On the Provision of International Public Goods in a Dynamic Global Economy*

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Abstract

This paper develops a dynamic two-country model with an international public good, the stock of which has a positive effect on the private sector's productivity in each country and the evolution of the stock is determined by each country's voluntary contribution. Two private goods exist in this economy, and it is shown that the country with a higher contribution technology becomes an exporter of the good which is more dependent on the stock of international public good. It is also shown that if the countries act cooperatively, the dynamics of the stock of international public good and its shadow price under free trade coincide with those under autarky, although the paths of each country's contribution level are different. Specifically, the contribution level in the country with a lower contribution technology becomes smaller under free trade than under autarky, and it is shown that this country unambiguously gains from trade. In the noncooperative regime, free trade achieves a larger steady-state stock of international public good than autarky.

Key Words: International public good; Two-country trade; Trade patterns; Cooperative solution; Open-loop Nash equilibrium

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1 Introduction

The world economy is benefited from various kind of public goods that are in the transnational or global dimension. Because there is no coercive authority that can enforce an efficient supply of such international public goods by collecting tax from sovereign countries to finance the cost of supply, the provision of such goods is implemented through voluntary contribution made by countries that are benefited from the goods in question. Examples of such international public goods include transnational communication systems such as the Internet, international organizations such as the United Nations, military alliances such as the North Atlantic Treaty Organization, and reduction of greenhouse gas emissions to prevent global warming.

Public goods, regardless of local, national, or transnational ones, have external effects that benefit the economy by raising consumers' utility levels directly or make the economy better off indirectly by augmenting private firms' productivity. With respect to the latter kind of public goods, i.e., public goods that have external effects on productivity, these goods generally have a characteristic of durable or capital goods; scientific knowledge, and transportation and communication infrastructures are typical examples. That is, it will be more reasonable to consider public goods that can be built up over time when the stock of such goods has positive external effects the production side.

This paper considers a global economy consisting of two countries in the presence of an international public good, the stock of which has a positive effect on the private sector's productivity in each country and the evolution of the stock is determined by each country's voluntary contribution. A distinctive feature when considering international rather than local or national dimension is that there can be trade in goods or factor movements between countries. It is of great interest whether and how the presence of international economic transactions affect each country's contribution behavior.[1] Therefore, I assume that two private goods exist in this world economy and these goods can be traded between countries. If the private goods are traded between countries, their relative price is endogenously determined in the world market. Under free trade, the countries take into consideration the effect of a change in the world price on the national welfare as well as the effect of a change in the stock of international public good and its shadow

[1] In static trade models with public goods, some interesting results may emerge. Suga and Tawada (2007) show that if the government of a country adopts the Lindahl pricing rule, the country may lose from trade. By contrast, Shimomura (2007) shows that the classical gains-from-trade proposition is still valid if the governments in the trading world behave strategically with respect to the provision of public goods. Long and Shimomura (2007) show that the well-known neutrality theorem, i.e., the neutrality of small redistribution of wealth on the Nash equilibrium allocation in the voluntary public-good contribution game, no longer holds if countries take into account the effect of their respective contributions on the world relative price of private goods.

price.

Section 2 provides a brief review of the literature related to the present paper. Section 3 sets up the two country, two private-goods dynamic model with an international public good. Section 4 considers an autarkic situation where there is no trade between countries. Both cooperative solution, where the countries jointly determine their contribution levels for the investment in the international public good in order to maximize the world welfare, and noncooperative equilibrium, where each country determines its contribution level in order to maximize it own national welfare, are derived. It is shown that under autarky the equilibrium price of the good which is more dependent on the stock international public good becomes lower in the country that has a higher contribution technology, regardless of whether the countries cooperatively or noncooperatively determine their contribution levels. Section 5 analyzes outcomes under free trade. It is shown that if the countries act cooperatively, the dynamics of the stock of international public good and its shadow price under free trade coincide with those under autarky, although the paths of each country's contribution level are different. More specifically, trade liberalization increases the optimal contribution level in the country with a higher contribution technology and reduces the contribution level in the country with a lower contribution technology. This implies that in the country with a lower contribution technology, the total labor available in the production of private goods becomes larger under free trade than under autarky, and it is shown that this country unambiguously gains from trade. By contrast, the country with a higher contribution technology may lose from trade. In the noncooperative regime, free trade achieves a larger steady-state stock of international public good than autarky, and the gains-from-trade result is reinforced in the country with a lower contribution technology.

2 Literature Review

As noted in the Introduction, this paper considers an economy consisting of sovereign countries, each of which can make a decision independently in contributing to the investment on an international public good. Thus, in order to formulate the situation that this paper considers, applying the models of dynamic voluntary or private provision of public good is suitable.

Fershtman and Nitzan (1991) develop a differential game model with a public good, the stock of which has a direct welfare effect on private agents who make donations to the investment on the accumulation of the public good. In this dynamic framework, the authors demonstrate the existence of a free-rider problem; compared to the Pareto optimal solution, each agent has an incentive to contribute less in the Nash equilibrium, which thus brings the economy about the lower stock of public good in the steady state. Moreover, the authors show that the free-rider problem becomes worse, compared with the case where agents are able to commit to a time path of contributions, when the agents use a strategy for their respective contribution level that is linearly dependent on the stock of public good. The Fershtman–Nitzan model is extended by Wirl (1996) and Itaya and Shimomura (2001). Wirl (1996) points out that the finding of Fershtman and Nitzan (1991) crucially depends on the chosen set of strategies, i.e., the restriction to the linear strategy, and shows that the outcome can be better when the agents use nonlinear strategies. Itaya and Shimomura (2001) consider a more general model than Fershtman and Nitzan (1991) and Wirl (1996), and elucidate the relationship between the steady state equilibria of a dynamic contribution game and conjectural variations equilibria of the corresponding static game.

A different kind of differential game model of voluntary public-good provision is presented by Shibata (2002), who develops an endogenous growth model with an infrastructure capital that is accumulated through voluntary investment by private agents, and analyzes how the presence of strategic interactions between the agents affects the long-run patterns of economic growth. He shows that the equilibrium dynamics are heavily dependent on the commitment behavior of agents; there exists a unique endogenous growth equilibrium if agents commit their announced paths of future actions, whereas multiple growth patterns emerge if agents condition their actions on the stock of infrastructure capital. Moreover, in the latter case, some of the long-run equilibria exhibit endogenous growth and others show no growth.

Although the above-mentioned studies do not explicitly take the international dimension into consideration, the models may be interpreted as ones with international public goods, where the economy consists of "countries" rather than individuals. However, such an interpretation seems hardly adequate because trade in goods between countries is assumed away in these models.

The present paper is also closely related to the literature on international trade in the presence of public intermediate goods that have positive effect on productivity in private production. There have been a number of studies dealing with this issue (Manning and McMillan, 1979; Tawada and Okamoto, 1983; Tawada and Abe, 1984; Ishizawa, 1988; Abe, 1990; Altenburg, 1992; Suga and Tawada, 2007). However, these studies are confined to a static framework. Dynamic models in which productivity effects of the stock of a public intermediate good exists and the national government determines the optimal path of the stock of public intermediate good are analyzed by McMillan (1978) and Yanase and Tawada (2012a, b).

McMillan (1978) shows that the stock of public intermediate good determines the slope of the production possibility frontier and thus determines the pattern of international trade. By re-examining McMillan's model, Yanase and Tawada (2012a) show the possibility of multiple steady states and history-dependent dynamic paths. In addition, Yanase and Tawada (2012a) discuss whether trade is gainful or not in McMillan's model. In both McMillan (1978) and Yanase and Tawada (2012a), the stock of public intermediate good is assumed to have an impact similar to the "creation of atmosphere" type externality classified by Meade (1952), where the technology of each private sector exhibits constant returns to scale in primary factors of production only. There is another class of public intermediate goods, which can be interpreted as "unpaid factors of production," again according to Meade's terminology (1952), where the production function of each private sector is characterized by constant returns to scale in all inputs, including the public intermediate good. Yanase and Tawada (2012b) develop a dynamic trade model with a stock of public intermediate good of this type. However, these studies consider a small open economy, where the price of goods are exogenously given, and take no account of international public goods. In other words, interactions between countries have not been analyzed.

3 Model

I consider a world economy consisting of two countries, home and foreign, in which two private goods, goods 1 and 2, are produced by using a single primary factor, labor. There is also an international public good, the stock of which has a positive external effect on the productivity of these goods. The investment on the accumulation of international public good is made by the government in each country, which takes the responses of the private sector and markets into consideration. It is assumed that firms and households are price takers, and total labor endowment in each country is given and constant over time.

3.1 Production side

Let us focus on the home country. The foreign country, whose variables are denoted with an asterisk (*), has a similar economic structure.

The production function of each private sector is assumed to take the following form:

$$Y_i = R^{\alpha_i} L_i^{1-\alpha_i}, \quad 0 \leq \alpha_i < 1, \quad i = 1, 2, \tag{1}$$

where Y_i is the output of good i, R is the stock of international public good, and L_i is the labor input in sector i. The parameter $\alpha_i \in [0, 1)$ denotes the production elasticity of the international public good in each sector: $\alpha_i = (\partial Y_i / \partial R) \cdot (R/Y_i)$.

In the following analysis, I make the following assumption regarding the impact of the stock of international public good to industries:

Assumption 1 $\alpha_1 > \alpha_2$, i.e., sector 1 is more dependent on the stock of international public good than sector 2.

Let us denote the labor contributed to the accumulation of international public good in the home country by L_R. Then, at each moment of time, the economy must face the following full employment constraint on labor:

$$L_1 + L_2 + L_R = L, \tag{2}$$

where $L > 0$ is labor endowment and is assumed to be given and constant over time.

Let $l \equiv L - L_R$ is the total labor inputs in the private sectors. Letting good 2 be a numeraire, the production side of the economy is characterized by the following GDP function:

$$G(p, R, l) = \max_{L_1, L_2} \left\{ pR^{\alpha} L_1^{1-\alpha} + R^{\alpha_2} L_2^{1-\alpha_2} \quad \text{s.t.} \quad L_1 + L_2 = l \right\}, \tag{3}$$

where p is the price of good 1. By applying the envelope theorem to the GDP function, it follows that the GDP function satisfies the following properties:[2]

$$G_p = Y_1, \quad G_R = \frac{\alpha_1 p Y_1 + \alpha_2 Y_2}{R}, \quad G_l = w, \tag{4}$$

where w denotes the wage. Appendix A.1 gives additional properties of the GDP function.

3.2 Consumption Side

The consumption side of the economy is described by a representative household, whose lifetime utility is given by:

$$U = \int_0^{\infty} e^{-\rho t} \left[\gamma \ln C_1 + (1 - \gamma) \ln C_2 \right] dt, \tag{5}$$

where C_i is consumption of good i ($i = 1, 2$), ρ is the rate of time preference, and $\gamma \in (0, 1)$ is a parameter. Let us denote the household's total expenditure at each moment of time by E, and assume that no borrowing or lending is permitted. Then, the household's optimal consumption must satisfy $C_1 = \gamma E/p$ and $C_2 = (1 - \gamma)E$.

With no international borrowing and lending, national income must equal total expenditure at all points in time: $E = G(p, R, l)$. Substituting the household's optimal consumption into the lifetime utility (5), its indirect lifetime utility is derived as

$$V = \int_0^{\infty} e^{-\rho t} \left\{ \ln[G(p, R, L - L_R)] - \gamma \ln p + \ln[\gamma^{\gamma}(1 - \gamma)^{1-\gamma}] \right\} dt. \tag{6}$$

The national welfare can be measured by the lifetime indirect utility (6).

[2]The subscripts denote partial derivatives: $G_p = \partial G / \partial p$, and so on.

3.3 International Public Good

Given the initial stock $R_0 > 0$, the international public good is assumed to accumulate over time according to the following differential equation:[3]

$$\dot{R} = \phi L_R + \phi^* L_R^* - \delta R, \tag{7}$$

where $\phi > 0$ denotes the technology of contribution to the accumulation of international public good in the home country and $\phi^* > 0$ is the foreign country's counterpart, and $\delta > 0$ is the depreciation rate of the stock of international public good.

4 Autarky

In this section, I assume that the two countries do not trade the goods, and derive the cooperative and noncooperative solutions under autarky.

4.1 Market equilibrium

Under autarky, at each moment in time, the demand for for each private good must be equal to the supply in each country. The market-clearing condition for good 1 in the home country, $C_1 = Y_1$, can be rewritten as

$$\frac{\gamma}{p} G(p, R, l) = G_p(p, R, l). \tag{8}$$

From (8), the autarkic equilibrium price of good 1 in the home country for a given pair of R and l is derived as $p_a = P^a(R, l)$, with the following properties:

$$P_R^a = \frac{\gamma G_R - pG_{pR}}{(1-\gamma)G_p + pG_{pp}}, \quad P_l^a = \frac{\gamma G_l - pG_{pl}}{(1-\gamma)G_p + pG_{pp}}. \tag{9}$$

I assume that the two countries share the identical preferences, meaning that $\rho = \rho^*$ and $\gamma = \gamma^*$, and the identical technologies for producing private goods, i.e., $\alpha_i = \alpha_i^*$, $i = 1, 2$. These assumptions imply that the autarkic equilibrium price in the foreign country is derived as $P_a^* = P^a(R, l^*)$.

4.2 Cooperative Solution

In this subsection, I derive a cooperative solution where the governments in both countries determine the paths of L_R and L_R^* in order to maximize the sum of these countries' welfare subject to the dynamics of international public good (7).

[3] A dot over a variable denotes the time derivative. To avoid unnecessary complication in the notation, we omit time arguments when no confusion arises from doing so.

the current value Hamiltonian associated with the world-welfare maximization problem is defined as

$$\begin{aligned}\mathcal{H}^w &= \ln[G(P^a(R, L - L_R), R, L - L_R)] - \gamma \ln[P^a(R, L - L_R)] \\ &\quad + \ln[G(P^a(R, L^* - L_R^*), R, L^* - L_R^*)] - \gamma \ln[P^a(R, L^* - L_R^*)] \\ &\quad + 2\ln[\gamma^\gamma(1-\gamma)^{1-\gamma}] + \theta^w(\phi L_R + \phi^* L_R^* - \delta R),\end{aligned}$$

where θ^w can be interpreted as the shadow price of the international public good in the world-welfare maximization problem. The first-order conditions for maximizing $\mathcal{H}^w$ are

$$\frac{\partial \mathcal{H}^w}{\partial L_R} = 0 \quad \Rightarrow \quad \frac{G_l(P^a(R, L - L_R), R, L - L_R)}{G(P^a(R, L - L_R), R, L - L_R)} = \theta^w \phi, \tag{10}$$

$$\frac{\partial \mathcal{H}^w}{\partial L_R^*} = 0 \quad \Rightarrow \quad \frac{G_l(P^a(R, L^* - L_R^*), R, L^* - L_R^*)}{G(P^a(R, L^* - L_R^*), R, L^* - L_R^*)} = \theta^w \phi^*, \tag{11}$$

where the market-clearing conditions in each country (8) are utilized. The adjoint equation is derived as

$$\begin{aligned}\dot{\theta}^w &= \rho\theta^w - \frac{\partial \mathcal{H}^w}{\partial R} \\ &= (\rho + \delta)\theta^w - \frac{G_R(p_a, R, L - L_R)}{G(p_a, R, L - L_R)} - \frac{G_R(p_a^*, R, L^* - L_R^*)}{G(p_a^*, R, L^* - L_R^*)},\end{aligned} \tag{12}$$

where $p_a = P^a(R, L - L_R)$ and $p_a^* = P^a(R, L^* - L_R^*)$, and the transversality condition is given by

$$\lim_{t\to\infty} e^{-\rho t}\theta^w(t)R(t) = 0. \tag{13}$$

From (10), the optimal contribution level in the home country can be expressed as $L_R = \lambda^a(\theta^w; \phi, L)$, with the derivatives[4]

$$\lambda_\theta^a = \frac{\phi G}{\left(\frac{G_l G_p}{G} - G_{lp}\right) P_l^a + \frac{(G_l)^2}{G} - G_{ll}}, \tag{14a}$$

$$\lambda_\phi^a = \frac{\theta^w G}{\left(\frac{G_l G_p}{G} - G_{lp}\right) P_l^a + \frac{(G_l)^2}{G} - G_{ll}}, \tag{14b}$$

$$\lambda_L^a = 1. \tag{14c}$$

As shown in Appendix A.2, both λ_ϕ^a and λ_L^a are positive, indicating that the optimal contribution level becomes larger in a country with higher contribution technology and/or with larger labor endowment. Substituting these expressions into (7), the optimal path of the stock of international public good is derived as

$$\dot{R} = \phi\lambda^a(\theta^w; \phi, L) + \phi^*\lambda^a(\theta^w; \phi^*, L^*) - \delta R. \tag{15}$$

[4]It can be verified that the optimal level of L_R is independent of R. See Appendix A.2.

The adjoint equation (12) is also rewritten as

$$\dot{\theta}^w = (\rho+\delta)\theta^w - \frac{G_R(P^a(R, L-\lambda^a(\theta^w;\phi,L)), R, L-\lambda^a(\theta^w;\phi,L))}{G(P^a(R, L-\lambda^a(\theta^w;\phi,L)), R, L-\lambda^a(\theta^w;\phi,L))} - \frac{G_R(P^a(R, L^*-\lambda^a(\theta^w;\phi^*,L^*)), R, L^*-\lambda^a(\theta^w;\phi^*,L^*))}{G(P^a(R, L^*-\lambda^a(\theta^w;\phi^*,L^*)), R, L^*-\lambda^a(\theta^w;\phi^*,L^*))}. \tag{16}$$

The evolution of the world economy under international cooperation is characterized by the system of differential equations (15) and (16).

Let us denote the autarkic steady-state solutions for R and θ^w under international cooperation by R_a^C and θ_a^w. It can be verified that the steady state, if it exists, is unique and saddle-point stable.[5]

4.3 Noncooperative Equilibrium

Let us turn to the situation where the governments in each country is self-interested and thus determines the path of its contribution level in order to maximize its national welfare. Because each country's contribution level affects the accumulation equation of the stock of international public good and this stock level in turn affects each country's welfare, this situation is characterized as a differential game.

There are two equilibrium concepts frequently employed in applications of differential game theory in economics; one is the open-loop Nash equilibrium, in which each player's equilibrium strategy is a simple function independent of the current state of the system, and the other is the Markov perfect Nash equilibrium, in which each player designs its optimal strategy as a feedback decision rule dependent only on the state variable. Both equilibrium concepts satisfy time consistency, but the only the Markov perfect Nash equilibrium satisfies subgame perfectness (see, for example, Long, 2010). However, I focus on the open-loop Nash equilibrium because of its tractability. This strategy concept requires that governments can commit themselves to particular strategy paths at the beginning of the game, and I simply assume that the commitment is credible. Formally, the open-loop Nash equilibrium of this dynamic contribution game is defined as a pair of time paths $\{(L_R(t), L_R^*(t))\}_{t=0}^{\infty}$, such that $\{L_R(t)\}_{t=0}^{\infty}$ maximizes the home country's national welfare subject to the dynamics of R given by (7), taking $\{L_R^*(t)\}_{t=0}^{\infty}$ as given, and $\{L_R^*(t)\}_{t=0}^{\infty}$ maximizes the foreign country's national welfare subject to the dynamics of R, taking $\{L_R(t)\}_{t=0}^{\infty}$ as given.

Let us define the home country's current value Hamiltonian as follows:

$$\mathcal{H} = \ln[G(P^a(R, L-L_R), R, L-L_R)] - \gamma \ln[P^a(R, L-L_R)] + \ln[\gamma^\gamma(1-\gamma)^{1-\gamma}] + \theta(\phi L_R + \phi^* L_R^* - \delta R),$$

[5] See Appendix A.3.

where θ can be interpreted as the shadow price of the international public good in the home country. The optimality conditions are given by

$$\frac{\partial \mathcal{H}}{\partial L_R} = 0 \quad \Rightarrow \quad \frac{G_l(P^a(R, L - L_R), R, L - L_R)}{G(P^a(R, L - L_R), R, L - L_R)} = \theta\phi, \tag{17}$$

$$\dot{\theta} = \rho\theta - \frac{\partial \mathcal{H}}{\partial R} = (\rho + \delta)\theta - \frac{G_R(P^a(R, L - L_R), R, L - L_R)}{G(P^a(R, L - L_R), R, L - L_R)}, \tag{18}$$

$$\lim_{t\to\infty} e^{-\rho t}\theta(t)R(t) = 0. \tag{19}$$

Notice that the first-order condition (17) is the same as (10) except that θ^w in (10) is replaced by θ. Therefore, from (17), the home country's optimal contribution level is expressed as $L_R = \lambda^a(\theta; \phi, L)$. The foreign country's optimality conditions can be derived analogously.

Substituting (17) and the foreign country's counterpart into (7), the dynamic path of the stock of international public good is derived as

$$\dot{R} = \phi\lambda^a(\theta; \phi, L) + \phi^*\lambda^a(\theta^*; \phi^*, L^*) - \delta R. \tag{20}$$

The home country's adjoint equation (12) and its foreign counterparts are also rewritten as

$$\dot{\theta} = (\rho + \delta)\theta - \frac{G_R(P^a(R, L - \lambda^a(\theta; \phi, L)), R, L - \lambda^a(\theta; \phi, L))}{G(P^a(R, L - \lambda^a(\theta; \phi, L)), R, L - \lambda^a(\theta; \phi, L))}, \tag{21}$$

$$\dot{\theta}^* = (\rho + \delta)\theta^* - \frac{G_R(P^a(R, L^* - \lambda^a(\theta^*; \phi^*, L^*)), R, L^* - \lambda^a(\theta^*; \phi^*, L^*))}{G(P^a(R, L^* - \lambda^a(\theta^*; \phi^*, L^*)), R, L^* - \lambda^a(\theta^*; \phi^*, L^*))}. \tag{22}$$

The evolution of the world economy in the open-loop Nash equilibrium is characterized by the system of differential equations (20), (21), and (22).

Let us denote the autarkic steady-state solutions for R, θ, and θ^* in the open-loop Nash equilibrium by R_a^N, θ_a, and θ_a^*. It can be verified that the steady state is a saddle point.[6]

4.4 Comparative Advantage

By substituting the optimal contribution levels in the case of international cooperation, the autarkic equilibrium prices of good 1 are $p_a = P^a(R, L - \lambda^a(\theta^w; \phi, L))$ in the home country and $p_a^* = P^a(R, L - \lambda^a(\theta^w; \phi^*, L^*))$ in the foreign country. Analogously for the case of noncooperative equilibrium, the autarkic equilibrium prices are given by $p_a = P^a(R, L - \lambda^a(\theta; \phi, L))$ and $p_a^* = P^a(R, L - \lambda^a(\theta^*; \phi^*, L^*))$. The properties of autarkic equilibrium price in each country are characterized by the following proposition.

[6] See Appendix A.3.

Proposition 1 *Suppose that there is no trade in goods between these countries and both countries choose positive contribution levels along the optimal path. Then, given the contribution regime (cooperative or noncooperative), along the dynamic path, (i) the autarkic equilibrium price of good 1 is lower in the country with a higher contribution technology, and (ii) labor endowment has no effect on the autarkic equilibrium price.*

Proof. Suppose first the case of international cooperation. Along the optimal path, both countries faces the same R and θ^w, and thus the differences in the contribution technologies and labor endowments determine the difference in the autarkic equilibrium prices. From (8), (9), and (14), it follows that

$$\frac{\partial p_a}{\partial \phi} = -P_l^a \lambda_\phi^a = \frac{p \cdot \left(G_{pl} - \frac{G_p G_l}{G}\right)}{(1-\gamma)G_p + pG_{pp}} \lambda_\phi^a, \tag{23}$$

$$\frac{\partial p_a}{\partial L} = P_l^a \cdot (1 - \lambda_L^a) = 0. \tag{24}$$

From (A.12), it follows that $\partial p_a / \partial \phi < 0$ under Assumption 1.

Next consider the case of noncooperative equilibrium. By differentiating (21) with respect to ϕ and using the derivatives of G derived in Appendices A.1 and A.2, it follows that

$$\begin{aligned}\frac{\partial \dot{\theta}}{\partial \phi} &= \frac{\lambda_\phi^a}{G} \left\{ \left(G_{pR} - \frac{G_R G_p}{G} \right) P_l^a + G_{Rl} - \frac{G_R G_l}{G} \right\} \\ &= \left(G_{pR} - \frac{G_R G_p}{G} \right) \left(\frac{G_R G_p}{G} - G_{pR} \right) \frac{p}{(1-\gamma)G_p + pG_{pp}} + G_{Rl} - \frac{G_R G_l}{G} \\ &= 0. \end{aligned} \tag{25}$$

Moreover, in light of (14), it holds that $\partial \dot{\theta} / \partial L = 0$. Therefore, for a given R, any changes in ϕ and L do not affect the equilibrium path of θ. This implies that both countries face the same paths of R and its shadow price. Then, as in the case of international cooperation, it follows that along the equilibrium path $p_a < p_a^*$ holds if $\phi > \phi^*$ and $p_a = p_a^*$ if $\phi = \phi^*$ (even though $L \neq L^*$). □

Proposition 1 (i) indicates that along the optimal or equilibrium path, the country with a higher contribution technology has a comparative advantage in a good that is more dependent on the stock of international public good. Intuitively, a higher ϕ implies a larger contribution level L_R, and in turn, the total labor available in the private sectors l becomes smaller. A decrease in l reduces both the output and consumption of good 1, but the reduction in C_1 is larger than that in Y_1. This implies that for a given

p the excess demand for good 1 becomes smaller, and thus the autarkic equilibrium price of good 1 becomes lower.

Proposition 1 (ii) indicates that two countries that differ only in their labor endowments faces the same autarkic equilibrium price along the optimal or equilibrium path, and in other words, that the difference in labor endowments cannot be a source of comparative advantage. This is because an increase in the labor endowment L increases L_R by the same amount, and thus the total labor available in the private sectors does not change. Therefore, the difference the labor endowment has no effect on output or consumption of the private goods.

Notice that Proposition 1 holds for any parameter values for $\alpha_i \in [0, 1)$, $i = 1, 2$, as long as both countries choose positive contribution levels. Therefore, in what follows, I put the following assumption as a substitute for Assumption 1:

Assumption 1′ $\alpha_1 = \alpha > 0 = \alpha_2$.

Under Assumption 1′, the GDP function is explicitly derived as

$$G(p, R, l) = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \alpha p^{\frac{1}{\alpha}} R + l, \tag{26}$$

and using this, the autarkic equilibrium price is derived as

$$P^a(R, l) = \left[\frac{\gamma l}{(1-\alpha\gamma)(1-\alpha)^{\frac{1-\alpha}{\alpha}} R} \right]^{\alpha}. \tag{27}$$

It follows that the optimal contribution levels under international cooperation are derived as[7]

$$\lambda^a(\theta^w; \phi, L) = L - \frac{1-\alpha\gamma}{\theta^w \phi} \equiv \lambda_a(\theta^w), \quad \lambda^a(\theta^w; \phi^*, L^*) = L^* - \frac{1-\alpha\gamma}{\theta^w \phi^*} \equiv \lambda_a^*(\theta^w), \tag{28}$$

and each country's optimal contribution levels in the open-loop Nash equilibrium are $\lambda_a(\theta)$ and $\lambda_a(\theta^*)$. The dynamic system of the cooperative solution path consisting of (15) and (16) is rewritten as

$$\dot{R} = \phi L + \phi^* L^* - \frac{2(1-\alpha\gamma)}{\theta^w} - \delta R, \tag{29}$$

$$\dot{\theta}^w = (\rho + \delta)\theta^w - \frac{2\alpha\gamma}{R}, \tag{30}$$

and the steady-state stock of international public good is derived as

$$R_a^C = \frac{\alpha\gamma(\phi L + \phi^* L^*)}{(1-\alpha\gamma)(\rho+\delta) + \alpha\gamma\delta}. \tag{31}$$

[7]Conditions under which the countries choose positive contribution levels are discussed in Appendix A.4.

Analogously, the open-loop Nash equilibrium path consisting of (20), (21), and (22) is rewritten as

$$\dot{R} = \phi L + \phi^* L^* - \frac{1-\alpha\gamma}{\theta} - \frac{1-\alpha\gamma}{\theta^*} - \delta R, \tag{32}$$

$$\dot{\theta} = (\rho+\delta)\theta - \frac{\alpha\gamma}{R}, \tag{33}$$

$$\dot{\theta}^* = (\rho+\delta)\theta^* - \frac{\alpha\gamma}{R}. \tag{34}$$

The steady-state stock of international public good is derived as

$$R_a^N = \frac{\alpha\gamma(\phi L + \phi^* L^*)}{2(1-\alpha\gamma)(\rho+\delta) + \alpha\gamma\delta}. \tag{35}$$

Comparing (31) and (35), it is easily verified that $R_a^C > R_a^N$. That is, under noncooperative behavior, the international public good is under-supplied in the long run. This is because, under noncooperative behavior, each country does not take account of the positive externality of the international public good on the other country in determining the path of its contribution level.

Substituting the steady-state solutions into $P^a(R,l)$, the autarkic steady-state equilibrium prices of good 1 in each country are derived as

$$p_a^C = \left[\frac{\rho+\delta}{2(1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha\phi}\right]^\alpha, \quad p_a^{*C} = \left[\frac{\rho+\delta}{2(1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha\phi^*}\right]^\alpha \tag{36}$$

in the case of cooperative solution and

$$p_a^N = \left[\frac{\rho+\delta}{(1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha\phi}\right]^\alpha, \quad p_a^{*N} = \left[\frac{\rho+\delta}{(1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha\phi^*}\right]^\alpha \tag{37}$$

in the case of noncooperative equilibrium, respectively. These expressions are consistent with Proposition 1. Moreover, the following proposition is obtained.

Proposition 2 *The autarkic steady-state equilibrium price of good 1 in each country is lower under cooperative regime than under noncooperative regime.*

Proposition 2 comes from the fact that the excess supply of good 1 is increasing in R (see the market clearing condition (8)) and $R_a^C > R_a^N$. That is, the steady-state stock of international public good is larger in the cooperative solution than in the noncooperative equilibrium, and thus the good 1, the output of which depends on the stock of international public good, can be supplied more efficiently.

5 Free Trade

5.1 Market Equilibrium

Under free trade, the total demand for for each private good must be equal to the total supply in the world market at each moment in time. The market-clearing condition for good 1, $C_1 + C_1^* = Y_1 + Y_1^*$, can be rewritten as

$$\frac{\gamma}{p}[G(p,R,l) + G^*(p,R,l^*)] = G_p(p,R,l) + G_p^*(p,R,l^*). \tag{38}$$

In light of (26), the market-clearing condition (38) derives the equilibrium price of good 1 in the world market as follows:

$$p_f = \left[\frac{\gamma(l+l^*)}{2(1-\alpha\gamma)(1-\alpha)^{\frac{1-\alpha}{\alpha}}R}\right]^{\alpha} \equiv P^f(R,l,l^*). \tag{39}$$

5.2 Cooperative Solution

The current value Hamiltonian associated with the world-welfare maximization problem is defined as

$$\begin{aligned}\mathcal{H}^w &= \ln[G(P^f(R, L-L_R, L^*-L_R^*), R, L-L_R)] - 2\gamma\ln[P^f(R, L-L_R, L^*-L_R^*)]\\ &+ \ln[G(P^f(R, L-L_R, L^*-L_R^*), R, L^*-L_R^*)] + 2\ln[\gamma^\gamma(1-\gamma)^{1-\gamma}]\\ &+ \theta^w(\phi L_R + \phi^* L_R^* - \delta R).\end{aligned}$$

In light of (26) and (27), the first-order conditions are given by

$$\begin{aligned}\frac{\partial\mathcal{H}^w}{\partial L_R} &= -\frac{2-\alpha\gamma}{\alpha\gamma(L-L_R+L^*-L_R^*) + 2(1-\alpha\gamma)(L-L_R)}\\ &- \frac{\alpha\gamma}{\alpha\gamma(L-L_R+L^*-L_R^*) + 2(1-\alpha\gamma)(L^*-L_R^*)}\\ &+ \frac{2\alpha\gamma}{L-L_R+L^*-L_R^*} + \theta^w\phi = 0, \qquad (40)\\ \frac{\partial\mathcal{H}^w}{\partial L_R^*} &= -\frac{\alpha\gamma}{\alpha\gamma(L-L_R+L^*-L_R^*) + 2(1-\alpha\gamma)(L-L_R)}\\ &- \frac{2-\alpha\gamma}{\alpha\gamma(L-L_R+L^*-L_R^*) + 2(1-\alpha\gamma)(L^*-L_R^*)}\\ &+ \frac{2\alpha\gamma}{L-L_R+L^*-L_R^*} + \theta^w\phi^* = 0. \qquad (41)\end{aligned}$$

The adjoint equation and the transversality condition are shown to be the same as those under autarky.

The temporary optimal solutions for L_R and L_R^* can be derived by solving the nonlinear equations (40) and (41). These equations derive a rather complicated solution pair for L_R and L_R^*, but adding up (40) multiplied by $L - L_R$ and (41) multiplied by $L^* - L_R^*$, it follows that $\phi L_R + \phi^* L_R^* = \phi L + \phi^* L^* - 2(1-\alpha\gamma)/\theta^w$. In light of (7), this means that the dynamics of the stock of international public good under free trade is given by (29), as in the cooperative solution under autarky. Thus, the following proposition is established.

Proposition 3 *Suppose that the governments in both countries act cooperatively and both countries choose positive contribution levels. Then, the optimal trajectories of the stock of international public good and its shadow price under free trade coincides with those under autarky, and hence the steady-state stock of international public good under free trade,* R_f^C*, coincides with* R_a^C*.*

In contrast with the autarkic case, the welfare effect of a change in the international price of good 1 is asymmetric between countries. Suppose that $\phi > \phi^*$ and thus the home (foreign) country exports (imports) good 1 and that p increases. Then, other things being equal, the increase in p makes the home country better off because of an improvement in its terms of trade, whereas the foreign country becomes worse off. Nevertheless, in the cooperative solution where both the home and foreign countries jointly maximize the world welfare, these positive and negative welfare effects induced by the price change are offset, and thus the outcome will be the same as that under autarky.

Although the paths of R and θ^w under free trade is the same as those under autarky, each country's optimal contribution level under free trade is different from that under autarky. Evaluating the left-hand side terms of (40) and (41) at $L_R = \lambda_a(\theta^w)$ and $L_R^* = \lambda_a^*(\theta^w)$ defined by eqn:Autarky-Cooperative-FOC-LR, it follows that

$$\left.\frac{\partial \mathcal{H}^w}{\partial L_R}\right|_{L_R=\lambda_a(\theta^w), L_R^*=\lambda_a^*(\theta^w)} = \theta^w \phi \Phi(\phi - \phi^*) > 0,$$

$$\left.\frac{\partial \mathcal{H}^w}{\partial L_R^*}\right|_{L_R=\lambda_a(\theta^w), L_R^*=\lambda_a^*(\theta^w)} = \theta^w \phi^* \Phi(\phi^* - \phi) < 0,$$

where

$$\Phi \equiv \frac{\alpha\gamma[(2-\alpha\gamma)(\phi^2 + \phi^{*2}) + 2\alpha\gamma\phi\phi^*]}{[(2-\alpha\gamma)\phi + \alpha\gamma\phi^*][(2-\alpha\gamma)\phi^* + \alpha\gamma\phi](\phi + \phi^*)} > 0.$$

From (40) and (41), the temporary optimal levels of L_R and L_R^* under free trade can be expressed as $\lambda_f(\theta^w)$ and $\lambda_f^*(\theta^w)$. Then, under assumption that $\phi > \phi^*$, it holds that $\lambda_f(\theta^w) > \lambda_a(\theta^w)$ and $\lambda_f^*(\theta^w) < \lambda_a^*(\theta^w)$. That is, trade liberalization induces the home country, which has a higher contribution technology, to contribute more to the investment on the international public good, whereas the foreign country to contribute less.

Proposition 4 *Suppose that the governments in both countries act cooperatively. Then, trade liberalization increases the optimal contribution level in the country with a higher contribution technology and reduces the contribution level in the country with a lower contribution technology.*

Because labor endowment in each country is assumed to be constant over time, Proposition 4 implies that under free trade, less labor is available to private sectors in the home country whereas the foreign country enjoys increased labor allocation in the private sectors. In other words, trade liberalization increases the foreign country's level of "free ride" on the home country's contribution effort.

As shown in Proposition 1, under the assumption that $\phi > \phi^*$, the home country has a comparative advantage in good 1. Under free trade, the home country actually exports good 1. That is, denoting the equilibrium price of good 1 under autarky and free trade for a given pair of R and θ^w by (p_a, p_a^*) and p_f, respectively, it holds that $p_a < p_f < p_a^*$. This can be verified as follows. Since the paths of R and θ^w under free trade coincides with those under autarky, (27) and (39) imply the following:

$$\frac{p_a}{p_f} = \left\{ \frac{2[L - \lambda(\theta_a^w)]}{L - \lambda_f(\theta^w) + L^* - \lambda_f^*(\theta^w)} \right\}^{\alpha}.$$

As shown above, $\lambda_f(\theta^w) > \lambda_a(\theta^w)$ and $\lambda_f^*(\theta^w) < \lambda_a^*(\theta^w)$ hold. It was also shown that $L - \lambda_a(\theta^w) < L^* - \lambda_a^*(\theta^w)$. Putting these inequalities together yields

$$L - \lambda_f(\theta^w) < L - \lambda_a(\theta^w) < L^* - \lambda_a^*(\theta^w) < \lambda_f^*(\theta^w). \tag{42}$$

Moreover, since $\phi\lambda_a(\theta^w) + \phi^*\lambda_a^*(\theta^w) = \phi\lambda_f(\theta^w) + \phi^*\lambda_f^*(\theta^w)$, the following inequality holds:

$$\phi > \phi^* \quad \Rightarrow \quad \lambda_f(\theta^w) - \lambda_a(\theta^w) < \lambda_a^*(\theta^w) - \lambda_f^*(\theta^w). \tag{43}$$

In light of (42) and (43), it holds that

$$\begin{aligned} \frac{2[L - \lambda_a(\theta^w)]}{L - \lambda_f(\theta^w) + L^* - \lambda_f^*(\theta^w)} &< \frac{L - \lambda_a(\theta^w) + L^* - \lambda_a^*(\theta^w)}{L - \lambda_f(\theta^w) + L^* - \lambda_f^*(\theta^w)} \\ &= \frac{L + L^* - [\lambda_a(\theta^w) + \lambda_a^*(\theta^w)]}{L + L^* - [\lambda_f(\theta^w) + \lambda_f^*(\theta^w)]} < 1, \end{aligned}$$

and thus $p_a/p_f < 1$. Analogously, $p_a^*/p_f > 1$ can be verified.

As mentioned above, when the governments cooperatively determine their respective contribution levels, free trade achieves the same paths of the international public good and its shadow price as autarky, but the contribution level in each country, and in turn, the total labor available in the production of private goods, differs between autarky and free trade. Specifically, under the assumption that $\phi > \phi^*$, trade liberalization reduces $L - L_R$

and increases $L^* - L_R^*$ for a given stock level of R. This implies that the production possibility frontier for a given R contracts in the home country and expands in the foreign country. Because the foreign country enjoys larger GDP under free trade than under autarky, this country will gain from trade.

Proposition 5 *Suppose that the governments in both countries act cooperatively. Then, the country with a lower contribution technology unambiguously enjoys the higher level of national welfare under free trade than under autarky.*

Proof. Let us define the expenditure function as

$$E(p,u) = \min_{C_1,C_2} \left\{ pC_1 + C_2 \quad \text{s.t.} \quad \gamma \log C_1 + (1-\gamma)\log C_2 \geq u \right\}.$$

It is easily verified that $E_u > 0$. Let us also denote the foreign country's utility levels under autarky and free trade for a given pair of R and θ^w by u_a^* and u_f^*, respectively. Because $Y_i = C_i$ holds under autarky for $i = 1,2$ and $E(p,u) = G(p,R,L-L_R)$ holds under free trade, the following expression is obtained:

$$\begin{aligned} &E(p_f, u_f^*) - E(p_f, u_a^*) \\ &= G(p_f, R, L^* - \lambda_f^*(\theta^w)) - G(p_f, R, L^* - \lambda_a^*(\theta^w)) + (p_f C_{1a}^* + C_{2a}^*) - E(p_f, u_a^*), \end{aligned} \tag{44}$$

where C_{ia}^* is foreign country's autarkic consumption level of good $i = 1,2$. From the definition of the expenditure function, it holds that $p_f C_{1a}^* + C_{2a}^* \geq E(p_f, u_a^*)$. Moreover, from (26), (27), and (39), it follows that

$$\begin{aligned} &G(p_f, R, L^* - \lambda_f^*(\theta^w)) - G(p_f, R, L^* - \lambda_a^*(\theta^w)) \\ &= \frac{\alpha\gamma[L - \lambda_f(\theta^w)] + (2-\alpha\gamma)[L^* - \lambda_f^*(\theta^w)]}{2(1-\alpha\gamma)} - \left\{ \frac{\alpha\gamma[L - \lambda_f(\theta^w) + L^* - \lambda_f^*(\theta^w)]}{2(1-\alpha\gamma)} + L^* - \lambda_a^*(\theta_a^w) \right\} \\ &= \lambda_a^*(\theta^w) - \lambda_f^*(\theta^w). \end{aligned} \tag{45}$$

As discussed above, $\lambda_a^*(\theta^w) > \lambda_f^*(\theta^w)$ holds if $\phi > \phi^*$. Then, it follows that the sign of (44) is unambiguously positive, and thus $u_f^* > u_a^*$. □

Contrary to the foreign country, the home country faces a reduction in its GDP level evaluated at $p = p_f$:

$$G(p_f, R, L - \lambda_f(\theta^w)) - G(p_f, R, L - \lambda_a(\theta^w)) = \lambda_a(\theta^w) - \lambda_f(\theta^w) < 0.$$

Even though the home consumer benefits from the improvement in consumption possibility under free trade, the national income in the home country decreases, and if the negative effect of the reduction in national income outweighs the positive effect of the improvement in the economy's consumption possibility the home country may lose from trade.

5.3 Noncooperative Equilibrium

The home country's current value Hamiltonian is given by

$$\mathcal{H} = \ln[G(P^f(R, L - L_R, L^* - L_R^*), R, L - L_R)] - \gamma \ln[P^f(R, L - L_R, L^* - L_R^*)] \\ + \ln[\gamma^\gamma (1-\gamma)^{1-\gamma}] + \theta^w(\phi L_R + \phi^* L_R^* - \delta R).$$

The first-order condition is given by

$$\frac{\partial \mathcal{H}}{\partial L_R} = -\frac{2 - \alpha\gamma}{\alpha\gamma(L - L_R + L^* - L_R^*) + 2(1-\alpha\gamma)(L - L_R)} \\ + \frac{\alpha\gamma}{L - L_R + L^* - L_R^*} + \theta\phi = 0. \tag{46}$$

The adjoint equation and the transversality condition are same as those under autarky. Analogously for the foreign country, defining the current value Hamiltonian $\mathcal{H}^*$ and deriving the first-order condition, it follows that

$$\frac{\partial \mathcal{H}^*}{\partial L_R^*} = -\frac{2 - \alpha\gamma}{\alpha\gamma(L - L_R + L^* - L_R^*) + 2(1-\alpha\gamma)(L^* - L_R^*)} \\ + \frac{\alpha\gamma}{L - L_R + L^* - L_R^*} + \theta^*\phi^* = 0. \tag{47}$$

Solving (46) and (47) for L_R and L_R^*, the Nash equilibrium contribution levels are derived, which are dependent on θ and θ^*. It is of interest whether trade liberalization increases the equilibrium contribution level in each country. As shown later, the equilibrium paths of R and its shadow price in each country under free trade do not coincide with those under autarky. Therefore, let us suppose that the stock of international public good is fixed at the steady-state level under autarky: $R = R_a^N$ and thus $\theta_a = \theta_a^* = \alpha\gamma/[(\rho+\delta)R_a^N]$. Evaluating the left-hand side terms of (46) and (47) at the autarkic steady-state levels $L_R = \lambda_a(\theta_a)$ and $L_R^* = \lambda_a^*(\theta_a^*)$, it follows that under the assumption $\phi > \phi^*$,

$$\left.\frac{\partial \mathcal{H}}{\partial L_R}\right|_{L_R=\lambda_a(\theta_a), L_R^*=\lambda_a^*(\theta_a^*)} = \frac{\alpha^2\gamma^2\phi^2(\phi - \phi^*)}{(\phi+\phi^*)[\alpha\gamma\phi + (2-\alpha\gamma)\phi^*](\rho+\delta)R_a^N} > 0,$$

$$\left.\frac{\partial \mathcal{H}^*}{\partial L_R^*}\right|_{L_R=\lambda_a(\theta_a), L_R^*=\lambda_a^*(\theta_a^*)} = \frac{\alpha^2\gamma^2\phi^{*2}(\phi^* - \phi)}{(\phi+\phi^*)[(2-\alpha\gamma)\phi + \alpha\gamma\phi^*](\rho+\delta)R_a^N} < 0.$$

That is, as with the cooperative case, the home country has an incentive to contribute more under free trade than under autarky, whereas the foreign country has an incentive to contribute less under free trade.

Adding up (46) multiplied by $(L - L_R)/\theta$ and (47) multiplied by $(L^* - L_R^*)/\theta^*$, and substituting $\theta = \theta^* = \alpha\gamma/[(\rho+\delta)R]$, it follows that

$$\phi L_R + \phi^* L_R^* = \phi L + \phi^* L^* - \frac{(\rho+\delta)R}{\alpha\gamma}[(2-\alpha\gamma)\Psi - \alpha\gamma],$$

where

$$\Psi \equiv \frac{L - L_R}{(2-\alpha\gamma)(L - L_R) + \alpha\gamma(L^* - L_R^*)} + \frac{L^* - L_R^*}{\alpha\gamma(L - L_R) + (2-\alpha\gamma)(L^* - L_R^*)}.$$

Since

$$\Psi - 1$$
$$= -\frac{\alpha\gamma(1-\alpha\gamma)[L - L_R - (L^* - L_R^*)]^2}{[(2-\alpha\gamma)(L - L_R) + \alpha\gamma(L^* - L_R^*)][\alpha\gamma(L - L_R) + (2-\alpha\gamma)(L^* - L_R^*)]} < 0,$$

it holds that $(2-\alpha\gamma)\Psi - \alpha\gamma < 2(1-\alpha\gamma)$. Then, comparing the steady-state condition under free trade

$$\dot{R} = \phi L + \phi^* L^* - \left\{ \frac{[(2-\alpha\gamma)\Psi - \alpha\gamma](\rho + \delta)}{\alpha\gamma} + \delta \right\} R = 0$$

with the steady-state condition under autarky

$$\dot{R} = \phi L + \phi^* L^* - \left\{ \frac{2(1-\alpha\gamma)(\rho + \delta)}{\alpha\gamma} + \delta \right\} R = 0,$$

it holds that the steady-state stock of international public good, R_f^N, is larger than the autarkic level R_a^N (see also Figure 1).

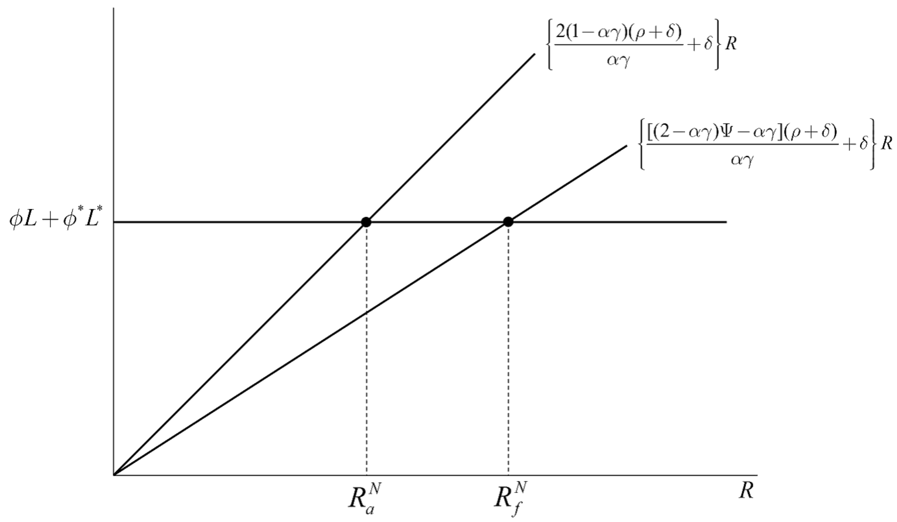

Figure 1: Comparison of steady-state stocks of international public good

To sum up, the following proposition is established.

Proposition 6 *Suppose that the governments in both countries noncooperatively determine their respective contribution levels by using the open-loop strategy. Then, the steady-state contribution level under free trade becomes larger in the country with a higher contribution technology, whereas it is smaller in the country with a lower contribution technology. The steady-state stock of international public good under free trade is larger than the autarkic steady-state level.*

In the absence of international cooperation over the provision of international public good, each country has a strategic incentive to control the contribution level so as to maximize its own welfare. The optimality condition for national welfare maximization implies that the marginal benefit from the international public good should be equal to the marginal cost of contribution in each country. From the assumption that $\phi > \phi^*$, the home country increases its contribution level under free trade compared to autarky, whereas the foreign country reduces its contribution level. Moreover, $\phi > \phi^*$ implies that the home country has a superior contribution technology compared to the foreign country, and thus the increase in L_R caused by trade liberalization outweighs the reduction in L_R^*. Therefore, in comparison with autarky, there is a net increase in the sum of the change in each country's contribution level multiplied by its contribution technology, i.e., $\phi L_R + \phi^* L_R^*$, under free trade. Consequently, free trade achieves the larger stock of international public good than the autarkic steady-state level.

In the cooperative regime, it was shown that even though the steady-state stock of international public good remains unchanged under free trade compared to autarky, the country with a lower contribution technology unambiguously gains from trade in the long run. In the noncooperative regime, $R_a^N < R_f^N$ holds and the total labor available in the country with a lower contribution technology increases under free trade compared to the autarkic steady state. Therefore, in the noncooperative regime, the gains-from-trade result in the country with a lower contribution technology is reinforced.

6 Concluding Remarks

In this paper I developed a dynamic two-country model with an international public good, the stock of which has a positive effect on the private sector's productivity in each country and the evolution of the stock is determined by each country's voluntary contribution. I derived both cooperative solution and noncooperative Nash equilibrium in open-loop strategies, under both autarky and free trade. As far as I know, there are no studies that consider international trade and the provision of international public goods in a unified dynamic trade model. I showed that under autarky the equilibrium price of the good which is more dependent on the stock international public good becomes lower in the country that has a higher contribution technology, regardless of whether the countries cooperatively or noncooperatively determine their contribution levels. I also showed that if the countries act cooperatively, the dynamics of the stock of international public good and its shadow price under free trade coincide with those under autarky, although the paths of each country's contribution level are different. More specifically, trade liberalization increases the optimal contribution level in the country with a higher contribution technology and reduces the contribution level in the country with a lower contribution technology. This implies that in the country with a lower contribution technology, the total labor available in the production of private goods becomes larger under free trade than under autarky, and it was shown that this country unambiguously gains from trade. By contrast, the country with a higher contribution technology may lose from trade. In the noncooperative regime, free trade achieves a larger steady-state stock of international public good than autarky, and the gains-from-trade result is reinforced in the country with a lower contribution technology.

Some of the welfare effects of trade liberalization are still not yet understood. For example, the condition for gains or loses from trade in the country with a higher contribution technology is not derived yet. Moreover, in this paper I assumed that in the noncooperative regime, countries use open-loop strategies to determine their respective contribution levels. However, the open-loop Nash equilibrium generally lacks subgame perfectness, and deriving the Markov perfect Nash equilibrium in this game would be more appropriate. Furthermore, this paper assumed that the international public good is accumulated only through the public investment. However, for example, the accelerated growth of the Internet is not only due to the efforts of public sectors but also private sectors. Therefore, it will also be interesting to extend the model to a case in which the international public good is accumulated through the contributions of private firms as well as national governments. These issues are remained for future research.

Appendix

A.1 Properties of the GDP function

Let us define the Lagrangian:

$$\mathcal{L}(L_1, L_2, w, p, R, l) = pR^{\alpha} L_1^{1-\alpha} + R^{\alpha_2} L_2^{1-\alpha_2} + w(l - L_1 - L_2),$$

where the Lagrangian multiplier w can be interpreted as the wage. The first-order conditions for maximizing $\mathcal{L}$ are $(1-\alpha_1)pY_1/L_1 = w = (1-\alpha_2)Y_2/L_2$ and $L_1 + L_2 = l$. Given these conditions and the production function (1), it follows that

$$Y_1 = R^{\alpha_1}\left[\frac{(1-\alpha_1)pY_1}{w}\right]^{1-\alpha_1}, \tag{A.1}$$

$$Y_2 = R^{\alpha_2}\left[\frac{(1-\alpha_2)Y_2}{w}\right]^{1-\alpha_2}, \tag{A.2}$$

$$(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2 = wl. \tag{A.3}$$

Totally differentiating eqs.(A.1), (A.1), and (A.3) gives

$$\begin{bmatrix} \frac{\alpha_1}{Y_1} & 0 & \frac{1-\alpha_1}{w} \\ 0 & \frac{\alpha_2}{Y_2} & \frac{1-\alpha_2}{w} \\ (1-\alpha_1)p & 1-\alpha_2 & -l \end{bmatrix} \begin{bmatrix} dY_1 \\ dY_2 \\ dw \end{bmatrix} = \begin{bmatrix} \frac{\alpha_1}{R}dR + \frac{1-\alpha_1}{p}dp \\ \frac{\alpha_2}{R}dR \\ -(1-\alpha_1)Y_1 dp + wdl \end{bmatrix}. \tag{A.4}$$

From the solutions of (A.4), the second-order derivatives of the GDP function are obtained as follows:

$$G_{pp} = \frac{\partial Y_1}{\partial p} = \frac{(1-\alpha_1)(1-\alpha_2)Y_1Y_2}{\{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2\}p} > 0, \tag{A.5}$$

$$\begin{aligned} G_{RR} &= \frac{\left(\alpha p\frac{\partial Y_1}{\partial R} + \alpha_2\frac{\partial Y_2}{\partial R}\right)R - (\alpha pY_1 + \alpha_2 Y_2)}{R^2} \\ &= -\frac{\alpha_1\alpha_2(wl)^2}{\{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2\}R^2} < 0, \end{aligned} \tag{A.6}$$

$$G_{ll} = \frac{\partial w}{\partial l} = -\frac{\alpha_1\alpha_2 w^2}{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2} < 0, \tag{A.7}$$

$$\begin{aligned} G_{pR} &= \frac{\partial Y_1}{\partial R} \\ &= \frac{\{(1-\alpha_1)\alpha_1\alpha_2 pY_1 + (1-\alpha_2)[\alpha_1 - (1-\alpha_1)\alpha_2]Y_2\}Y_1}{\{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2\}R}, \end{aligned} \tag{A.8}$$

$$G_{pl} = \frac{\partial Y_1}{\partial l} = \frac{w(1-\alpha_1)\alpha_2 Y_1}{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2} > 0, \tag{A.9}$$

$$\begin{aligned} G_{Rl} &= \frac{\alpha_1}{R}p\frac{\partial Y_1}{\partial l} + \frac{\alpha_2}{R}\frac{\partial Y_2}{\partial l} \\ &= \frac{\alpha_1\alpha_2 w^2 l}{\{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2\}R} > 0. \end{aligned} \tag{A.10}$$

Moreover, the following calculation results, which are useful in the subsequent analysis, are obtained:

$$G_{pR} - \frac{G_p G_R}{G} = \frac{(\alpha_1 - \alpha_2) wlY_1Y_2}{\{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2\}(pY_1 + Y_2)R}, \tag{A.11}$$

$$G_{pl} - \frac{G_p G_l}{G} = -\frac{(\alpha_1 - \alpha_2) wY_1Y_2}{\{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2\}(pY_1 + Y_2)}, \tag{A.12}$$

$$G_{Rl} - \frac{G_R G_l}{G} = -\frac{(\alpha_1 - \alpha_2)^2 wpY_1Y_2}{\{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2\}(pY_1 + Y_2)R} < 0. \tag{A.13}$$

If $\alpha_1 > \alpha_2$, the sign of (A.11) is positive, whereas the sign of (A.12) is negative.

A.2 Derivatives of $\lambda^a(\theta^w; \phi, L)$

In light of (8) and (9), the denominator of the derivatives in (14) is rewritten as

$$\begin{aligned} &\left(\frac{G_l G_p}{G} - G_{lp}\right) P_l^a + \frac{(G_l)^2}{G} - G_{ll} \\ &= \left(\frac{G_l G_p}{G} - G_{lp}\right)^2 \frac{p}{(1-\gamma)G_p + pG_{pp}} + \frac{(G_l)^2}{G} - G_{ll}. \end{aligned} \tag{A.14}$$

Since $G_{pp} > 0$ and $G_{ll} < 0$ as derived in Appendix A.1, the sign of the above expression is unambiguously positive. Thus, λ^a_θ and λ^a_ϕ have positive signs.

Suppose that the optimal level of L_R that satisfies (10) depends on R, and solve for $\partial\lambda^a/\partial R$. The denominator of $\partial\lambda^a/\partial R$ is equal to (A.14), and its numerator is, in light of the derivatives of G derived in Appendix A.1, rewritten as

$$\begin{aligned} &\left(\frac{G_l G_p}{G} - G_{lp}\right) P_R^a + \frac{G_l G_R}{G} - G_{lR} \\ &= \left(\frac{G_l G_p}{G} - G_{lp}\right)\left(\frac{G_p G_R}{G} - G_{pR}\right) \frac{p}{(1-\gamma)G_p + pG_{pp}} + \frac{G_l G_R}{G} - G_{lR} \\ &= \frac{(\alpha_1 - \alpha_2)^2 wpY_1Y_2}{\{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2\}^2 (pY_1 + Y_2)\{(1-\gamma)G_p + pG_{pp}\}} \\ &\quad \times \{[(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2](pY_1 + Y_2)[(1-\gamma)G_p + pG_{pp}] - wlY_1Y_2\}. \end{aligned} \tag{A.15}$$

However, since[8]

$$
\begin{aligned}
&\{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2\}(pY_1+Y_2)\{(1-\gamma)G_p + pG_{pp}\} \\
&= \{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2\}Y_1Y_2\left\{1+\frac{(1-\alpha_1)(1-\alpha_2)(pY_1+Y_2)}{(1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2}\right\} \\
&= Y_1Y_2\{(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2\} = Y_1Y_2 w(L_1+L_2),
\end{aligned}
$$

the terms in the curly brackets in (A.15) become zero, and thus $\partial\lambda^a/\partial R = 0$.

A.3 Uniqueness and stability of autarkic steady-state solutions

Cooperative solution Let $\lambda_a(\theta^w) \equiv \lambda^a(\theta^w;\phi,L)$ and $\lambda_a^*(\theta^w) \equiv \lambda^a(\theta^w;\phi^*,L^*)$. From the steady-state condition $\dot{R} = \phi\lambda_a(\theta^w) + \phi^*\lambda_a^*(\theta^w) - \delta R = 0$, it holds that

$$
\left.\frac{d\theta^w}{dR}\right|_{\dot{R}=0} = \frac{\delta}{\phi\lambda_a' + \phi^*\lambda_a^{*\prime}} > 0. \tag{A.16}
$$

From the steady-state condition $\dot{\theta}^w = 0$,

$$
\left.\frac{d\theta^w}{dR}\right|_{\dot{\theta}^w=0} = -\frac{\partial\dot{\theta}^w/\partial R}{\partial\dot{\theta}^w/\partial\theta^w}. \tag{A.17}
$$

Straightforward calculations yield

$$
\begin{aligned}
\frac{\partial\dot{\theta}^w}{\partial R} &= \frac{\left(\frac{G_pG_R}{G} - G_{pR}\right)P_R^a + \frac{G_R^2}{G} - G_{RR}}{G} + \frac{\left(\frac{G_p^*G_R^*}{G^*} - G_{pR}^*\right)P_R^{a*} + \frac{G_R^{*2}}{G^*} - G_{RR}^*}{G^*} \\
&= \frac{p\left(\frac{G_pG_R}{G} - G_{pR}\right)^2}{\{(1-\gamma)G_p + pG_{pp}\}G} + \frac{\frac{G_R^2}{G} - G_{RR}}{G} + \frac{p\left(\frac{G_p^*G_R^*}{G^*} - G_{pR}^*\right)^2}{\{(1-\gamma)G_p^* + pG_{pp}^*\}G^*} + \frac{\frac{G_R^{*2}}{G^*} - G_{RR}^*}{G^*},
\end{aligned} \tag{A.18}
$$

which is unambiguously positive because $G_{pp} > 0$ and $G_{RR} < 0$. Moreover, in light of (A.15) it holds that

$$
\begin{aligned}
&\left(G_{Rp} - \frac{G_RG_p}{G}\right)P_l^a + G_{Rl} - \frac{G_RG_l}{G} \\
&= \left(G_{Rp} - \frac{G_RG_p}{G}\right)\left(\frac{G_lG_p}{G} - G_{lp}\right)\frac{p}{(1-\gamma)G_p + pG_{pp}} + G_{Rl} - \frac{G_RG_l}{G} = 0,
\end{aligned}
$$

and thus

$$
\begin{aligned}
\frac{\partial\dot{\theta}^w}{\partial\theta^w} &= \rho + \delta + \lambda_a'\frac{\left(G_{Rp} - \frac{G_RG_p}{G}\right)P_l^a + G_{Rl} - \frac{G_RG_l}{G}}{G} + \lambda_a^{*\prime}\frac{\left(G_{Rp}^* - \frac{G_R^*G_p^*}{G}\right)P_l^{a*} + G_{Rl}^* - \frac{G_R^*G_l^*}{G^*}}{G^*} \\
&= \rho + \delta > 0.
\end{aligned} \tag{A.19}
$$

[8] In the derivation of this equation, the first-order conditions for maximizing the GDP and the market-clearing condition (8) are utilized.

Therefore, the sign of (A.17) is unambiguously negative. It follows that the steady state, if it exists, is uniquely determined.

Linearizing the dynamic system (15) and (16) around the steady state (R_a^C, θ_a^w), it follows that

$$\begin{bmatrix} \dot{R} \\ \dot{\theta}^w \end{bmatrix} = \begin{bmatrix} -\delta & \phi\lambda_a'(\theta_a^w) + \phi^*\lambda_a^{*\prime}(\theta^w) \\ \Theta_a^C & \rho+\delta \end{bmatrix} \begin{bmatrix} R - R_a^C \\ \theta^w - \theta_a^w \end{bmatrix}, \tag{A.20}$$

where Θ_a^C is the value of (A.18) evaluated at $(R, \theta^w) = (R_a^C, \theta_a^w)$. The determinant of the Jacobian matrix in (A.20) is negative, and thus the dynamic system has one positive and one negative eigenvalues. Since the dynamical system has one predetermined variable R, it follows that the steady state is locally saddle-point stable.

Noncooperative equilibrium The steady-state condition $\dot{R} = 0$ is rewritten as $R = [\phi\lambda_a(\theta) + \phi^*\lambda_a^*(\theta^*)]/\delta$. Substituting this into (21) and (22), it follows that

$$\dot{\theta} = (\rho+\delta)\theta - \frac{G_R(P^a(\frac{\phi\lambda_a(\theta)+\phi^*\lambda_a^*(\theta^*)}{\delta}, L-\lambda_a(\theta)), \frac{\phi\lambda_a(\theta)+\phi^*\lambda_a^*(\theta^*)}{\delta}, L-\lambda_a(\theta))}{G(P^a(\frac{\phi\lambda_a(\theta)+\phi^*\lambda_a^*(\theta^*)}{\delta}, L-\lambda_a(\theta)), \frac{\phi\lambda_a(\theta)+\phi^*\lambda_a^*(\theta^*)}{\delta}, L-\lambda_a(\theta))},$$

$$\dot{\theta}^* = (\rho+\delta)\theta^* - \frac{G_R(P^a(\frac{\phi\lambda_a(\theta)+\phi^*\lambda_a^*(\theta^*)}{\delta}, L^*-\lambda_a^*(\theta^*)), \frac{\phi\lambda_a(\theta)+\phi^*\lambda_a^*(\theta^*)}{\delta}, L^*-\lambda_a^*(\theta^*))}{G(P^a(\frac{\phi\lambda_a(\theta)+\phi^*\lambda_a^*(\theta^*)}{\delta}, L^*-\lambda_a^*(\theta^*)), \frac{\phi\lambda_a(\theta)+\phi^*\lambda_a^*(\theta^*)}{\delta}, L^*-\lambda_a^*(\theta^*))}.$$

Then, it follows that

$$\left.\frac{d\theta^*}{d\theta}\right|_{\dot{R}=\dot{\theta}=0} = -\frac{\rho+\delta+\frac{\phi\lambda_a'}{\delta G}\left\{\left(\frac{G_R G_p}{G} - G_{Rp}\right)P_R^a + \frac{G_R^2}{G} - G_{RR}\right\}}{\frac{\phi^*\lambda_a^{*\prime}}{\delta G}\left\{\left(\frac{G_R G_p}{G} - G_{Rp}\right)P_R^a + \frac{G_R^2}{G} - G_{RR}\right\}}, \tag{A.21}$$

$$\left.\frac{d\theta^*}{d\theta}\right|_{\dot{R}=\dot{\theta}^*=0} = -\frac{\frac{\phi\lambda_a'}{\delta G^*}\left\{\left(\frac{G_R^* G_p^*}{G^*} - G_{Rp}^*\right)P_R^{*a} + \frac{G_R^{*2}}{G^*} - G_{RR}^*\right\}}{\rho+\delta+\frac{\phi^*\lambda_a^{*\prime}}{\delta G^*}\left\{\left(\frac{G_R^* G_p^*}{G^*} - G_{Rp}^*\right)P_R^{*a} + \frac{G_R^{*2}}{G^*} - G_{RR}^*\right\}}, \tag{A.22}$$

and thus

$$\left.\frac{d\theta^*}{d\theta}\right|_{\dot{R}=\dot{\theta}^*=0} < \left.\frac{d\theta^*}{d\theta}\right|_{\dot{R}=\dot{\theta}=0} < 0.$$

Therefore, the steady state of the open-loop Nash equilibrium, if it exists, is uniquely determined.

Linearizing the dynamic system (20), (21), and (22) around the steady state $(R_a^N, \theta_a, \theta_a^*)$, it follows that

$$\begin{bmatrix} \dot{R} \\ \dot{\theta} \\ \dot{\theta}^* \end{bmatrix} = \begin{bmatrix} -\delta & \phi\lambda_a'(\theta_a) & \phi^*\lambda_a^{*\prime}(\theta_a^*) \\ \Theta_a^N & \rho+\delta & 0 \\ \Theta_a^{*N} & 0 & \rho+\delta \end{bmatrix} \begin{bmatrix} R - R_a^N \\ \theta - \theta_a \\ \theta^* - \theta_a^* \end{bmatrix}, \tag{A.23}$$

where Θ_a^N and Θ_a^{*N} are the values of $\partial\dot{\theta}/\partial R$ and $\partial\dot{\theta}^*/\partial R$, respectively, evaluated at $(R, \theta, \theta^*) = (R_a^N, \theta_a, \theta_a^*)$. The eigenvalues of the Jacobian matrix in (A.23) are $\rho+\delta$ and $\{\rho \pm \sqrt{\rho^2 + 4[(\rho+\delta)\delta + \Theta_a^N \phi \lambda_a' + \Theta_a^{*N} \phi^* \lambda_a^{*\prime}]}\}/2$. Since the dynamic system has two positive and one negative eigenvalues, it follows that the steady state is locally saddle-point stable.

A.4 Conditions for positive contribution levels

Consider the case in which the countries are under autarky and cooperatively determine their contribution levels. Under Assumption $1'$, the interior solutions for the optimal contribution levels are given by (28). Suppose that only the foreign country make a contribution to the investment in the accumulation of the international public good: $\lambda_a(\theta^w) = 0$ and $\lambda_a^*(\theta^w) > 0$. $\lambda_a(\theta^w) = 0$ holds if

$$L < \frac{1-\alpha\gamma}{\theta^w \phi}. \tag{A.24}$$

Substituting $\lambda_a(\theta^w) = 0$ and $\lambda_a^*(\theta^w) > 0$ into (7), it follows that $\dot{R} = \phi^* L^* - (1-\alpha\gamma)/\theta^w - \delta R$. Given this and (30), the steady-state solution for θ^w is calculated as $\theta_a^w = [(1-\alpha\gamma)\rho + (1+\alpha\gamma)\delta]/[\rho+\delta)\phi^* L^*]$. Substituting this into (A.24), the condition under which only the foreign country make a contribution in the steady state is given by

$$\frac{\phi^* L^*}{\phi L} > \frac{(1-\alpha\gamma)\rho + (1+\alpha\gamma)\delta}{(1-\alpha\gamma)(\rho+\delta)} > 1. \tag{A.25}$$

Analogously, the condition under which only the home country make a contribution in the steady state is derived as

$$\frac{\phi^* L^*}{\phi L} < \frac{(1-\alpha\gamma)(\rho+\delta)}{(1-\alpha\gamma)\rho + (1+\alpha\gamma)\delta} < 1. \tag{A.26}$$

From (A.25) and (A.26), the condition under which both countries choose positive contribution levels is

$$\frac{(1-\alpha\gamma)(\rho+\delta)}{(1-\alpha\gamma)\rho + (1+\alpha\gamma)\delta} \leq \frac{\phi^* L^*}{\phi L} \leq \frac{(1-\alpha\gamma)\rho + (1+\alpha\gamma)\delta}{(1-\alpha\gamma)(\rho+\delta)}. \tag{A.27}$$

Then, it follows that if the difference between ϕL and $\phi^* L^*$ is not so large, both countries choose positive contribution levels in the autarkic steady state under international cooperation, as illustrated in Figure A.1.

In the case where the countries noncooperatively choose their contribution levels, the condition under which both L_R and L_R^* are positive in the autarkic steady state is derived in a similar manner:

$$\frac{(1-\alpha\gamma)(\rho+\delta)}{(1-\alpha\gamma)\rho + \delta} \leq \frac{\phi^* L^*}{\phi L} \leq \frac{(1-\alpha\gamma)\rho + \delta}{(1-\alpha\gamma)(\rho+\delta)}. \tag{A.28}$$

Comparing the above condition with (A.27), it follows that the region in which both $L_R > 0$ and $L_R^* > 0$ holds become narrower in the noncooperative equilibrium.

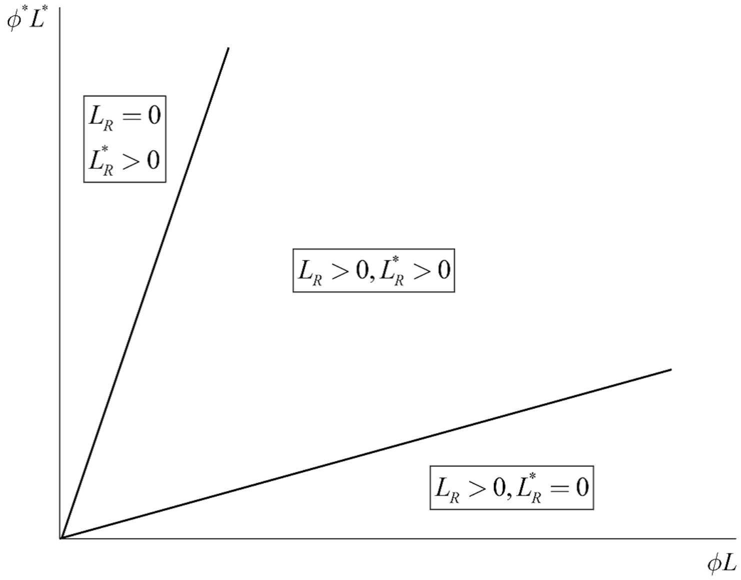

Figure A.1: Conditions for positive contribution levels in the autarkic steady state

References

[1] Abe, K. (1990), A Public Input As a Determinant of Trade, *Canadian Journal of Economics* 23, 400–407.

[2] Altenburg, L. (1992), Some Trade Theorems with a Public Intermediate Good, *Canadian Journal of Economics* 25, 310–332.

[3] Fershtman, C. and S. Nitzan (1991), Dynamic Voluntary Provision of Public Goods, *European Economic Review* 35, 1057–1067.

[4] Ishizawa, S. (1988), Increasing Returns, Public Inputs, and International Trade, *American Economic Review* 78, 794–795.

[5] Itaya, J.-I. and K. Shimomura (2001), A Dynamic Conjectural Variations Model in the Private Provision of Public Goods: A Differential Game Approach, *Journal of Public Economics* 81, 153–172.

[6] Long, N.V. (2010), *A Survey of Dynamic Games in Economics*, World Scientific.

[7] Long, N.V. and K. Shimomura (2007), Voluntary Contributions to a Public Good: Non-Neutrality Results, *Pacific Economic Review* 12, 153–170.

[8] Manning, R. and J. McMillan (1979), Public Intermediate Goods, Production Possibilities, and International Trade, *Canadian Journal of Economics* 12, 243–257.

[9] McMillan, J. (1978), A Dynamic Analysis of Public Intermediate Goods Supply in Open Economy, *International Economic Review* 19, 665–678.

[10] Meade, J.E. (1952), External Economies and Diseconomies in a Competitive Situation, *Economic Journal* 62, 54–67.

[11] Shibata, A. (2002), Strategic Interactions in a Growth Model with Infrastructure Capital, *Metroeconomica* 53, 434–460.

[12] Shimomura, K. (2007), Trade Gains and Public Goods, *Review of International Economics* 15, 948–954.

[13] Suga, N. and M. Tawada (2007), International Trade with a Public Intermediate Good and the Gains from Trade, *Review of International Economics* 15, 284–293.

[14] Tawada, M. and H. Okamoto (1983), International Trade with a Pubic Intermediate Good, *Journal of International Economics* 17, 101–115.

[15] Tawada, M. and K. Abe (1984), Production Possibilities and International Trade with a Public Intermediate Good, *Canadian Journal of Economics* 17, 232–248.

[16] Wirl, F. (1996), Dynamic Voluntary Provision of Public Goods: Extension to Nonlinear Strategies, *European Journal of Political Economy* 12, 555–560.

[17] Yanase, A. and M. Tawada (2012a), History-Dependent Paths and Trade Gains in a Small Open Economy with a Public Intermediate Good, *International Economic Review* 53, 303–314.

[18] Yanase, A. and M. Tawada (2012b), Trade Patterns and Trade Gains in a Dynamic Open Economy with a Semi-public Intermediate Good, unpublished manuscript.