Numerical Analysis of Noncontributors and Contributors with International Public Good: Post-Kyoto Protocol and ODA Policy*

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Abstract

This paper develops a numerical algorithm to calculate the percentage of noncontributors in the presence of voluntary contributions within an international public good model. The algorithm is used to investigate the effects of the number of countries, the diffusion of income, prices and preference parameters, and income transfers on the percentage of noncontributors, which has received considerable attention in the post-Kyoto Protocol. It is also used to analyze the best official development assistance method: provision of public goods or income subsidies to all countries.

Keywords: international public good; algorithm; noncontributors; post-Kyoto Protocol; ODA policy
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1. Introduction

Bergstrom, Blume and Varian (1986, 1992), who examine the noncooperative Nash equilibrium of voluntary contributions to public goods, prove the existence and uniqueness of the equilibrium, assuming the same productivity of producing the public good in each country. A large number of papers have extended their model, including Chan et al(1997),Sonnemans et al (1998), Andreoni (2007) and Besley and Ghatak (2007).

Having received considerable attention recently, the international public good model was developed by Ihori (1996) and extended by Boadway and Hayashi (1999), Arce and Sandler (2001), Kim and Shim (2006), Cornes and Hartley (2007) and Lei, Tucker and Vesely (2007). The difference between international public good models and public good models is that the former considers: (i) an "*international public good*" that is nonrivalrous and nonexcludable across borders and (ii) different productivities of producing the international public good in each country. Examples of international public goods include "vast forests to prevent global warming" or "weapons" held by the allies of the North Atlantic Treaty Organization. Such features of international public good models enable policy analysis to be broadly applicable, i.e., global warming or international security. Therefore, in particular, it is interesting to consider the hot issue of noncontributors (the free-rider problem) in an international public good model.

To date, no previous studies have analyzed noncontributors (who do not contribute to public goods). Therefore, the following are important questions. Do noncontributors and contributors coexist under voluntary contributions? Who becomes a noncontributor? How can welfare be improved? Previous studies have excluded noncontributors through assumption. However, we believe that it is important to analyze the effects of noncontributors because many noncontributors present in numerical experiment of parameters in this paper. Recent proposals associated with the post-Kyoto Protocol (proposals in each country after the Kyoto Protocol[1]) suggest that all countries should be contributors to international public goods, emphasizing the undesirability of free riding by noncontributors.[2] This was reinforced in the speech "Invitation to Cool Earth 50" given on May 24, 2007 by the Japanese Prime Minister Abe. In this sense, it is important to analyze the characteristics of noncontributors.

There are two types of official development assistance (ODA): money and goods. The goods are international public goods, the main categories of which are health, education, water and sanitation. For details, see Japan's Official Development Assistance Charter.[3] Which assistance method is better for developing countries: international public goods or money?[4] To our knowledge, this question has not been considered in academic research. However, recent large fiscal deficits for all developed countries make a small ODA budget efficient. In fact, the longstanding target for ODA as a share of the national income of donor countries is 0.7 percent of gross national income (GNI), as agreed by

[1] Its full name is the Kyoto Protocol to the United Nations Framework Convention on Climate Change.
[2] We do not discuss the mechanism design , which resolves the first-order and the second-order dilemma of public goods with an inefficient equilibrium and nonwelled mechanism. Useful surveys are Okada (2008) and Okada et al. (2009).
[3] http://www.mofa.go.jp/policy/oda/reform/charter.html
[4] Ministry of Foreign Affairs of Japan: http://www.mofa.go.jp/policy/oda/reform/charter.html

countries at the UN in 1971. However, there was a large gap between commitment and payment of ODA in 2009. This question is an important issue for international public good models.[5]

The purpose of this paper is to develop a numerical algorithm for a solution to an international public good model including both noncontributors and contributors under voluntary contribution. We then apply the algorithm to investigate the effects of the number of countries, diffusion of income, prices and preferences parameters, and income transfers on the percentage of noncontributors in all countries, which has received considerable attention in the post-Kyoto Protocol. It is also applied to analyze the better ODA method: provision of public goods or income subsidies to all countries.

The organization of this paper is as follows. Section 2 outlines the international public good model. Section 3 specifies the Cobb–Douglas utility function and provides the numerical solution algorithm in the presence of both noncontributors and contributors, given exogenous values of each countries incomes, prices and preferences. Section 4 applies the algorithm to investigate select issues in the post-Kyoto Protocol: effects of the number of countries, diffusion of income, prices and preference parameters, and income transfers on the percentage of noncontributors. Furthermore, we discuss how to increase the percentage of contributors. Section 5 applies the algorithm to determine the best ODA method: provision of public goods or income subsidies to all countries. Section 6 provides concluding remarks.

2. The model

Framework

Consider a model where there is one public good, one private good and n countries ($i = 1,2,\ldots,n$). Country i consumes an amount x_i of the private good and provides an amount g_i to the supply of the international public good. The total supply of the international public good, G, is simply the sum of g_i provided by each country. Country i's utility is given by $U_i = U_i(x_i, G)$, where U_i is strictly increasing and quasi-concave. Country i's budget constraint is given by $x_i + p_i g_i = w_i$, where $w_i > 0$ is the exogenously given national income of country i and $p_i > 0$ is the relative price (cost of production) of public goods in terms of private consumption in country i. A low (high) p_i means a high (low) productivity of producing the public good. We also make the Nash assumption that each country believes that the contributions of others are independent of its own. Then, we let $G_{-i} = \sum_{j \neq i} g_j$ denote the sum of g_j provided by countries j other than i and $G = g_i + G_{-i}$. Implicitly, each country is choosing not only

[5] Payment of ODA from the DAC countries (members of the OECD's Development Assistance Committee) in 2009 totaled US$119.6 billion (measured in 2009 US dollars), equivalent to 0.31 percent of developed countries' combined national income. Therefore, the difference between the funds committed and paid in 2009 was US$152.6 billion or 0.39 percent of developed countries' GNI. The data source is "Where are the gaps?" by the United Nations:
http://www.un.org/millenniumgoals/pdf/GAP_FACTS_2010_EN.pdf

their gift, but also the equilibrium level of G itself. When country i does not make a gift $g_i = 0$ (G = G_{-i}), it is called a noncontributor, and when it makes a gift $g_i > 0$ (G > G_{-i}), it is called a contributor.

Definition 1. *A Nash equilibrium in this model is such that for each i,* (x_i^*, G^*) *i = 1,…,n solves:*

$$\max_{x_i, G} U_i = U_i(x_i, G)$$
$$s.t. \quad x_i + p_i G = w_i + p_i G_{-i} \;, \quad G \geq G_{-i} \;, \quad G \geq 0, \quad x_i \geq 0 \,. \tag{1}$$

Existence and uniqueness of Nash equilibrium

Let $f_i(w_i + p_i G_{-i})$ be consumer i's demand function for the public good G, representing the value of G that country i would choose as a function of the right-hand side of the above budget constraint (1).

Assumption 1. $0 < p_i$ *for all i.*

Assumption 2. *There is a single-valued demand function for the public good,* $f_i(w_i + p_i G_{-i})$*, which is a differentiable function of income* w_i*. The marginal propensity to consume the public good is greater than zero and less than one so that 0 <* $p_i f_i'(w_i + p_i G_{-i})$ *< 1 for all i = 1,…,n.*

The latter assumption simply requires that both the public and the private good be normal goods for all countries.[6] We use Nash "reaction functions" for the proofs.

The demand function $f_i(w_i + p_i G_{-i})$ ignores the inequality constraint ($G \geq G_{-i}$ in (1)). Then country i's demand for the public good, taking the inequality constraint into account, is simply:

$$G = \max\{f_i(w_i + p_i G_{-i}), G_{-i}\}. \qquad i = 1,2,\dots,n. \tag{2}$$

Subtracting G_{-i} from both sides of this equation, we have country i's optimal response, that is, the Nash reaction functions:

$$g_i = \max\{f_i(w_i + p_i G_{-i}) - G_{-i},\, 0\}. \qquad i = 1,2,\dots,n. \tag{3}$$

Theorem 1. *A Nash equilibrium exists.*

Theorem 2. *There is a unique Nash equilibrium, with a uniquely determined quantity of the public good and a unique set of contributing countries.*

Cornes and Hartley (2007) and Miyakoshi and Suzuki (2010) previously developed this international public good model with proofs of existence and uniqueness. The latter extends the familiar proofs of Bergstrom et al.'s (1986, 1992) public good model to international public good models, while the former proved it without requiring

[6] Bergstrom et al.'s (1986) public good model set $p_i = 1$ for all i in Assumptions 1 and 2. International public good models extend this to $0 < p_i$ for all i.

the use of a fixed point theorem but with expressing a transparent geometric representation, different from Bergstrom et al. (1986, 1992).

3. Numerical analysis for noncontributors and contributors

3.1. Algorithm

We analyze the model numerically and then provide an algorithm for the Nash equilibrium under the condition that country *i*'s utility function is the following Cobb–Douglas type: $U_i(x_i,G)=x_i^{\alpha_i}\bullet G^{\beta_i};\ \alpha_i>0,\ \beta_i>0,\ 1\geq\alpha_i+\beta_i$.[7] The Nash equilibrium solution in (1) is rewritten as the set of $\{G^*, x_1^*, x_2^*,.., x_n^*\}$ satisfying the following condition:

$$G^* = \arg\max_G \left\{U_i(w_i + p_i G_{-i} - p_i G,\ G) \mid 0 \leq G_{-i} \leq G,\ 0 \leq x_i\right\},\ i = 1,2,...,n \quad (4)$$

where Warr (1983, p. 209) shows that x_i^* is uniquely decided by G^*.

Given G_{-i}, differentiating (4) with respect to G, the solution G^* can be solved as:[8]

$$G^* = \begin{cases} \dfrac{\beta_i(w_i + p_i G_{-i})}{p_i(\alpha_i + \beta_i)}, & G^* - G_{-i} > 0,\ i.e., \dfrac{\beta_i w_i}{p_i \alpha_i} > G_{-i} \\ G_{-i}\qquad , & \dfrac{\beta_i w_i}{p_i \alpha_i} \leq G_{-i} \end{cases}. \quad (5)$$

Using $r_i \equiv \dfrac{\alpha_i}{\alpha_i + \beta_i}$, $q_i \equiv \dfrac{\beta_i w_i}{p_i(\alpha_i+\beta_i)}$, $\hat{w}_i \equiv \dfrac{w_i}{p_i}\dfrac{\beta_i}{\alpha_i}$, the solution can be rewritten as:

$$G^* = \begin{cases} q_i + (1-r_i)G_{-i}, & \hat{w}_i > G_{-i} \\ G_{-i}\qquad , & \hat{w}_i \leq G_{-i} \end{cases}. \qquad i = 1,2,...,n. \quad (6)$$

We rewrite (6) in terms of g_i^*:[9]

$$g_i^* = \begin{cases} \dfrac{q_i - r_i G^*}{1-r_i} = \dfrac{w_i}{p_i} - \dfrac{\alpha_i}{\beta_i}G^*, & \hat{w}_i > G^* \\ 0\quad , \quad \hat{w}_i \leq G^* \end{cases} \qquad ,i = 1,2,...,n \quad (7)$$

$$where \quad \frac{r_i}{1-r_i} = \frac{\alpha_i/(\alpha_i+\beta_i)}{1-\alpha_i/(\alpha_i+\beta_i)} = \frac{\alpha_i}{\beta_i},\ \ \frac{q_i}{1-r_i} = \frac{w_i}{p_i}$$

When only *k* of the *n* countries contribute to the public good, $G_k^* = g_1^* + g_2^* +,...,+ g_k^*$, then because of (7), G_k* is:

[7] We can apply our methodology to the CES function.

[8] The second order condition is obviously satisfied.

[9] As shown in Warr (1983, p. 209), g_i^* is uniquely decided by G_k* through x_i^*. In the case of the Cobb–Douglas utility function, g_i^* is a linear function of G_k*.

$$G_k^* = \sum_{i=1}^{k} g_i^* = \sum_{i=1}^{k} \frac{w_i}{p_i} - G_k^* \sum_{i=1}^{k} \frac{\alpha_i}{\beta_i} \quad , \quad hence\ G_k^* = \frac{\sum_{i=1}^{k} w_i / p_i}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} = \nu_k \pi_k , \qquad (8)$$

$$where\ \nu_k = \frac{1}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i}\ and\ \pi_k = \sum_{i=1}^{k} w_i / p_i$$

Therefore, using (7), (8) and the budget constraint in (1), g_i^*, x_i^* and utility for contributor *i* are uniquely decided by G_k^* as follows:

$$g_i^* = \frac{w_i}{p_i} - \left(\frac{\sum_{i=1}^{k} w_i / p_i}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} \right) \frac{\alpha_i}{\beta_i} > 0 , \quad x_i^* = p_i \left(\frac{\sum_{i=1}^{k} w_i / p_i}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} \right) \frac{\alpha_i}{\beta_i} , \quad U_i(x_i^*, G_k^*) . \ (9)$$

The g_i^* for the contributor is increasing with real income w_i / p_i and decreasing with preferences α_i / β_i. On the other hand, g_i^*, x_i^* and utility for noncontributors *i* are as follows:

$$g_i^* = 0, \quad x_i^* = w_i , \quad U_i(w_i, G_k^*) . \qquad (10)$$

The utility of the contributors and noncontributors is increasing together with *G*.

How do we search for *k* to decide G^*? First, we have to provide the following theorem.

Theorem 3. *Suppose* $\hat{w}_k \geq \hat{w}_{k+1}$. *When* $g_k^* = 0$, $g_{k+1}^* = 0$. *However, when* $g_{k+1}^* > 0$, $g_k^* > 0$.

Proof: When $g_k^* = 0$, $\nu_k \pi_k = G_k^* \geq \hat{w}_k$ because of (7). Then, because of $\hat{w}_k \geq \hat{w}_{k+1}$,

$$\frac{\sum_{i=1}^{k} w_i / p_i}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} \equiv \nu_k \pi_k \equiv G_k^* \geq \hat{w}_k \geq \hat{w}_{k+1} \equiv \frac{w_{k+1} / p_{k+1}}{\alpha_{k+1} / \beta_{k+1}}$$

. Rearranging both sides in this inequality,

$$\frac{\sum_{i=1}^{k} w_i / p_i + w_{k+1} / p_{k+1}}{w_{k+1} / p_{k+1}} \geq \frac{1 + \sum_{i=1}^{k} \alpha_i / \beta_i + \alpha_{k+1} / \beta_{k+1}}{\alpha_{k+1} / \beta_{k+1}} . \text{ Then } \frac{\sum_{i=1}^{k+1} w_i / p_i}{w_{k+1} / p_{k+1}} \geq \frac{1 + \sum_{i=1}^{k+1} \alpha_i / \beta_i}{\alpha_{k+1} / \beta_{k+1}} , \text{ i.e., } \pi_{k+1} \nu_{k+1} \geq \hat{w}_{k+1} .$$

Then, $G_{k+1}^* = \nu_{k+1} \pi_{k+1} \geq \hat{w}_{k+1}$ and hence, because of (7), $g_{k+1}^* = 0$. Furthermore, when $g_{k+1}^* > 0$, $\hat{w}_{k+1} > G_{k+1}^* = \nu_{k+1} \pi_{k+1}$ and then $g_k^* > 0$. **Q.E.D.**

Next, the algorithm of *k* is as follows.

Algorithm of *k*

Step 0: Assume the subscript *k* is attached to $\hat{w}_i$ in ascending order:

$\hat{w}_1 \geq \hat{w}_2 \geq ... \geq \hat{w}_k \geq \hat{w}_{k+1} .... \geq \hat{w}_n$. Suppose k = 1.

Step 1: Solve G_k^* in (8).

Step 2: Stop if the condition ($\hat{w}_k > G_k^* \equiv \nu_k \pi_k \geq \hat{w}_{k+1}, i.e., \hat{w}_k > G_k^* \equiv \nu_k \pi_k \text{ and } G_{k+1}^* \equiv \nu_{k+1}\pi_{k+1} \geq \hat{w}_{k+1}$) is satisfied.[10] Otherwise $k = k + 1$ and repeat Step 1.

We will rationalize the algorithm of k. For any given k, in Step 2, when $\hat{w}_k > G_k^*$, $g_k^* > 0$ because of (7). Applying Theorem 3 gives us $g_{k-1}^* > 0$ and then $g_i^* > 0$ for all i from 1 to k–2. Moreover, when $G_{k+1}^* \equiv \nu_{k+1}\pi_{k+1} \geq \hat{w}_{k+1}$, $g_{k+1}^* = 0$ because of (7). Applying Theorem 3 leads to $g_{k+2}^* = g_{k+3}^* = ... = g_n^* = 0$. The algorithm stops at k. Finally, we must confirm that this algorithm for k leads to the equilibrium. The algorithm of k obviously provides the equilibrium in (1). Moreover, the equilibrium is unique for a given set of parameters, as proved by Theorem 2. The schematic diagram for this logic is illustrated in Figure 1.

[Insert Figure 1 here]

Economic rationale of the algorithm

We now provide an economic interpretation of this algorithm, focusing on Step 2. What is the economic meaning of $\hat{w}_k > G_k^*$ in (7)? The marginal utility of G in (4) is $G^{\beta_k - 1}(w_k + p_k G_{-k} - p_k G)^{\alpha_k - 1}[\beta_k(w_k + p_k G_{-k} - p_k G) - \alpha_k p_k G]$. Then, when the third term in this marginal utility is positive (i.e., $\beta_k w_k - \alpha_k p_k G > \beta_k p_k g_k$), this marginal utility with $g_i > 0$ becomes positive. That is, a positive g_k is equivalent to $\beta_k w_k - \alpha_k p_k G_k^* > 0, i.e., \hat{w}_k > G_k^*$ (which shows that the marginal utility is positive). Given G_{-k}, the $g_k > 0$ can be decided until the marginal utility of $g_i \equiv G - G_{-i}$ is zero. Assume $\hat{w}_1 > ... > \hat{w}_{k-1} > \hat{w}_k > G_k^* \equiv \nu_k \pi_k > \hat{w}_{k+1} > ... > \hat{w}_n$. Then, because the marginal utility from country 1 to k is positive, these countries contribute to the public good. However, the negative marginal utility of countries k + 1 to k + n means that these countries do not contribute. This is because $\hat{w}_k > G_k^* \equiv \nu_k \pi_k \geq \hat{w}_{k+1}, i.e., \hat{w}_k > G_k^* \equiv \nu_k \pi_k \text{ and } G_{k+1}^* \equiv \nu_{k+1}\pi_{k+1} \geq \hat{w}_{k+1}$, as proved in Step 2.

Characteristics of the algorithm

First, the solutions for G^* and x_i^* for all i depend on the parameters of countries 1 to k (contributors), not the noncontributors: see (8), (9) and (10). Second, k (the number

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[10] Because $G_k^* \equiv \nu_k \pi_k = \dfrac{\sum_{i=1}^k w_i / p_i}{1 + \sum_{i=1}^k \alpha_i / \beta_i} \geq \hat{w}_{k+1} = \dfrac{w_{k+1}/p_{k+1}}{\alpha_{k+1}/\beta_{k+1}}$,

$$\frac{\sum_{i=1}^k w_i / p_i + w_{k+1}/p_{k+1}}{w_{k+1}/p_{k+1}} \geq \frac{1 + \sum_{i=1}^k \alpha_i / \beta_i + \alpha_{k+1}/\beta_{k+1}}{\alpha_{k+1}/\beta_{k+1}}$$ and then

$$\frac{\sum_{i=1}^{k+1} w_i / p_i}{w_{k+1}/p_{k+1}} \geq \frac{1 + \sum_{i=1}^{k+1} \alpha_i / \beta_i}{\alpha_{k+1}/\beta_{k+1}}, \; i.e., \; G_{k+1}^* \equiv \pi_{k+1}\nu_{k+1} \geq \hat{w}_{k+1}.$$

of contributors) is decided by all parameters of all countries through $\hat{w}_i \equiv \frac{w_i}{p_i}\frac{\beta_i}{\alpha_i} : i = 1, 2, ..n$. Third, when $\hat{w}_1 \ge \hat{w}_2 \ge ... \ge \hat{w}_k \ge \hat{w}_{k+1} .... \ge \hat{w}_n$, the top *k* countries are contributors. Then, the composition of $\hat{w}_i \equiv \frac{w_i}{p_i}\frac{\beta_i}{\alpha_i}$ suggests that the more income w_i the country has, the lower the price of the public good for the country, the more preferable the public good (β_i *much larger than* α_i) and thus the more likely the country is to be a contributor.

4. Percentage of noncontributors: Contribution by all countries in the post-Kyoto Protocol

We apply our model to proposed 'contributions by all countries' in the post-Kyoto Protocol. Most of the papers on international public good models consider only the contributors. However, we show the significant possibility of the appearance of noncontributors. Importantly, one of the recent proposals related to international public goods is "contribution by all countries" as seen in the post-Kyoto Protocol. In this sense, it is important to analyze the characteristics of noncontributors.

We investigate numerically the characteristics of switching between the noncontributors and contributors: the effects of the number of countries, diffusion of income, prices and preference parameters, and income transfers on the percentage of noncontributors. In addition, we find that the ratio of contributors can be increased by using income transfers. We obtain three new findings below.

4.1. Effects of the number of countries

First, we show that an increase in the number of countries decreases the percentage of contributors (contributors/countries). The income w_i for each country follows an identical independent uniform distribution, $U \sim U[0,1]$.[11] The income variable is defined as follows:

$$w_i^{pre} = 10 + 5U \Rightarrow normalized\ \ w_i = (w_i^{pre}) \bullet 10 / \sum_{j=1}^{n} w_j^{pre} . \qquad (11)$$

Then, the mean of the *n* sample countries is $E(\sum_{i=1}^{n} w_i) = 10$.[12] Similarly, the price variable and preference uniform distribution, $U \sim U[0,1]$, is defined as follows:

$$\begin{aligned} &p_i = 1.2 + 0.2U; E(p_i) = 1.3 \\ &\beta_i = 0.55 + 0.1U; E(\beta) = 0.6,\ \ \alpha = 1 - \beta . \end{aligned} \qquad (12)$$

We conduct the simulations under these distributions. We select each value of *U* randomly and independently from the uniform distribution $U \sim U[0,1]$. In total, we select 20 *U*s and hence one vector of $(w_1, p_1, \beta_1, w_2, p_2, \beta_2, ..., w_{20}, p_{20}, \beta_{20})$. Using this vector (trial 1), we calculate $(x_i, g_i, U_i(x_i, G))$ for each of the 20 countries and depict those values in Figure 2-a. The other cases for *n* = 100 are depicted in Figure 2-b. Under the

[11] We also used a normal distribution, yet the results did not change.
[12] We normalized this expression to get an average of 10.

uniform distribution, the ratio of the number of contributors *k* and sample countries *n* is *k*/*n* = 9/20, *k*/*n* = 20/100. The ratio of contributors (noncontributors) decreases (increases), depending on the increased number of sample countries.

[Insert Figure 2-a, 2-b here]

We check the robustness of this result using 200 trials. Note that we implement the trials for only one of the three parameters (w_i, p_i, β_i) at a time and the other two parameters are held constant at the starting values $w_i = 10, p_i = 1.2, \beta_i = 0.6$. For example, with *n* = 20, we select random values of w_i for each of the 20 countries, combining these values with $p_i = 1.2, \beta_i = 0.6$. Thus, we change only income w_i for each country. When we implement this trial, we can get $(w_1, 1.2, 0.6, w_2, 1.2, 0.6, ..., w_{20}, 1.2, 0.6)$, and calculate the number *k* of contributors. After this trial, we calculate the standard deviation of $(w_1, w_2, ..., w_n)$, which appears on the horizontal axis in Figure 3, and the number of contributors *k* appears on the vertical axis. Using this procedure, we run 200 trials. When we compare the average of *k* for *n* = 20 in each trial (Figure 3-a) and for *n* = 100 (Figure 3-b), we discover that the ratio of *k*/*n* is decreasing depending on the size of n, *k/n*=8.3/20 and 16.3/100. However, we change only the price or the preferences for each country, respectively. We get 200 trials with the values $(10, p_1, 0.6, 10, p_2, 0.6, ..., 10, p_n, 0.6)$ and $(10, 1.2, \beta_1, 10, 1.2, \beta_2, ..., 10, 1.2, \beta_n)$, respectively, and 200 trials for each n=20 and n=100. The same findings as w are found, while the figures are not shown in the text: the values of *k*/*n* are 13.2/20 and 34.2/100 for *p*, and 9.6/20 and 24.3/100 for β. Why? When the number of countries increases, the deviation of the randomly selected parameters increases. Several contributors that make large contributions toward the public good cause several countries to be noncontributors, as explained in the next paragraph.

4.2. Effects of diffusion of income, price and preference parameters

Second, we discover from Figure 3 that *k* decreases as the standard deviation of the income parameters $(w_1, w_2, ..., w_n)$ increases, for both 20 and 100 countries. The same fact is found for larger standard deviations of price and preference parameters. We can now confirm the reason for the first result. For example, even though the number of countries is fixed, the larger standard deviation of the income parameters leads to the following result: several contributors making large contributions to the public good increase the number of noncontributors.

[Insert Figure 3-a, 3-b here]

4.3. Income transfers

How do we increase the percentage of contributors using policy?

First, we consider this problem by using income transfers from less productive country *j* to more productive country *i* ($p_i < p_j$), which have been analyzed in previous studies, including Ihori (1996). Previous studies show that an income transfer *t* from *j* to *i* produces the following change:

$$\hat{w}_i \equiv \frac{w_i}{p_i}\frac{\beta_i}{\alpha_i} \ \ to\ \ \hat{w}_i' \equiv \frac{w_i + t}{p_i}\frac{\beta_i}{\alpha_i} \ \ for\ i, \quad \hat{w}_j \equiv \frac{w_j}{p_j}\frac{\beta_j}{\alpha_j} \ \ to\ \ \hat{w}_j' \equiv \frac{w_j - t}{p_j}\frac{\beta_j}{\alpha_j} \ \ for\ \ j, \tag{13}$$

at Step 2 in the algorithm for *k*. Moreover:

$$\pi_k \ to\ \pi_k' \equiv \pi_k + \frac{t}{p_i} - \frac{t}{p_j} > \pi_k \ \ and\ \ \nu_k \ to\ \nu_k, \ where\ \nu_k = \frac{1}{1+\sum_{i=1}^{k}\alpha_i/\beta_i} \ and\ \pi_k = \sum_{i=1}^{k}\frac{w_i}{p_i} \tag{14}$$

Previous studies assume that after this income transfer, the status of *j*, *i* and *k* are the same, e.g., contributors are contributors.[13]

$$\hat{w}_i, \ \hat{w}_j \gg \hat{w}_k > \pi_k \nu_k \equiv G_k^* \ \Rightarrow \ \hat{w}_i', \ \hat{w}_j' \gg \hat{w}_k > \pi_k' \nu_k \equiv G_k'^*, \tag{15}$$

where obviously $\pi_k' \nu_k \geq \pi_k \nu_k \geq \hat{w}_{k+1}$ because of (14).

Because $G_k'^* = \pi_k' \nu_k > \pi_k \nu_k = G_k^*$ in (15), the consumption of G_k^* *and* x_i^* of the public good and the private good by contributors including *i* and *j* increases and then their utility increases because of (8) and (9).[14] The utility and public good consumption of noncontributors also increases because of (10), while the consumption of the private good is constant and equal to its income.

The following income transfer *t* rationalizes the assumption that *k* is still a contributor in (15).

$$\hat{w}_k > \pi_k' \nu_k \equiv G_k'^*, \ \ that\ \ is, \quad \hat{w}_k / \nu_k - \pi_k > t(1/p_i - 1/p_j) \,. \tag{16}$$

The income transfer increases more. Then, as shown at Step 2 of the algorithm for *k*, the *k*–1 is a contributor but *k* becomes a noncontributor when:

$$\hat{w}_{k-1} > G_k'^* = \pi_{k-1}' \nu_{k-1} = (\pi_{k-1} + t(1/p_i - 1/p_j))\nu_{k-1} \geq \hat{w}_k \,. \tag{17}$$

As a result, utility increases more for all countries, while the number of contributors decreases in terms of their income transfer.

Second, as similarly considered, the income transfer from productive country *i* to less productive country *j* increases the number of contributors, decreasing the utility. Finally, the previously considered income transfer cannot lead to a situation where the utility and the ratio of contributors increase.

5. Provision of public goods and subsidies in an ODA policy

We apply our model to consider whether the provision of public goods or subsidies in an ODA policy is better. There are two types of ODA methods: subsidies

[13] We have already analyzed income transfers from noncontributors to contributors and from noncontributors to noncontributors. There results were not changed significantly.

[14] However, the provision of public good g_i for income receiver *i* increases but the provision of g_j for income sender *j* decreases as follows:

$$g_i^* = \frac{w_i + t}{p_i} - \left(\frac{(\sum_{i=1}^{k} w_i / p_i) + t(1/p_i - 1/p_j)}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} \right) \frac{\alpha_i}{\beta_i}, \quad \frac{\partial g_i^*}{\partial t} = (1/p_i)(1 - \nu_k \frac{\alpha_i}{\beta_i}) + (1/p_j)\nu_k \frac{\alpha_i}{\beta_i} > 0$$

$$g_j^* = \frac{w_j - t}{p_j} - \left(\frac{(\sum_{i=1}^{k} w_i / p_i) + t(1/p_i - 1/p_j)}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} \right) \frac{\alpha_j}{\beta_j}, \quad \frac{\partial g_j^*}{\partial t} = (1/p_j)(-1 + \nu_k \frac{\alpha_j}{\beta_j}) - (1/p_j)\nu_k \frac{\alpha_j}{\beta_j} < 0$$

(money) and goods. The main categories of assistance in goods are health, education, water and sanitation, which are international public goods. Which assistance method is better for developing countries: international public goods or money?

Exogenous provision *t* of public goods

Suppose public good *t* is provided exogenously. Equation (7) is as follows:

$$U_i(w_i - p_i g_i, G): G \equiv g_i + G_{-i} + t$$

$$g_i^* = \begin{cases} \dfrac{w_i}{p_i} - \dfrac{\alpha_i G}{\beta_i}, & \hat{w}_i > G \\ 0 \quad , & \hat{w}_i \leq G \end{cases} \tag{18}$$

Therefore, using (18) solves $G_k'^*$ as follows:

$$G = t + \sum_{i=1}^{k} g_i^* = t + \sum_{i=1}^{k} \frac{w_i}{p_i} - G \sum_{i=1}^{k} \frac{\alpha_i}{\beta_i}, \text{ then } G_k'^* = v_k(t + \pi_k). \tag{19}$$

Then, by inserting (19) into (18) and using (1), the g_i^*, x_i^* and utility are uniquely determined by $G_k'^*$. The results are similar to those for (8), (9) and (10). However, the difference is at $G_k'^* = v_k(t + \pi_k)$ in (19) and $G_k^* = v_k \pi_k$ in (8). We focus on the last contributor *k*. Before contributing to the public good, $\hat{w}_k > v_k \pi_k \geq \hat{w}_{k+1}$. Afterwards, $v_k(t + \pi_k) > \hat{w}_{k+1}$. (i) When $\hat{w}_k > v_k(t + \pi_k) \geq \hat{w}_{k+1}$, *k* is still a contributor because of Step 2 at the algorithm for *k*. Moreover, by exogenous provision of the public good *t*, $G_k'^* = v_k(t + \pi_k) > v_k \pi_k = G_k^*$.

Then, all countries increase their consumption of public and private goods and utilities. However, (ii) when we consider a large exogenous provision of the public good, $v_k(\pi_k + t) > \hat{w}_k > \hat{w}_{k+1}$, which causes *k* to be a noncontributor. The contributors are from 1 to *k*–1.[15] However, the utilities of the contributors and noncontributors are increasing together with $G_k'^*$. In fact, $U_i(w_i - p_i g_i, G) = U_i(p_i \dfrac{\alpha_i G}{\beta_i}, G)$ for contributors because of (18).

Thus, the percentage of contributors decreases, and all countries increase their utility.

Exogenous provision of subsidies

Suppose a subsidy of $p_i \alpha_i t / \beta_i$ is provided exogenously to all countries. Equation (7) is as follows:

$$U_i(w_i + p_i \alpha_i t / \beta_i - p_i g_i, G): G \equiv g_i + G_{-i}$$

$$g_i^* = \begin{cases} \dfrac{\beta_i (w_i + p_i \alpha_i t / \beta_i) - \alpha_i p_i G}{\beta_i p_i}, & \hat{w}_i + t > G \\ 0 \quad , & \hat{w}_i + t \leq G \end{cases} \tag{20}$$

[15] When $t \geq \hat{w}_1$, there are no contributors.

Therefore, using (20) solves $G_k'^*$ as follows:

$$G=\sum_{i=1}^{k} g_i^* = \sum_{i=1}^{k}\frac{w_i}{p_i}+(t-G)\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i},\ then\ G^*=\nu_k(t\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}+\pi_k)\ . \tag{21}$$

Then, by inserting (21) into (20) and using (1), g_i^*, x_i^* and utility are uniquely decided by $G_k'^*$. One of the conditions for the last contributor *k* is the following, as shown at Step 2 of the algorithm for k:

$$\hat{w}_k+t>G_k^* \Leftrightarrow \hat{w}_k+t>\nu_k(t\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}+\pi_k)\Leftrightarrow \hat{w}_k>t(\nu_k\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}-1)+\nu_k\pi_k=\nu_k\pi_k-t\nu_k\ .\text{[16]} \tag{22}$$

Moreover, another condition for it is as follows:

$$G_k^*=\nu_k(t\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}+\pi_k)\geq \hat{w}_{k+1}+t\text{ , i.e., } \pi_k-\hat{w}_{k+1}/\nu_k\geq t\ .\text{[17]} \tag{23}$$

We rewrite the relations of (22) and (23) as follows:

$$\hat{w}_k+t>G_k^*=\nu_k(t\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}+\pi_k)\geq \hat{w}_{k+1}+t,\ that\ is,\ \pi_k-\hat{w}_{k+1}/\nu_k\geq t>\pi_k-\hat{w}_k/\nu_k\ . \tag{24}$$

However, the subsidy *t* increases gradually until $t\geq \pi_k-\hat{w}_{k+1}/\nu_k=\pi_{k+1}-\hat{w}_{k+1}/\nu_{k+1}$,[18] and then the subsidy *t* is located in the following:

$$\pi_{k+1}-\hat{w}_{k+2}/\nu_{k+1}>t\geq\pi_{k+1}-\hat{w}_{k+1}/\nu_{k+1},\ i.e.,\ \hat{w}_{k+1}+t>G_{k+1}^*=\nu_{k+1}(t\sum_{i=1}^{k+1}\frac{\alpha_i}{\beta_i}+\pi_{k+1})\geq\hat{w}_{k+2}+t\ ,\text{[19]} \tag{25}$$

where the contributors are the countries from 1 to *k* + 1. Therefore, the number of contributors increases.[20]

What are the values of $x_i^*, g_i^*, G_k^*\ and\ utility\ for\ \ i$? Because of (20):

$$g_i^*=\frac{\beta_i(w_i+\alpha_i p_i t/\beta_i)-\alpha_i p_i G_k^*}{\beta_i p_i},\quad x_i^*=w_i+p_i\alpha_i t/\beta_i-p_i g_i^*=\frac{\alpha_i}{\beta_i}G_k^*. \tag{26}$$

When *k* is still the last contributor, G_k^* increases together with the increase in *t* because of (21) and then x_i^* increases because of (26).This fact shows that the utility increases for contributors. For noncontributors, it is obvious that their utility increases. Furthermore, the g_i^* for contributors increases because of (20), while those of noncontributors are constant.

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[16] $t-t\nu_k\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}=t[\,1-(\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i})/(1+\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}\ )\,]=t\nu_k\ .$

[17] The same relation as footnote 11 is used.

[18] $$\begin{aligned}&\pi_{k+1}-\hat{w}_{k+1}/\nu_{k+1}=\pi_k+w_{k+1}/p_{k+1}-(w_{k+1}/p_{k+1})(\beta_{k+1}/\alpha_{k+1})(1/\nu_k+\alpha_{k+1}/\beta_{k+1})\\&=\pi_k-\hat{w}_{k+1}/\nu_k+w_{k+1}/p_{k+1}-w_{k+1}/p_{k+1}=\pi_k-\hat{w}_{k+1}/\nu_k\end{aligned}.$$

[19] The equivalence of this relation is induced because of (24).

[20] When $t\geq\pi_{n-1}-\hat{w}_n/\nu_{n-1}$, all countries are contributors.

When k + 1 becomes a new contributor, what happens? Set t to produce G^*_{k+1} and then set $t - \alpha$ less than t ($\alpha > 0$) to produce G^*_k because of (24) and (25). The difference between total public goods G^*_k and G^*_{k+1} under contributor k and contributor k + 1 is:

$$\begin{aligned}
&\nu_{k+1}(t\sum_{i=1}^{k+1}\frac{\alpha_i}{\beta_i}+\pi_{k+1})-\nu_k((t-\alpha)\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}+\pi_k)\\
&=t(\nu_{k+1}\sum_{i=1}^{k+1}\frac{\alpha_i}{\beta_i}-\nu_k\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i})+\nu_{k+1}\pi_{k+1}-\nu_k\pi_k+\nu_k\alpha\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}=t\nu_{k+1}\nu_k\frac{\alpha_{k+1}}{\beta_{k+1}}-(\nu_k\pi_k-\nu_{k+1}\pi_{k+1})+\nu_k\alpha\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}\\
&=t-\frac{\beta_{k+1}}{\alpha_{k+1}}(\frac{\pi_k}{\nu_{k+1}}-\frac{\pi_{k+1}}{\nu_k})+\nu_k\alpha\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}=t-\frac{\beta_{k+1}\pi_k}{\alpha_{k+1}}(\frac{1}{\nu_{k+1}}-\frac{1}{\nu_k})-\frac{\beta_{k+1}}{\alpha_{k+1}}\frac{\frac{w_{k+1}}{p_{k+1}}}{\nu_k}+\nu_k\alpha\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}\\
&=t-\frac{\pi_k\beta_{k+1}}{\alpha_{k+1}}\frac{\alpha_{k+1}}{\beta_{k+1}}-\frac{\beta_{k+1}}{\alpha_{k+1}}\frac{\frac{w_{k+1}}{p_{k+1}}}{\nu_k}+\nu_k\alpha\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}=t-(\pi_k-\hat{w}_{k+1}/\nu_k)+\nu_k\alpha\sum_{i=1}^{k}\frac{\alpha_i}{\beta_i}
\end{aligned} \quad . \quad (27)$$

This difference is positive, because of the condition for the last contributor k + 1 seen in (25).

Then, the total public good G^* is increasing together with the number of contributors, and then the utility for all countries increases.

The common and contrast effects between the exogenous provision of public goods and subsidies from outside are as follows. Both ODA policies increase the utility for all countries. However, the policy of exogenous subsidies increases the number of contributors, while the policy of exogenous provision of public goods decreases it.

6. Concluding remarks

This paper developed an algorithm to calculate the percentage of noncontributors in the presence of both noncontributors and contributors for an international public good model with voluntary contribution. We applied this analysis to various issues in the post-Kyoto Protocol, which emphasizes that all countries should be contributors: effects of the number of countries, the diffusion of income, prices, preference parameters and income transfers among countries on the percentage of contributors. We also applied it to the ODA policy to determine the better assistance method: exogenous provision of public goods or exogenous income subsidies to all countries.

The application of the model provided the following findings about the issue of all countries being contributors in the post-Kyoto Protocol. First, the ratio of noncontributors (free riders) increases with increases in the number of sample countries. This is because when the number of countries increases, the deviation of the parameters selected randomly increases. Several contributors that provide large contributions toward the public good cause several countries to be noncontributors. However, the percentage of noncontributors among all countries increases with larger standard deviations of income, prices and preference parameters. Second, an income transfer from a less productive country to a more productive country increases the utility of all countries, while the number of contributors decreases. Third, regarding the ODA policy, the provision of international public goods and income subsidies both increase the utility for all countries. Fourth, income subsidies increase the number of contributors, while provision of public goods decreases it. The principal of all countries being contributors in the post-Kyoto Protocol is consistent with income subsidies. These findings hold for international public good models with voluntary contributions.

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Figure 1

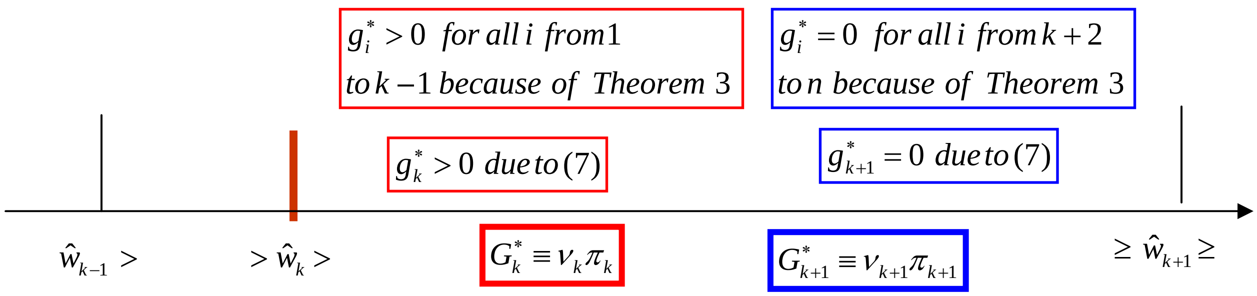

$g_i^* > 0$ for all i from 1 to $k-1$ because of Theorem 3
$g_i^* = 0$ for all i from $k+2$ to n because of Theorem 3
$g_k^* > 0$ due to (7)
$g_{k+1}^* = 0$ due to (7)
$\hat{w}_{k-1} >$
$> \hat{w}_k >$
$G_k^* \equiv v_k \pi_k$
$G_{k+1}^* \equiv v_{k+1} \pi_{k+1}$
$\geq \hat{w}_{k+1} \geq$

Figure 2-a

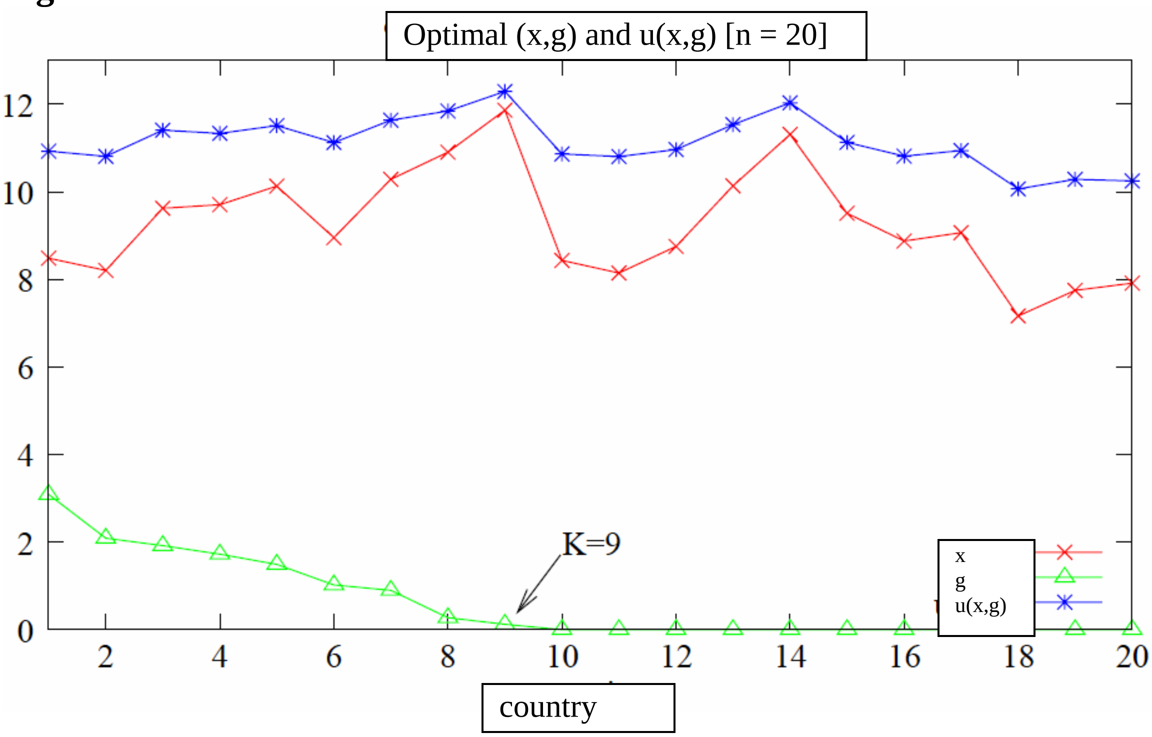
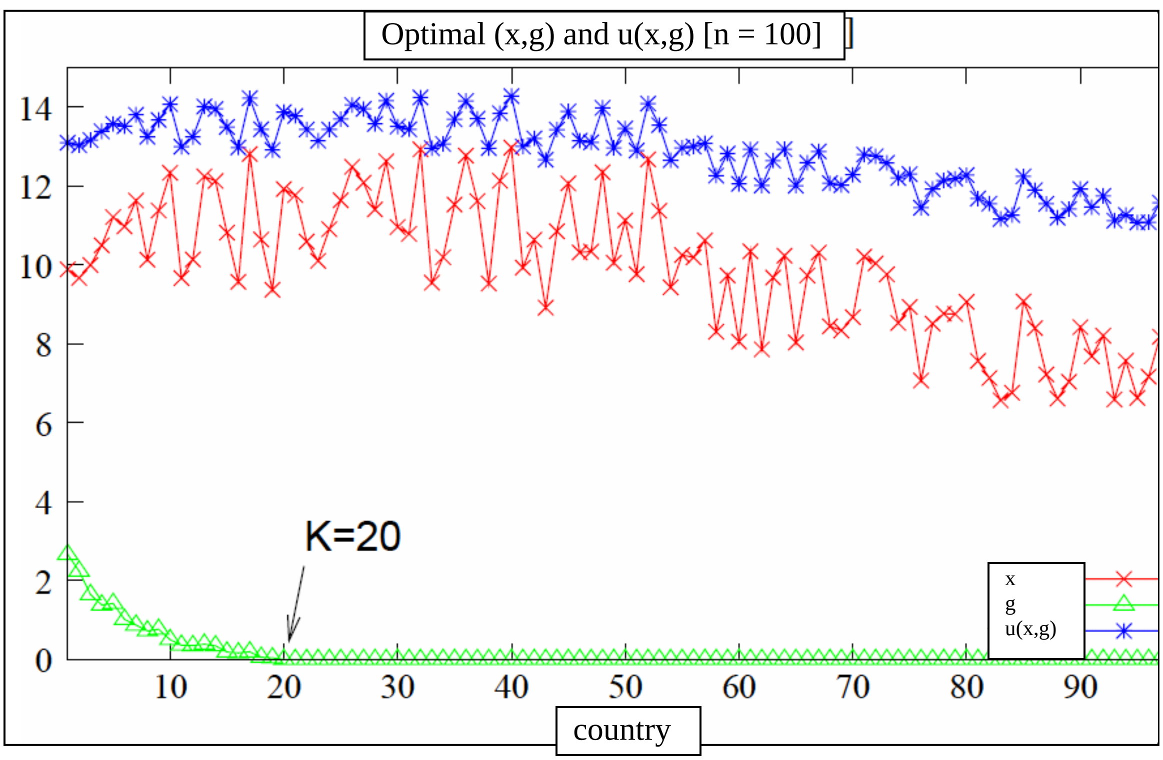


Figure 2-b



Note: x, g and u(x,g) denote provision of public good, consumption and utility of each country, respectively: u(x,g) $\equiv u(x_i, g_i + \sum_{i \neq j} g_j)$.

Figure 3-a

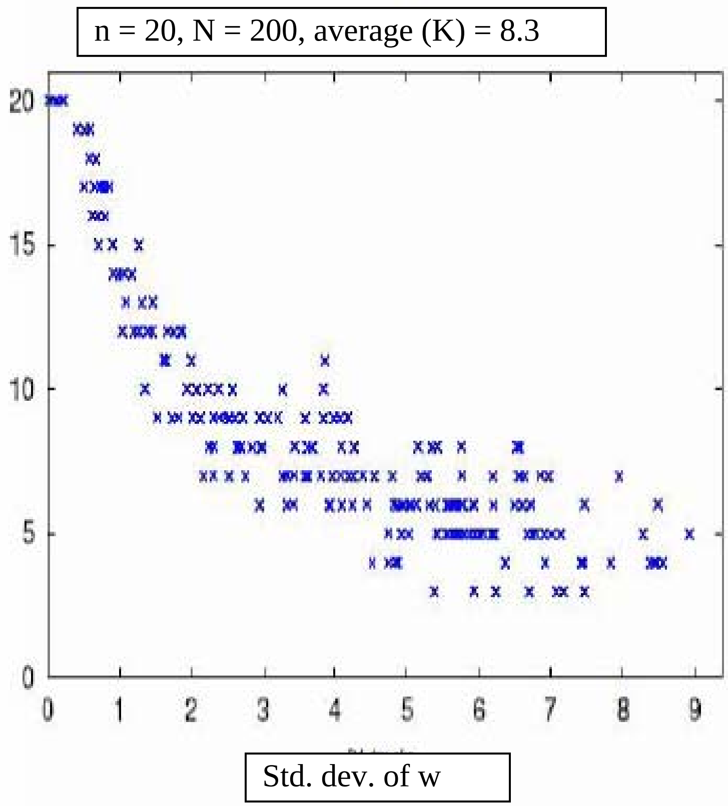
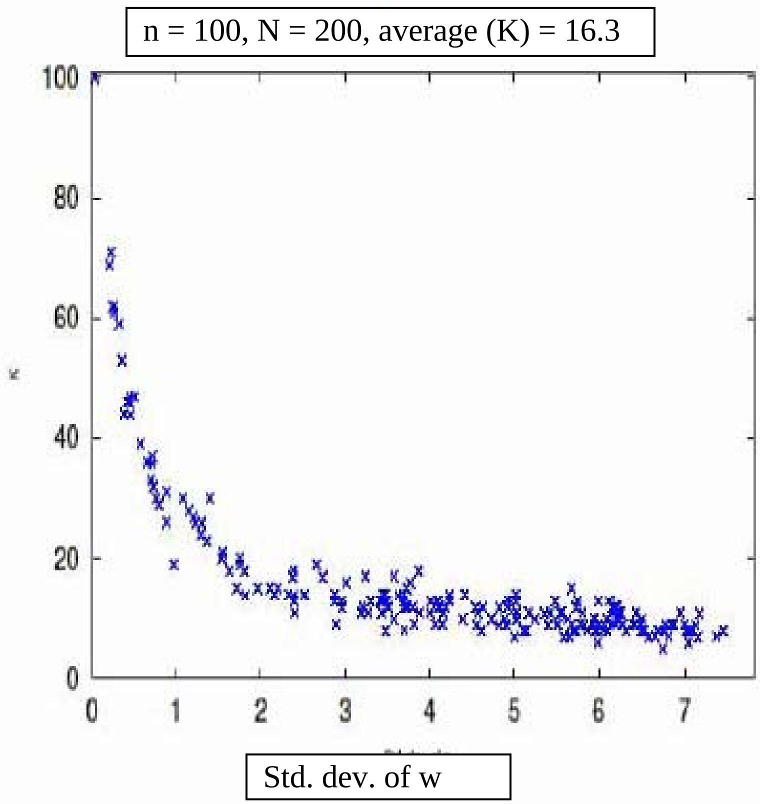


Figure 3-b



Note: Std. dev, n, N, and average (K) denote standard deviation, number of countries, number of simulations and average number of contributors in each simulation, respectively.