slope of each firm's reaction function is negative, i.e. $r_{x^*} < 0$ and $r_x^* < 0$ which hold if and only if $E < \min\{\frac{1}{s}, \frac{1}{s^*}\}$.

The above set of equations governs the equilibrium output of each firm, denoted by $\hat{x}(c, \bar{c}^*)$ for the home firm and $\hat{x}^*(c, \bar{c}^*)$ for the foreign firm. To make our comparative statics in the succeeding discussion sensible, we assume that the equilibrium is globally stable under the standard Cournot output adjustment process, which requires

$$\Delta \stackrel{\text{def}}{=} 1 - r_{x^*}(x^*, c)r_x^*(x, \bar{c}^*) > 0. \quad (16)$$

Since we may rewrite it as

$$\Delta \stackrel{\text{def}}{=} 1 - r_{x^*}r_x^* = \frac{3 - E}{(2 - sE)(2 - s^*E)}$$

and Assumption 3 holds, the assumption of global stability for the equilibrium given the tariff package is equivalent to

Assumption 4 *For all the relevant equilibria, there holds $E < 3$.*

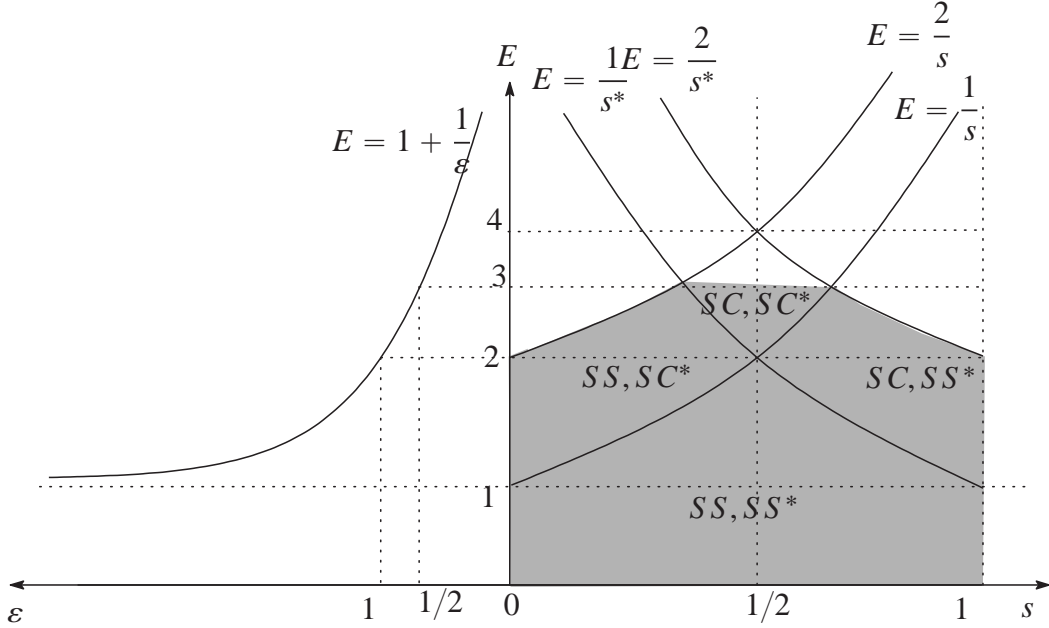


Figure 3: Market Shares of the Firms and the Elasticities of the Demand and its Slope

Thus in view of Assumption 4, all the possibilities for strategic substitution and complementarity can be summarized in Figure 3. The stability condition coupled with the concavity of the profit function in the own output (i. e., Assumption 3) implies that the relevant pairs of the market share of the home firm s and the elasticity of the slope of the demand function E should be in the shaded region on the right panel of the figure. SS , for example, means that the output of the home firm is a strategic substitute to the foreign firm's, and SC that it is a strategic complement. When the demand function is linear in the price, i.e., $E = 0$, each firm's output is a strategic substitute to the other's.

Hereafter, to sharpen the results, as in the standard Cournot-quantity competition, we assume

Assumption 5 *Each firm's output is a strategic substitute to the other's, i.e., $E < \min \left\{ \frac{1}{s}, \frac{1}{s^*} \right\}$.*

When the demand is iso-elastic, we may use the relation

$$0 = \frac{d \ln \varepsilon}{d \ln X} = \frac{d \ln P(X)}{d \ln X} - \frac{d \ln (-P'(X))}{d \ln X} - 1 = E - 1 - \frac{1}{\varepsilon}. \quad (17)$$

where use was made of $\varepsilon = -\frac{P(X)}{XP'(X)}$ and $E = -\frac{XP''(X)}{P'(X)}$. Thus when the price

elasticity of demand is constant, there holds

$$E = 1 + \frac{1}{\varepsilon},$$

which is shown by the downward sloping curve on the left panel. Assumption 4 requires that the price elasticity of demand ε should be at least 1/2. Figure 3 shows that the strategic substitution and complementarity depends not only on the price elasticity of demand but also on the market share of the home firm.

Based on these assumptions, let us first undertake the discussion for Step 1.

4.2 Changes in the Unit Costs and the Outputs

Introduction of the tariff package as well as its change affects the unit costs of both firms. Thus in this Step 1, we thus undertake the comparative statics for the final good market with respect to the unit cost of each firm.

Put the equilibrium output function of each firm, $\hat{x}(\cdot)$ and $\hat{x}^*(\cdot)$, into the equilibrium condition.

$$\begin{aligned}\hat{x}(c, \bar{c}^*) &= r(\hat{x}^*(c, \bar{c}^*), c), \\ \hat{x}^*(c, \bar{c}^*) &= r^*(\hat{x}(c, \bar{c}^*), \bar{c}^*).\end{aligned}\tag{18}$$

Application of the implicit function theorem yields

$$\frac{\partial \ln \hat{x}}{\partial \ln c} = \frac{r_c c}{\Delta x} = \frac{2 - s^* E}{p'(3 - E)} \frac{c}{x} = -\frac{(2 - s^* E)(\varepsilon - s)}{s(3 - E)} < 0,\tag{19}$$

$$\frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} = r_{x^*} \frac{r_{\bar{c}^*}^* \bar{c}^*}{\Delta x} = -\frac{1 - sE}{p'(3 - E)} \frac{\bar{c}^*}{x} = \frac{(1 - sE)(\varepsilon - s^*)}{s(3 - E)} > 0,\tag{20}$$

where use was made of Assumption 2-5 and the following relations

$$\begin{aligned}\frac{\partial \hat{x}}{\partial c} &= \frac{r_c}{\Delta} < 0, & \frac{\partial \hat{x}^*}{\partial c} &= r_x^* \frac{\partial x}{\partial c} > 0, \\ \frac{\partial \hat{x}}{\partial \bar{c}^*} &= r_{x^*} \frac{\partial \hat{x}^*}{\partial \bar{c}^*} > 0, & \frac{\partial \hat{x}^*}{\partial \bar{c}^*} &= \frac{r_{\bar{c}^*}^*}{\Delta} < 0.\end{aligned}$$

The associated equilibrium total output, denoted by $\hat{X}(c, \bar{c}^*)$, has a noteworthy property that it depends only on the sum of the unit costs over the industry. Sum the first-order conditions for profit maximization (15) over the firms, and obtain

$$0 = 2P(X) + XP'(X) - (c + \bar{c}^*).$$

The associated equilibrium market price, denoted by $\hat{P}(c, \bar{c}^*)$, thus depends also only on the sum of the unit costs over the industry. Since we often resort to these

results in the succeeding discussion result, we express the relations of the total output and the market price with the sum of the unit costs over the industry by $\bar{X}(c_T)$ and $\bar{P}(c_T)$ where $c_T \stackrel{\text{def}}{=} c + \bar{c}^*$. And it is summarized in the following proposition for the convenience of reference in the succeeding discussion.

Proposition 3 *When the unit cost of each firm is constant, then the equilibrium total output and market price depends only on the sum of the unit costs over the industry, i.e.,*

$$\hat{X}(c, \bar{c}^*) = \bar{X}(c + \bar{c}^*), \quad \hat{P}(c, \bar{c}^*) = \bar{P}(c + \bar{c}^*).$$

By virtue of (19) and (20), the changes in the total output are expressed by

$$\begin{aligned} \frac{\partial \hat{X}}{\partial c} &= \frac{\partial \hat{X}}{\partial \bar{c}^*} = \frac{d\bar{X}}{dc_T} = \frac{1}{3P'(X) + XP''(X)} = \frac{1}{P'(3-E)} < 0 \\ \frac{\partial \hat{P}}{\partial c} &= \frac{\partial \hat{P}}{\partial \bar{c}^*} = \frac{d\bar{P}}{dc_T} = \frac{P'(X)}{3P'(X) + XP''(X)} = \frac{1}{3-E} > 0. \end{aligned}$$

where $c_T = c + \bar{c}^*$. In the logarithmic terms, the above results are rewritten as

$$\frac{d \ln \bar{P}}{d \ln c_T} = \frac{\partial \ln \hat{P}}{\partial \ln c} + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \quad (21)$$

$$\frac{\partial \ln \hat{P}}{\partial \ln c} = \frac{\partial \ln P}{\partial \ln X} \frac{\partial \ln \hat{X}}{\partial \ln c} = \frac{1}{3-E} \left(1 - \frac{s}{\varepsilon}\right) \quad (22)$$

$$\frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} = \frac{\partial \ln P}{\partial \ln X} \frac{\partial \ln \hat{X}}{\partial \ln \bar{c}^*} = \frac{1}{3-E} \left(1 - \frac{s^*}{\varepsilon}\right) \quad (23)$$

As is already familiar in the studies on indirect taxation in oligopoly, an increase in the unit costs may lead to less than a proportional raise in the market price. The cause of such over-incidence of cost increase is captured by noting the relation (17). Substitute this relation into (22) and (23) above, and obtain

$$\begin{aligned} \frac{\partial \ln \hat{P}}{\partial \ln c} &= \frac{\left(1 - \frac{s}{\varepsilon}\right)}{\left(1 - \frac{s}{\varepsilon}\right) + \left(1 - \frac{s^*}{\varepsilon}\right) - \frac{d \ln \varepsilon}{d \ln X}}, \\ \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} &= \frac{\left(1 - \frac{s^*}{\varepsilon}\right)}{\left(1 - \frac{s}{\varepsilon}\right) + \left(1 - \frac{s^*}{\varepsilon}\right) - \frac{d \ln \varepsilon}{d \ln X}}. \end{aligned}$$

Thus insofar as the price elasticity of demand for the final good is non-increasing in the total output, i.e., $\frac{d \ln \varepsilon}{d \ln X} \leq 0$, an increase in the unit cost of either firm raises the market price less than proportionately. Throughout the following discussion, we assume this condition to hold.

Assumption 6 *The price elasticity of demand for the final good is non-increasing in the total output, i.e., $\frac{d \ln \varepsilon(X)}{d \ln X} \leq 0$.*

There are three remarks in order here. First, the above assumption assures that the rate of increase in the market price not to exceed the rate of increase in the unit cost of either firm, i.e., there holds $\frac{\partial \ln \hat{P}}{\partial \ln c}, \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \in (0, 1)$.

Second, the assumption assures that the increase in the market price due to the proportional increase in the unit costs over the industry does not exceed the rate of increase in the unit costs. This is ascertained by noting

$$\frac{\partial \ln \hat{P}}{\partial \ln c} + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} = \frac{(1 - \frac{s}{\varepsilon}) + (1 - \frac{s^*}{\varepsilon})}{(1 - \frac{s}{\varepsilon}) + (1 - \frac{s^*}{\varepsilon}) - \frac{d \ln \varepsilon}{d \ln X}}.$$

And lastly, the condition stated in the assumption holds for both linear and iso-elastic demand for the final good.

4.3 Changes in the Unit Costs and the Labor Demand

The changes in the unit costs due to the introduction of a tariff package affects the output configuration in the final good market, thus leading to a change in the home firm's demand for the labor specific to the industry in question. To see this, we first note that the labor demand by the home firm is given by.

$$L^d(w_L, w_M, \bar{c}^*) = c_L(w_L, w_M) \hat{x}(c(w_L, w_M), \bar{c}^*).$$

As a preliminary step for characterizing the equilibrium wage rate, we inquire into how the labor demand responds to the changes in the factor prices as well as the effective unit cost of the foreign firm. And for this purpose, the following decomposition of the change in the labor demand will turn to be very useful.

$$d \ln L^d = d \ln c_L + d \ln \hat{x},$$

where the first term on the right-hand side shows the factor substitution effect and the second the output effect.

Let us first explore the effect of the changes in the factor prices. Logarithmic differentiation of the labor demand function yields,

$$\frac{\partial \ln L^d}{\partial \ln w_L} = \frac{\partial \ln c_L}{\partial \ln w_L} + \theta_L \frac{\partial \ln \hat{x}}{\partial \ln c} < 0 \quad (24)$$

$$\frac{\partial \ln L^d}{\partial \ln w_M} = -\frac{\partial \ln c_L}{\partial \ln w_L} + \theta_M \frac{\partial \ln \hat{x}}{\partial \ln c} \quad (25)$$

where $c_{LL} < 0$, $\frac{\partial \hat{x}}{\partial c} < 0$. These results have the following straightforward intuition.

When the wage rate of the domestic labor gets higher, the substitution effect decreases the labor demand and the output decrease of the home resulting from the wage hike leads to the negative output effect lowering the labor demand. Thus the total effect is clearly negative as shown by (24). On the other hand, when the domestic price of the intermediate good rises, the substitution of the intermediate good for the domestic labor increases the labor demand while the resulting increase in the home firm's unit cost gives rise to a negative output effect through the competition over the final-good market. Thus the net effect is generally ambiguous as shown by (25).

However when the domestic wage and the domestic price of the intermediate good rise equiproportionately, their relative price stays constant canceling the two substitution effects discussed above and only a negative output effect remains, i.e.,

$$\frac{\partial \ln L^d}{\partial \ln w_L} + \frac{\partial \ln L^d}{\partial \ln w_M} = \frac{\partial \ln \hat{x}}{\partial \ln c} < 0. \quad (26)$$

The demand for the industry-specific labor should definitely decrease in this case.

Lastly, when the effective unit cost of the foreign rival firm increases, the labor demand changes only through the output effect. Since the foreign rival's output definitely decreases, the change in the home firm's output depends on whether the home firm's output is a strategic substitute or complement to the foreign rival's. In fact we obtain

$$\frac{\partial \ln L^d}{\partial \ln \bar{c}^*} = \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} > 0, \quad (27)$$

where use was made of the assumption of strategic substitution, i.e., Assumption 5.

The problem is how the labor demand changes when there are equiproportionate changes in the factor prices and the effective unit cost of the foreign firm. (24), (25) and (27) leads to

$$\frac{\partial \ln L^d}{\partial \ln w_L} + \frac{\partial \ln L^d}{\partial \ln w_M} + \frac{\partial \ln L^d}{\partial \ln \bar{c}^*} = \frac{\partial \ln \hat{x}}{\partial \ln c} + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*}.$$

That is, the change in the labor demand hinges on whether the equiproportionate increases in the unit costs of both firms decrease the home firm's equilibrium output. Although the direction of the output change is ambiguous in general, our intuition would suggest that the direct effect of the own unit cost increase should dominate the indirect effect of the rival's unit cost.³ Thus we impose

³When the demand is either linear in price or iso-elastic, this result in fact holds. See the discussion in Appendix Section C.

Assumption 7 *The equiproportionate increases in the unit costs of both firms decrease the equilibrium output of each firm, i.e.,*

$$\frac{\partial \ln \hat{x}}{\partial \ln c} + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} < 0, \quad \frac{\partial \ln \hat{x}^*}{\partial \ln c} + \frac{\partial \ln \hat{x}^*}{\partial \ln \bar{c}^*} < 0,$$

so that the equiproportionate increases in the factor prices and the effective unit cost of the foreign firm decreases the home firm's demand for the labor.

4.4 Equilibrium Wage

As expressed by (24), the domestic labor demand is strictly decreasing in the wage rate, so that application of the implicit function theorem implies that the equilibrium wage rate is a function of the domestic price of the intermediate good and the effective unit cost of the foreign. We express this relation by

$$w_L = w_L^e(w_M, \bar{c}^*).$$

The implicit function theorem also yields

$$\begin{aligned} \frac{\partial \ln w_L^e}{\partial \ln w_M} &= -\frac{\partial \ln L^d / \partial \ln w_M}{\partial \ln L^d / \partial \ln w_L} < 1, \\ \frac{\partial \ln w_L^e}{\partial \ln \bar{c}^*} &= -\frac{\partial \ln L^d / \partial \ln \bar{c}^*}{\partial \ln L^d / \partial \ln w_L} > 0, \end{aligned}$$

where use was made of (26) when deriving the inequality on the rightest-hand side of the first equation and Assumption 5 for the second inequality. Thus as implied by (25), the first equation means that an increase in the domestic price of the intermediate good may raise the domestic wage rate, but the resulting wage increase must be less than the increase in the domestic price of the intermediate good.^{4 5}

⁴If the wage rate rises more than the domestic price of the intermediate good, both the substitution effect and the output effect work negatively to the labor demand. The labor market will be in excess supply to make the wage rate to decrease.

⁵As shown in Appendix Section D, even when the production technology is of a CES type, the equilibrium wage rate increases along with the domestic price of the intermediate good if and only if the following condition holds.

$$\frac{s}{\varepsilon} > \frac{2 - s^*E}{\sigma_{LM}(3 - E) + (2 - s^*E)}$$

That is, the stated result holds, only when the elasticity of factor substitution and the market share of the domestic firm are sufficiently large.

In the succeeding discussion, we take the case of no factor substitution as our benchmark case, for we want to inquire into whether Corden's result for the short-run effect of the tariff package still holds on imperfectly competitive markets. When $\sigma_{LM} = 0$ (so that $c_{LL} = 0$) holds, the above results can be rewritten as

$$\begin{aligned} \frac{\partial \ln w_L^e}{\partial \ln w_M} \Big|_{\sigma_{LM}=0} &= -\frac{\theta_M}{\theta_L} < 0 \\ 0 < \frac{\partial \ln w_L^e}{\partial \ln \bar{c}^*} \Big|_{\sigma_{LM}=0} &= -\frac{1}{\theta_L} \frac{\partial \ln \hat{x} / \partial \ln \bar{c}^*}{\partial \ln \hat{x} / \partial \ln c} < \frac{1}{\theta_L}, \end{aligned}$$

where use was made of Assumption 7.

4.5 Effective and Nominal Protection

Let us now explore the relation between the effective and nominal rates of protection. Since the government of the home country imposes tariffs simultaneously on the final good and the intermediate good, we must be careful to identify the effects of differential taxes on those goods. Let us suppose that the government employs a tariff escalation package (t_F, t_M) satisfying $0 < t_M < t_F$, and inquire into the effect of infinitesimal departure from free trade toward the tariff package. Such infinitesimal departure can be captured by introducing a parameter α and consider the following gross tariff package parameterized by α

$$\begin{aligned} \tau_M(\alpha) &= 1 + \alpha t_M, \\ \tau_F(\alpha) &= 1 + \alpha t_F, \end{aligned}$$

where $\alpha \in [0, 1]$. All the equilibrium variables depend on the tariff-package parameter α . More specifically, we employ the following notations to express those equilibrium values.

$$\begin{aligned} w_M^\dagger(\alpha) &= w_M = \tau_M(\alpha) w_M^* \\ \bar{c}^{*\dagger}(\alpha) &= \bar{c}^* = \tau_F(\alpha) c^* \\ w_L^\dagger(\alpha) &= w_L^e(w_M(\alpha), \bar{c}^*(\alpha)) \\ c^\dagger(\alpha) &= c(w_L^\dagger(\alpha), w_M^\dagger(\alpha)) \\ x^\dagger(\alpha) &= \hat{x}(c^\dagger(\alpha), \bar{c}^{*\dagger}(\alpha)) \\ P^\dagger(\alpha) &= \hat{P}(c^\dagger(\alpha), \bar{c}^{*\dagger}(\alpha)) \\ \phi^\dagger(\alpha) &= \frac{w_M^\dagger(\alpha) c_M(w_M^\dagger(\alpha), w_L^\dagger(\alpha))}{P^\dagger(\alpha)} \end{aligned}$$

What we want to find is the condition for the revenue share of the intermediate good $\phi^\dagger(\alpha)$ after enforcing the tariff escalation policy. There are two factors which govern the change in this revenue share, as is shown by the rewritten equation

$$\phi^\dagger(\alpha) = \frac{c^\dagger(\alpha)}{P^\dagger(\alpha)} \theta_M \left(w_L^\dagger(\alpha), w_M^\dagger(\alpha) \right).$$

The first is the change in the price-cost ratio p/c , which we call the *market power effect*. This effect works through the oligopolistic competition in the final good market when the cost conditions of the two firms change under the given tariff escalation policy.

The second is the change in the cost factor share of the imported intermediate good θ_M , which we call the *factor substitution effect*. Since the change is expressed by ⁶

$$\begin{aligned} d \ln \theta_M &= -\frac{\frac{w_L c_L}{w_M c_M}}{\frac{w_L c_L}{w_M c_M} + 1} d \ln \left(\frac{w_L c_L}{w_M c_M} \right) \\ &= -\frac{w_L c_L}{c} [d \ln (w_L/w_M) + d \ln (c_L/c_M)] \\ &= -\theta_L (1 - \sigma_{LM}) d \ln (w_L/w_M). \end{aligned} \quad (28)$$

The factor substitution effect is determined by the size of the change in the relative factor price and the elasticity of factor substitution between the domestic labor and the imported intermediate good.

In the following we will explore the two effects by finding how the costs and the prices of the final good and factors change. But at this juncture, it is straightforward to derive

$$\left. \frac{d \ln w_M^\dagger}{d \alpha} \right|_{\alpha=0} = \left. \frac{d \ln \tau_M}{d \alpha} \right|_{\alpha=0} = \left. \frac{t_M}{\tau_M} \right|_{\alpha=0} = t_M \quad (29)$$

$$\left. \frac{d \ln \bar{c}^{*\dagger}}{d \alpha} \right|_{\alpha=0} = \left. \frac{d \ln \tau_F}{d \alpha} \right|_{\alpha=0} = \left. \frac{t_F}{\tau_F} \right|_{\alpha=0} = t_F. \quad (30)$$

In order to explore the effects governing the change of the intermediate-good-cost-price ratio, we examine the following changes.

⁶Use was made of $\theta_M = \frac{w_M c_M}{c} = \frac{w_M c_M}{w_L c_L + w_M c_M} = \frac{1}{\frac{w_L c_L}{w_M c_M} + 1}$.

4.5.1 Domestic Labor Wage w_L

As a preliminary step for finding how the relative factor price changes, let us see how the domestic wage alters after introduction of the tariff escalation policy. Using (29) and (30) as well as

$$w_L^\dagger(\alpha) = w_L^e(w_M(\alpha), \bar{c}^*(\alpha)),$$

one can easily derive the change of w_L as follows

$$\begin{aligned} \left. \frac{d \ln w_L^\dagger}{d\alpha} \right|_{\alpha=0} &= \frac{\partial \ln w_L^e}{\partial \ln w_M} t_M + \frac{\partial \ln w_L^e}{\partial \ln \bar{c}^*} t_F \\ &= -\frac{1}{\frac{\partial \ln L^d}{\partial \ln w_L}} \left[\frac{\partial \ln L^d}{\partial \ln w_M} t_M + \frac{\partial \ln L^d}{\partial \ln \bar{c}^*} t_F \right] \\ &= -\frac{\frac{\partial \ln L^d}{\partial \ln \bar{c}^*}}{\frac{\partial \ln L^d}{\partial \ln w_L}} t_M \left(\frac{\frac{\partial \ln L^d}{\partial \ln w_M}}{\frac{\partial \ln L^d}{\partial \ln \bar{c}^*}} + \frac{t_F}{t_M} \right) \\ &= -\frac{\frac{\partial \ln L^d}{\partial \ln \bar{c}^*}}{\frac{\partial \ln L^d}{\partial \ln w_L}} t_M \left(\frac{t_F}{t_M} + \theta_M \frac{\frac{\partial \ln \hat{x}}{\partial \ln c}}{\frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*}} - \frac{\frac{\partial \ln c_L}{\partial \ln w_L}}{\frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*}} \right) \end{aligned} \quad (31)$$

(32)

Thus we have established

Proposition 4 *The tariff escalation policy raises the domestic wage if and only if*

$$\frac{t_F}{t_M} + \theta_M \frac{\frac{\partial \ln \hat{x}}{\partial \ln c}}{\frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*}} - \frac{\frac{\partial \ln c_L}{\partial \ln w_L}}{\frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*}} > 0.$$

As stated in the above result, the wage of the industry-specific labor always increases when a rise in the intermediate good boosts its demand. However it is generally ambiguous whether the condition in question holds. The result becomes a little sharper for our benchmark case of no factor substitution, i.e., $\sigma_{LM} = 0$. We restate it as a corollary.⁷

Corollary 3 *Given no factor substitution between the domestic industry-specific labor and the intermediate good, the tariff escalation policy raises the domestic wage if and only if*

$$\frac{t_F}{t_M} + \theta_M \frac{\frac{\partial \ln \hat{x}}{\partial \ln c}}{\frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*}} > 0.$$

⁷In the case of strategic complementarity, the domestic wage rate always gets lower by enforcing the tariff package.

4.5.2 Relative Factor Price (w_L/w_M)

As we discussed in the competitive case, the change in the relative factor price w_L/w_M plays an important role in governing the effective rate of protection. (29) and (32) yields

$$\begin{aligned} \frac{d \ln(w_L^\dagger/w_M^\dagger)}{d\alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) &= \left(\frac{\partial \ln L^d}{\partial \ln w_M} + \frac{\partial \ln L^d}{\partial \ln w_L} \right) t_M + \frac{\partial \ln L^d}{\partial \ln \bar{c}^*} t_F \\ &= \frac{\partial \ln \hat{x}}{\partial \ln c} t_M + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F \\ &= \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_M \left(\frac{t_F}{t_M} + \frac{\partial \ln \hat{x} / \partial \ln c}{\partial \ln \hat{x} / \partial \ln \bar{c}^*} \right), \end{aligned}$$

which establishes

Proposition 5 *The domestic wage becomes relatively higher than the intermediate good price after enforcing the tariff escalation policy if and only if $\frac{t_F}{t_M} + \frac{\partial \ln \hat{x} / \partial \ln c}{\partial \ln \hat{x} / \partial \ln \bar{c}^*} > 0$ holds.*

Again, as in the case of the domestic wage, the relative factor price rises only if the tariff rate on the final good is sufficiently higher than on the intermediate good.⁸ In fact, when the demand function is linear, the condition stated in the above result is expressed by⁹

$$\frac{t_F}{t_M} > \frac{2c^\dagger(0)}{c^*},$$

which implies that the tariff rate on the final good required to raise the relative factor price is really high enough.

Corollary 4 *When the demand function is linear in the price, the tariff escalation policy raises the domestic wage relative to the intermediate good price if and only if $\frac{t_F}{t_M} > \frac{2c^\dagger(0)}{c^*}$ holds.*

⁸When the home firm's output is a strategic complement to the foreign rival's, the relative factor price w_L/w_M always declines.

⁹See the comparative statics for the linear demand in Appendix Section C.

Now the factor substitution effect of the tariff escalation policy is straightforward. We obtain by using (28)

$$\begin{aligned}
\frac{d \ln \theta_M}{d \alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) &= \theta_L (\sigma_{LM} - 1) \frac{d \ln(w_L/w_M)}{d \alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= -\theta_L (1 - \sigma_{LM}) \left(\frac{\partial \ln \hat{x}}{\partial \ln c} t_M + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F \right) \\
&= -\theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_M (1 - \sigma_{LM}) \left(\frac{t_F}{t_M} + \frac{\partial \ln \hat{x} / \partial \ln c}{\partial \ln \hat{x} / \partial \ln \bar{c}^*} \right).
\end{aligned}$$

Thus we have established

Proposition 6 *The tariff escalation policy lowers the factor cost share of the intermediate good if and only if $(1 - \sigma_{LM}) \left(\frac{t_F}{t_M} + \frac{\partial \ln \hat{x} / \partial \ln c}{\partial \ln \hat{x} / \partial \ln \bar{c}^*} \right) > 0$. As for the benchmark case of $\sigma_{LM} = 0$, the result holds if and only if $\frac{t_F}{t_M} + \frac{\partial \ln \hat{x} / \partial \ln c}{\partial \ln \hat{x} / \partial \ln \bar{c}^*} > 0$.*

The following is a corollary for the linear demand for the final good.

Corollary 5 *When the demand for the final good is linear in the price, the tariff escalation policy lowers the factor cost share of the intermediate good if and only if $\frac{t_F}{t_M} > \frac{2c^\dagger(0)}{c^*}$ holds.*

4.5.3 Unit Cost and Final-Good Price

As a preliminary step for characterizing the market power effect, let us see how the unit cost of the home firm changes after enforcing the tariff escalation policy. As already defined, the unit cost of the domestic firm is a function of α , i.e.,

$$c^\dagger(\alpha) = c(w_L, w_M) = c\left(w_L^\dagger(\alpha), w_M^{*\dagger}(\alpha)\right).$$

Logarithmic differentiation of $c^\dagger(\alpha)$ yields

$$\begin{aligned}
&\frac{d \ln c^\dagger}{d \alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \left(\theta_L \frac{d \ln w_L}{d \alpha} \Big|_{\alpha=0} + \theta_M t_M \right) \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \theta_L \frac{d \ln w_L}{d \alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) + \theta_M t_M \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F - \frac{\partial \ln c_L}{\partial \ln w_L} t_M > 0, \tag{33}
\end{aligned}$$

where use was made of (31) and Assumption 5 of strategic substitution.

As with the change in the equilibrium market price, logarithmic differentiation of $p = \hat{P}(c, \bar{c}^*) = P^\dagger(\alpha)$ yields

$$\frac{d \ln P^\dagger}{d\alpha} \Big|_{\alpha=0} = \frac{\partial \ln \hat{P}}{\partial \ln c} \frac{d \ln c}{d\alpha} + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} t_F > 0 \quad (34)$$

We can summarize the above results as follows.

Proposition 7 *The tariff escalation policy raises the unit cost of the home firms as well as the equilibrium market price, i.e.,*

$$\frac{d \ln c^\dagger}{d\alpha} \Big|_{\alpha=0} > 0 \quad , \quad \frac{d \ln P^\dagger}{d\alpha} \Big|_{\alpha=0} > 0$$

Then as with the cost-price ratio, Assumption 6 and 7 coupled with (33) and (34) yields

$$\begin{aligned} & \frac{d \ln(c/p)}{d\alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\ &= \left(\frac{d \ln c^\dagger}{d\alpha} - \frac{d \ln P^\dagger}{d\alpha} \right) \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\ &= \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{d \ln c}{d\alpha} \Big|_{\alpha=0} + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \frac{\partial \ln L^d}{\partial \ln w_L} t_F \\ &= \left[\theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \frac{\partial \ln L^d}{\partial \ln w_L} \right] t_F - \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \ln c_L}{\partial \ln w_L} t_M. \end{aligned}$$

Thus when the production technology allows factor substitution, i.e., $\sigma_{LM} > 1$, the above equation can be rewritten as

$$\begin{aligned} & \frac{d \ln(c/p)}{d\alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\ &= - \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \ln c_L}{\partial \ln w_L} t_F \left\{ \frac{t_M}{t_F} - \frac{\theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \frac{\partial \ln L^d}{\partial \ln w_L}}{\left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \ln c_L}{\partial \ln w_L}} \right\}. \end{aligned}$$

Proposition 8 *When there is factor substitution between the domestic industry-specific labor and the intermediate good, the tariff escalation policy lowers the cost-price ratio (or strengthens the market power) of the home firm if and only if*

$$\frac{t_M}{t_F} < \frac{\theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \frac{\partial \ln L^d}{\partial \ln w_L}}{\left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \ln c_L}{\partial \ln w_L}} \text{ holds.}$$

But as in the benchmark case, when $\sigma_{LM} = 0$ holds, we may rewrite the equation as

$$\begin{aligned}
& \left. \frac{d \ln(c/p)}{d\alpha} \right|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) = \theta_L t_F \left[\left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \frac{\partial \ln \hat{x}}{\partial \ln c} \right] \\
& = \theta_L t_F \frac{\bar{c}^*}{x} \left\{ \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \hat{x}}{\partial \bar{c}^*} + \frac{\partial \ln \hat{P}}{\partial \ln c} \frac{\partial \hat{x}}{\partial c} \right\} \\
& = \theta_L t_F \frac{\bar{c}^*}{x} \left\{ \left(1 - \frac{1 - \frac{s}{\varepsilon}}{3 - E} \right) \left(\frac{1 - sE}{-p'(3 - E)} \right) + \frac{1 - \frac{s}{\varepsilon}}{3 - E} \left(\frac{2 - s^*E}{p'(3 - E)} \right) \right\} \\
& = \theta_L t_F \frac{\bar{c}^*}{x} \left\{ \frac{s}{-p'(3 - E)} \left(\frac{1}{\varepsilon} - E \right) \right\}.
\end{aligned}$$

Thus we have established

Proposition 9 *In the absence of factor substitution, the tariff escalation policy lowers the cost-price ratio of the home firm if and only if $\frac{1}{\varepsilon} < E$ or equivalently $\frac{d \ln \varepsilon}{d \ln X} > -1$ holds.*

The condition stated in the above proposition holds for the iso-elastic demand case, but it does not for the linear demand. In the latter case, the tariff escalation policy always increases the cost-price ratio and thus the market power of the home firm.

4.5.4 Final good outputs \hat{x} and \hat{x}^*

The change of the output of the domestic firm is expressed as below.

$$\begin{aligned}
& \left. \frac{d \ln \hat{x}}{d\alpha} \right|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
& = \frac{\partial \ln \hat{x}}{\partial \ln c} \frac{d \ln c}{d\alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
& = \frac{\partial \ln \hat{x}}{\partial \ln c} \left(\theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F - \frac{\partial \ln c_L}{\partial \ln w_L} t_M \right) + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F \left(-\frac{\partial \ln c_L}{\partial \ln w_L} - \theta_L \frac{\partial \ln \hat{x}}{\partial \ln c} \right) \\
& = -\frac{\partial \ln c_L}{\partial \ln w_L} \left(\frac{\partial \ln \hat{x}}{\partial \ln c} t_M + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F \right) \\
& = -\frac{\partial \ln c_L}{\partial \ln w_L} t_M \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \left(\frac{t_F}{t_M} + \frac{\partial \ln \hat{x} / \partial \ln c}{\partial \ln \hat{x} / \partial \ln \bar{c}^*} \right)
\end{aligned}$$

which use was made of (24)(33).

Proposition 10 *The tariff escalation policy raises the output of the domestic firm if and only if $\frac{t_F}{t_M} + \frac{\partial \ln \hat{x}/\partial \ln c}{\partial \ln \hat{x}/\partial \ln \bar{c}^*} > 0$.*¹⁰

Note the above result coincides with the results when the tariff escalation raises the factor price ratio w_L/w_M or the cost share of the intermediate good θ_M in the benchmark case.

The change of the output of the foreign firm is expressed as following.

$$\begin{aligned}
& \left. \frac{d \ln \hat{x}^*}{d\alpha} \right|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \frac{\partial \ln \hat{x}^*}{\partial \ln c} \left. \frac{d \ln c}{d\alpha} \right|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) + \frac{\partial \ln \hat{x}^*}{\partial \ln \bar{c}^*} t_F \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \frac{\partial \ln \hat{x}^*}{\partial \ln c} \left(\theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F - \frac{\partial \ln c_L}{\partial \ln w_L} t_M \right) + \frac{\partial \ln \hat{x}^*}{\partial \ln \bar{c}^*} t_F \left(-\frac{\partial \ln c_L}{\partial \ln w_L} - \theta_L \frac{\partial \ln \hat{x}}{\partial \ln c} \right) \\
&= \theta_L \left(\frac{\partial \ln \hat{x}^*}{\partial \ln c} \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} - \frac{\partial \ln c}{\partial \ln c} \frac{\partial \ln \hat{x}^*}{\partial \ln \bar{c}^*} \right) - \frac{\partial \ln c_L}{\partial \ln w_L} \left(\frac{\partial \ln \hat{x}^*}{\partial \ln c} t_M + \frac{\partial \ln \hat{x}^*}{\partial \ln \bar{c}^*} t_F \right) \\
&< 0
\end{aligned}$$

which use was made of (19)(20)(24)(33) and Assumption 7. Summarize the above results as follows.

Proposition 11 *The output of the foreign firm would unambiguously decrease when enforcing the tariff escalation policy.*

4.5.5 Revenue Share of the Intermediate Good ϕ^\dagger

Let us integrate what we have obtained for the benchmark case of no factor substitution. First, the factor substitution effect lowers the factor cost share of the intermediate good if and only there holds

$$\frac{t_F}{t_M} + \frac{\partial \ln \hat{x}/\partial \ln c}{\partial \ln \hat{x}/\partial \ln \bar{c}^*} > 0.$$

Second, the market power effect lowers the cost-price ratio for the home firm if and only if there holds

$$\frac{1}{\varepsilon} < E, \quad \text{or equivalently} \quad \frac{d \ln \varepsilon}{d \ln X} > -1.$$

When the two conditions hold simultaneously, the tariff escalation policy lowers the revenue share of the intermediate good, thus leading to the effective rate of protection larger than the nominal one.

¹⁰In the case of strategic complementarity, the result is reversed.

Proposition 12 *Given no factor substitution, the effective rate of protection exceeds the nominal one when there hold $\frac{d \ln \varepsilon}{d \ln X} > -1$ and $\frac{t_F}{t_M} + \frac{\partial \ln \hat{x} / \partial \ln c}{\partial \ln \hat{x} / \partial \ln \bar{c}^*} > 0$.*

However there is an alternative direct way to obtain the condition for $R_e > R_n$ in the benchmark case. That is, there holds ¹¹

$$\begin{aligned}
& \left. \frac{d \ln \phi_f}{d \alpha} \right|_{\alpha=0} \left(- \frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \theta_L \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F + \theta_L \frac{\bar{c}^*}{c} \frac{\partial \ln \hat{P}}{\partial \ln c} \frac{\partial \ln \hat{x}}{\partial \ln c} t_F - \theta_L \left(\frac{\partial \ln \hat{x}}{\partial \ln c} t_M + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F \right) \\
&= \theta_L \left[\frac{\partial \ln \hat{P}}{\partial \ln c} \left(\frac{\bar{c}^*}{c} \frac{\partial \ln \hat{x}}{\partial \ln c} - \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right) t_F - \frac{\partial \ln \hat{x}}{\partial \ln c} t_M \right] \\
&= - \theta_L t_M \frac{\partial \ln \hat{P}}{\partial \ln c} \left(\frac{\bar{c}^*}{c} \frac{\partial \ln \hat{x}}{\partial \ln c} - \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right) \left(\lambda - \frac{t_F}{t_M} \right)
\end{aligned}$$

where

$$\lambda \stackrel{\text{def}}{=} \frac{\frac{\partial \ln \hat{x}}{\partial \ln c}}{\frac{\partial \ln \hat{P}}{\partial \ln c} \left(\frac{\bar{c}^*}{c} \frac{\partial \ln \hat{x}}{\partial \ln c} - \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right)}$$

and use was made of (19)(20)(22). Therefore the revenue share of the intermediate good declines if and only if $\frac{t_F}{t_M} > \lambda$ holds. Here λ can be rewritten as follows

$$\begin{aligned}
\lambda &= \frac{\frac{\partial \ln \hat{x}}{\partial \ln c}}{\frac{\partial \ln \hat{P}}{\partial \ln c} \left(\frac{\bar{c}^*}{c} \frac{\partial \ln \hat{x}}{\partial \ln c} - \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right)} \\
&= \frac{p \frac{\partial \hat{x}}{\partial c}}{\bar{c}^* \frac{\partial \hat{P}}{\partial c} \left(\frac{\partial \hat{x}}{\partial c} - \frac{\partial \hat{x}}{\partial \bar{c}^*} \right)} \\
&= \frac{p}{\bar{c}^*} \cdot \frac{1}{P'(X) (1 + r_x^*) \left(\frac{\partial \hat{x}}{\partial c} - \frac{\partial \hat{x}}{\partial \bar{c}^*} \right)} \\
&= \frac{p}{\bar{c}^* (1 + r_x^*)} \cdot \frac{1}{\frac{P'(X)}{\Delta} (r_c - r_x^* r_{c^*}^*)} \\
&= \frac{p}{\bar{c}^* (1 + r_x^*)} \cdot \frac{1}{\frac{P'(2-sE)(2-s^*E)}{(3-E)} \left(\frac{1}{P'(2-sE)} + \frac{(1-sE)}{(2-sE)} \cdot \frac{1}{P'(2-s^*E)} \right)} \\
&= \frac{p}{\bar{c}^* (1 + r_x^*)},
\end{aligned}$$

where use was made of (16), (19), and (20). Thus we have established

¹¹We examine the case with factor substitution in Appendix Section B.

Proposition 13 *In the absence of factor substitution, the tariff escalation policy leads to the effective rate of protection larger than the nominal one if and only if*

$$\frac{t_F}{t_M} > \frac{P^\dagger(0)}{c^*} \frac{1}{1 + r_x^*} \text{ holds.}$$

The above proposition shows the possibility that the result in the competitive case will be likely reversed in the duopoly case.

5 Conclusions

In this paper we show the correlation of the effective and nominal rate of protection under factor substitution in different market structure. When the market is in perfect competition, the higher tariff imposition on the final good than on the intermediate good leads to the higher protection of the domestic industry if the factor substitution between the domestic labor and the imported intermediate good is inelastic. Such result is also satisfied in the case of more intermediate goods.

Appendix

A Competitive Case with More Than Two Imported Intermediate Goods

Let us extend the results in the previous section to the case in which the final good industry needs two or more intermediate goods. Let us assume that it requires n types of imported intermediate goods. Let w_i denote the domestic price of the i -th intermediate good and $\mathbf{w}_M \stackrel{\text{def}}{=} (w_1, \dots, w_n)$. Then the unit cost function in the previous discussion is now replaced with the newly defined one $c(w_L, \mathbf{w}_M)$, and we may follow the same approach. All the necessary basic relations are summarized by the following set of equations.

$$p = c(w_L, \mathbf{w}_M) \quad (\text{A.1})$$

$$L = c_L(w_L, \mathbf{w}_M)x \quad (\text{A.2})$$

$$0 = \theta_L d \ln c_L + \sum_i \theta_i d \ln c_i \quad (\text{A.3})$$

$$d \ln c_i - d \ln c_L = \sigma_{Li} (d \ln w_L - d \ln w_i) \quad (\text{A.4})$$

where $c_i \stackrel{\text{def}}{=} \frac{\partial c(w_L, \mathbf{w}_M)}{\partial w_i}$ denotes the input coefficient of the i -th intermediate good and σ_{Li} the elasticity of factor substitution between labor and the i -th intermediate good. The first equation shows equality between the price and the unit cost in perfect competition, the second the necessary costs for cost-minimization, and the last the definition of the substitution elasticities.

Solve (A.4) for $d \ln c_i$ and put the result into (A.3). Then one gets

$$d \ln c_L = - \sum_i \theta_i \sigma_{Li} d \ln(w_L/w_i), \quad (\text{A.5})$$

while (A.1) yields

$$d \ln(w_L/p) = - \frac{1}{\theta_L} \sum_i \theta_i d \ln(w_i/p) \quad (\text{A.6})$$

Changes in the prices of the final and intermediate goods gives rise to the

following change in the labor-cost share in the final good industry.

$$\begin{aligned}
d \ln \theta_L &= d \ln(w_L/p) + d \ln c_L \\
&= d \ln(w_L/p) - \sum_i \theta_i \sigma_{Li} d \ln(w_L/w_i) \quad (\because \text{(A.5)}) \\
&= d \ln(w_L/p) - \sum_i \theta_i \sigma_{Li} d \ln(w_L/p) - \sum_i \theta_i \sigma_{Li} d \ln(p/w_i) \\
&= \left\{ \frac{1}{\theta_L} \left(1 - \sum_i \theta_i \sigma_{Li} \right) \sum_i \theta_i - \sum_i \theta_i \sigma_{Li} \right\} d \ln(p/w_i) \quad (\because \text{(A.6)}) \\
&= \frac{1}{\theta_L} \sum_i \theta_i \left\{ 1 - \left(\theta_L \sigma_{Li} + \sum_j \theta_j \sigma_{Lj} \right) \right\} d \ln(p/w_i). \quad (\text{A.7})
\end{aligned}$$

To see that the above equation gives a generalization of the results in the previous section, let $w_i = w_M$ for all $i = 1, \dots, n$ and assume that the tariff rates are the same for all the intermediate goods. Then the above equation becomes

$$\begin{aligned}
d \ln \theta_L &= \frac{1}{\theta_L} \left\{ \sum_i \theta_i - \sum_i \theta_i \sigma_{Li} \right\} d \ln(p/w_M) \\
&= \frac{\theta_M}{\theta_L} \left(1 - \frac{\sum_i \theta_i \sigma_{Li}}{\theta_M} \right) d \ln(p/w_M) \quad (\text{A.8})
\end{aligned}$$

where $\theta_M \stackrel{\text{def}}{=} \sum_i \theta_i$ and one should note that $\frac{\sum_i \theta_i \sigma_{Li}}{\theta_M}$ represents the weighted-average elasticity of factor substitution between the labor and the intermediate goods. Since the relative price p/w_M increases if and only if the tariff rate is higher on the final good than on the intermediate good, we obtain the exactly same result as in the previous section of a single intermediate good.

Thus Proposition 2 can be extended to the case of two or more than two intermediate goods as follows.

Proposition 14 *When the tariff rates over the intermediate goods are all the same, the higher tariff rate on the final good than on the intermediate goods leads to the effective rate of protection greater than the nominal one if and only if the weighted average elasticity of factor substitution between the domestic labor and the intermediated goods is smaller than unity in the competitive final good and intermediate good industry.*

Change in the output when the tariff rates over the intermediate goods $w_i = w_M$

$$\begin{aligned}
d \ln x &= - \left(\frac{w_L c_{LL} d \ln w_L}{c_L} + \frac{\sum w_i c_{Li} d \ln w_i}{c_L} \right) \\
&= - \frac{w_L c_{LL}}{c_L} d \ln(w_L/w_M) \quad (\text{A.9})
\end{aligned}$$

which yields the same result as the case of one intermediate good.

$$\begin{aligned}
& \frac{d \ln(c/p)}{d\alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \left(\frac{d \ln c}{d\alpha} - \frac{d \ln P}{d\alpha} \right) \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \ln c}{d\alpha} \Big|_{\alpha=0} + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \frac{\partial \ln L^d}{\partial \ln w_L} t_F \\
&= \left[\theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \frac{\partial \ln L^d}{\partial \ln w_L} \right] t_F - \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \ln c_L}{\partial \ln w_L} \theta_M t_M \\
&< \left[\left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \left(\theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} - \frac{\partial \ln c_L}{\partial \ln w_L} \right) + \frac{\bar{c}^*}{c} \frac{\partial \ln \hat{P}}{\partial \ln c} \frac{\partial \ln L^d}{\partial \ln w_L} \right] t_F \\
&= \left(\theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} - \frac{\partial \ln c_L}{\partial \ln w_L} \right) + \frac{\partial \ln \hat{P}}{\partial \ln c} \left[\frac{c_T}{c} \frac{\partial \ln c_L}{\partial \ln w_L} + \theta_L \left(\frac{\bar{c}^*}{c} \frac{\partial \ln \hat{x}}{\partial \ln c} - \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right) \right] \\
&< 0 \quad \text{if} \quad \frac{\partial \ln \hat{P}}{\partial \ln c} > \frac{\frac{\partial \ln c_L}{\partial \ln w_L} - \theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*}}{\frac{c_T}{c} \frac{\partial \ln c_L}{\partial \ln w_L} + \theta_L \frac{\bar{c}^*}{x} \left(\frac{\partial \hat{x}}{\partial c} - \frac{\partial \hat{x}}{\partial \bar{c}^*} \right)}
\end{aligned}$$

In the benchmark case as $\sigma_{LM} = 0$

$$\begin{aligned}
& \frac{d \ln(c/p)}{d\alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \theta_L \left[\left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \frac{\partial \ln \hat{x}}{\partial \ln c} \right] t_F \\
&= \theta_L \frac{\bar{c}^*}{x} \left[\left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \hat{x}}{\partial \bar{c}^*} + \frac{\partial \ln \hat{P}}{\partial \ln c} \frac{\partial \hat{x}}{\partial c} \right]
\end{aligned}$$

where in the above equation

$$\begin{aligned}
& \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \frac{\partial \hat{x}}{\partial \bar{c}^*} + \frac{\partial \ln \hat{P}}{\partial \ln c} \frac{\partial \hat{x}}{\partial c} \\
&= \left(1 - \frac{1 - \frac{s}{\varepsilon}}{3 - E} \right) \left(\frac{1 - sE}{-p'(3 - E)} \right) + \frac{1 - \frac{s}{\varepsilon}}{3 - E} \left(\frac{2 - s^*E}{p'(3 - E)} \right) \\
&= \frac{s}{-p'(3 - E)} \left(\frac{1}{\varepsilon} - E \right)
\end{aligned}$$

Thus we get the following result in the benchmark case.

$$\frac{1}{\varepsilon} \begin{matrix} \geq \\ \leq \end{matrix} E \quad \iff \quad \frac{d \ln(c/p)}{d\alpha} \Big|_{\alpha=0} \begin{matrix} \geq \\ \leq \end{matrix} 0$$

B the value of $d \ln \phi^\dagger$

Substitute the result of $d \ln(c/p)$ and $d \ln \theta_M$ into $d \ln \phi_f$, it yields

$$\begin{aligned}
& \frac{d \ln \phi^\dagger}{d\alpha} \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \left(\frac{d \ln(c/p)}{d\alpha} + \frac{d \ln \theta_M}{d\alpha} \right) \Big|_{\alpha=0} \left(-\frac{\partial \ln L^d}{\partial \ln w_L} \right) \\
&= \left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \left(-\frac{\partial \ln c_L}{\partial \ln w_L} t_M + \theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F \right) + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \frac{\partial \ln L^d}{\partial \ln w_L} t_F \\
&\quad - \theta_L (1 - \sigma_{LM}) \left(\frac{\partial \ln \hat{x}}{\partial \ln c} t_M + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} t_F \right) \\
&< \left[\left(1 - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \left(-\frac{\partial \ln c_L}{\partial \ln w_L} + \theta_L \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right) + \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} \frac{\partial \ln L^d}{\partial \ln w_L} - \theta_L (1 - \sigma_{LM}) \left(\frac{\partial \ln \hat{x}}{\partial \ln c} + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right) \right] t_F \\
&= \left[\left(\frac{\partial \ln \hat{P}}{\partial \ln c} + \frac{\bar{c}^*}{c} \frac{\partial \ln \hat{P}}{\partial \ln c} - 1 \right) \left(\theta_L \frac{\partial \ln \hat{x}}{\partial \ln c} + \frac{\partial \ln c_L}{\partial \ln w_L} \right) + \theta_L \left(\sigma_{LM} - \frac{\partial \ln \hat{P}}{\partial \ln c} \right) \left(\frac{\partial \ln \hat{x}}{\partial \ln c} + \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right) \right] t_F \\
&= \left\{ \frac{\partial \ln \hat{P}}{\partial \ln c} \left[\frac{c_T}{c} \frac{\partial \ln c_L}{\partial \ln w_L} + \theta_L \left(\frac{\bar{c}^*}{c} \frac{\partial \ln \hat{x}}{\partial \ln c} - \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right) \right] \right. \\
&\quad \left. + \theta_L \left[(\sigma_{LM} - 1) \frac{\partial \ln \hat{x}}{\partial \ln c} + \sigma_{LM} \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right] - \frac{\partial \ln c_L}{\partial \ln w_L} \right\} t_F \\
&< 0 \quad \text{if} \quad \frac{\partial \ln \hat{P}}{\partial \ln c} > \frac{\frac{\partial \ln c_L}{\partial \ln w_L} - \theta_L \left[(\sigma_{LM} - 1) \frac{\partial \ln \hat{x}}{\partial \ln c} + \sigma_{LM} \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} \right]}{\frac{c_T}{c} \frac{\partial \ln c_L}{\partial \ln w_L} + \theta_L \frac{\bar{c}^*}{x} \left(\frac{\partial \hat{x}}{\partial c} - \frac{\partial \hat{x}}{\partial \bar{c}^*} \right)}
\end{aligned}$$

C Linear Demand Function

Assume the inverse demand function as $p = A - (x + x^*)$.

Profit function

$$\begin{aligned}
\pi &= (A - (x + x^*) - c)x \\
\pi^* &= (A - (x + x^*) - \bar{c}^*)x^*
\end{aligned}$$

Optimum output and market price

$$\hat{x} = \frac{A - 2c + \bar{c}^*}{3} \quad \hat{x}^* = \frac{A - 2\bar{c}^* + c}{3} \quad \hat{p} = \frac{A + c + \bar{c}^*}{3}$$

Then we have the following results.

$$\begin{aligned}\frac{\partial \ln \hat{x}}{\partial \ln c} &= \frac{-2c}{A - 2c + \bar{c}^*} & \frac{\partial \ln \hat{x}}{\partial \ln \bar{c}^*} &= \frac{\bar{c}^*}{A - 2c + \bar{c}^*} \\ \frac{\partial \ln \hat{P}}{\partial \ln c} &= \frac{c}{A + c + \bar{c}^*} & \frac{\partial \ln \hat{P}}{\partial \ln \bar{c}^*} &= \frac{\bar{c}^*}{A + c + \bar{c}^*}\end{aligned}$$

D CES Production Technology

Assume the production function as following.

$$F(L, M) = (L^\rho + M^\rho)^{\frac{1}{\rho}}$$

As for the unit cost minimization

$$\begin{aligned}\min_{c_L, c_M} & w_L c_L + w_M c_M \\ \text{s.t.} & c_L^\rho + c_M^\rho = 1\end{aligned}$$

The standard result yields

$$\begin{aligned}c_L &= w_L^{\frac{1}{\rho-1}} \left(w_L^{\frac{\rho}{\rho-1}} + w_M^{\frac{\rho}{\rho-1}} \right)^{-\frac{1}{\rho}} \\ c_M &= w_M^{\frac{1}{\rho-1}} \left(w_L^{\frac{\rho}{\rho-1}} + w_M^{\frac{\rho}{\rho-1}} \right)^{-\frac{1}{\rho}}\end{aligned}$$

The elasticity of substitution

$$\sigma_{LM} = -\frac{d \ln(c_L/c_M)}{d \ln(w_L/w_M)} = \frac{1}{1 - \rho}$$

The unit cost function

$$c(w_L, w_M) = \left(w_L^{\frac{\rho}{\rho-1}} + w_M^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}}$$

The we have the following results.

$$\begin{aligned}\theta_L &= \frac{w_L c_L}{c} = w_L^{\frac{\rho}{\rho-1}} \left(w_L^{\frac{\rho}{\rho-1}} + w_M^{\frac{\rho}{\rho-1}} \right)^{-1} \\ \theta_M &= \frac{w_M c_M}{c} = w_M^{\frac{\rho}{\rho-1}} \left(w_L^{\frac{\rho}{\rho-1}} + w_M^{\frac{\rho}{\rho-1}} \right)^{-1} \\ \frac{\partial \ln c_L}{\partial \ln w_L} &= -\frac{\partial \ln c_L}{\partial \ln w_M} = \frac{1}{\rho - 1} w_M^{\frac{\rho}{\rho-1}} \left(w_L^{\frac{\rho}{\rho-1}} + w_M^{\frac{\rho}{\rho-1}} \right)^{-1} = -\sigma_{LM} \theta_M \\ \frac{\partial \ln c_M}{\partial \ln w_M} &= -\frac{\partial \ln c_M}{\partial \ln w_L} = \frac{1}{\rho - 1} w_L^{\frac{\rho}{\rho-1}} \left(w_L^{\frac{\rho}{\rho-1}} + w_M^{\frac{\rho}{\rho-1}} \right)^{-1} = -\sigma_{LM} \theta_L\end{aligned}$$

As $\phi_f = w_M c_M / p$, the change of ϕ_f can be rewritten as below.

$$\begin{aligned} \left. \frac{d \ln \phi_f}{d\alpha} \right|_{\alpha=0} &= \frac{d \ln w_M}{d\alpha} + \left(\frac{\partial \ln c_M}{\partial \ln w_L} \frac{d \ln w_L}{d\alpha} + \frac{\partial \ln c_M}{\partial \ln w_M} \frac{d \ln w_M}{d\alpha} \right) - \frac{\partial \ln p}{d\alpha} \\ &= \left(1 + \frac{\partial \ln c_M}{\partial \ln w_M} \right) t_M + \frac{\partial \ln c_M}{\partial \ln w_L} \frac{d \ln w_L}{d\alpha} - \frac{d \ln p}{d\alpha} \end{aligned} \quad (\text{A.10})$$

We can infer the following results

$$\text{if } \begin{cases} 1 + \frac{\partial \ln c_M}{\partial \ln w_M} \leq 0 \\ \frac{d \ln w_L}{d\alpha} \leq 0 \end{cases} \implies \sigma_{LM} \leq 1/\theta_L \implies d\phi_f < 0 \implies R_e > R_n$$

We can also have the weaker condition even if $\frac{d \ln w_L}{d\alpha} > 0$, when the second and third terms in (A.10) $\frac{\partial \ln c_M}{\partial \ln w_L} \frac{d \ln w_L}{d\alpha} - \frac{d \ln p}{d\alpha} < 0$, we can have the same result.

$$\begin{aligned} &\frac{\partial \ln c_M}{\partial \ln w_L} \frac{d \ln w_L}{d\alpha} - \frac{d \ln p}{d\alpha} \\ &= \theta_L \sigma_{LM} \frac{d \ln w_L}{d\alpha} - \frac{d \ln p}{d\alpha} \end{aligned}$$

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